CENTRA - Lisbon

PSEUDOSPECTRUM AND BLACK-HOLE QUASI-NORMAL MODE (IN)STABILITY

J. L. Jaramillo, L. Al Sheik

PRX 11 031003 (2021)

PRL 128 21 211102 (2002)

K. Destounis, RPM, E. Berti, V. Cardoso, J.L. Jaramillo

(Reissner Nørdstrom Spacetime)

PRD 104 084091 (2021)

Rodrigo Panosso Macedo

QUASINORMAL MODES

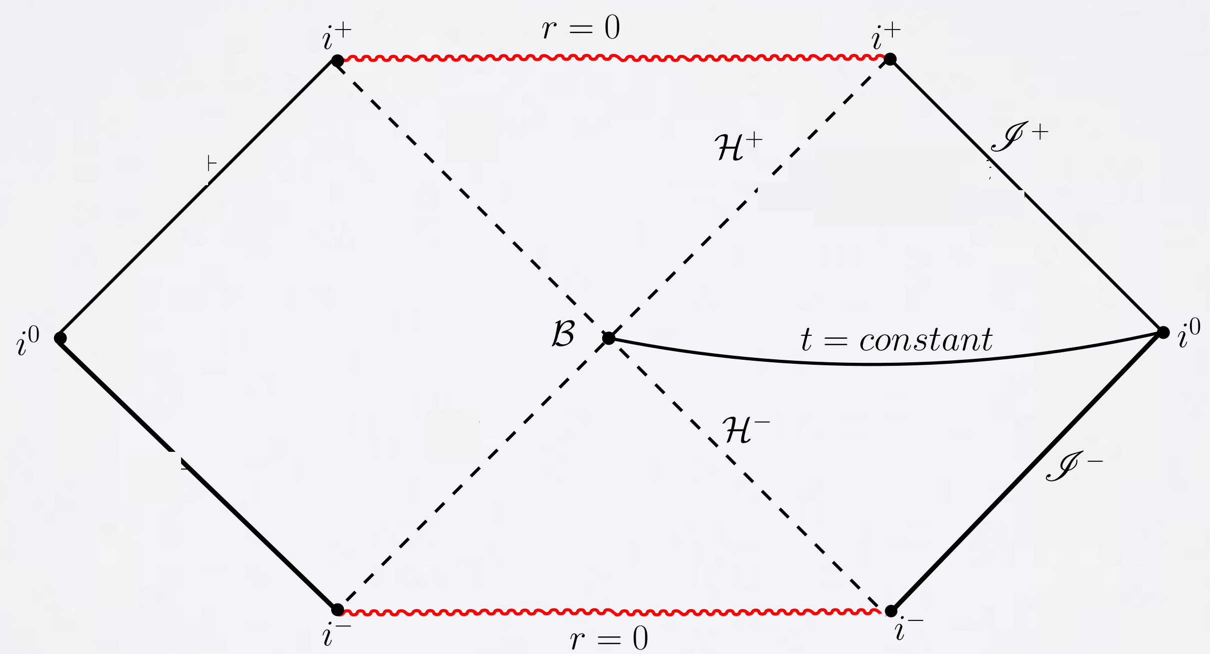
- **Wave equation on (spherically sym.) black-hole background**

$$-\Psi_{,\bar{t}\bar{t}} + \Psi_{,xx} - \mathcal{P}\Psi = 0$$

- **Phenomenological approach:**

Ansatz for time dependence (Fourier modes)

$$\Psi = e^{-i\omega\bar{t}}\psi(x) \qquad \Rightarrow \quad \left[\partial^2_{xx} - \mathcal{P}\right]\psi = -\omega^2\psi$$



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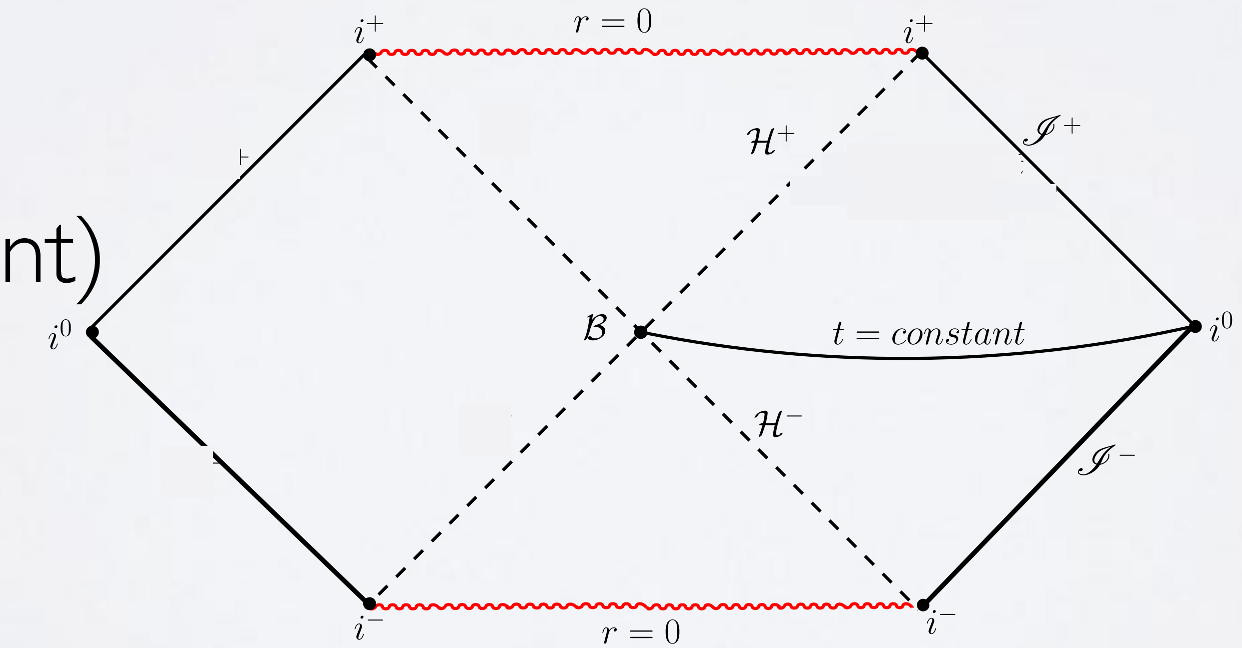
$\mathcal{B}$: Horizon
(bifurcation point)
$x \to -\infty$



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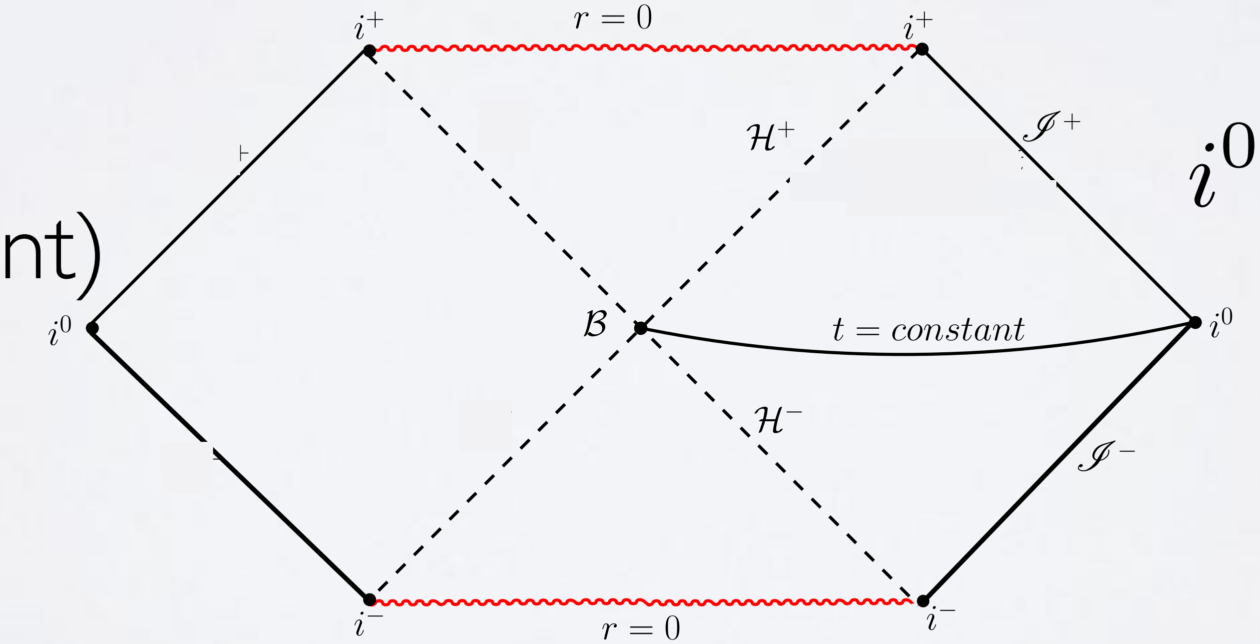

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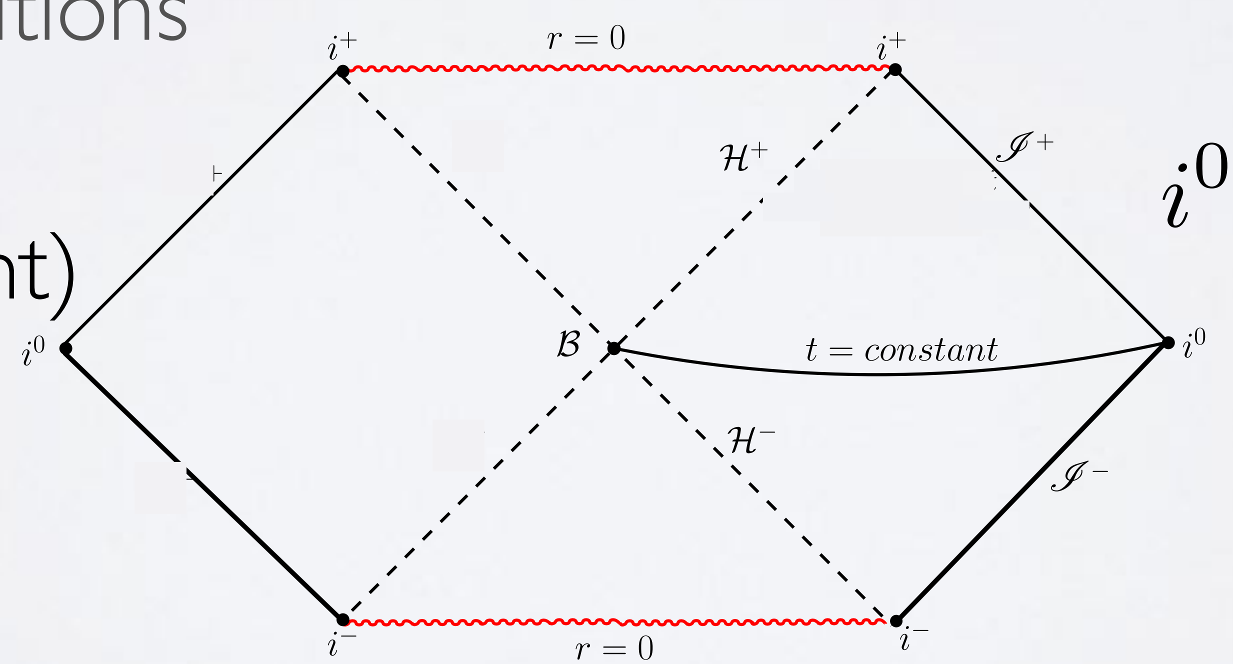

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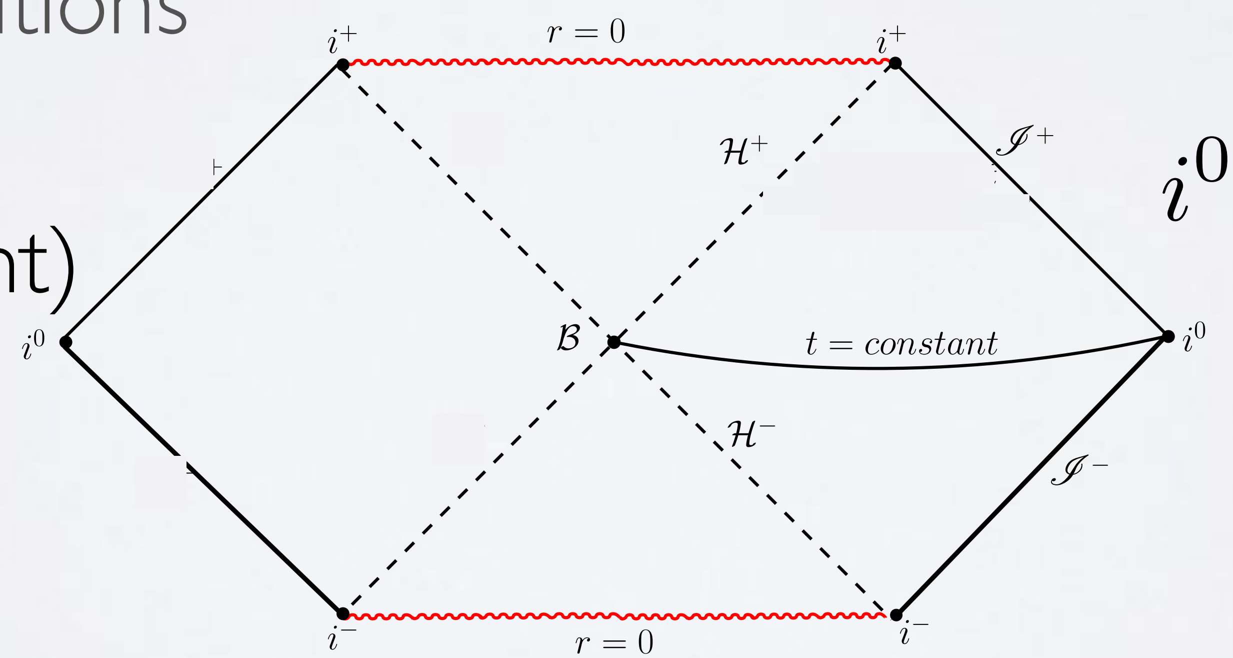

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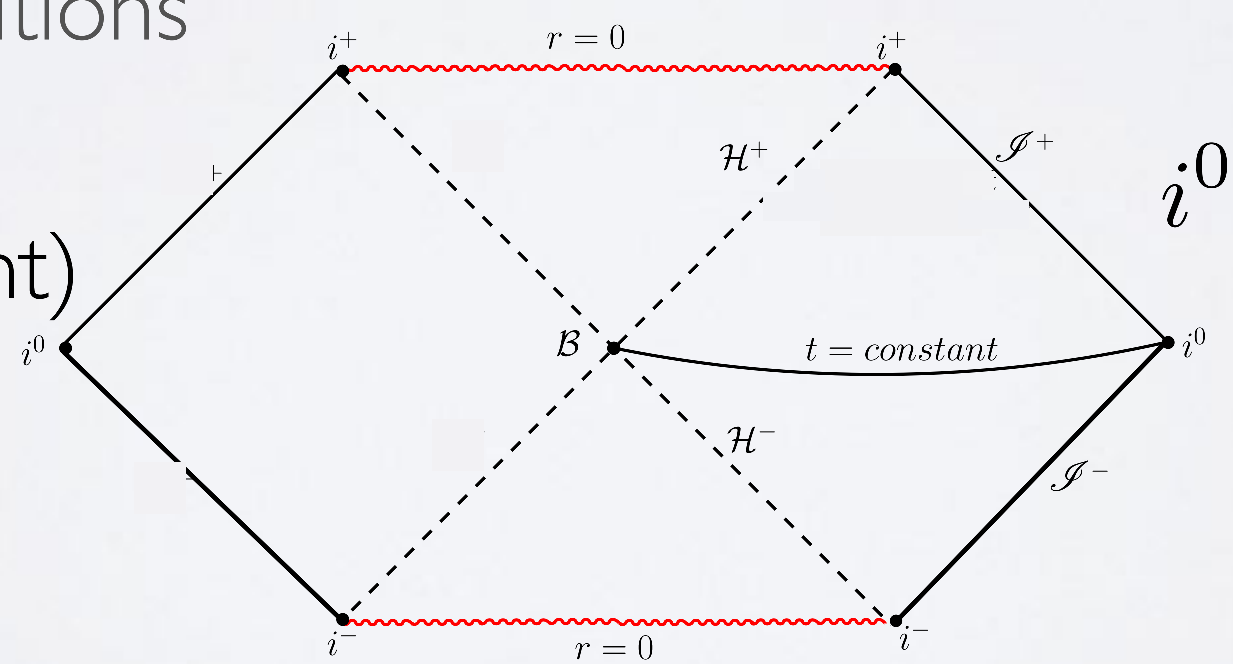

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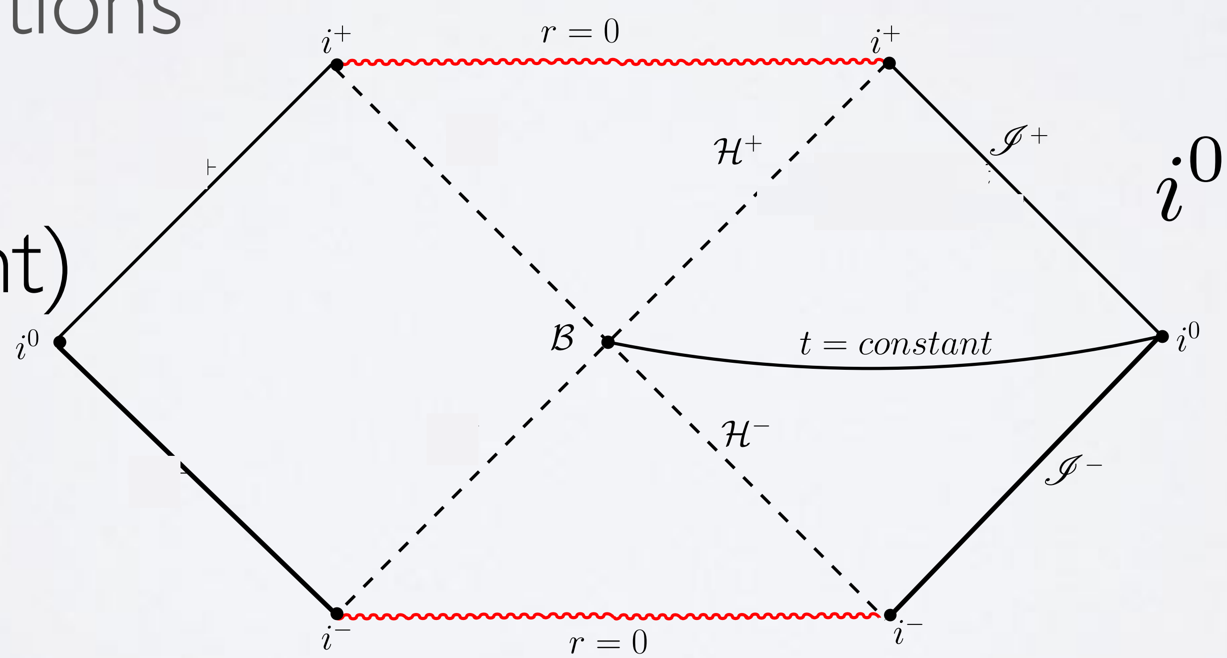

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- **Comparison to *normal* modes:**

eigenvalue problem

NORMAL OPERATORS: SPECTRAL THEOREMS

Def. *Given matrix* L *and its adjoint* $L^\dagger$*. Then* L *is* <u>*normal*</u> *iff* $[L, L^\dagger] = 0$

Ex. *Symmetric, hermitian, orthogonal, unitary…*

Spectral Theo. (matrices): L *is normal iff is unitarily diagonalisable*

Obs. *Theorem extends to normal operators in Hilbert (or Banach) space*

↓

Hermitian Physics (self adjoint operators):

Eigenvectors are orthogonal and form complete basis

Eigenvalues are stable $L \to L + \epsilon\delta L \Rightarrow \lambda \to \lambda + \epsilon\delta\lambda$

NON-NORMAL OPERATORS: NO SPECTRAL THEOREMS

If $[L, L^\dagger] \neq 0$

Completeness more difficult to study
Eigenvectors not necessarily orthogonal
Spectral Instabilities

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$$|\lambda_i(\epsilon) - \lambda_i| \leq \kappa_i \, \epsilon$$

<u>Eigenvalue Condition Number:</u> $\kappa_i = \dfrac{||r_i|| \, ||l_i||}{|r_i \cdot l_i|}$

$L \, r_i = \lambda_i \, r_i$ (Right eigenvector)

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Small perturbation in operator; small displacement in eigenvalue

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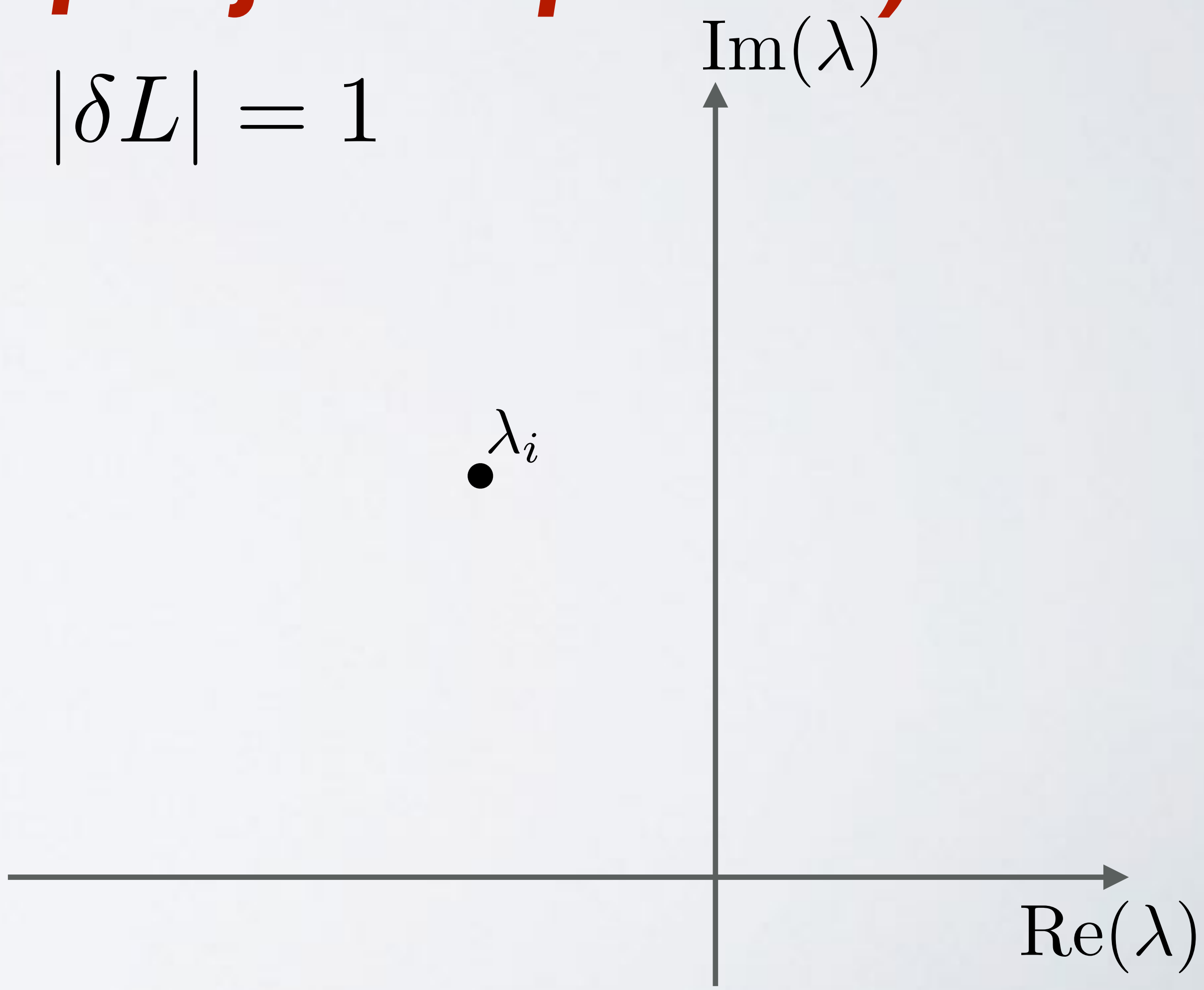

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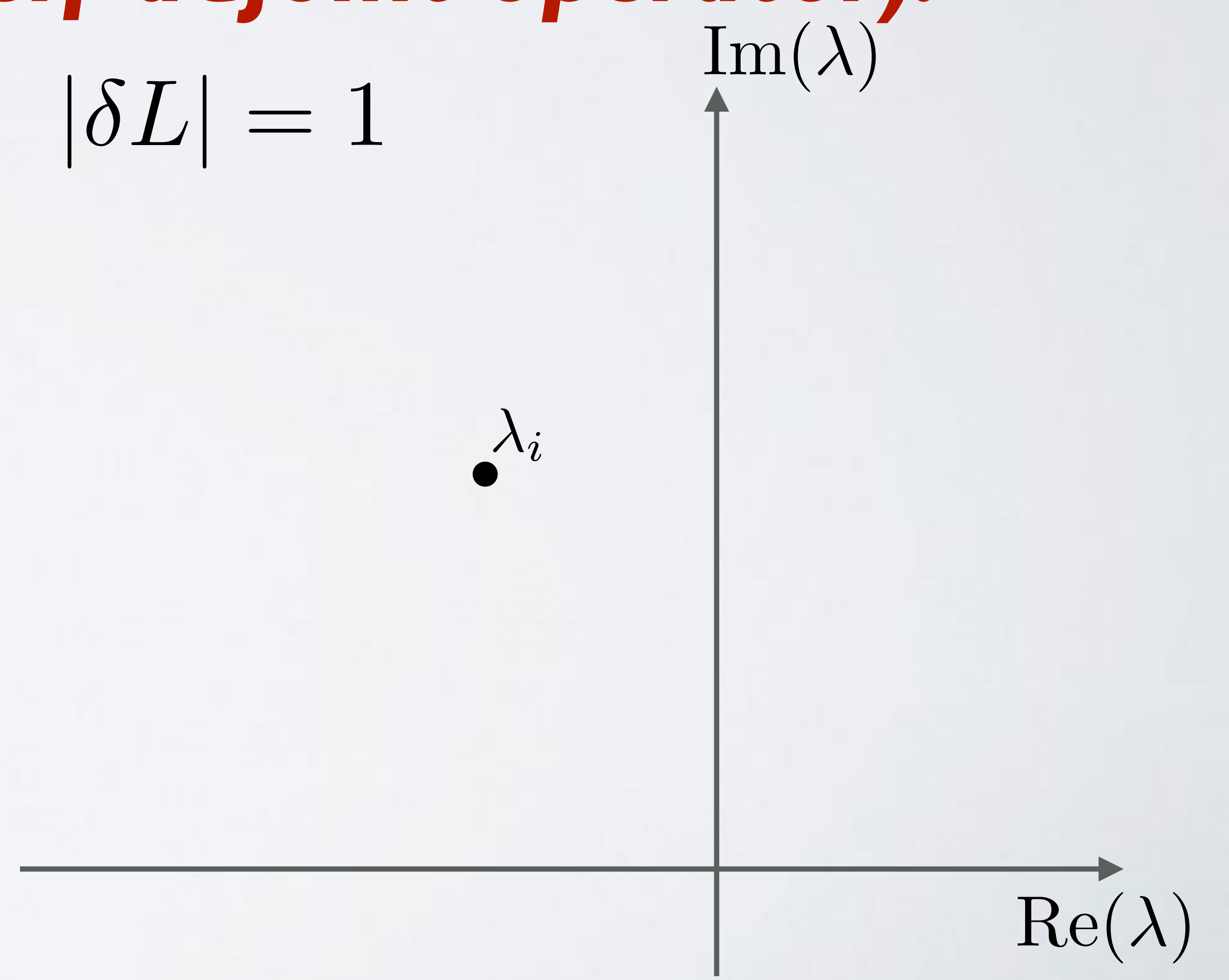

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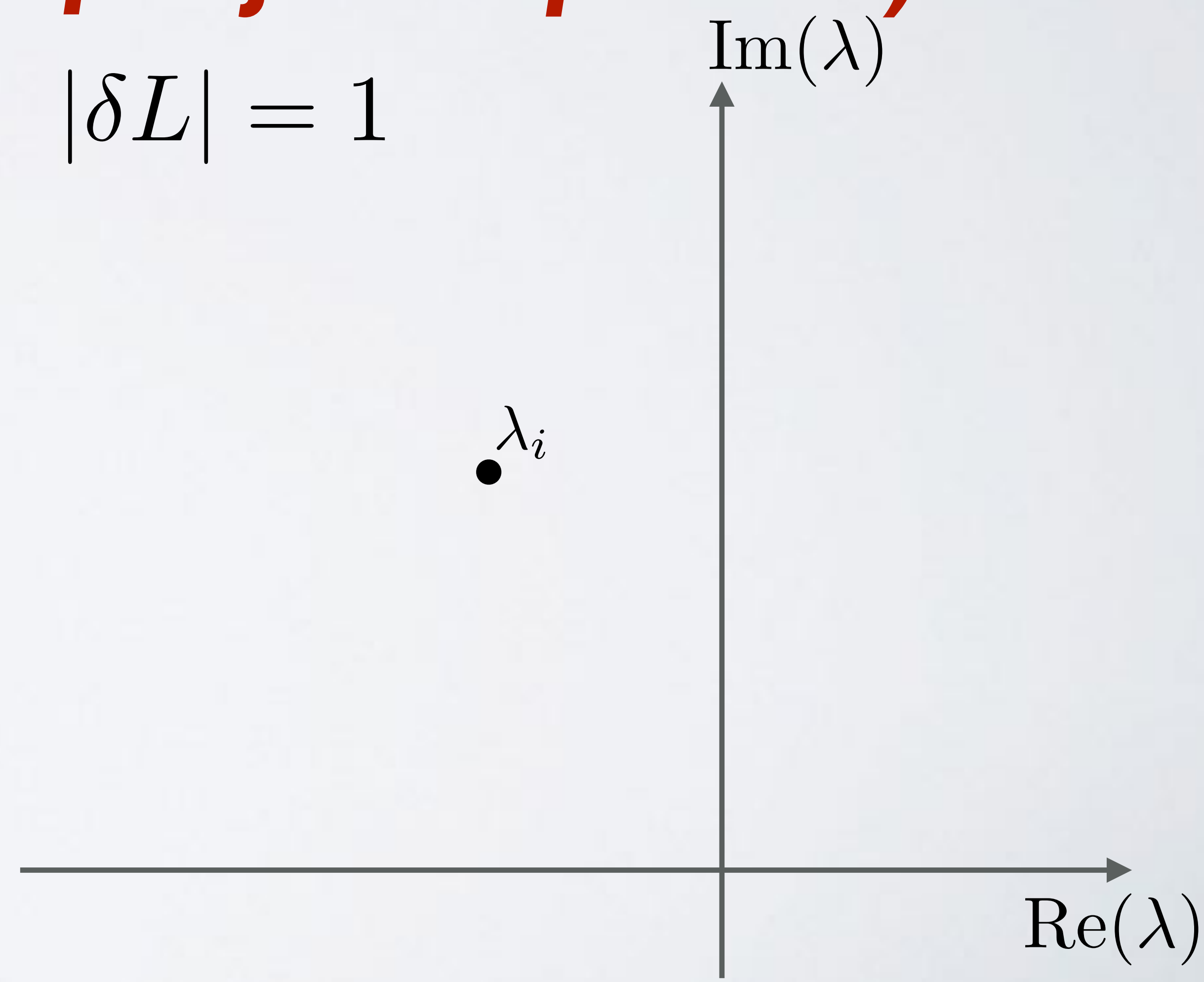

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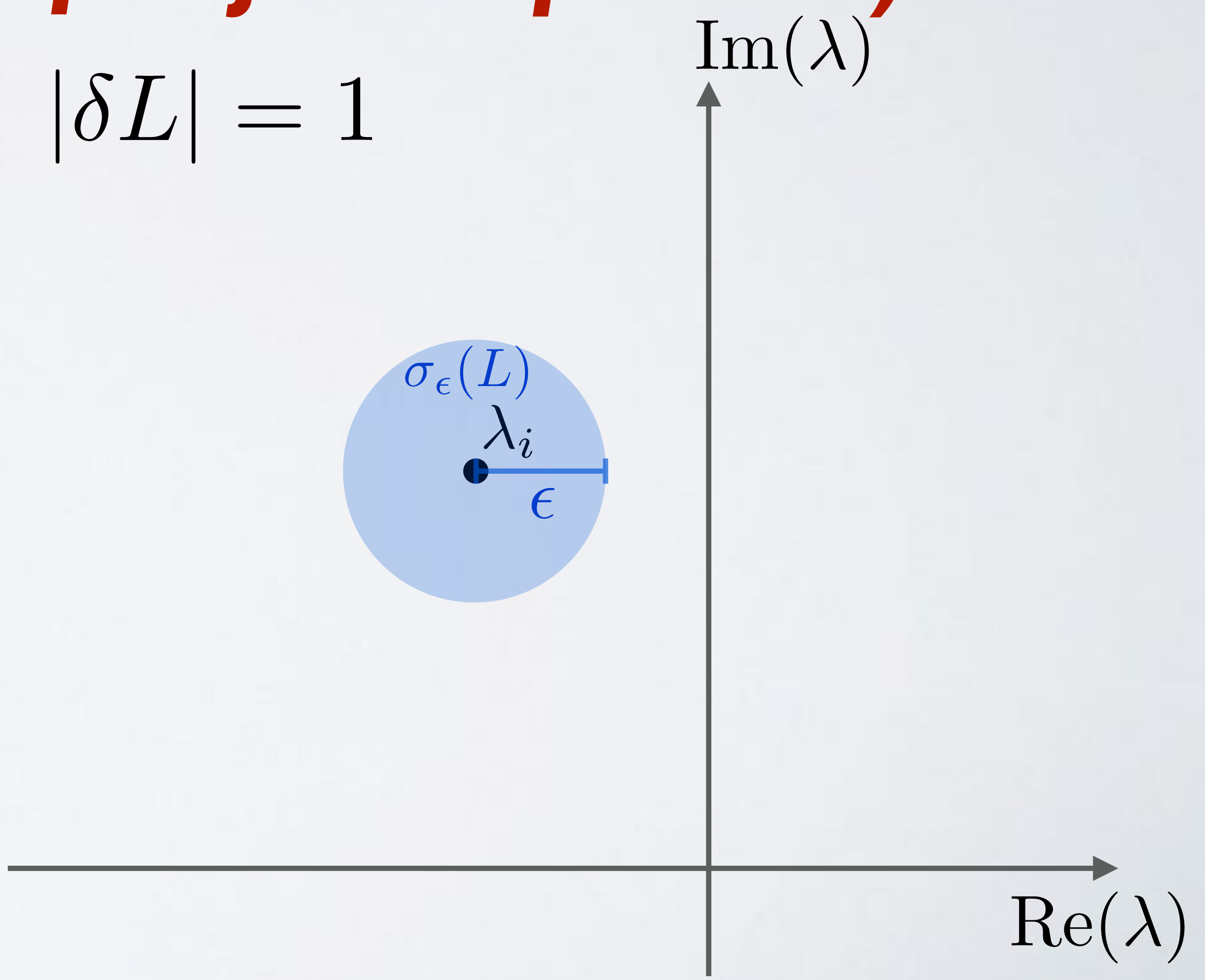

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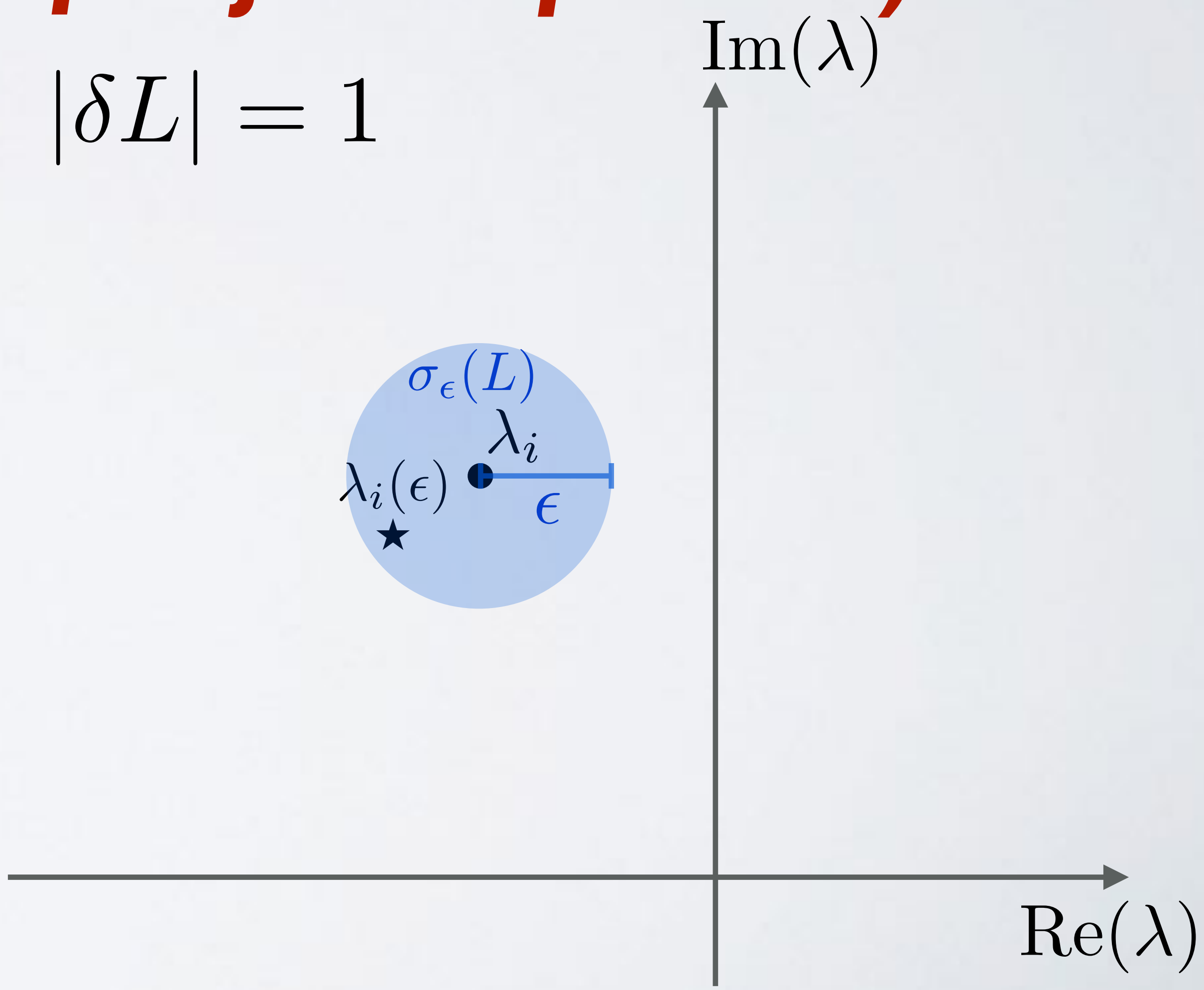

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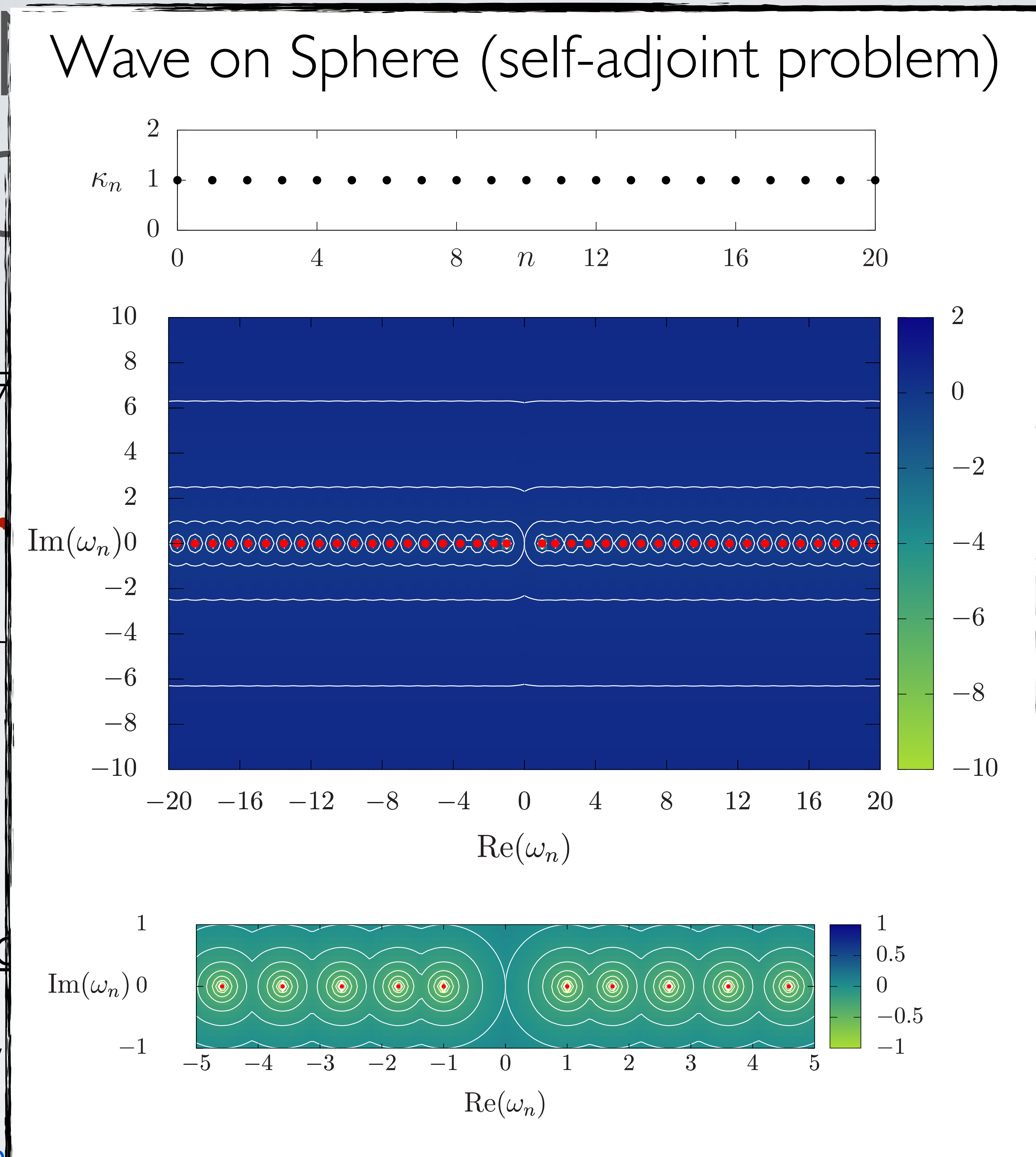

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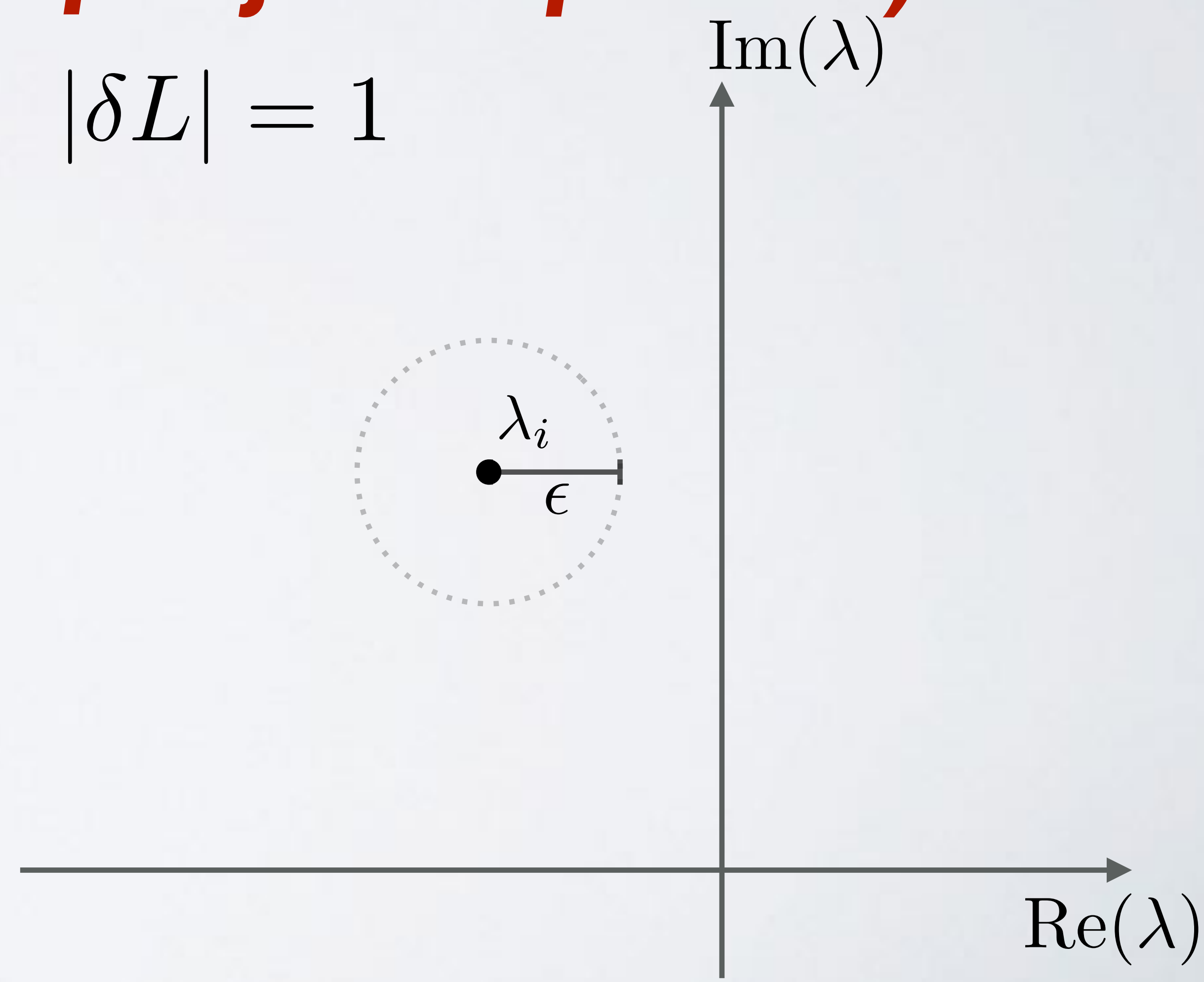

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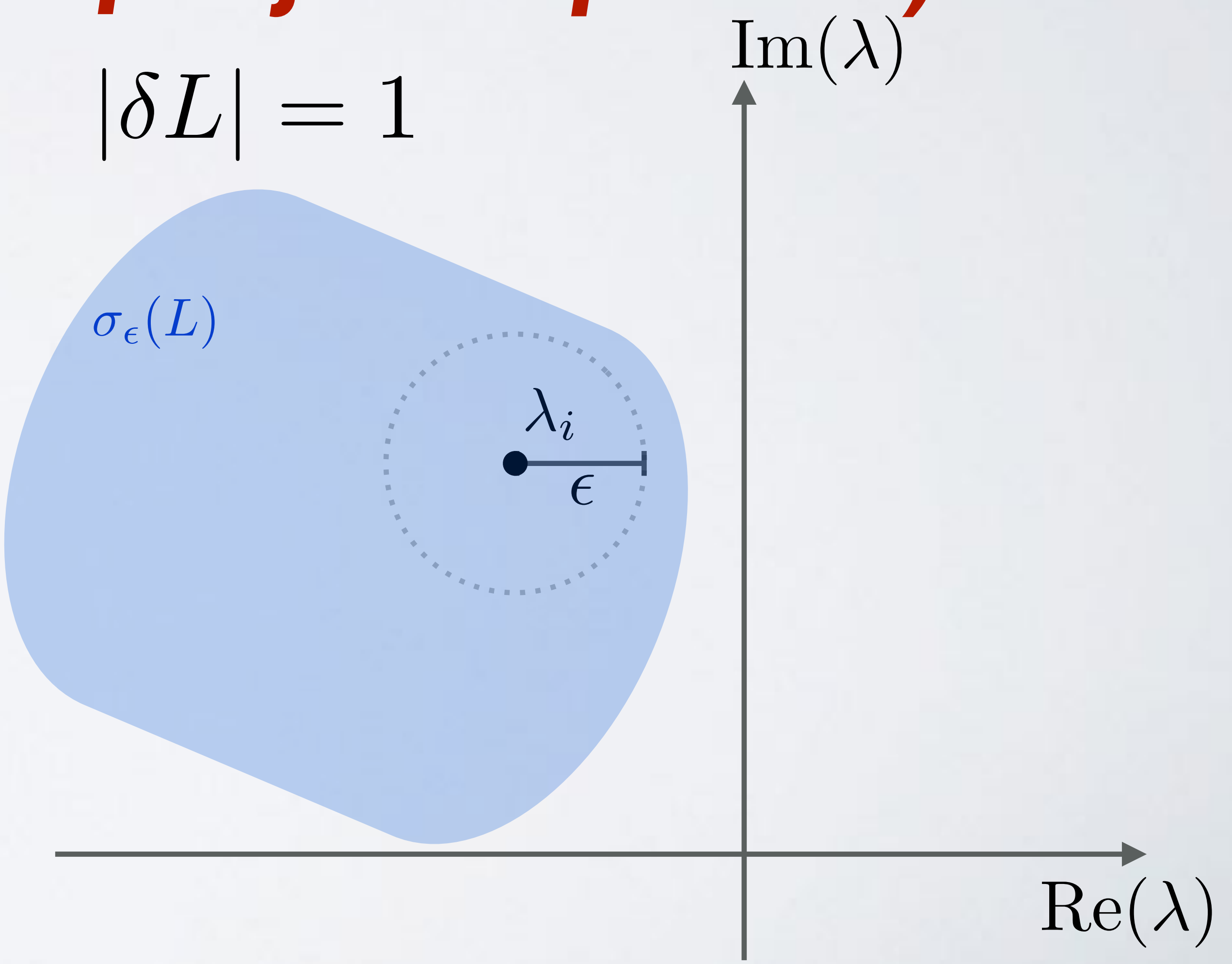

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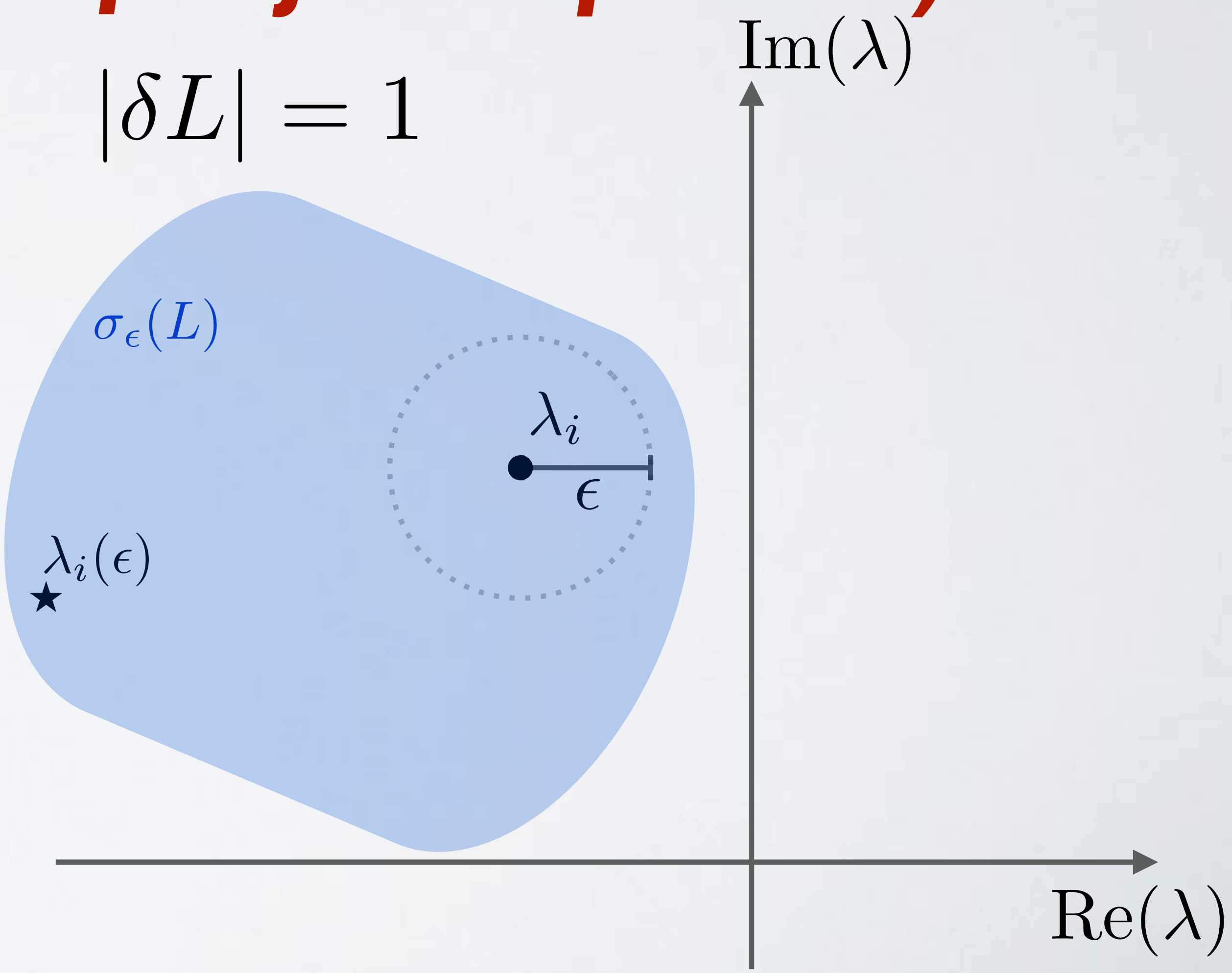

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Take away message:

- The <u>unperturbed</u> operator contains <u>all</u> pieces of information to assess potential spectral instabilities
- Tools to measure: condition number and pseudospectra

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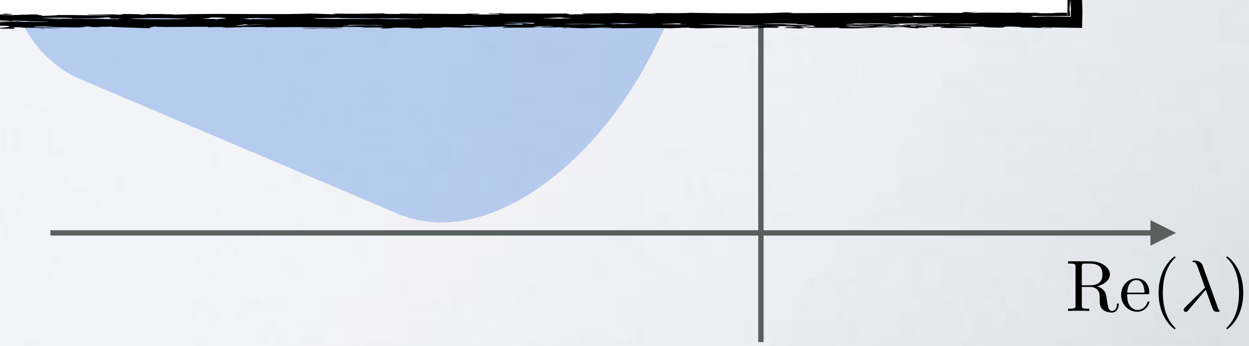

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Non-Hermitian Physics (non-self adjoint operator):

Flow of energy, particles and information to external degrees of freedom out of the Hilbert space

Y. Ashida, Z. Gong, M. Ueda Advances in Physics 69 3 (2020) - arXiv 2006.01837

Table 1. A wide variety of classical and quantum systems described by non-Hermitian matrices/operators together with their physical origins of non-Hermiticity, presented in order of appearance in the present review.

Systems / Processes	Physical origin of non-Hermiticity	Theoretical methods
Photonics	Gain and loss of photons	Maxwell equations [12, 13]
Mechanics	Friction	Newton equation [14, 15]
Electrical circuits	Joule heating	Circuit equation [16]
Stochastic processes	Nonreciprocity of state transitions	Fokker-Planck equation [17, 18]
Soft matter and fluid	Nonlinear instability	Linearized hydrodynamics [19–21]
Nuclear reactions	Radiative decays	Projection methods [4–6]
Mesoscopic systems	Finite lifetimes of resonances	Scattering theory [22, 23]
Open quantum systems	Dissipation	Master equation [24, 25]
Quantum measurement	Measurement backaction	Quantum trajectory approach [26–31]

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Gravity (Black hole theory)

Fields/Energy leaking away from the system:

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How to geometrically impose the notions of far distant

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Hyperboloidal Approach to BH perturbation theory

B. Schmidt "On relativistic stellar oscillations"
Gravity Research Foundation essay (1993)

A. Zenginoglu :
PRD 83 127502 (2011)

M. Ansorg, RPM PRD 93 124016 (2016)

RPM, J. L. Jaramillo, M. Ansorg
PRD 98 124005 (2018)

RPM CGQ 37 6 (2020)

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- Condition Number + Pseudospectral tools applied to Schwarzschild

$$\partial_\tau u = iLu$$

$$L\hat{u} = \omega\hat{u}$$

$$L = \frac{1}{i}\left(\begin{array}{c|c} 0 & 1 \\ \hline L_1 & L_2 \end{array}\right)$$

$$L_1 = (1+\sigma)^{-1}\left(\partial_\sigma\left[\sigma^2(1-\sigma)\partial_\sigma\right] - \underbrace{\left[\ell(\ell+1) + (1-s^2)\sigma\right]}_{q_\ell}\right)$$

$$L_2 = (1+\sigma)^{-1}\left((1-2\sigma^2)\partial_\sigma - 2\sigma\right) \qquad \sigma \in [0,1]$$

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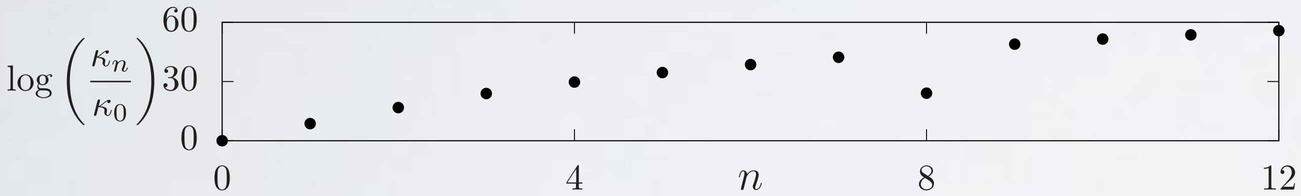
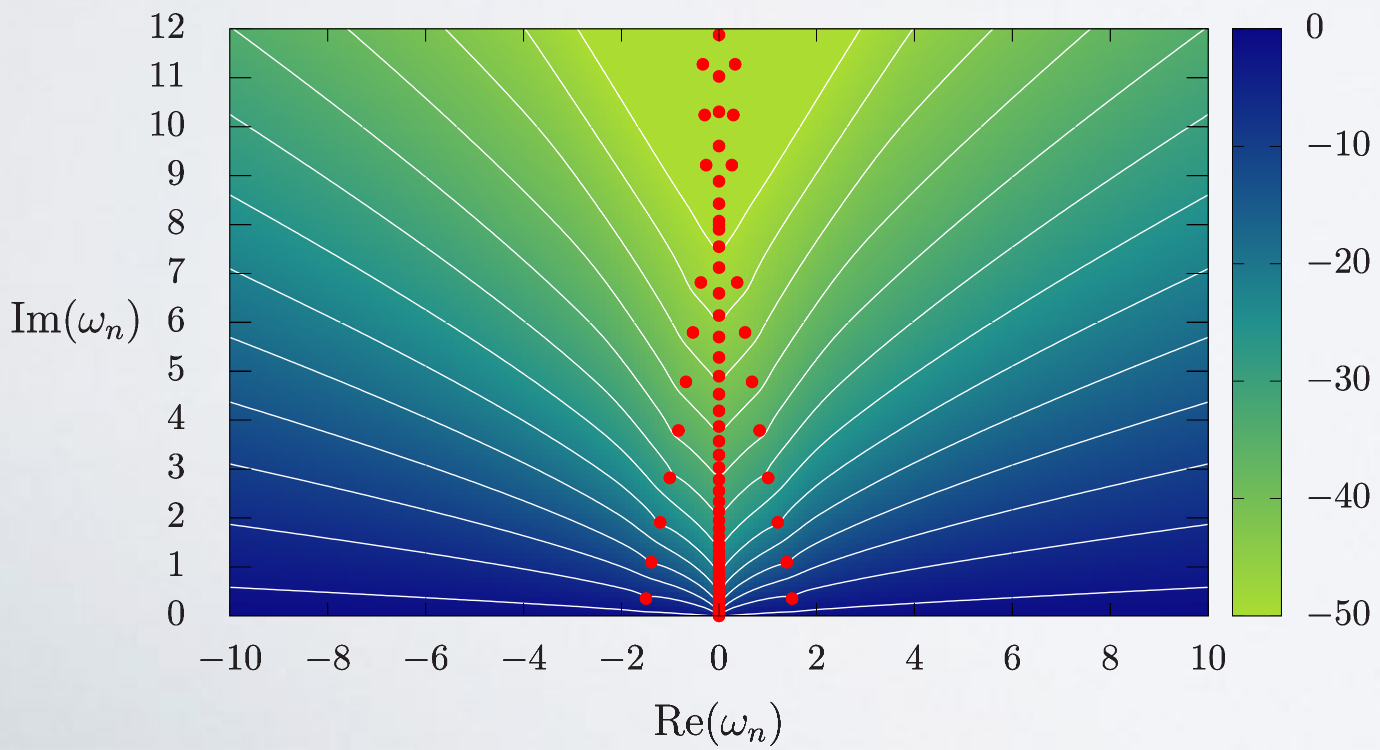




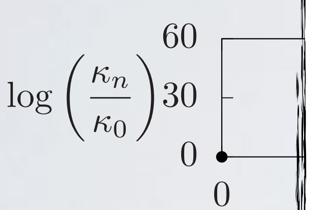
SPECTRAL INSTABILITY

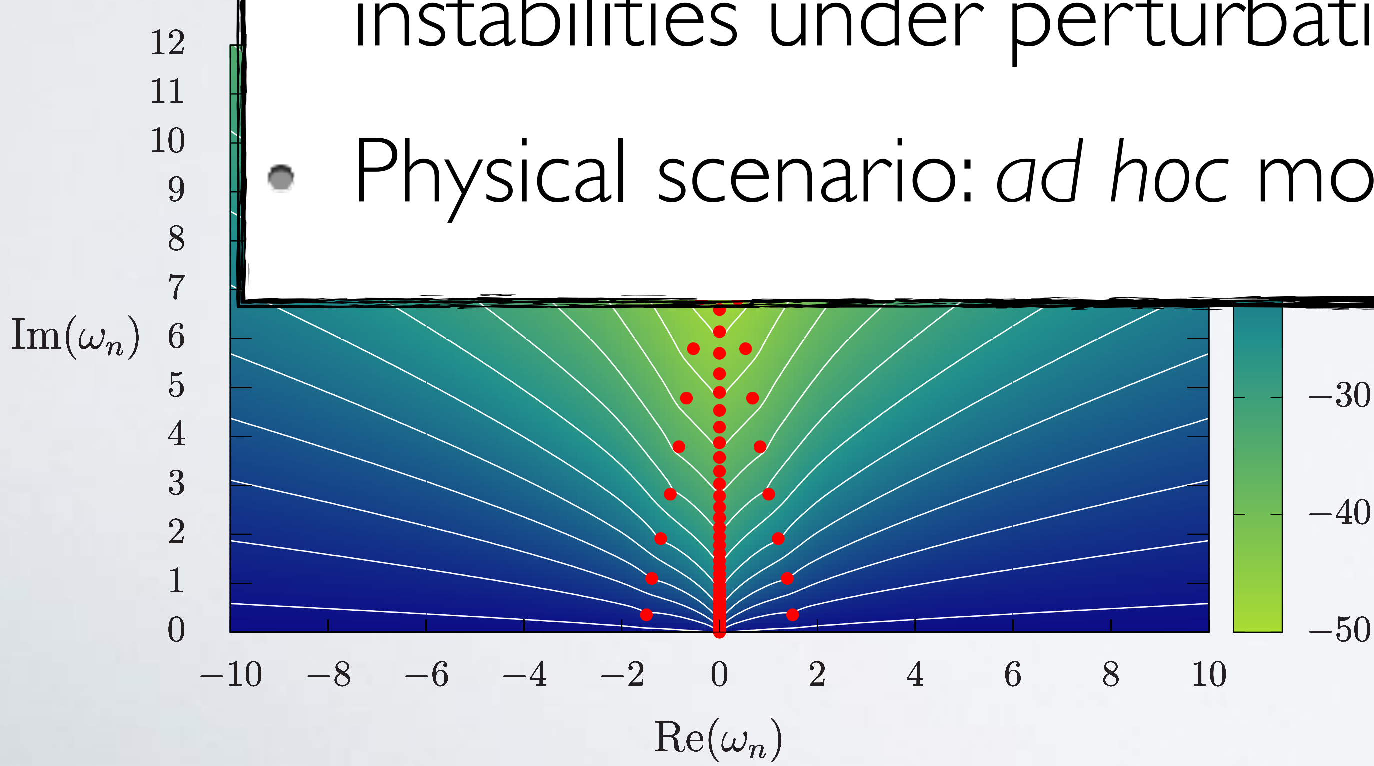
- Condition Number + Pseudospectral tools applied to Schwarzschild

$$\partial_\tau u = iLu \qquad L = \frac{1}{i}\left(\begin{array}{c|c} 0 & 1 \\ \hline L_1 & L_2 \end{array}\right) \qquad L_1 = (1+\sigma)^{-1}\left(\partial_\sigma\left[\sigma^2(1-\sigma)\partial_\sigma\right] - \left[\ell(\ell+1)+(1-s^2)\sigma\right]\right)$$

$$L\hat{u} = \omega\hat{u}$$

$$L_2 = (1+\sigma)^{-1}\left((1-2\sigma^2)\partial_\sigma - 2\sigma\right) \quad \sigma\in[0,1]$$





Partial Conclusions:

- Condition number and pseudospectra indicate potential instabilities under perturbation of the operator.
- Physical scenario: *ad hoc* modification of the potential

HISTORICAL INTERLUDE

C.V. Vishveshwara: "On the Black Hole Trail … : A Personal Journey" (1996)

> wave during the coalscence of binary black holes[18]. Recently Aguirregabiria and I have studied the sensitivity of the quasinormal modes to the scattering potential[19]. The motivation is to understand how any perturbing influence, such as another gravitating source, that might alter the effective potential would thereby affect the quasinormal modes. Interestingly, we find that the fundamental mode is, in general, insensitive to small changes in the potential, whereas the higher modes could alter drastically. The fundamental mode would therefore carry the imprint of the black hole, while higher modes might indicate the nature of the perturbing source.

J.M Aguirregabiria, C.V. Vishveshwara: Phys. Letter A 210 251 (1996)

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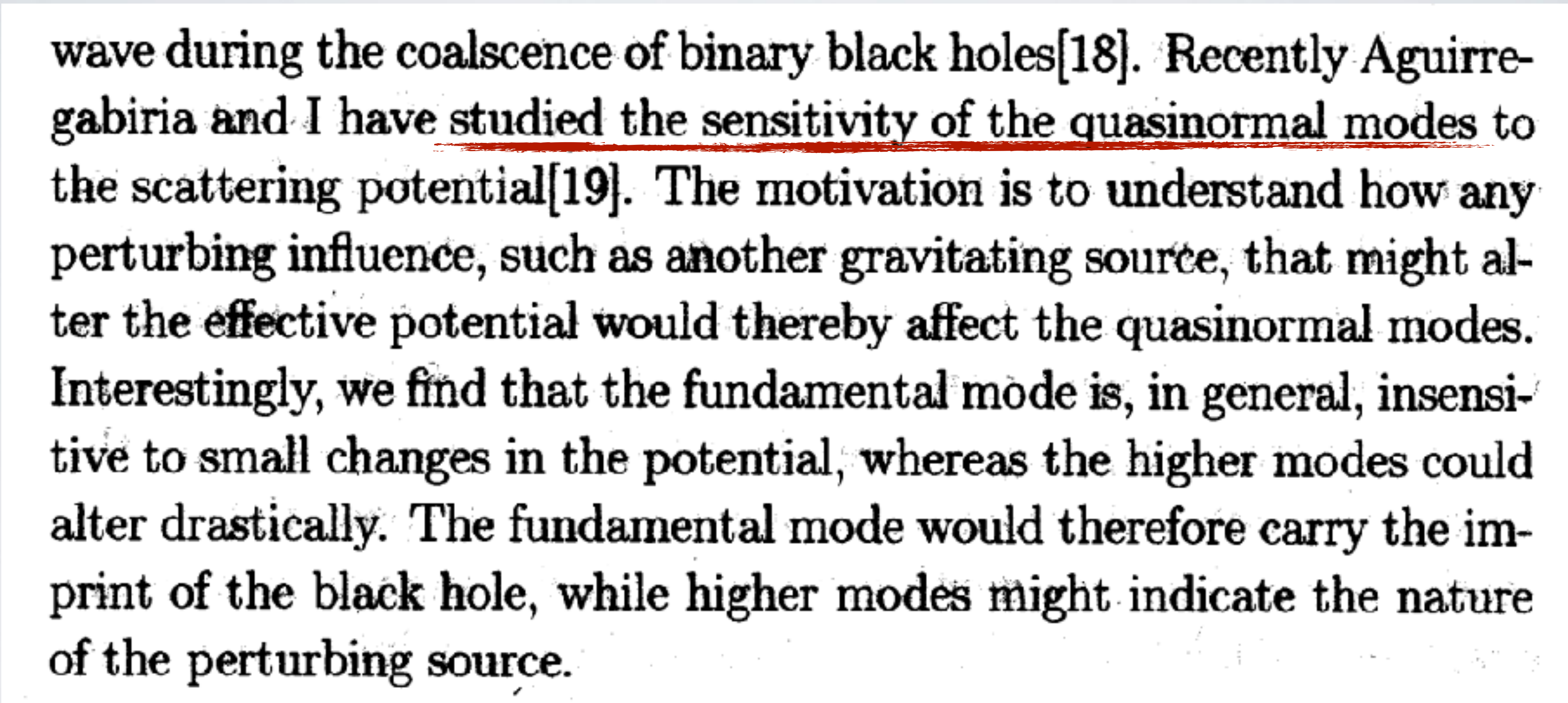
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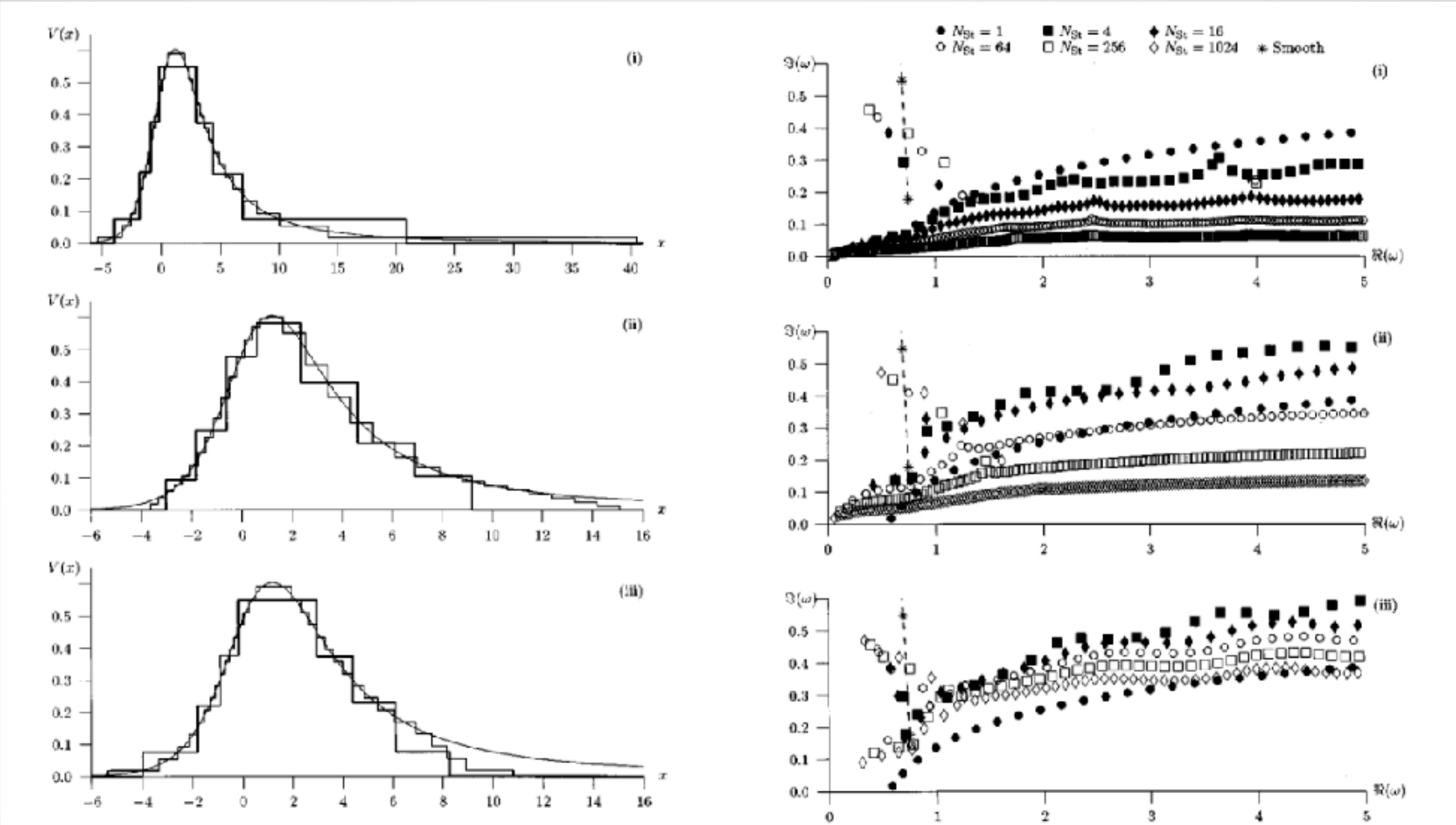
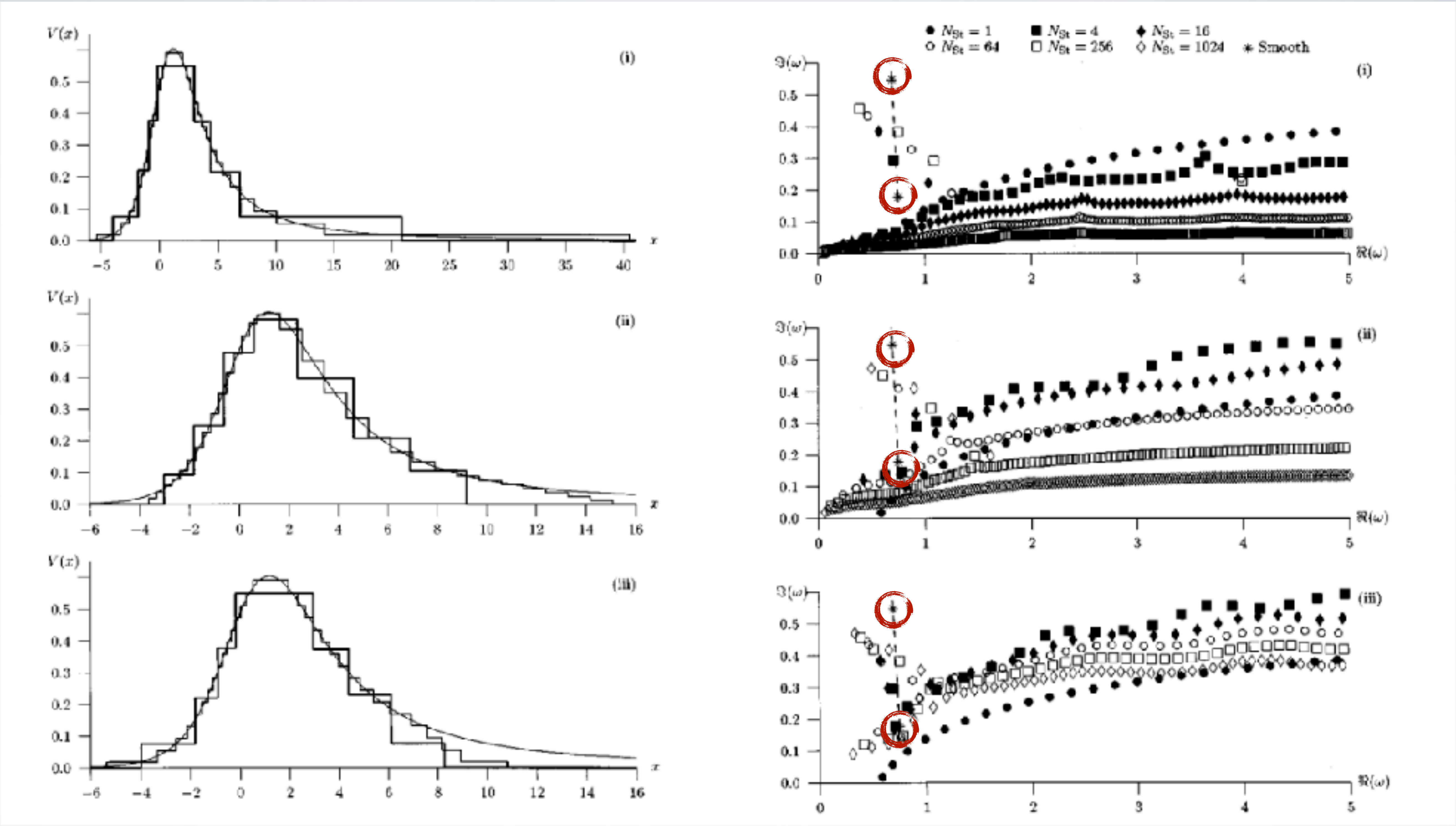


H.P. Nollert PRD 53 8 (1996)

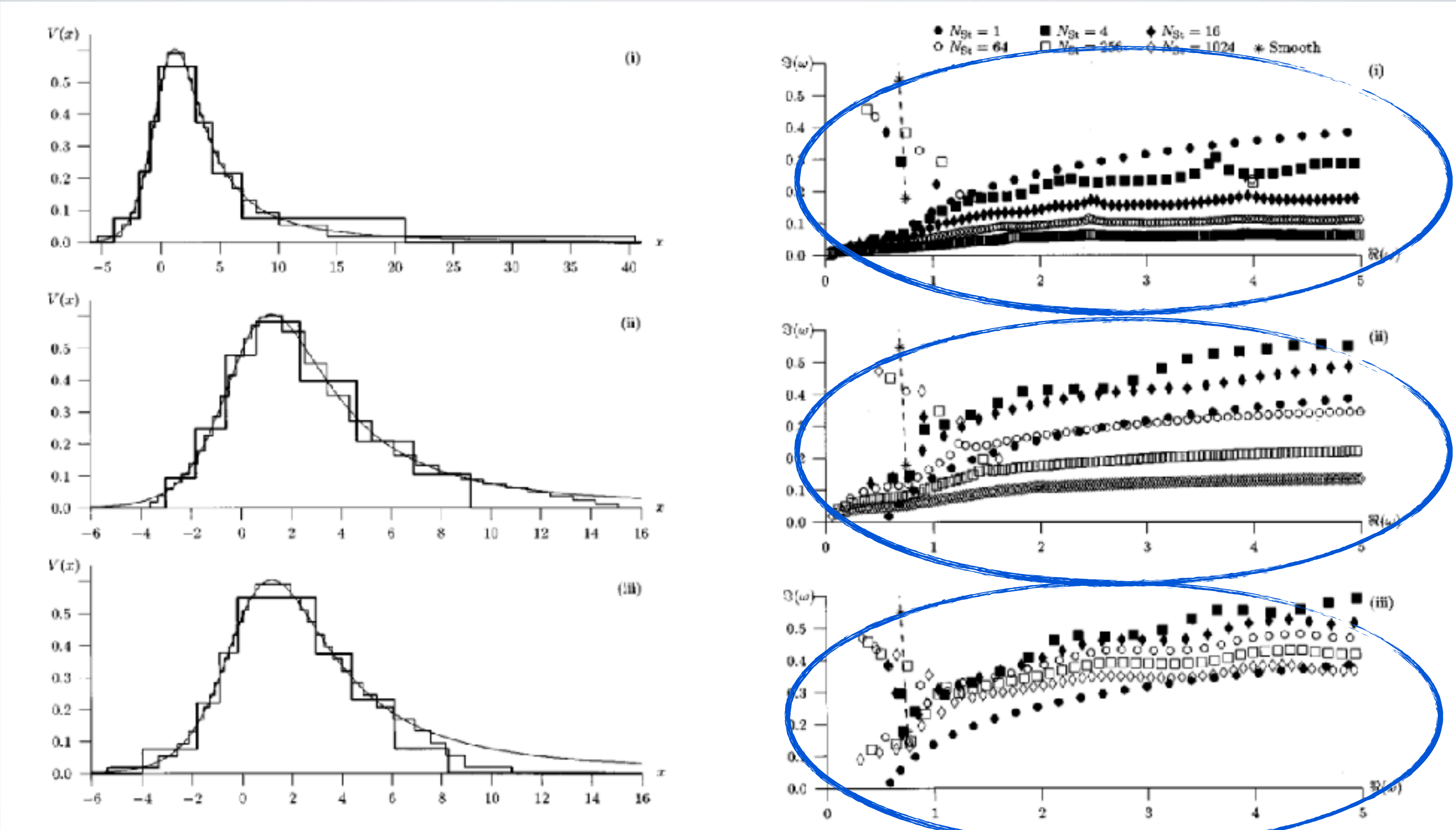
HISTORICAL INTERLUDE



- Original (unperturbed) QNMs

H.P. Nollert PRD 53 8 (1996)

HISTORICAL INTERLUDE

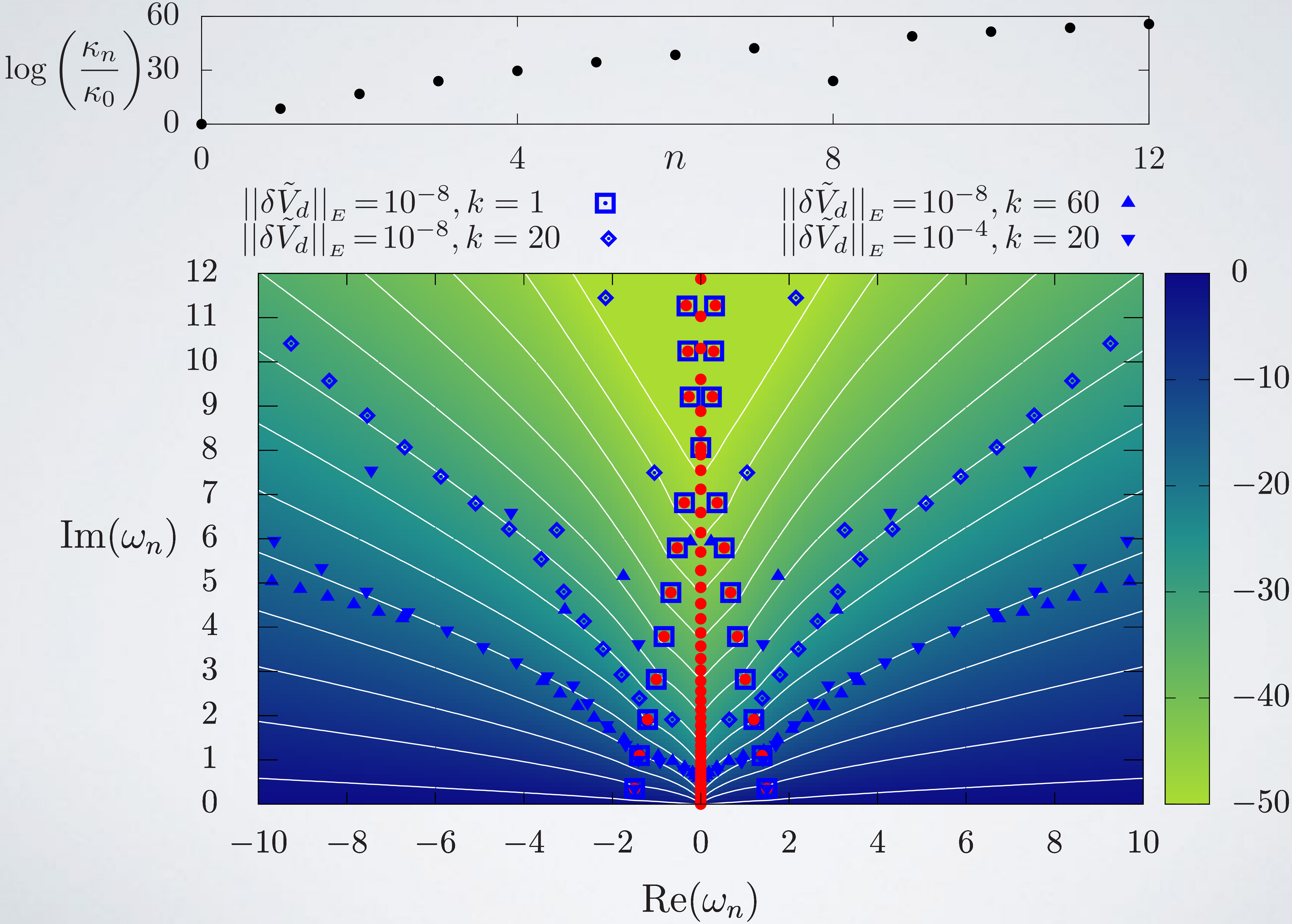


H.P. Nollert PRD 53 8 (1996)

- Original (unperturbed) QNMs
- Perturbed QNMs

SPECTRAL INSTABILITY

Perturbed potential: $q_\ell \to q_\ell + \delta V_{\rm d}$ $\delta V_{\rm d} \propto \cos(2\pi k\,\sigma)$



SPECTRAL INSTABILITY

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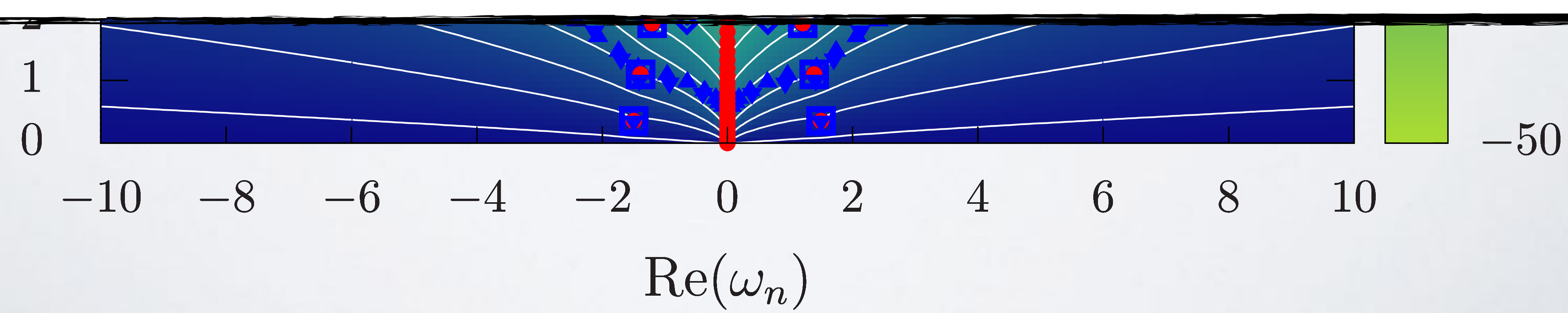
Partial Conclusions:

- Fundamental Mode is stable (potential perturbation respects asymptotic decays: ultra-violet effect)
- Overtones are stable under low wave number perturbation
- Overtones are unstable under high wave number perturbations
- Fundamental Mode is unstable (potential perturbation disturbs asymptotic decays: infra-red effect)

"the elephant and the flea effect- M. Cheung, K. Destounis, RPM, E. Berti, V. Cardoso. PRL 128 11 (2022)"



TIME X FREQUENCY DOMAIN

- Can we read, unveil, detect instability in the GW signal?

$\Psi_{\rm evol}$ (unpert. pot.) $\overline{\Psi}_{\rm evol}$ (pert. pot.)

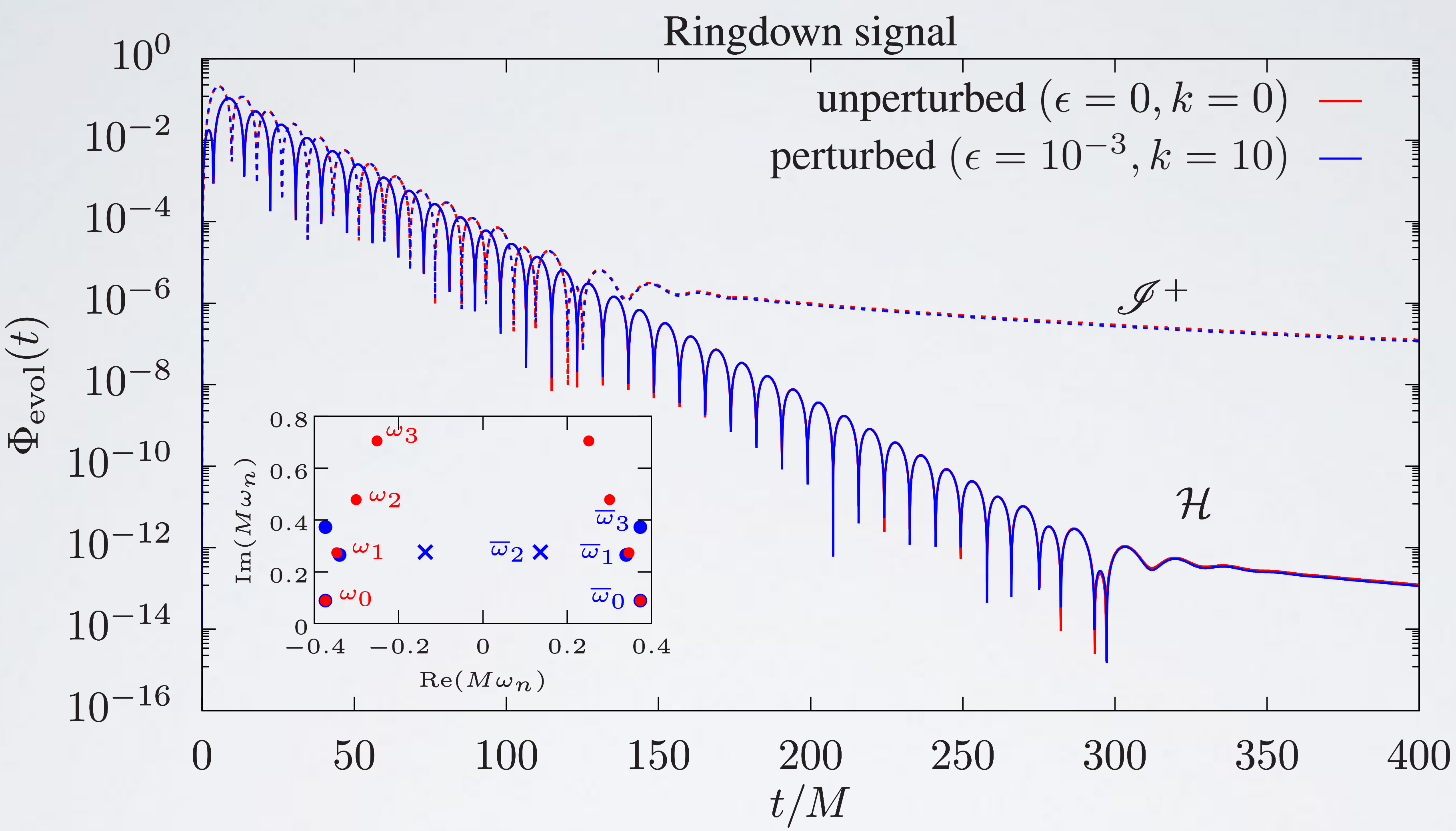


TABLE I. QNMs for unperturbed and perturbed Schwarzschild potentials via Prony's method. Crosses are QNMs not identified.

Unperturbed Potential ($\epsilon = 0, \quad k = 0$)				
QNMs	$M\omega_0$	$M\omega_1$	$M\omega_2$	$M\omega_3$
'Spectral'	$\pm 0.37367168 - 0.08896231\,i$	$\pm 0.3467110 - 0.2739149\,i$	$\pm 0.3010534 - 0.4782770\,i$	$\pm 0.2515049 - 0.7051482\,i$
Prony's Fit	$\pm 0.37367169 - 0.08896232\,i$	$\pm 0.34670 - 0.27392\,i$	$\pm 0.302 - 0.48\,i$	$\times\times\times$

Perturbed Potential ($\epsilon = 10^{-3}, \quad k = 10$)				
QNMs	$M\overline{\omega}_0$	$M\overline{\omega}_1$	$M\overline{\omega}_2$	$M\overline{\omega}_3$
'Spectral'	$\pm 0.37364032 - 0.08898850\,i$	$\pm 0.3401722 - 0.2648723\,i$	$\pm 0.1367705 - 0.2761794\,i$	$\pm 0.3735536 - 0.3723973\,i$
Prony's Fit	$\pm 0.37364030 - 0.08898850\,i$	$\pm 0.342 - 0.266\,i$	$\times\times\times$	$\pm 0.37 - 0.4\,i$

CONCLUSION

- **Hyperboloidal Framework for perturbation theory:**
- **Fundamental aspects of black-hole physics:**
  Use of tools from theory of scattering resonances and non-self adjoint operators — pseudospectra — to assess structural aspects of QNM spectra.
- **Perspectives:**
  Astrophysics and Cosmology: Black hole spectroscopy; universality of compact object QNM

  Fundamental gravitational physics: (Sub)planckian-scale physics; QNMs and strong cosmic censorship; spacetime semiclassical limits
  e.g. R. Konoplya et al. JCAP 10 091 (2021) - Asymptotically safe gravity

  Mathematical relativity: Formal statements of the numerical experiments

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SEE TALKS IN SEC. 7B: (20/12)

- R. Konoplya 14:30-14:45
- F. Duque: 14:45-15:00
- V. Boyanov: 15:15 - 15:30