Motion of S2 and bounds on scalar clouds around SgrA*

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XV Black Hole workshop @ Lisbon
19 Dec. 2022

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Motivation

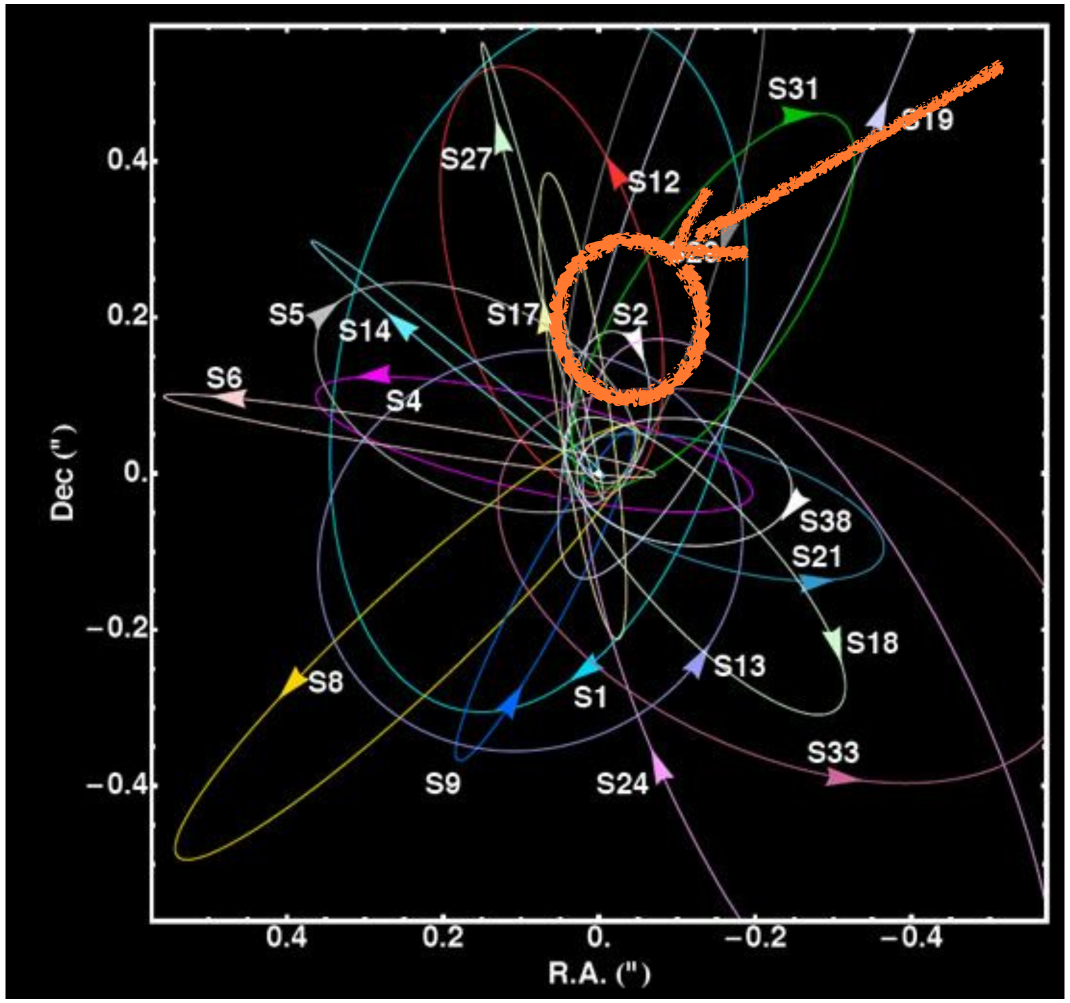
Idea: Constrain an ultralight scalar field cloud around the supermassive Black Hole (BH), ***Sagittarius A****, at the center of the Milky Way using orbital motion of S-stars.

We will focus on star S2.

Data: We have **astrometry** (positions in the sky) and **spectroscopy** (radial velocity measurements).

Motivation: Ultralight bosons are possible candidates for **Dark Matter** (DM).
DM may cluster around supermassive BHs (Sadeghian *et al*. 2013).

Several works used S-stars to obtain upper bounds on the extended mass around Sgr A*.



Credits to S. Gillessen, GRAVITY Coll., Max Planck Institute

Setup

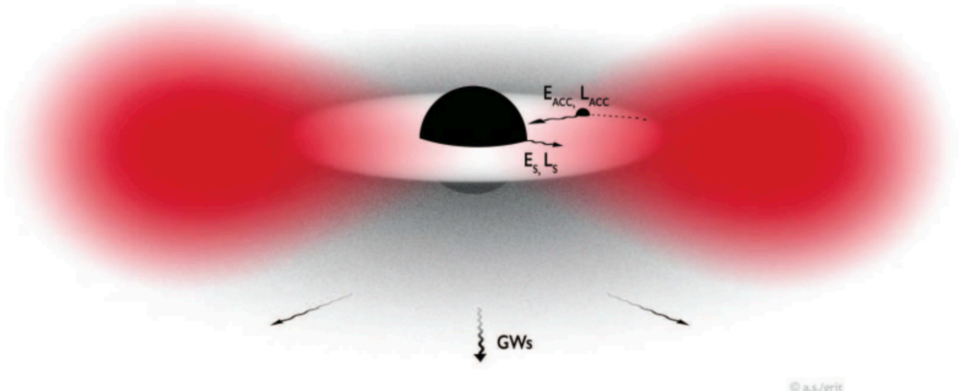
$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G c^{-4}} - \frac{1}{2} g^{\alpha\beta} \partial_\alpha \Psi^* \partial_\beta \Psi^* - \frac{\mu}{2} \Psi \Psi^* \right)$$

Mass coupling: $\alpha = r_g \mu = \left[\frac{GM_\bullet}{c^2}\right] \left[\frac{m_s c}{\hbar}\right]$

In the limit $\alpha \ll 1$, the fundamental mode of the field $(\ell = m = 1)$ is given by (Brito *et al.* 2015)

$$\Psi = A_0 e^{-i(\omega_R t - \varphi)}\, r\, \alpha^2 e^{-r\alpha^2/2} \sin\theta \qquad \text{where} \qquad A_0^2 = \Lambda \frac{\alpha^4}{64\,\pi}$$

$$\Lambda = \frac{M_{\rm cloud}}{M_\bullet}$$



Credits for image to Ana Carvalho, from *Brito et al. 2015*

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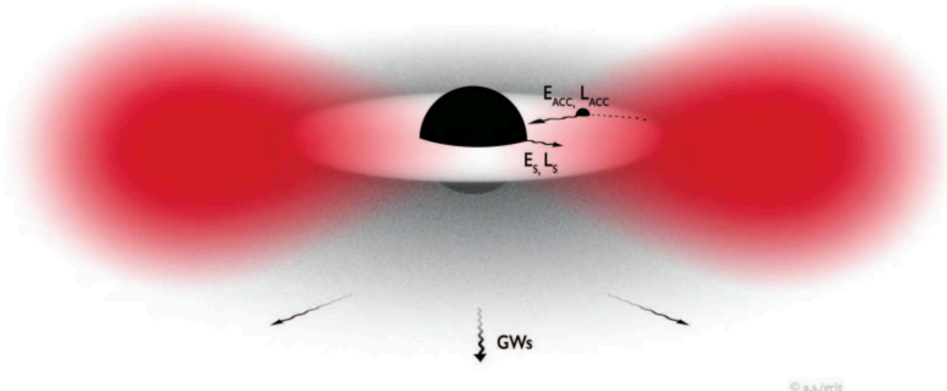
The energy density of the scalar field is:

$$\rho = \frac{m_s^2 c^2}{\hbar^2} |\Psi|^2 + \mathcal{O}\left(c^{-4}\right)$$



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Solving $\nabla^2 U_{\rm scalar} = 4\pi\rho$ we obtain the scalar potential:

$$U_{\rm scalar} = \sum_{\ell m} \frac{4\pi}{2\ell + 1} \left[q_{\ell m}(r) \frac{Y_{\ell m}(\theta, \varphi)}{r^{\ell+1}} + p_{\ell m}(r) r^\ell Y_{\ell m}(\theta, \varphi) \right] = \Lambda \left[P_1(r, \alpha) + P_2(r, \alpha) \cos^2\theta \right]$$

and the Lagrangian:

$$\mathcal{L} = \frac{1}{2}\left(\dot r^2 + r^2 \dot\theta^2 + r^2 \sin^2\theta \dot\phi\right) + \frac{M_\bullet}{r} + \Lambda\left(P_1(r, \alpha) + P_2(r, \alpha) \cos^2\theta\right)$$

Setup

Corrections to the Newtonian model

(GRAVITY Coll. 2018, Alexander 2005)

- **Newtonian effect**: the Roemer delay due to finite value of c.
- **Relativistic effects**: the Doppler shift and the gravitational redshift.
- **1 Post Newtonian (PN) correction**

Schwarzschild precession has been detected on S2 motion at 8σ confidence level (GRAVITY Coll. 2020)

$$a_{1\mathrm{PN}} = f_{\mathrm{SP}} \frac{M_\bullet}{r^2} \left[\left(\frac{4M_\bullet}{r} - v^2 \right) \frac{\boldsymbol{r}}{r} + 4\dot{r}\boldsymbol{v} \right]$$

where $f_{\mathrm{SP}} = 1, \boldsymbol{r} = r\hat{r}, \boldsymbol{v} = \left(\dot{r}\hat{r}, r\dot{\theta}\hat{\theta}, r\dot{\phi}\sin\theta\hat{\phi} \right), v = |\boldsymbol{v}|$

Method

First step: minimize the χ^2

Effective peak position of ρ

$$R_{\rm peak} = \frac{\int_0^\infty \rho \bar{r} d\bar{r}}{\int_0^\infty \rho d\bar{r}} = \frac{3M_\bullet}{\alpha^2}$$

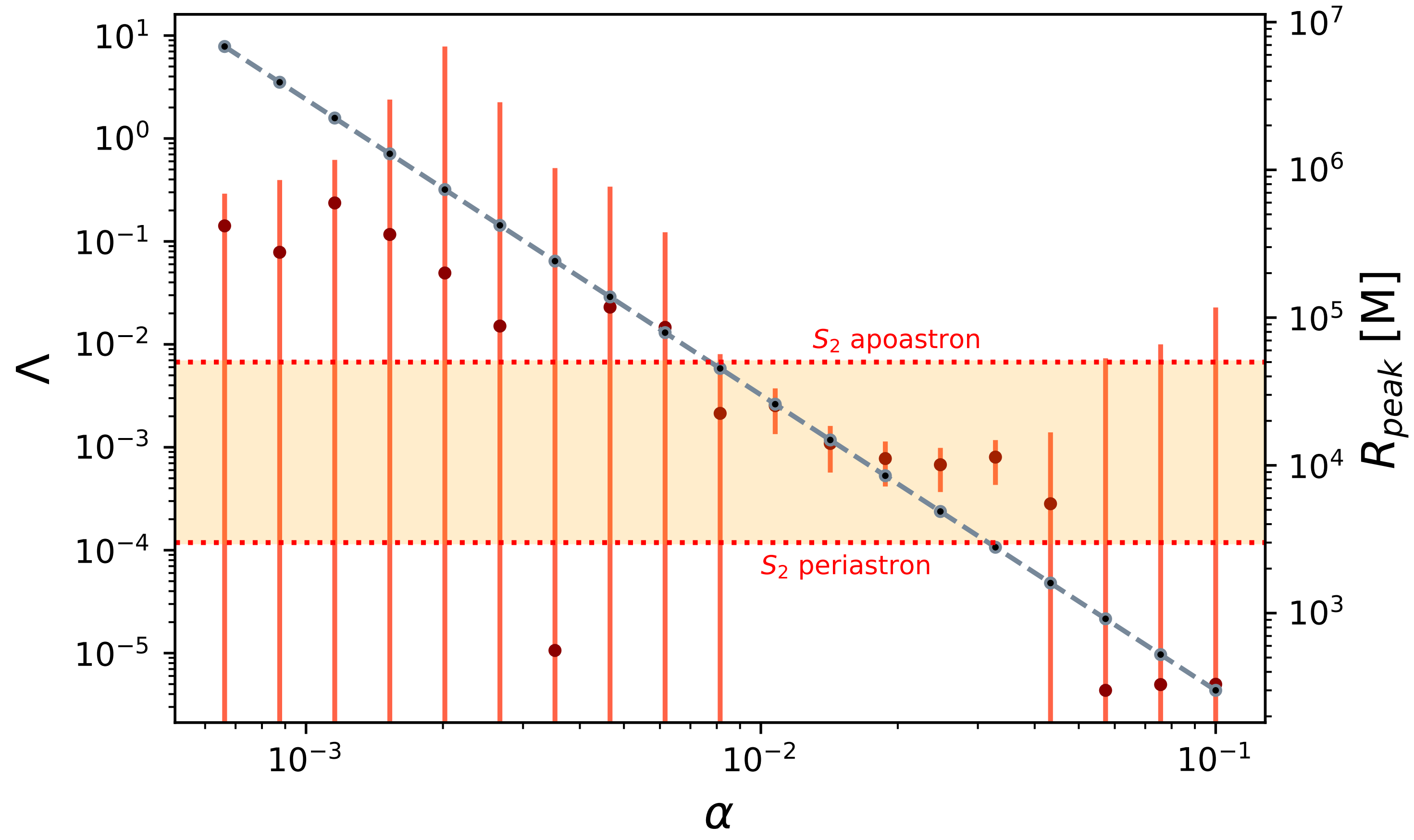
Smaller uncertainties in Λ for

$$0.01 \lesssim \alpha \lesssim 0.3$$

which (roughly) corresponds to

$$35\,M_\bullet \lesssim R_{\rm peak} \lesssim 30000\,M_\bullet$$

$$\left(3000\,M_\bullet \lesssim r_{S2} \lesssim 50000\,M_\bullet\right)$$



Method

Second step: applying Markov Chain Monte Carlo (MCMC) method using **emcee** (Foreman-Mackey *et al*. 2013) Python package

We need to *sample* $P(\theta\,|\,D) \propto P(D\,|\,\theta)P(\theta)$ for different fixed values of α

D = data set

$$\theta_i = \{e, a_{\text{sma}}, \Omega, i, \omega, t_p, R_0, M_\bullet, x_0, y_0, v_{x0}, v_{y0}, v_{z0}, \Lambda\}$$

Keplerian elements ($e, a_{\text{sma}}, \Omega, i, \omega, t_p$)

BH Mass and GC distance ($R_0, M_\bullet$)

Correction to NACO and RV data ($x_0, y_0, v_{x0}, v_{y0}, v_{z0}$)

Scalar field fractional mass (Λ)

$P(D\,|\,\theta)$ = Gaussian Likelihood

$P(\theta)$ = **Uniform** priors for physical parameters, **Gaussian** priors for $(x_0, y_0, v_{x0}, v_{y0}, v_{z0})$ (Plewa *et al*. 2015)

Method

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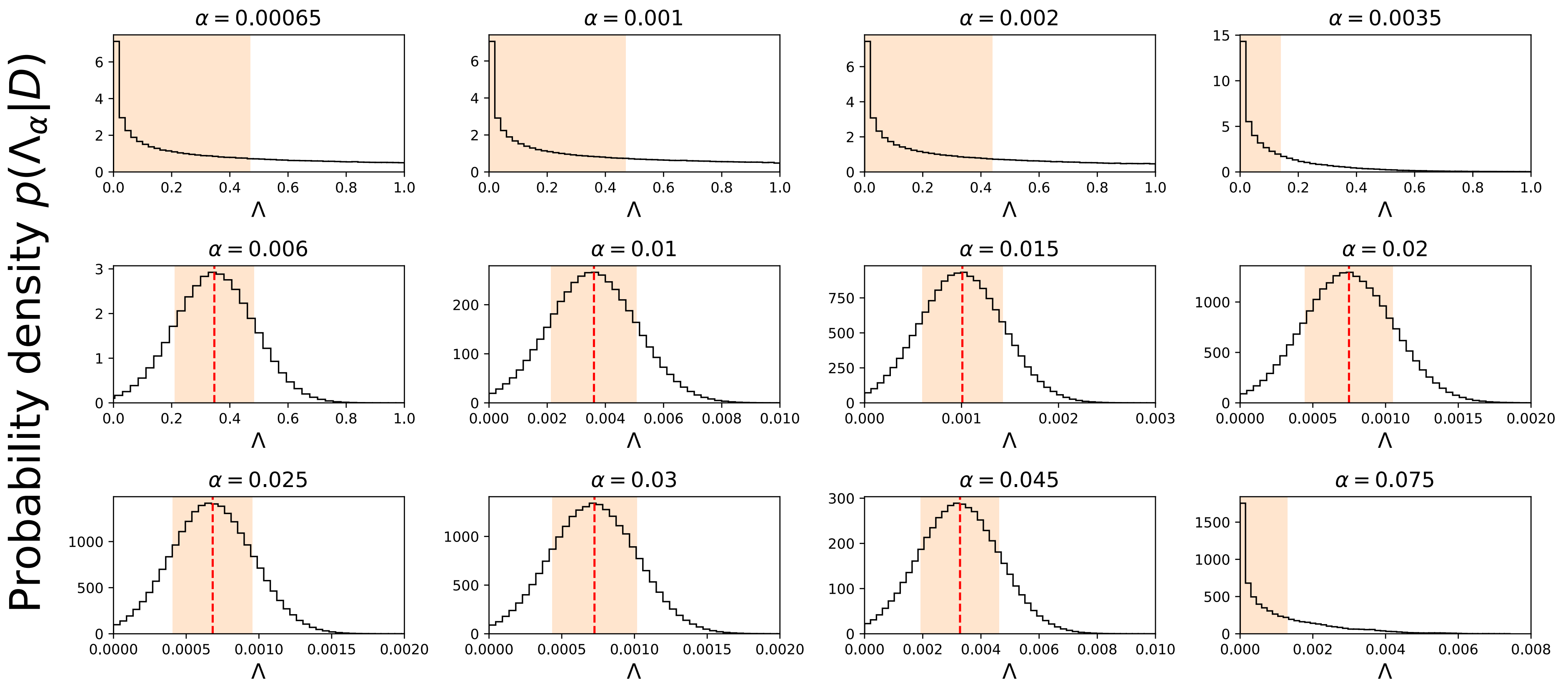
$\hat{\Lambda} = \arg\max \mathscr{L}(\Lambda_\alpha | \mathrm{D})$

α	$\hat{\Lambda}$	$\log_{10} K$
0.00065	$\lesssim (0.470\,, 0.980)$	0.09
0.001	$\lesssim (0.470\,, 0.980)$	0.08
0.002	$\lesssim (0.440\,, 0.978)$	-0.06
0.0035	$\lesssim (0.140\,, 0.780)$	-10.58
0.006	0.34671 ± 0.13666	1.44
0.01	0.00361 ± 0.00147	1.29
0.015	0.00101 ± 0.00042	1.24
0.02	0.00075 ± 0.00030	1.33
0.025	0.00068 ± 0.00028	1.35
0.03	0.00073 ± 0.00029	1.33
0.045	0.00328 ± 0.00135	1.27
0.075	$\lesssim (0.0013\,, 0.0052)$	0.0001



Orange bands: 1σ confidence interval, such that $P(\Lambda_\alpha < \Lambda_1 | D) \approx 68\,\%$ of $P(\Lambda_\alpha | D)$

Bayes' factor $$K = \frac{\mathscr{L}(\hat{\Lambda}_\alpha | D)}{\mathscr{L}(\Lambda = 0 | D)}$$

According to (Kass & Raftery 1995):

$1 \leq \log_{10} K \leq 2$ evidence is **strong**

$\log_{10} K > 2$ evidence is **decisive**

Conclusions, possible issues and future prospects

To summarize…

We used the astrometry and the radial velocity measurements of S2 to constrain the fractional mass $\Lambda = M_{\rm cloud}/M$ of a boson field cloud around Sgr A*.
Orbital range of S2 only allow us to constrain $0.01 \lesssim \alpha \lesssim 0.045$ and we found $\Lambda \lesssim 10^{-3}$ at 3σ confidence level.

Cloud formation process

Fluctuations of massive scalar fields can be exponentially amplified by **superradiance** (Brito *et al*. 2015).
However, (Kodama & Yoshino 2012) show that for $M_\bullet \sim 4 \cdot 10^6\, M_\odot$

$$m_s \geq 10^{-18}\,\mathrm{eV} \qquad\qquad (\alpha = 0.045,\;\; m_s \simeq 3 \cdot 10^{-18}\,\mathrm{eV})$$

However, we can assume DM existed by itself in the galaxy and the BH passes through it, leading to long-lived structures (Cardoso *et al*. 2022a, Cardoso *et al*. 2022b).

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Possible issues and future works

- S2 is orbiting on the equator of the BH, i.e. $\theta = \pi/2$ but there are no evidences. However, max difference in the astrometry and radial velocity with orbit at $\theta = 0$ is $\Delta\mathrm{DEC} \sim \Delta\mathrm{R.A.} \approx 25\,\%$ for $\alpha = 0.01$ and $\Delta V_R \approx 15\,\%$ for $\alpha = 0.045$. Difference would be smaller for any $\theta \in (0, \pi/2)$.

- No inclusion of BH's spin axis inclination with respect to observer frame. (GRAVITY Coll. 2019) showed that it plays important role in the effects the cloud has on S2 motion. Left for future works.

- Inclusion of other S-stars, and hence different orbital ranges, is needed in order to have stronger constraints - or even a detection!

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Thank you for your attention!

Back-up slides

Setup

Corrections to the Newtonian model

(GRAVITY 2018, Alexander 2005)

- **Newtonian effect**: the Roemer delay due to finite value of c

Roemer equation: $$t_{\rm obs} - t_{\rm em} + \frac{z_{\rm obs}(t_{\rm em})}{c} = 0$$

1st order expansion around $t_{\rm obs}$: $$t_{\rm em} \simeq t_{\rm obs} + \frac{z_{\rm obs}(t_{\rm obs})}{c - v_{z_{\rm obs}}(t_{\rm obs})}$$

On average on S2 orbit
$$\Delta t = t_{\rm em} - t_{\rm obs} \approx 8 \text{ days}$$

Setup

Corrections to the Newtonian model

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- **Relativistic effects**: the Doppler shift and the gravitational redshift $(G = c = 1)$ must be included when S2 reaches periastron with total space velocity $\beta \sim 10^{-2}$.

Doppler: $$z_D = \frac{1 + \beta \cos\theta}{\sqrt{1 - \beta^2}} - 1$$

Gravitational redshift: $$z_{\text{grav}} = \frac{1}{\sqrt{1 - 2M_\bullet / r_{\text{em}}}} - 1$$

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The equations of motion

From Euler-Lagrange equations: $$\frac{d}{dt}\left(\frac{\partial \mathscr{L}}{\partial \dot{q}}\right) - \frac{\partial \mathscr{L}}{\partial q} = 0$$

$$\longrightarrow \begin{cases} \ddot{r} - r\dot{\theta}^2 - r\sin^2\theta\dot{\phi}^2 + \dfrac{1}{r^2} - \Lambda\left(P_1'(r) + P_2'(r)\cos 2\theta\right) = 0 \\ 2r\dot{r}\sin^2\theta\dot{\phi} + 2r^2\cos\theta\sin\theta\dot{\theta}\dot{\phi} + r^2\sin^2\theta\ddot{\phi} = 0 \\ 2r\dot{r}\dot{\theta} + r^2\ddot{\theta} - r^2\cos 2\theta\dot{\theta}\dot{\phi}^2 + 2\Lambda P_2(r)\sin 2\theta\dot{\theta} = 0 \end{cases}$$

That we numerically integrate using an adaptive Runge-Kutta of order 4(5) and initial conditions given by the solution of Kepler's two body problem.

$$r(t_0) = \frac{1-e^2}{1+e\cos(\phi(t_0))}$$

$$\phi(t_0) = 2\arctan\left(\sqrt{\frac{1+e}{1-e}}\tan\frac{\mathscr{E}(t_0)}{2}\right)$$

$$\dot{r}(t_0) = \frac{2\pi e\sin(\mathscr{E}(t_0))}{1-e\cos(\mathscr{E}(t_0))}$$

$$\dot{\phi}(t_0) = \frac{2\pi(1-e)}{(e\cos(\mathscr{E}(t_0))-1)^2}\sqrt{\frac{1+e}{1-e}}$$

with

Kepler's equation

$$\mathscr{E} - e\sin\mathscr{E} - \mathscr{M} = 0$$

$$\mathscr{M} = \frac{2\pi}{P}\left(t_0 - t_p\right)$$

How emcee works

(Foreman-Mackey et al. 2013, Goodman & Weare 2010)

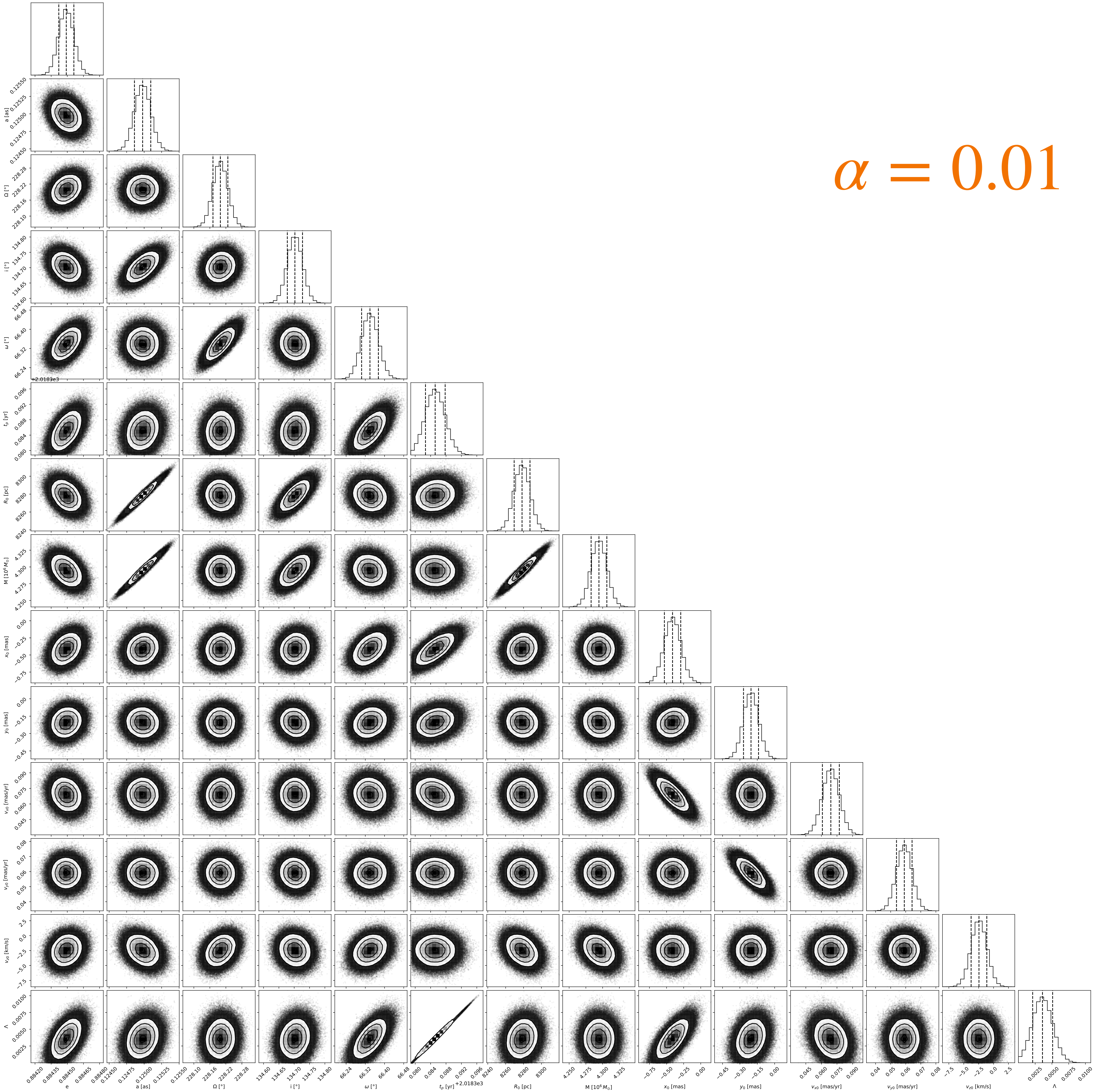
- **Step 1**. It generates K walkers around any initial value of the parameters θ_i^0 from $\mathcal{N}(\theta_i^0, \sigma)$ ($\sigma = 10^{-5}$);

- **Step 2.** To update the position of a walker at $X_k(t)$, a walker X_j is randomly extracted from the complementary ensemble $S_{[k]} = \left\{ X_j, \forall j \neq k \right\}$ and the new position is generated as $Y = X_j + Z\left[X_k(t) - X_j\right]$, where Z is drawn from $g(Z = z)$ defined as:

$$g(z) \propto \begin{cases} \frac{1}{\sqrt{z}} & \text{if } z \in \left[\frac{1}{a}, a\right] \\ 0 & \text{otherwise} \end{cases}$$

- **Step 3**. It computes $q = \min\left(1, Z^{N-1}\frac{p(Y)}{p\left(X_k(t)\right)}\right)$, where N is the number of parameters, for each walker.

- **Step 4**. It randomly extracts a variable $r \sim U\left[0, 1\right]$. If $r \leq q$ then the move is ***accepted*** and $X_k(t+1) \rightarrow Y$. If $r > q$ the move is ***rejected*** and $X_k(t+1) \rightarrow X_k(t)$.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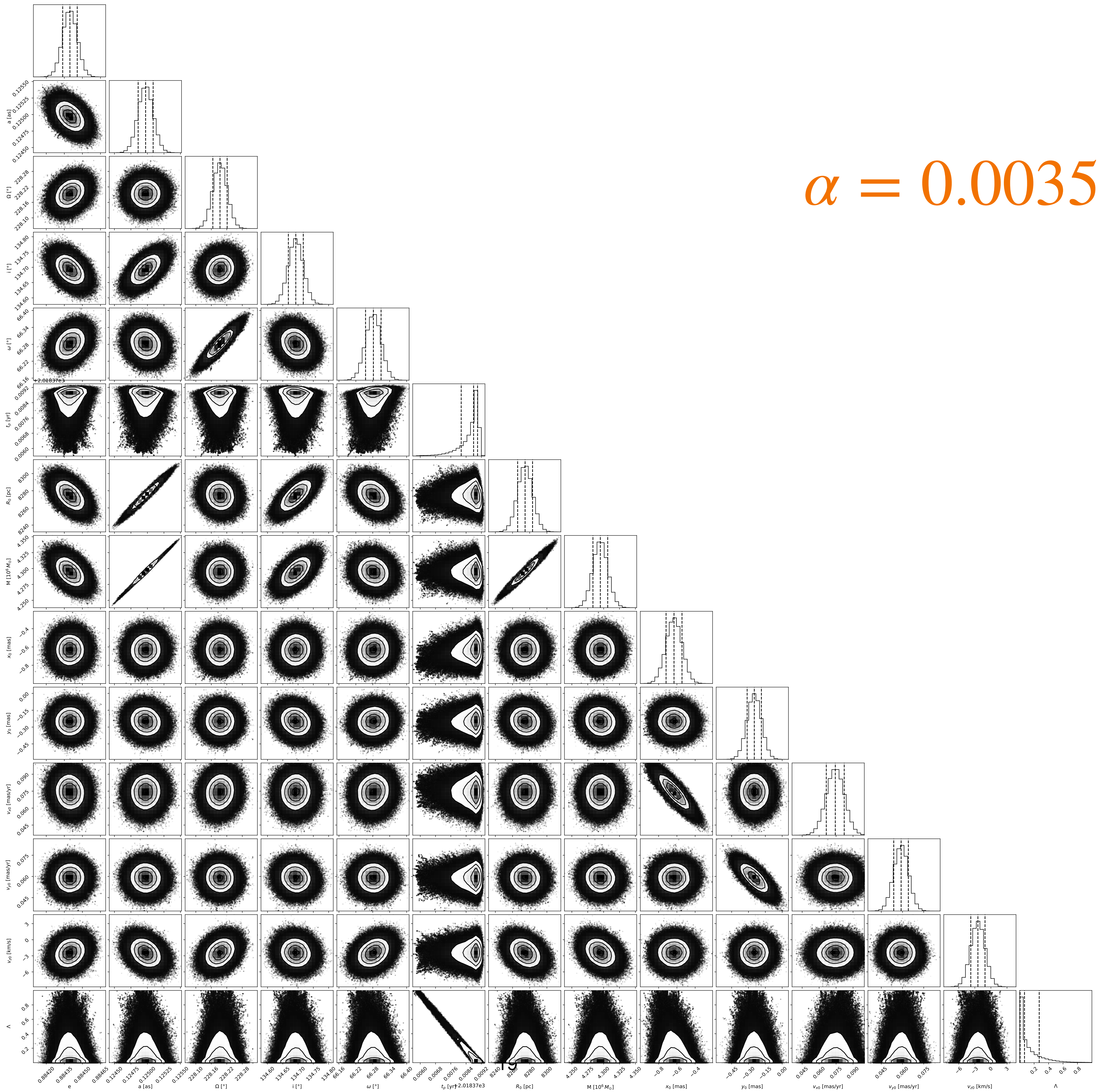
Λ

$\alpha = 0.01$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Λ

$\alpha = 0.0035$