Effective Models of Nonsingular Black Holes

Speaker: Andrea Pierfrancesco Sanna
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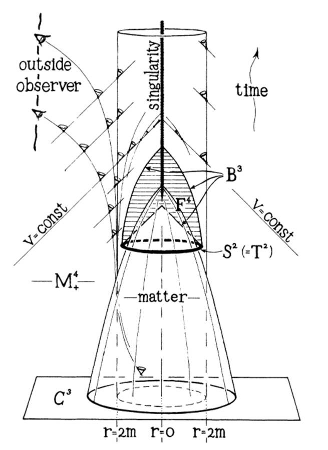
- Validity of Einstein's equations
- Energy conditions
- Global Hyperbolicity



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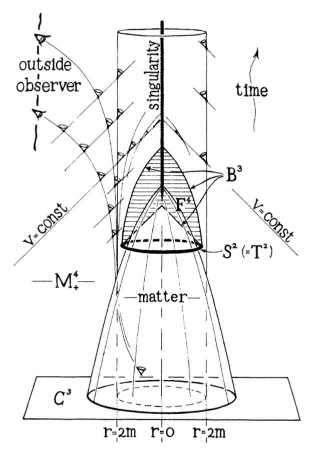

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Relaxing one (or more) assumptions allows to circumvent the theorem and obtain regular spacetimes[Carballo-Rubio et al. (2019)]

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- Corpuscular gravity[Dvali & Gomez (2013), Dvali et al. (2021, 2022)]

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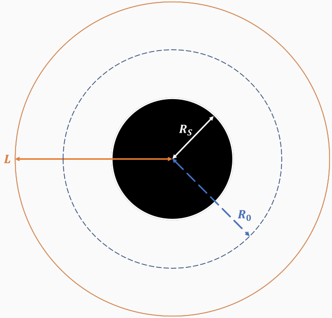
Schwarzschild-de Sitter solution

$$f(r) = 1 - \frac{R_S}{r} - \frac{r^2}{L^2}$$

R_S: baryonic-matter scale (**inner horizon**)

L: cosmological scale (**outer horizon**)

$R_0 = \sqrt{R_S L}$: dark-matter scale



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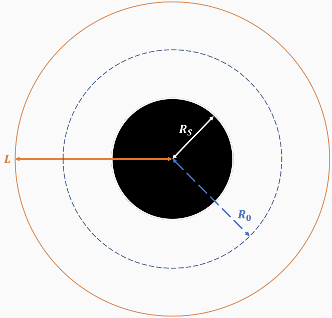

Effective model: GR sourced by an *anisotropic* fluid encoding *long-range quantum gravity effects*[Cadoni et al. (2018-2021)]

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Main idea: use the same (*but reversed!*) principle adopted for the cosmological solution to address the singularity problem

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Solution:

$$-e^{\nu(r)} = e^{-\lambda(r)} \equiv A(r) = 1 - \frac{2GM(r)}{r}\,, \qquad M(r) = 4\pi \int_0^r \rho\, \tilde{r}^2\, d\tilde{r}$$

Imposing a de Sitter core

To eliminate the singularity: glue together a Schwarzschild patch at great distances from the center with a de Sitter patch near $r \sim 0$

$$A(r) \underset{r \to 0}{\sim} 1 - \frac{r^2}{\hat{L}^2} \qquad\qquad A(r) \underset{r \to \infty}{\sim} 1 - \frac{R_S}{r}$$

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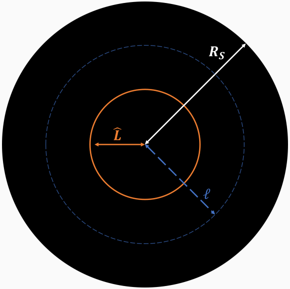

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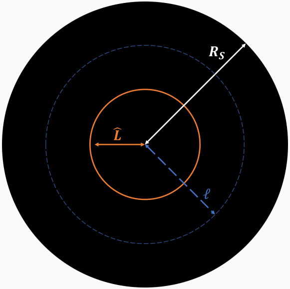

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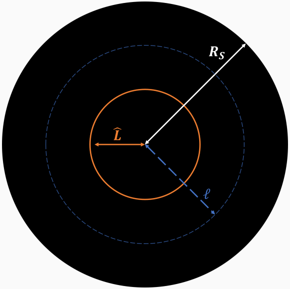

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ℓ is an additional **quantum hair**

Several models in the literature

- Bardeen model[Bardeen (1968)]
- Dymnikova model[Dymnikova (1992)]
- Hayward model[Hayward (2005)] [First proposed by Poisson & Israel (1988)]
- Gaussian-core model[Nicolini et al. (2006), Modesto et al. (2011)]
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In some models, the hair ℓ is an UV QG parameter (noncommutative geometry [Nicolini et al. (2006)], LQG effects[Modesto et al. (2011)], others[Rovelli et al. (2014) ,Frolov (2014)])
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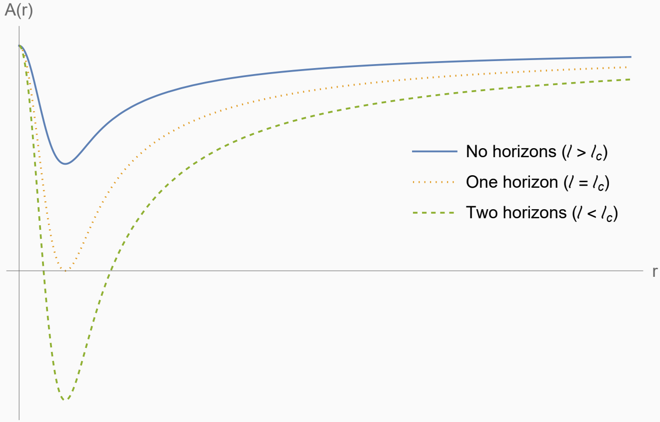
$$A(r) = 1 - \frac{R_S}{\ell}\mathscr{F}\left(\frac{r}{\ell}\right) \qquad \begin{cases} \mathscr{F}(y) \sim y^2 & \text{for } y \sim 0 \\ \mathscr{F}(y) \sim \frac{1}{y} & \text{for } y \to \infty \end{cases}$$


A(r)
r
No horizons ($l > l_c$)
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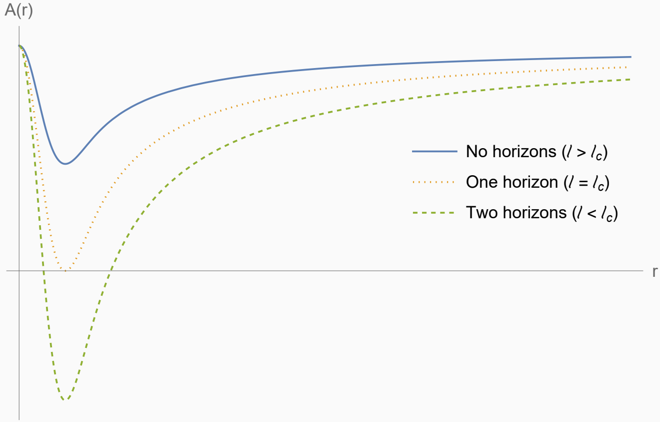

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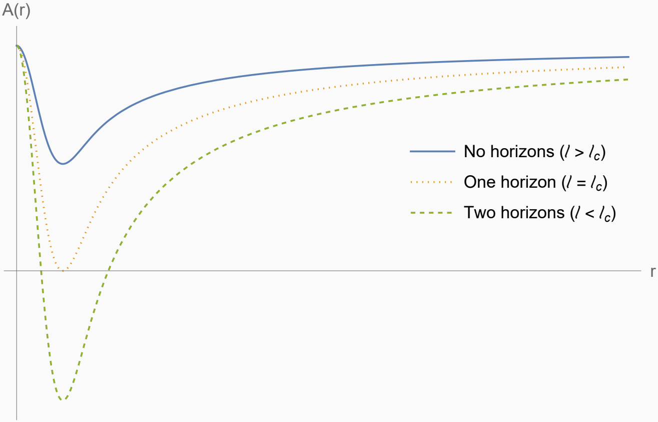

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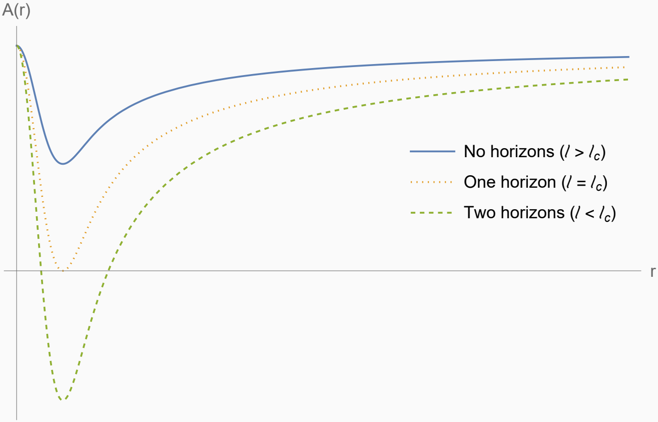

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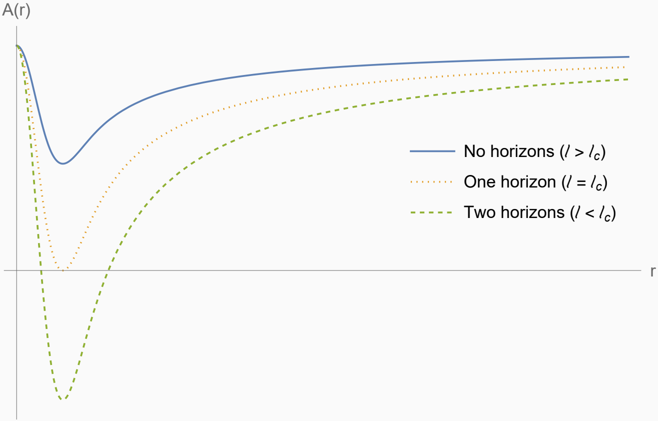

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Extremal model $\qquad \ell_c \sim R_S$

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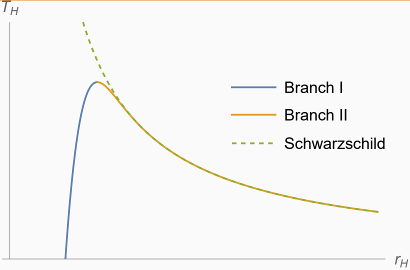

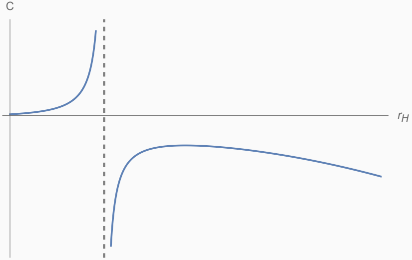

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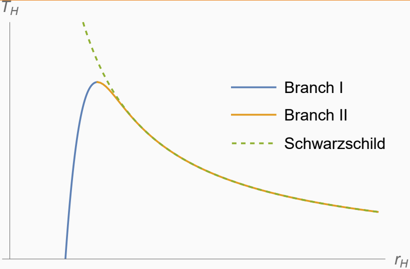

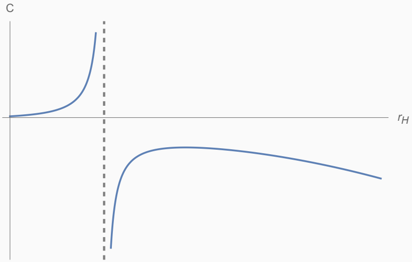

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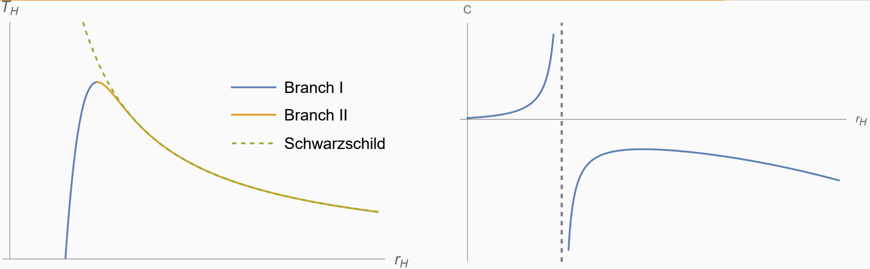

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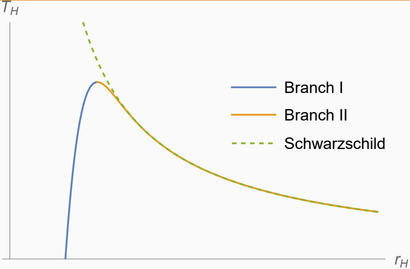

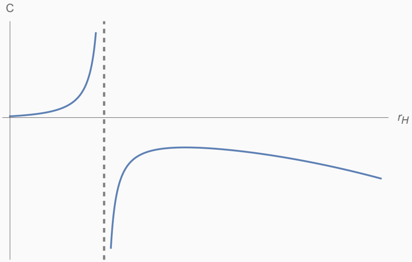

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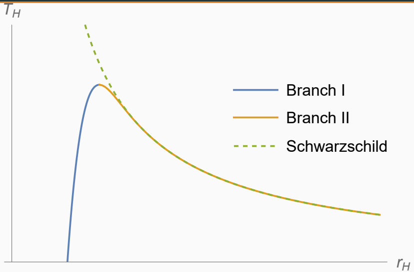

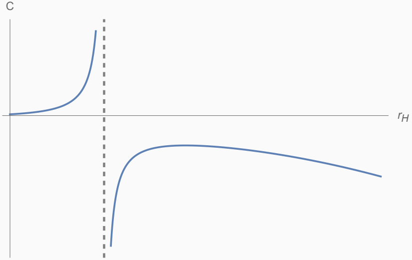

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For other interesting thermodynamic features, see
Cadoni, Oi, **APS**, arXiv:2204.09444

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(e.g. $\ell_c \simeq 0.296\, GM$)

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[Cadoni, De Laurentis, De Martino, Della Monica, Oi, **APS**, arXiv:2211.11585]

$$A(r) = 1 - \frac{2GMr^2}{(r+\ell)^3} \underset{r \gg GM}{\sim} 1 - \frac{2GM}{r} + 6GM\,\frac{\ell}{r^2}$$

With $M = M_{\mathrm{SgrA}^*}$ at $r \sim r_{\mathrm{S2}}$

$\ell \sim \ell_{\mathrm{P}} \quad \Rightarrow \quad \dfrac{6GM\ell}{r^2} \sim 10^{-53}$

$\ell \sim 0.2GM \Rightarrow \dfrac{6GM\ell}{r^2} \sim 10^{-9}$

(e.g. $\ell_c \simeq 0.296\,GM$)

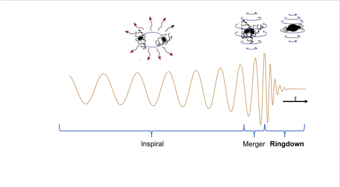
$\Rightarrow$

S2 Observational data

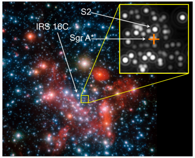
$\ell \leq 0.47\,GM$ with 95 % C. L.

[arXiv:2211.11585]

Conclusions



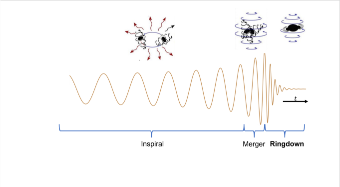
Credits: EHT Collaboration



Credits: ESO, Gillessen et al.

- The possibility of having IR QG corrections is an alternative interesting way to resolve the singularity problem
- Deviations from standard phenomenology are hopeless to be detected in the near future if $\ell \ll R_S$
  They are not if $\ell \sim R_S$

Conclusions



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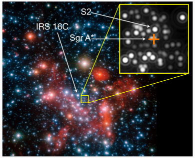

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THANKS FOR YOUR ATTENTION!