A little hair can make a big difference:
local thermodynamic stability of hairy black holes

Nuno M. Santos (Ph.D. candidate in Physics)

Instituto Superior Técnico
Universidade de Lisboa

Centre for Research and Development in Mathematics and Applications
Universidade de Aveiro

Phys. Rev. D **106** (2022) 12 (in collaboration with C. A. R. Herdeiro and E. Radu)

XV Black Holes Workshop — Lisbon, Dec. 19, 2022

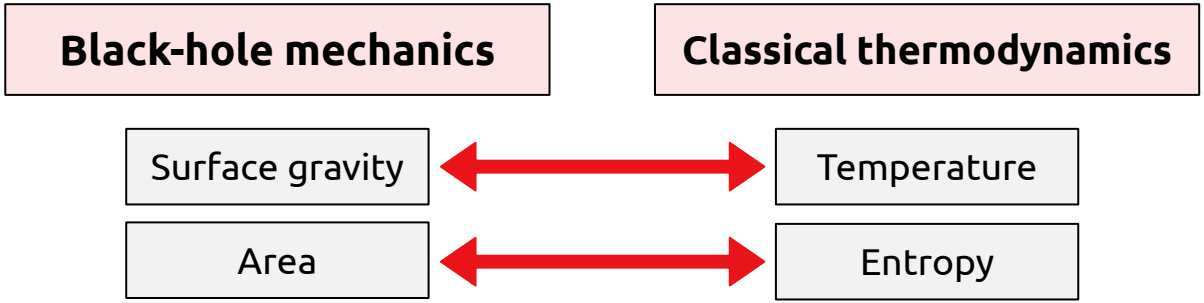
TÉCNICO LISBOA

CIDMA

FCT Fundação para a Ciência e a Tecnologia

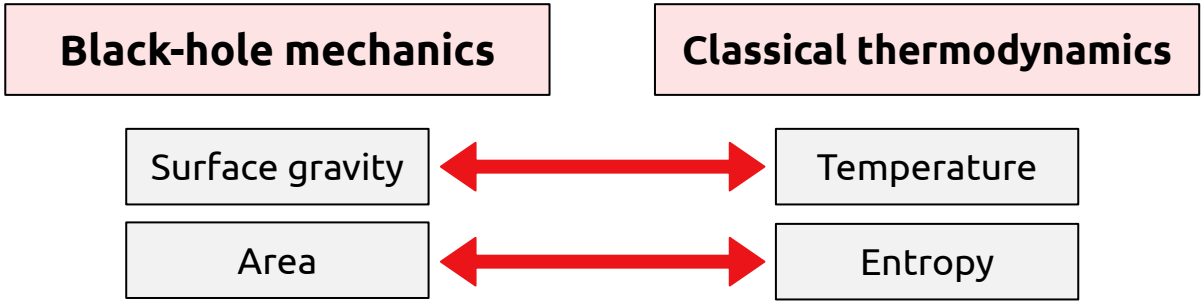
SFRH/BD/143407/2019

Black holes as thermodynamic systems



Bardeen, Carter & Hawking, *CMP* **31** (1973) 161

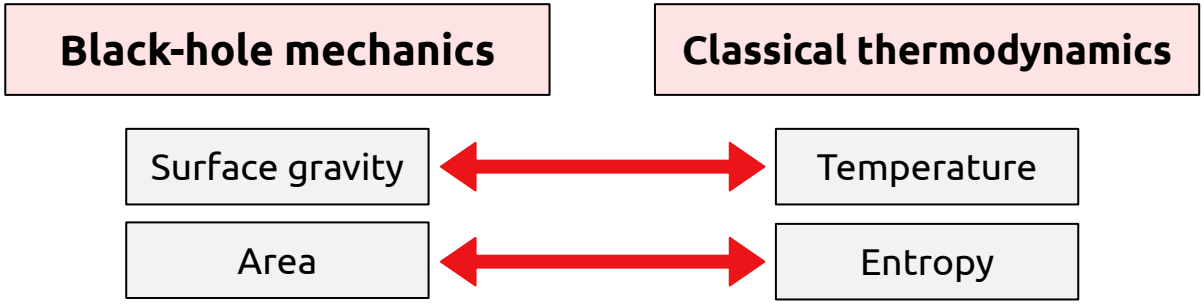
Black holes as thermodynamic systems



Bardeen, Carter & Hawking, *CMP* **31** (1973) 161

Are black holes thermodynamically stable?

Black holes as thermodynamic systems



Bardeen, Carter & Hawking, *CMP* **31** (1973) 161

Are black holes thermodynamically stable?

Local stability

Black holes as thermodynamic systems

Black-hole mechanics		Classical thermodynamics
Surface gravity	⟷	Temperature
Area	⟷	Entropy

Bardeen, Carter & Hawking, *CMP* **31** (1973) 161

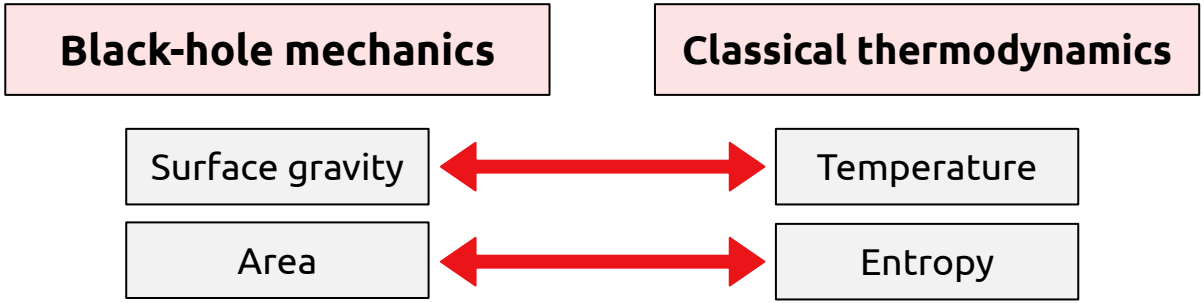
Are black holes thermodynamically stable?

Local stability

$$C = T\left(\frac{\partial S}{\partial T}\right)$$

C: specific heat; S: entropy; T: temperature

Black holes as thermodynamic systems



Bardeen, Carter & Hawking, *CMP* **31** (1973) 161

Are black holes thermodynamically stable?

Schwarzschild BHs

unstable

$$C = T\left(\frac{\partial S}{\partial T}\right) < 0$$

Black holes in the canonical ensemble

Local thermodynamic stability

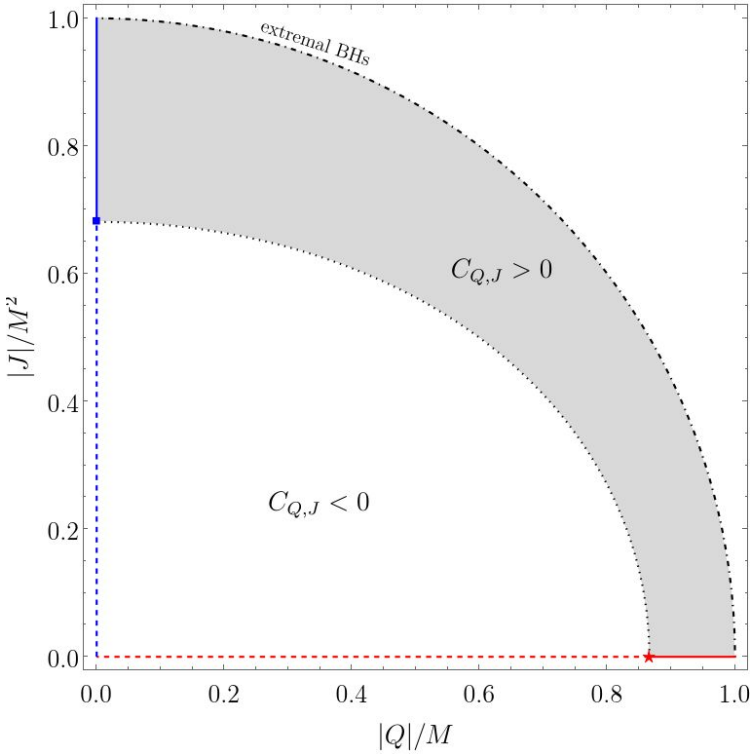
Kerr-Newman BHs

$$C_{Q,J} = T\left(\frac{\partial S}{\partial T}\right)_{Q,J} > 0$$

electric charge

angular momentum

$$J^4 + 6J^2M^4 + 4Q^2M^6 - 3M^8 > 0$$



Davies, *RSL A* **353** (1977) 499

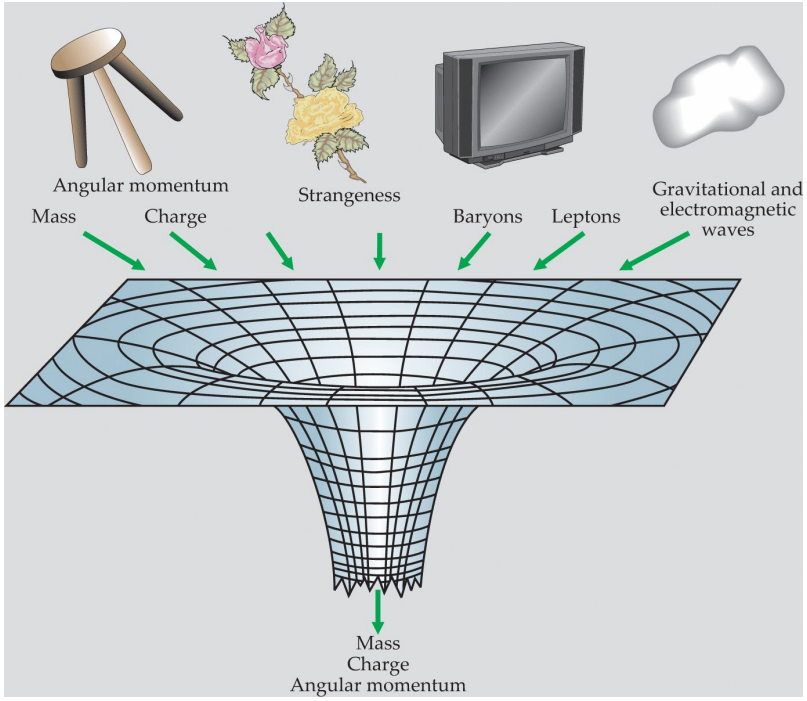
Are hairy BHs branching off from locally stable canonical BHs also locally stable?

(in the same statistical ensemble)

Hairy black holes

General relativity → Modify gravity

Particle Physics → Add new particle(s)

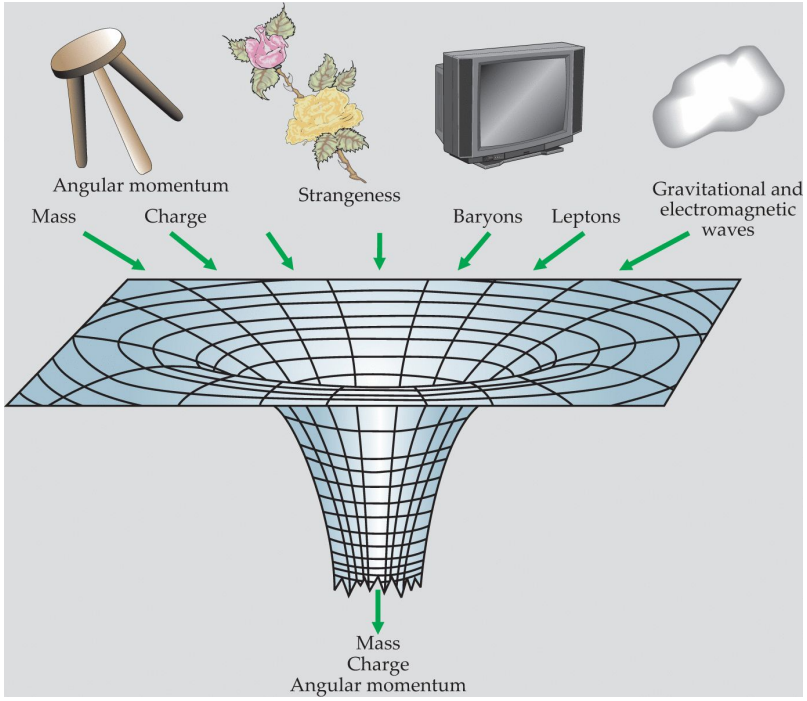


Ruffini & Wheeler, *Phys. Today* **24** (1971) 1, 30

Hairy black holes

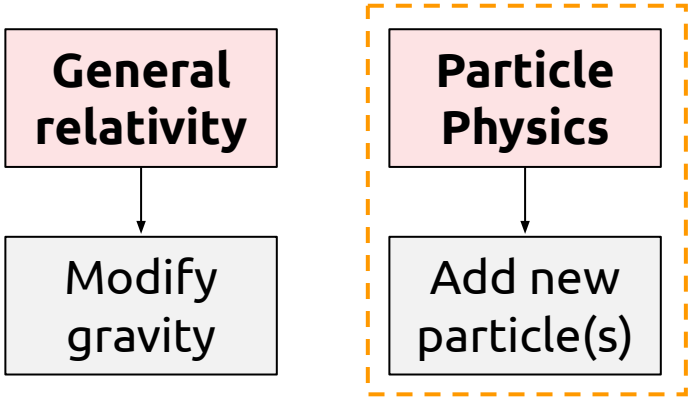
General relativity → Modify gravity

Particle Physics → Add new particle(s)



Ruffini & Wheeler, *Phys. Today* **24** (1971) 1, 30

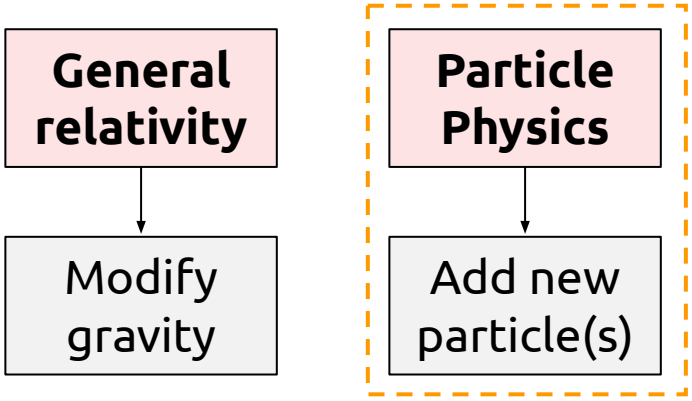
Hairy black holes



1. Scalarized black holes in Einstein-Maxwell-scalar theories
   bifurcate from Reissner-Nordström BHs

2. Black holes with synchronized hair
   bifurcate from Kerr BHs

Hairy black holes



1. Scalarized black holes in Einstein-Maxwell-scalar theories

   secondary hair

2. Black holes with synchronized hair

   primary hair

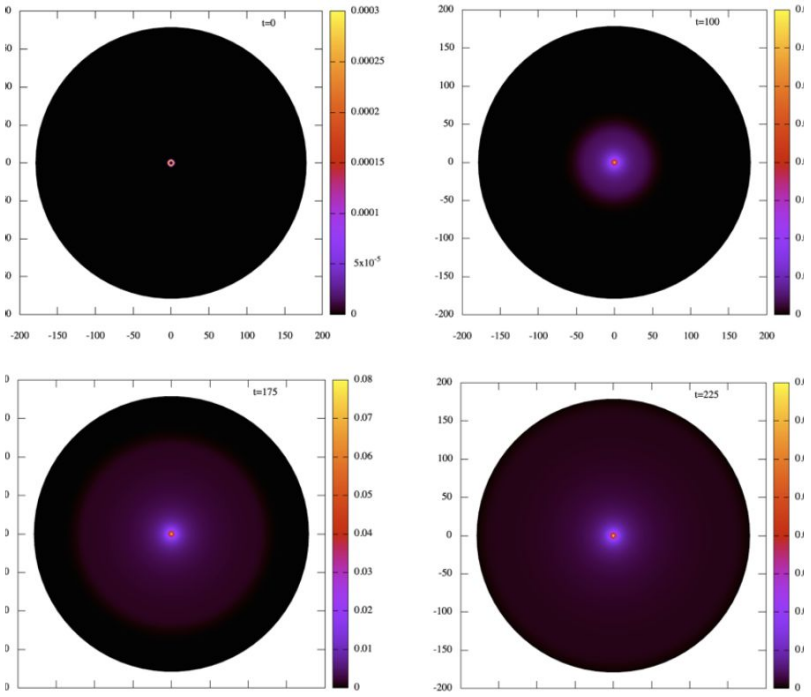
Scalarized black holes

Einstein-Maxwell-scalar theories

$$\mathcal{S} = \int \mathrm{d}^4x \left(\frac{R}{16\pi} - 2(\nabla_a\phi)(\nabla^a\phi) - f(\phi)F_{ab}F^{ab} \right)$$

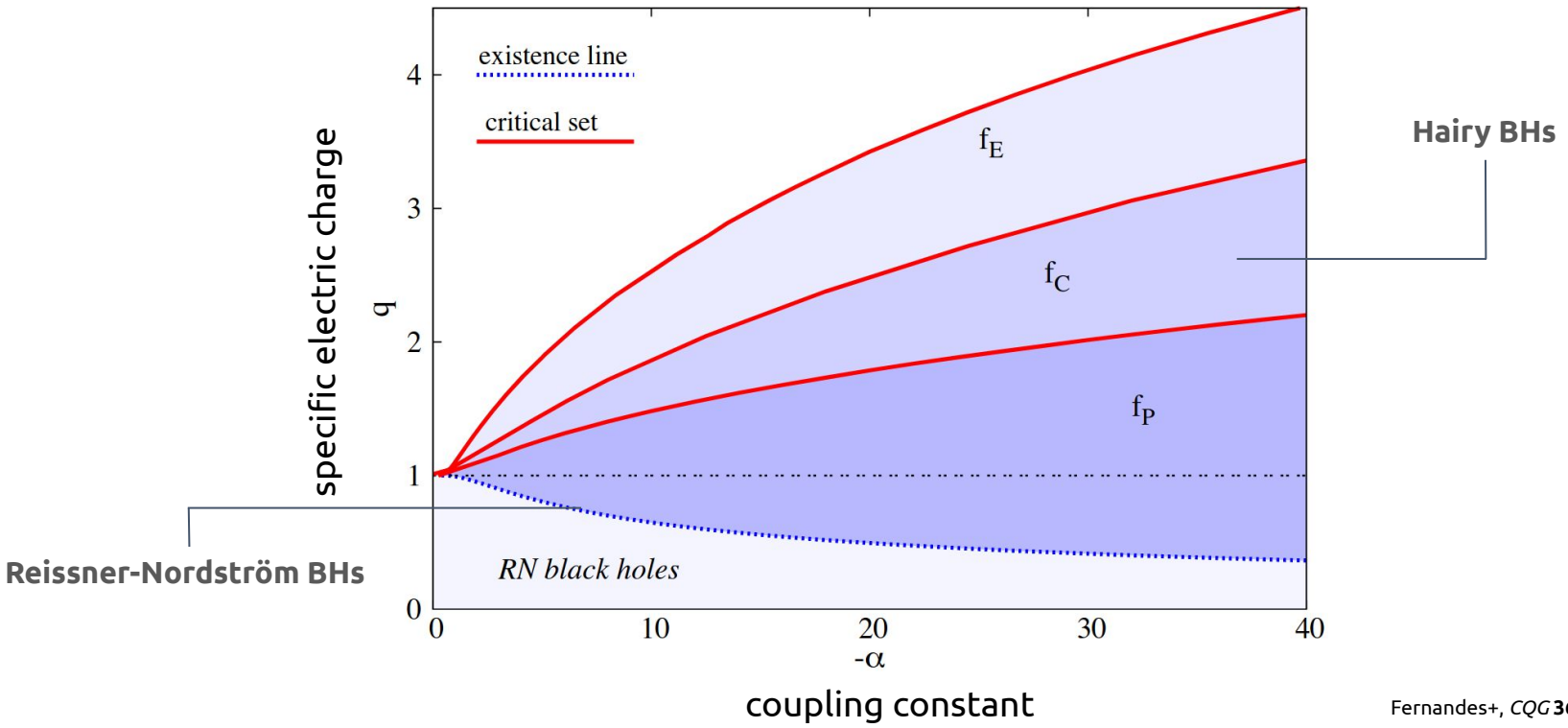
coupling function

Exponential	$f_E(\phi) = e^{-\alpha\phi^2}$
Hyperbolic	$f_C(\phi) = \cosh(\sqrt{-2\alpha}\phi)$
Power	$f_P(\phi) = 1 - \alpha\phi^2$



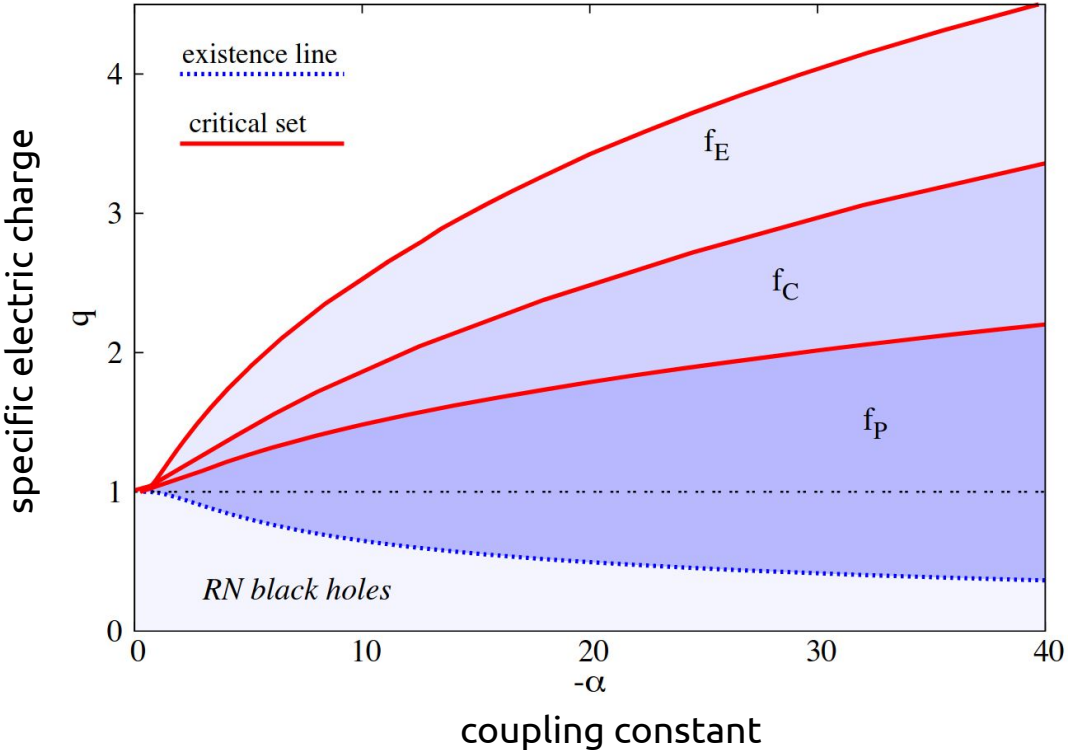
Fernandes+, *CQG* **36** (2019) 134002

Domain of existence



Fernandes+, *CQG* **36** (2019) 134002

Domain of existence



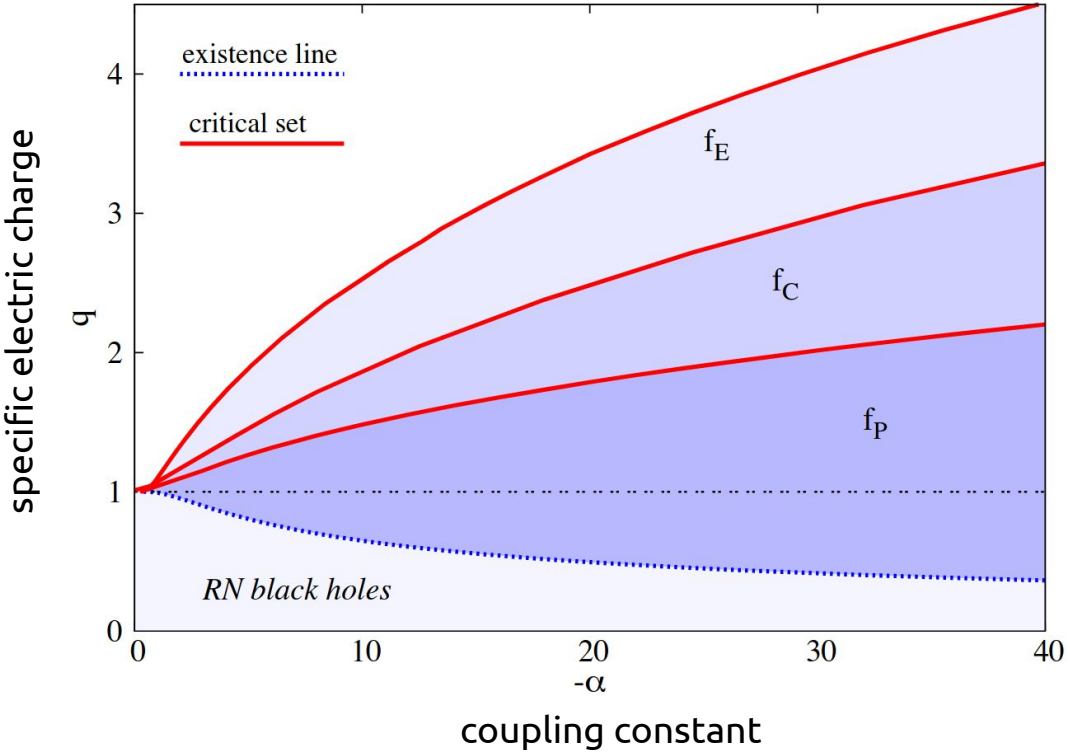
Reissner-Nordström BHs

stable

$$|Q|/M > \sqrt{3}/2$$

Fernandes+, *CQG* **36** (2019) 134002

Domain of existence



Reissner-Nordström BHs

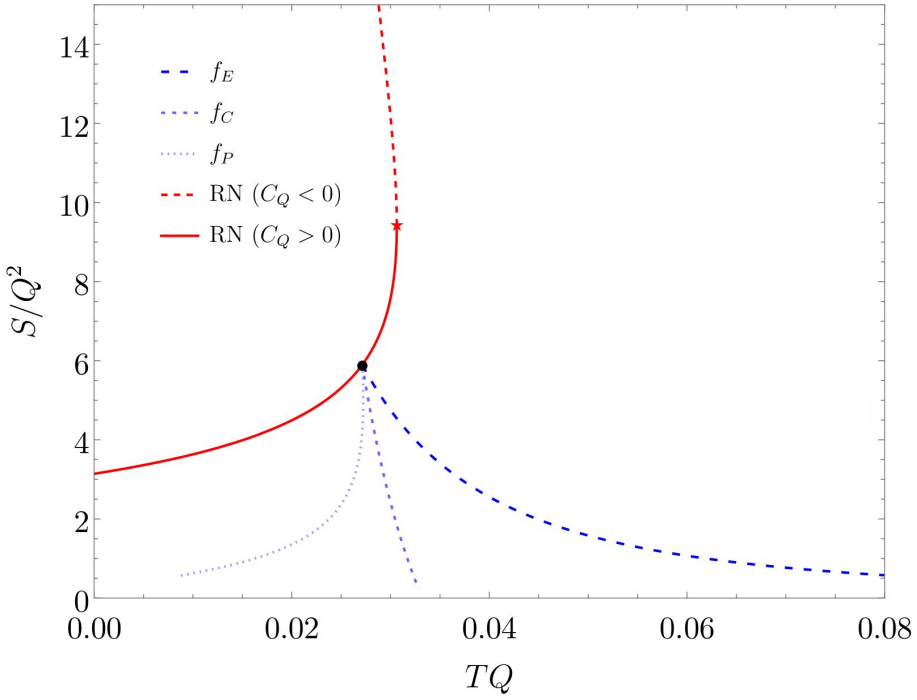
stable

$$|Q|/M > \sqrt{3}/2$$

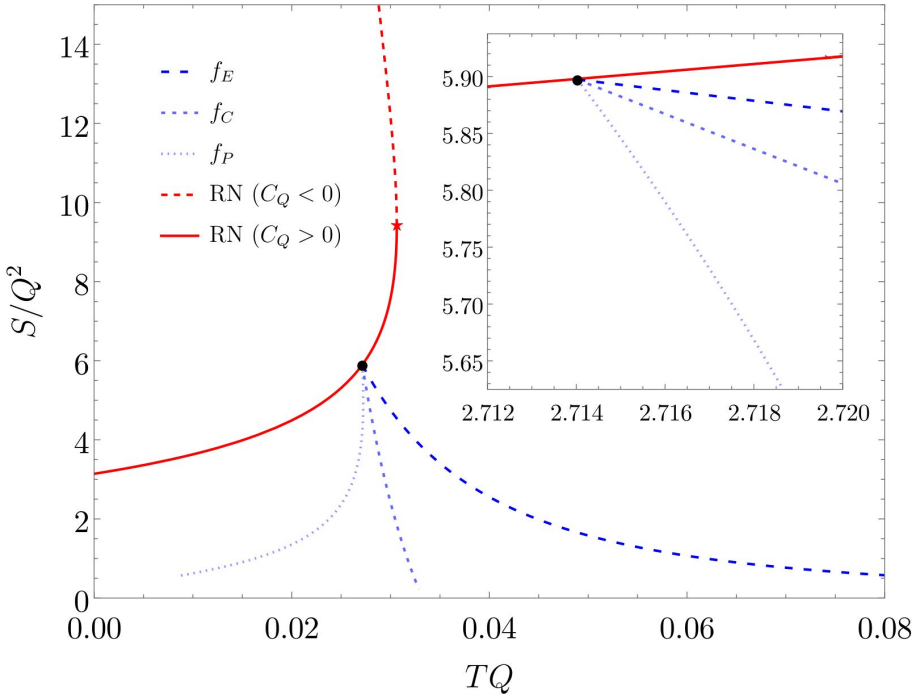
$$C_Q = T\left(\frac{\partial S}{\partial T}\right)_Q > 0$$

Fernandes+, *CQG* **36** (2019) 134002

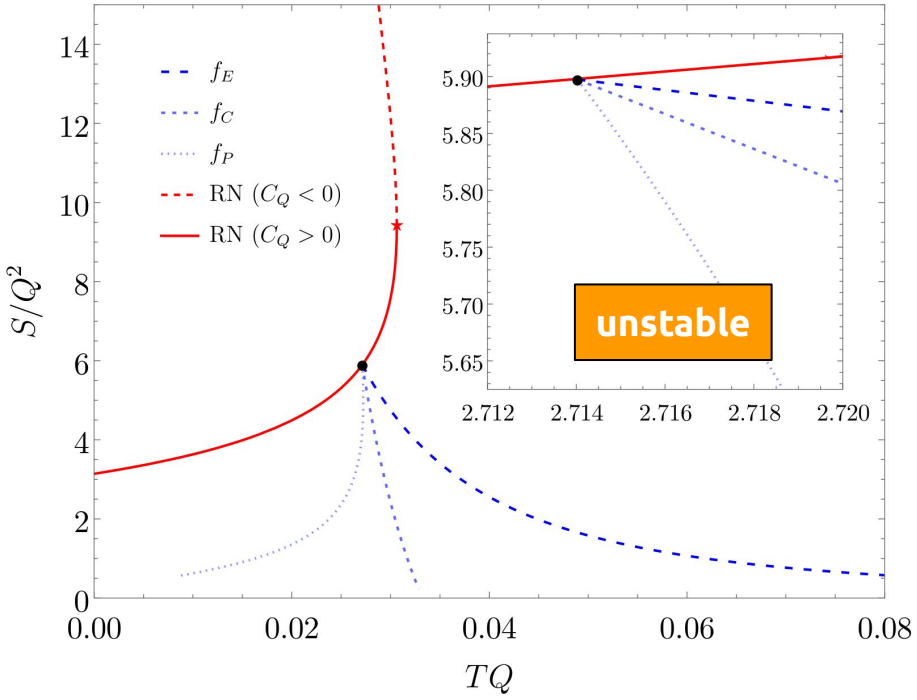
Local thermodynamic stability



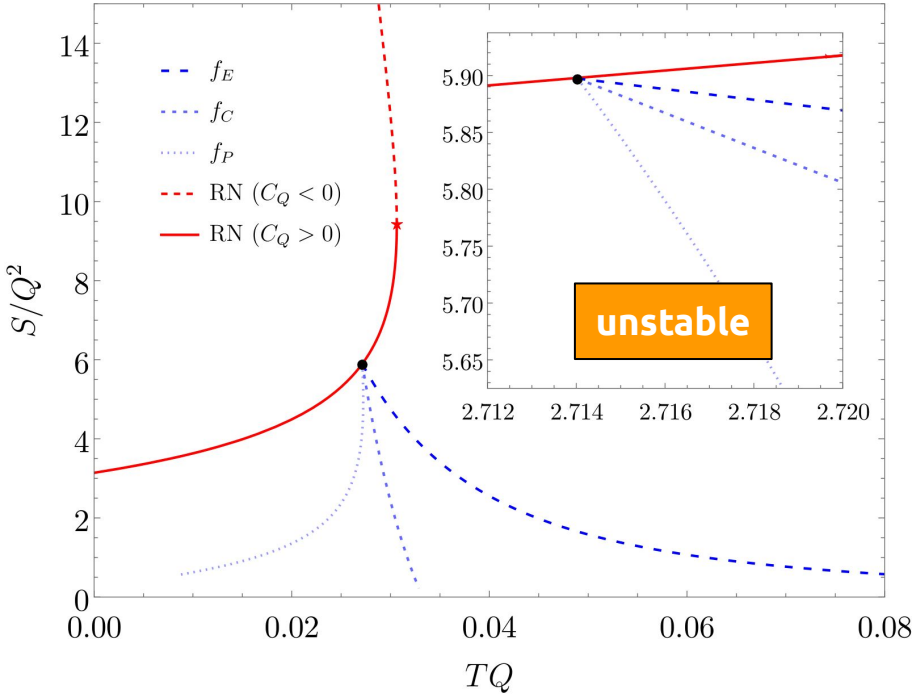
Local thermodynamic stability

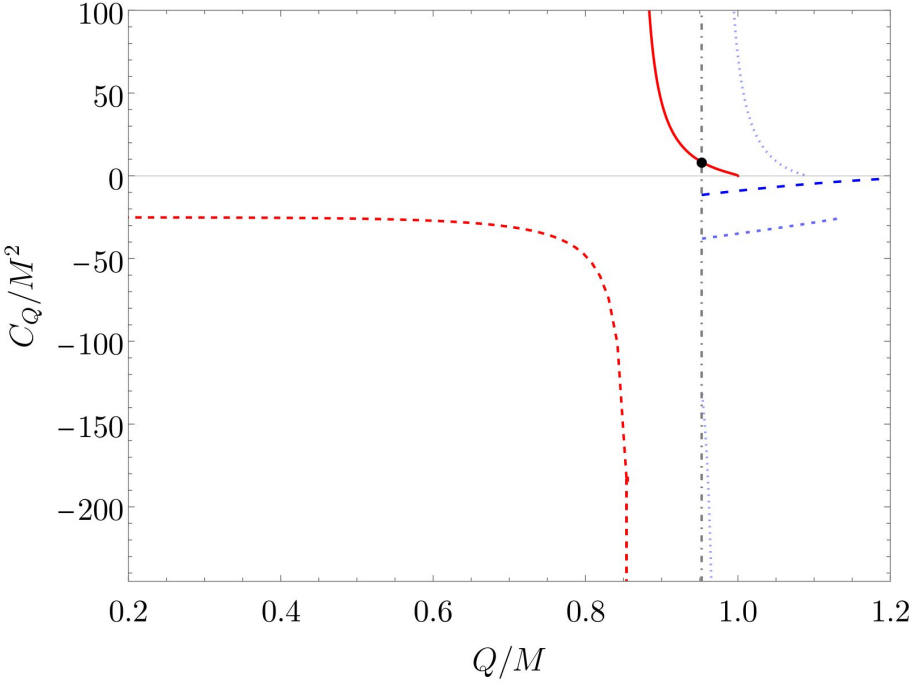


Local thermodynamic stability



Local thermodynamic stability

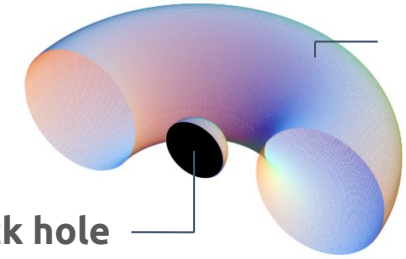




Black holes with synchronized hair

Scalar field

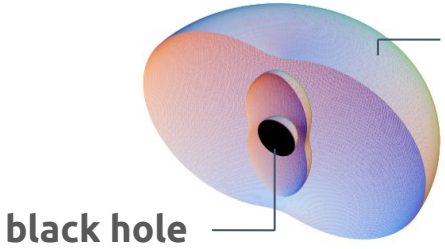
$$\mathcal{S} = \int \mathrm{d}^4x \left[\frac{R}{16\pi} - (\nabla_a \bar{\Psi})(\nabla^a \Psi) - \mu^2 |\Psi|^2 \right]$$



Herdeiro & Radu, *PRL* **112** (2014) 221101

Vector field

$$\mathcal{S} = \int \mathrm{d}^4x \left[\frac{R}{16\pi} - \frac{1}{4} F_{ab} \bar{F}^{ab} - \frac{1}{2} \mu^2 A_a \bar{A}^a \right]$$



Herdeiro & Radu, *CQG* **33** (2016) 154001

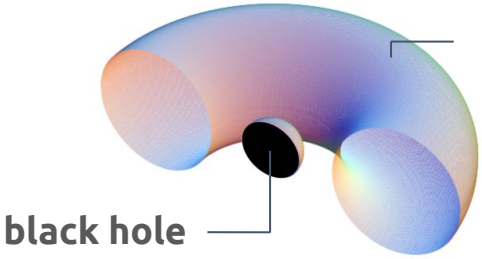
$$M\mu \sim 1$$

black hole size — field mass

Black holes with synchronized hair

Scalar field

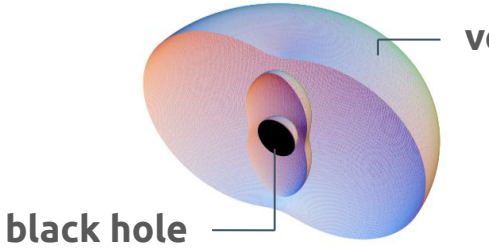
$$\mathcal{S} = \int \mathrm{d}^4x \left[\frac{R}{16\pi} - (\nabla_a \bar{\Psi})(\nabla^a \Psi) - \mu^2 |\Psi|^2 \right]$$



Herdeiro & Radu, *PRL* **112** (2014) 221101

Vector field

$$\mathcal{S} = \int \mathrm{d}^4x \left[\frac{R}{16\pi} - \frac{1}{4} F_{ab} \bar{F}^{ab} - \frac{1}{2} \mu^2 A_a \bar{A}^a \right]$$

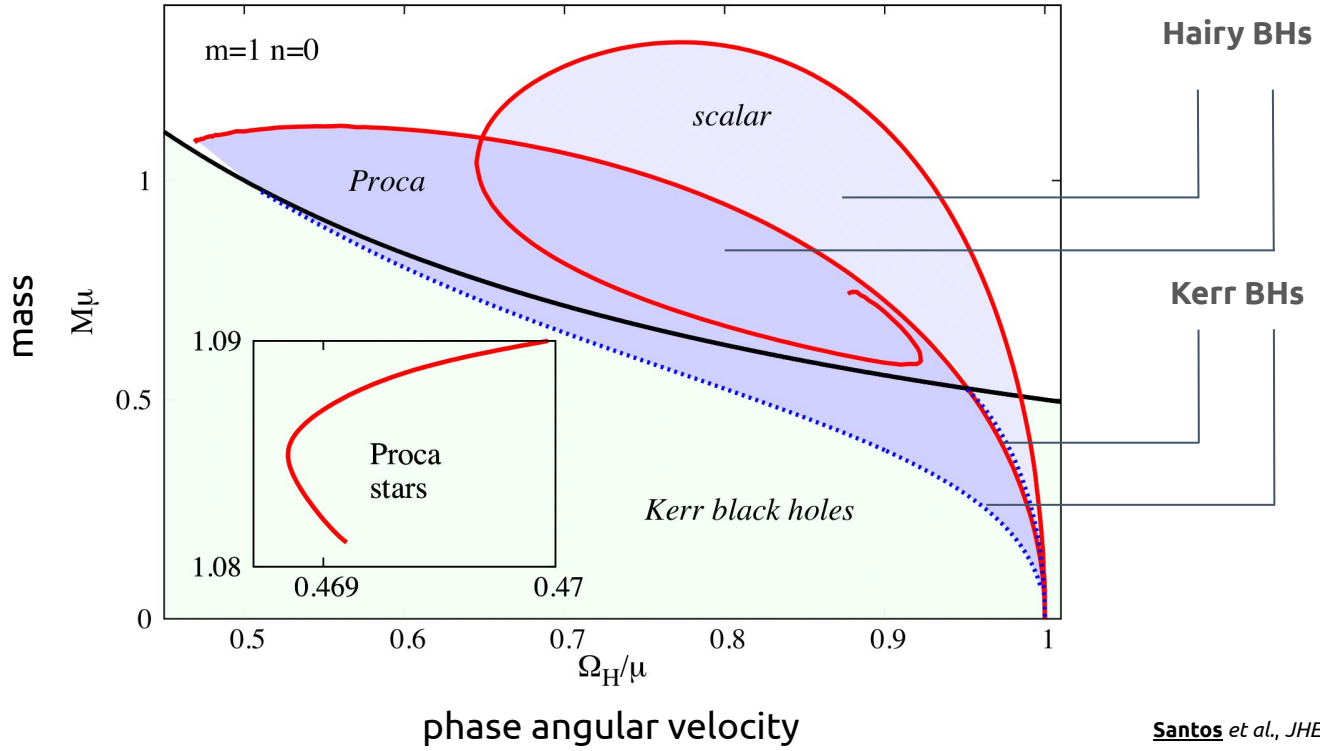


Herdeiro & Radu, *CQG* **33** (2016) 154001

synchronization

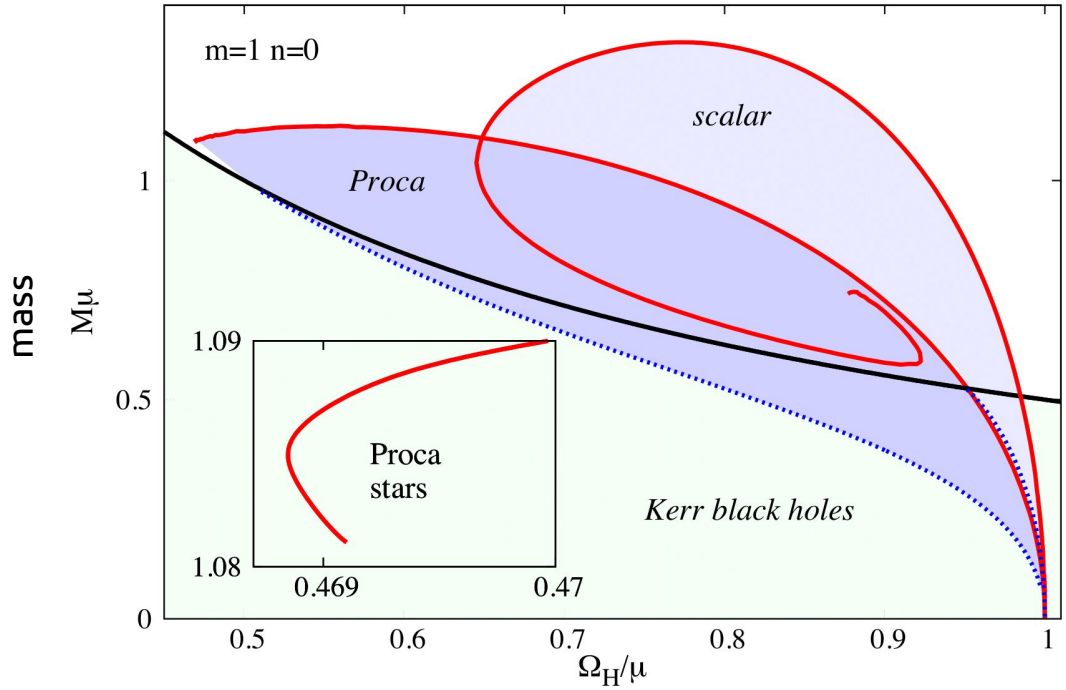
horizon angular velocity — $\Omega_H = \dfrac{\omega}{m}$ — phase angular velocity

Domain of existence



Santos *et al.*, *JHEP* **07** (2020) 010

Domain of existence



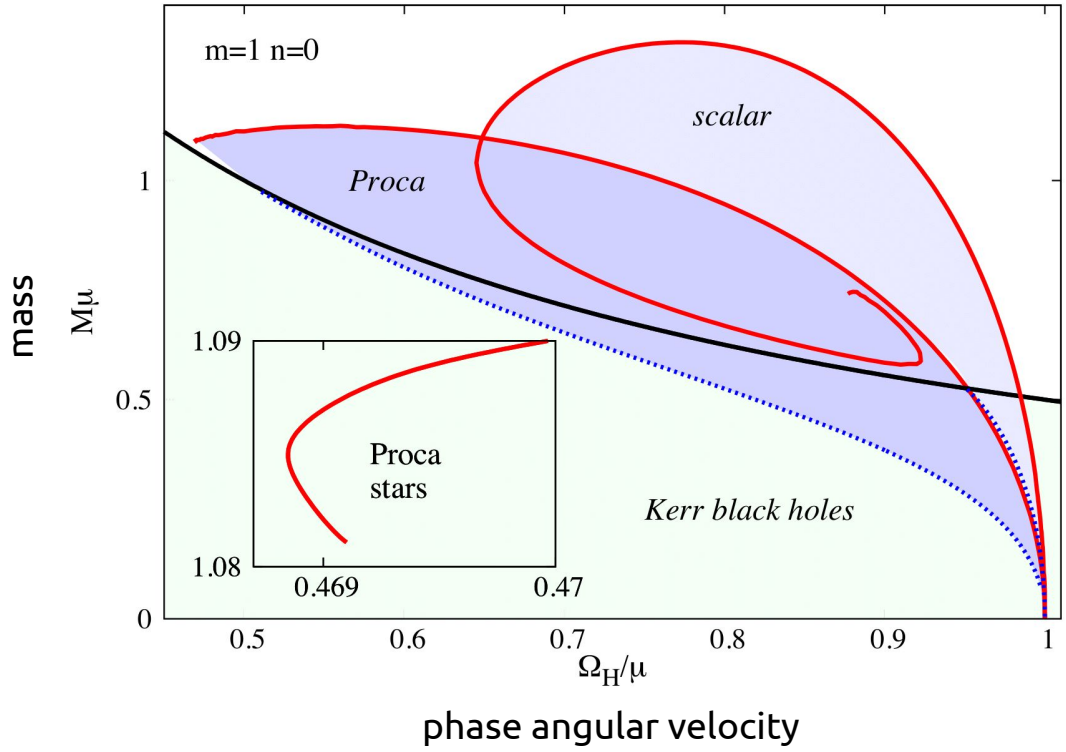
Kerr BHs

stable

$$|J|/M^2 > \sqrt{2\sqrt{3}-3}$$

Santos *et al.*, *JHEP* **07** (2020) 010

Domain of existence



Kerr BHs

stable

$$|J|/M^2 > \sqrt{2\sqrt{3}-3}$$

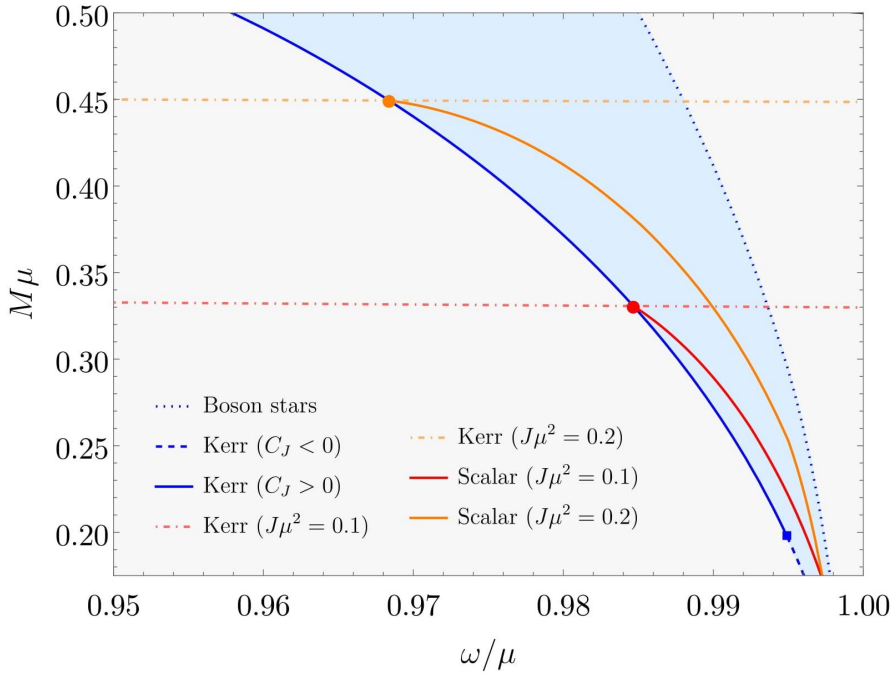
$$C_J = T\left(\frac{\partial S}{\partial T}\right)_J > 0$$

Santos *et al.*, *JHEP* **07** (2020) 010

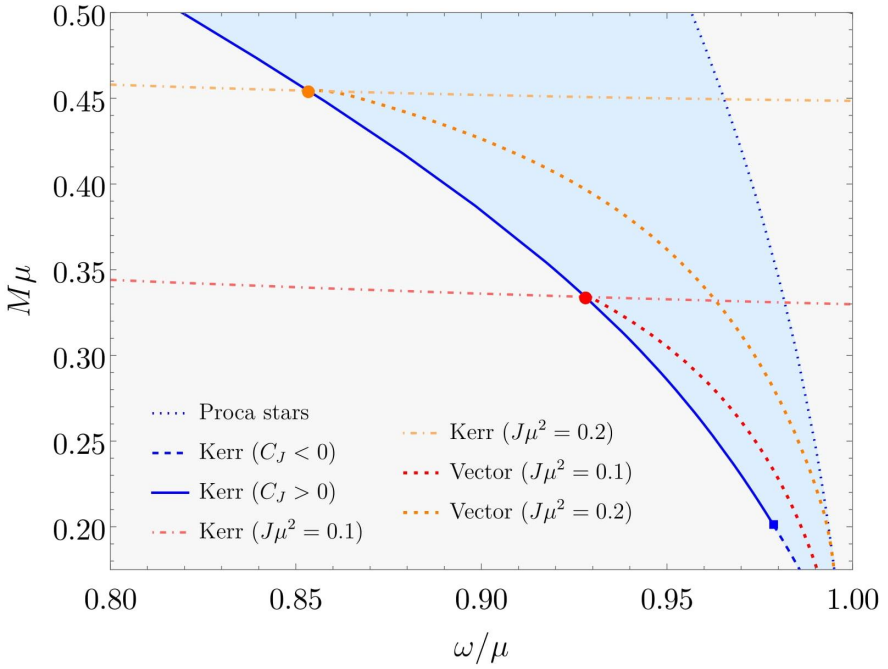
2

Local thermodynamic stability

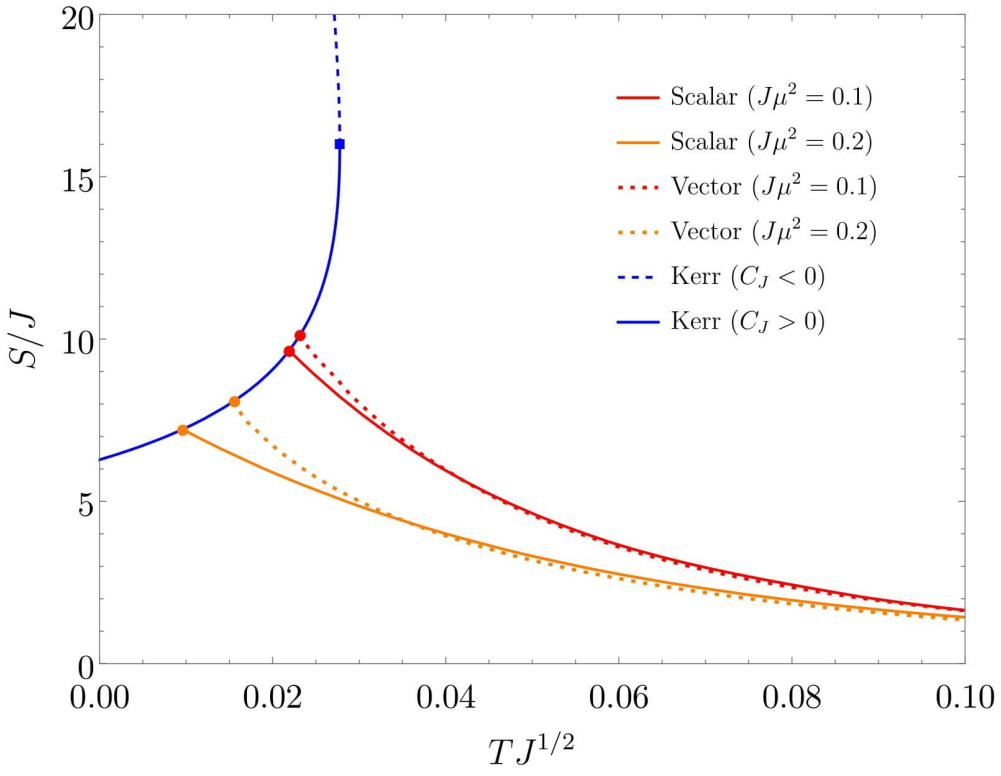
Scalar field



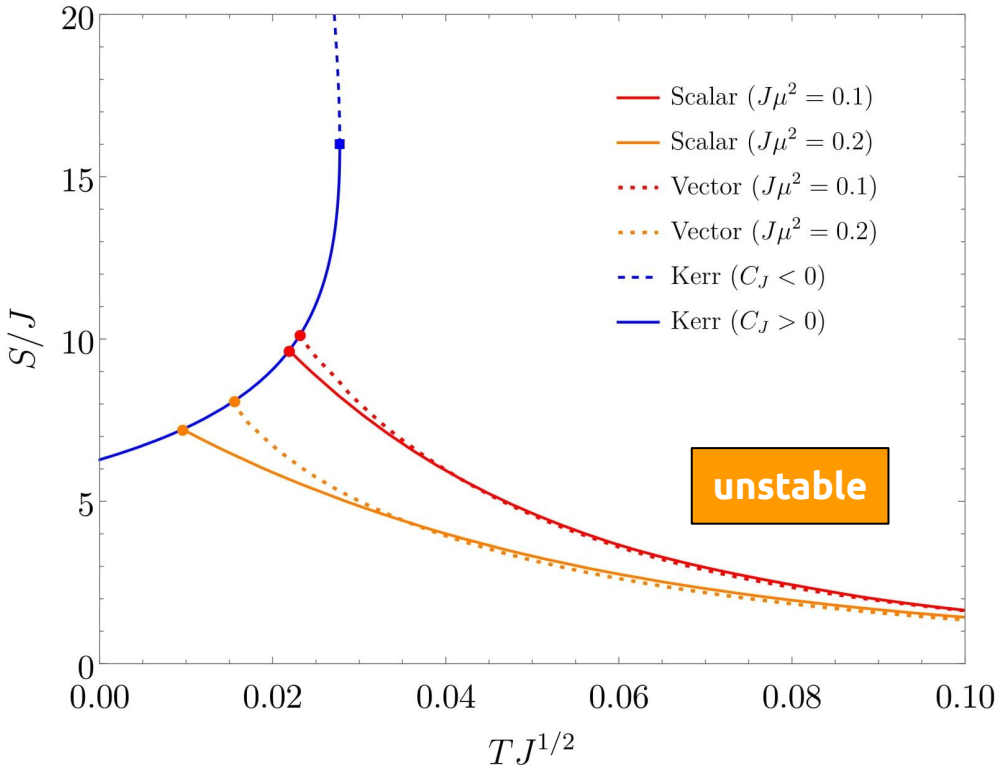
Vector field



Local thermodynamic stability



Local thermodynamic stability



Summary and future work

1. Scalarized black holes in Einstein-Maxwell-scalar theories

   secondary hair

2. Black holes with synchronized hair

   primary hair

- The addition of a bosonic field minimally coupled to Einstein's gravity can change the thermodynamic behaviour of black holes.

Summary and future work

1. Scalarized black holes in Einstein-Maxwell-scalar theories

   secondary hair

2. Black holes with synchronized hair

   primary hair

- The addition of a bosonic field minimally coupled to Einstein's gravity can change the thermodynamic behaviour of black holes.

A little hair can make a big difference!

Summary and future work

1. Scalarized black holes in Einstein-Maxwell-scalar theories

   secondary hair

2. Black holes with synchronized hair

   primary hair

- The addition of a bosonic field minimally coupled to Einstein's gravity can change the thermodynamic behaviour of black holes.

  A little hair can make a big difference!

- This might be a universal behavior.

Summary and future work

1. Scalarized black holes in Einstein-Maxwell-scalar theories

   secondary hair

2. Black holes with synchronized hair

   primary hair

- The addition of a bosonic field minimally coupled to Einstein's gravity can change the thermodynamic behaviour of black holes.

  A little hair can make a big difference!

- This might be a universal behavior.

- It would be interesting to study the thermodynamic stability of (asymptotically flat) hairy BHs in the grand-canonical ensemble.

A little hair can make a big difference:
local thermodynamic stability of hairy black holes

Nuno M. Santos (Ph.D. candidate in Physics)

Instituto Superior Técnico
Universidade de Lisboa

Centre for Research and Development in Mathematics and Applications
Universidade de Aveiro

Phys. Rev. D **106** (2022) 12 (in collaboration with C. A. R. Herdeiro and E. Radu)

XV Black Holes Workshop — Lisbon, Dec. 19, 2022

SFRH/BD/143407/2019