Elastic Compact Objects and Compactness Bounds

XV Black Holes Workshop– 19/12/2022

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arXiv: 2107.12272, arXiv: 2202.00043 + CQG (Jan.2023)

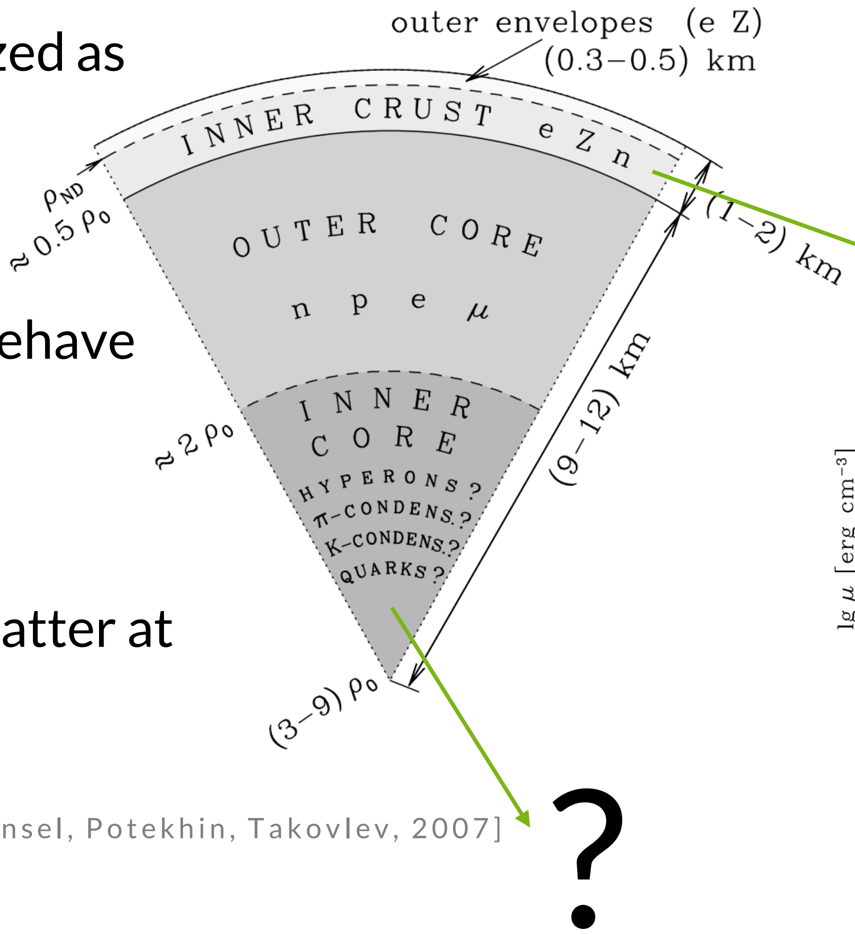
with Artur Alho, José Natário and Paolo Pani

Introduction

Why Elasticity in Compact Objects?

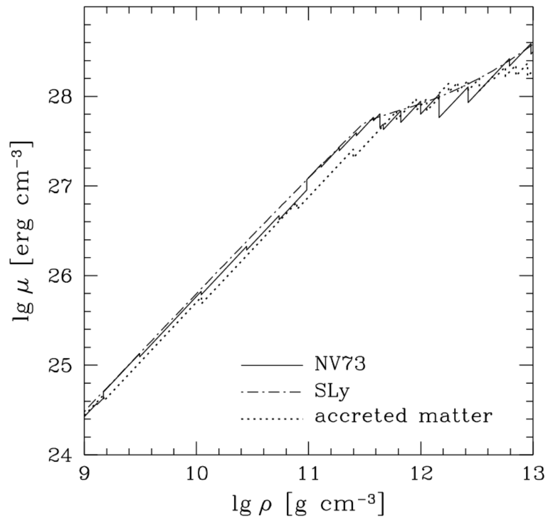
Neutron Stars

- Astrophysical objects are often idealized as self-gravitating perfect fluids.
- At relatively low densities, fermions behave as a weakly interacting gas;
- Challenging to understand phase of matter at high densities;



[Haensel, Potekhin, Takovlev, 2007]

Solid Crust



[Chamel, Haensel, 2008]

Introduction

Why Elasticity in Compact Objects?

Tests of the BH paradigm and Exotic Compact Objects

- **Buchdahl Limit:** (GR, fluids, isotropy)

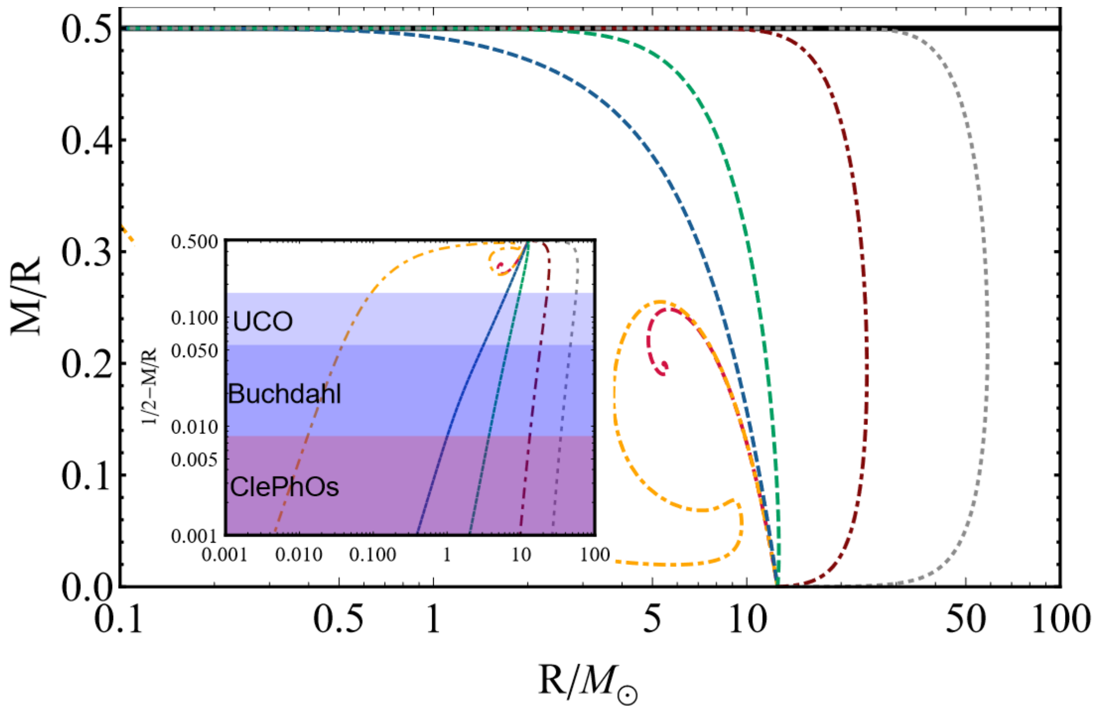
$$M/R < 4/9$$

- Several works have studied stars made of anisotropic fluid…

[Bowers, Liang, 1974; Dev, Gleiser, 2002; Mak, Harko, 2003; Raposo+, 2018]

… but, no physical motivation for the fluid model.

- **Elastic matter** naturally describes **anisotropies**



Introduction

Why Elasticity in Compact Objects?

Tests of the BH paradigm and Exotic Compact Objects

- **Beyond Buchdahl:** (GR, fluids, ~~iso~~tropy)

$$M/R \to 1/2$$

- Several works have studied stars made of anisotropic fluid…

[Bowers, Liang, 1974; Dev, Gleiser, 2002; Mak, Harko, 2003; Raposo+, 2018]

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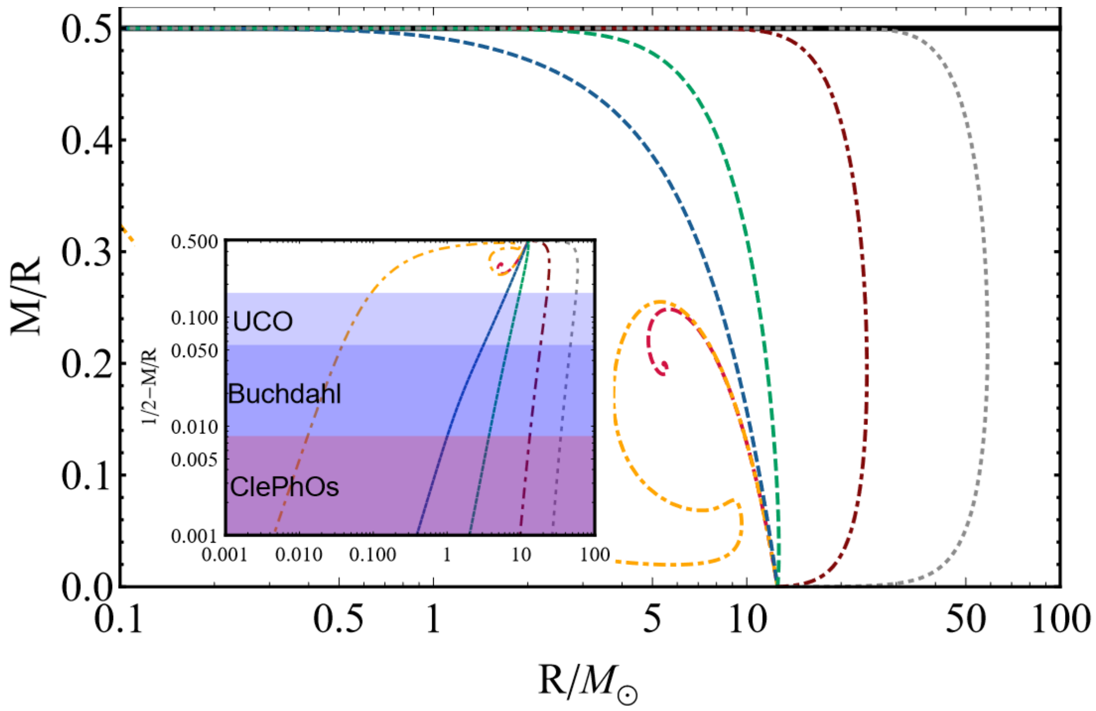


Compact Objects in GR

Spherically symmetric stars + Elastic Matter

$$ds^2 = -e^{2\alpha}dt^2 + \frac{dr^2}{1-2m/r} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2$$

$$T^\mu_\nu = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & P_r & 0 & 0 \\ 0 & 0 & P_t & 0 \\ 0 & 0 & 0 & P_t \end{bmatrix}$$

- ρ → Energy density
- P_r → Radial Pressure
- P_t → Tangential Pressure

Anisotropic TOV equations

$$m' = 4\pi r^2 \rho \qquad \alpha' = \frac{e^{2\beta}}{r}\left(\frac{m}{r} + 4\pi r^2 P_r\right) \qquad P_r' = \frac{2}{r}(P_t - P_r) - (P_r + \rho)\alpha'$$

Equation Of State

Perfect Fluids: One EOS	Anisotropic Fluids: Two EOS	Elastic Matter
$P_r = P_t = P$ $P = P(\rho)$	$P_r = P_r(\rho)$ $P_t = P_t(P_r, \rho)$	? Derived from theory of Relativistic Elasticity

Equation of State

Fluid Materials

Number density

$$\rho = \rho(n)$$

$$P = (n\frac{\partial \rho(n)}{\partial n} - \rho)$$

Elastic Materials

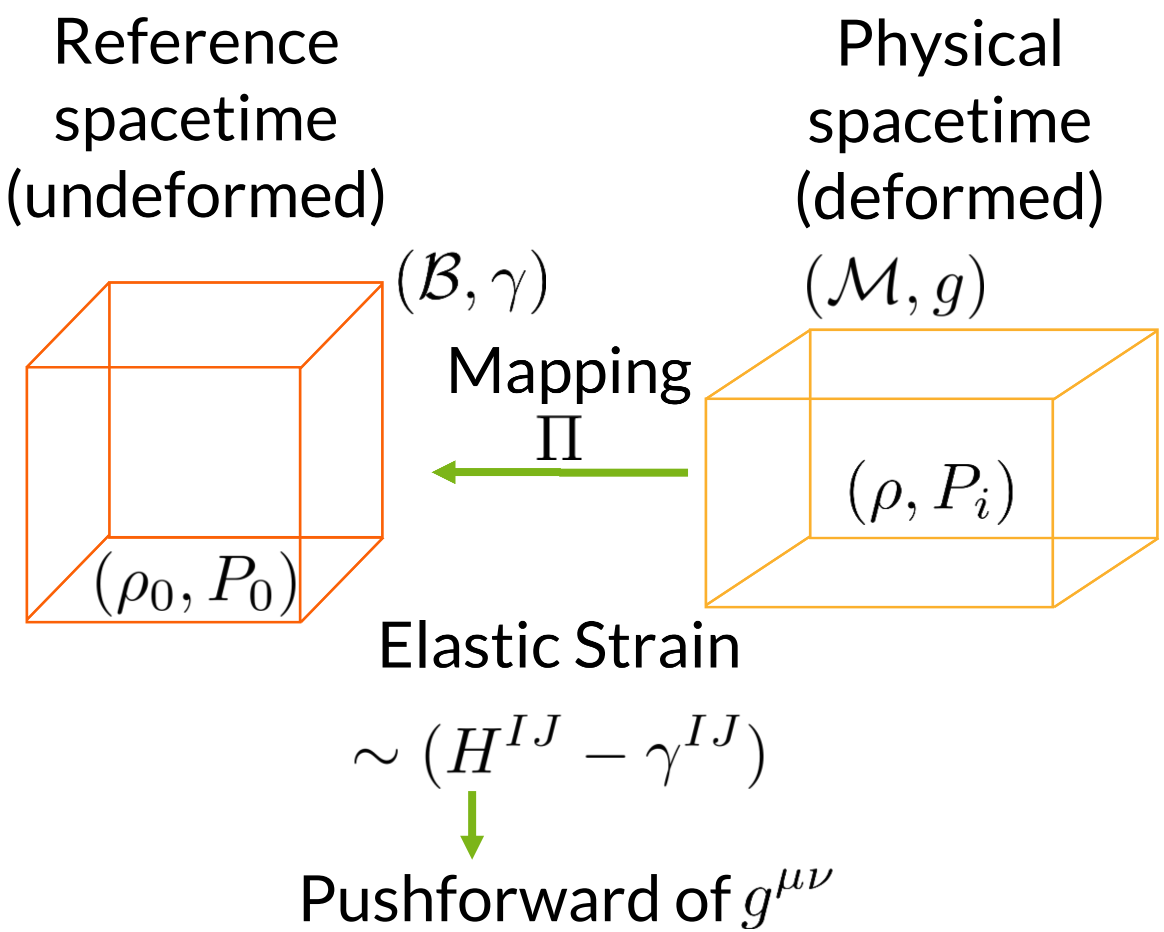
Linear density

$$\rho = \rho(n_1, n_2 = n_3)$$

$$P_i = (n_i\frac{\partial \rho}{\partial n_i} - \rho)$$

$$n_1(t,r) = e^{-\beta}R'(r), \qquad n_2(t,r) = n_3(t,r) = \frac{R(r)}{r}.$$

Elastic Theory in a nutshell



Example: Polytropic EOS

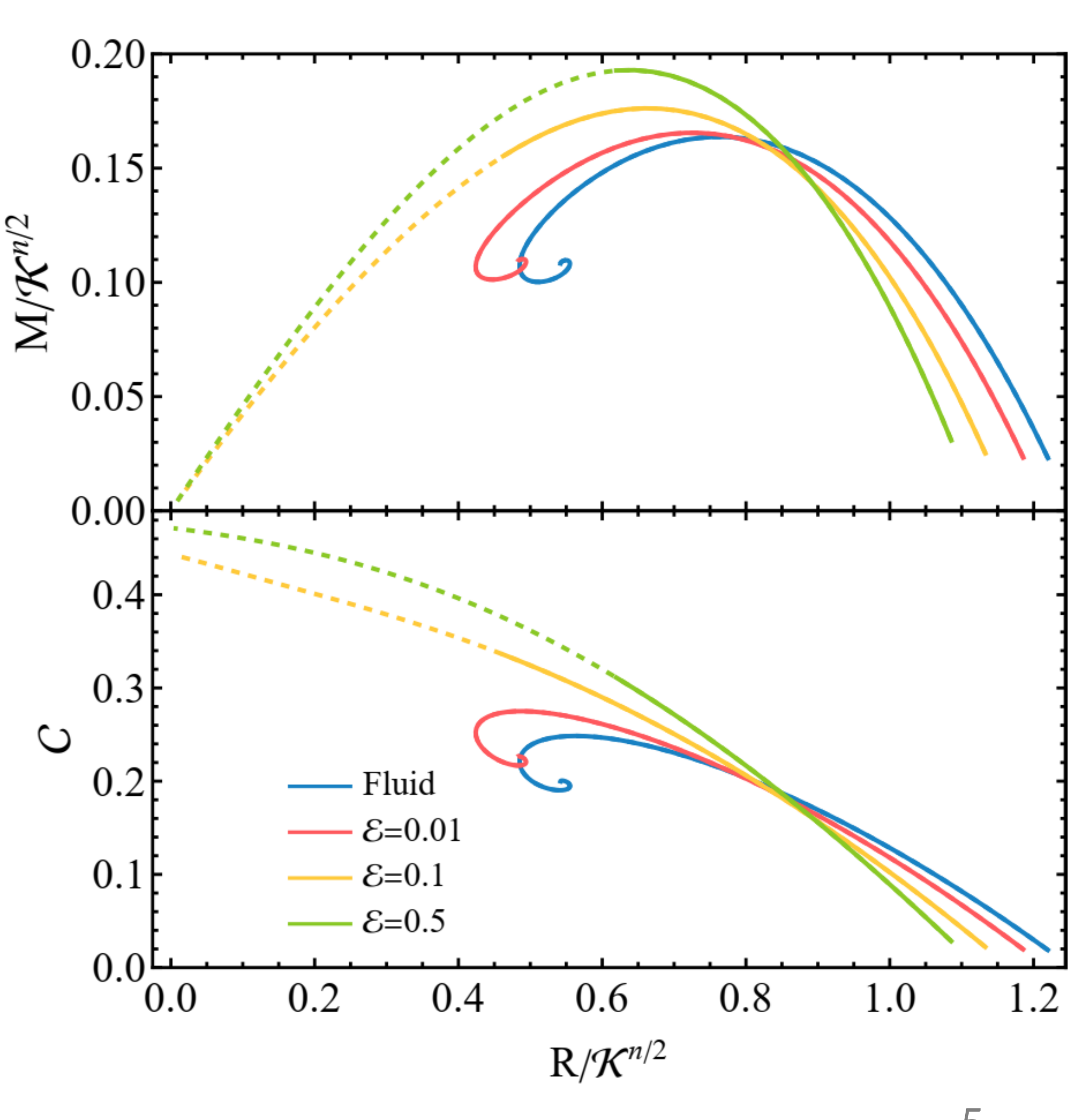
$$\widehat{\rho}(\delta,\eta) = \underbrace{\left(\rho_0 - \frac{n^2 K}{1+n}\right)\delta + \frac{n^2 K}{1+n}\delta^{1+1/n}}_{\text{Perfect-fluid Polytrope}} + \underbrace{\frac{2\mu}{3}\eta^2\left(1-\frac{\delta}{\eta}\right)^2}_{\text{Quadratic Correction}}$$

Effects of Elasticity

1. **Increase the maximum mass and compactness of star:**
   Ultracompact and beyond-Buchdahl configurations;

2. **New wave propagation modes:**
   5 modes (elastic) vs 1 mode (fluid)

3. **Unphysical branch at high densities;**



Compactness Bounds in GR

How compact can a "non-exotic" compact object be?

Incompressible Fluid EOS

$$\rho(r) = \rho_0$$

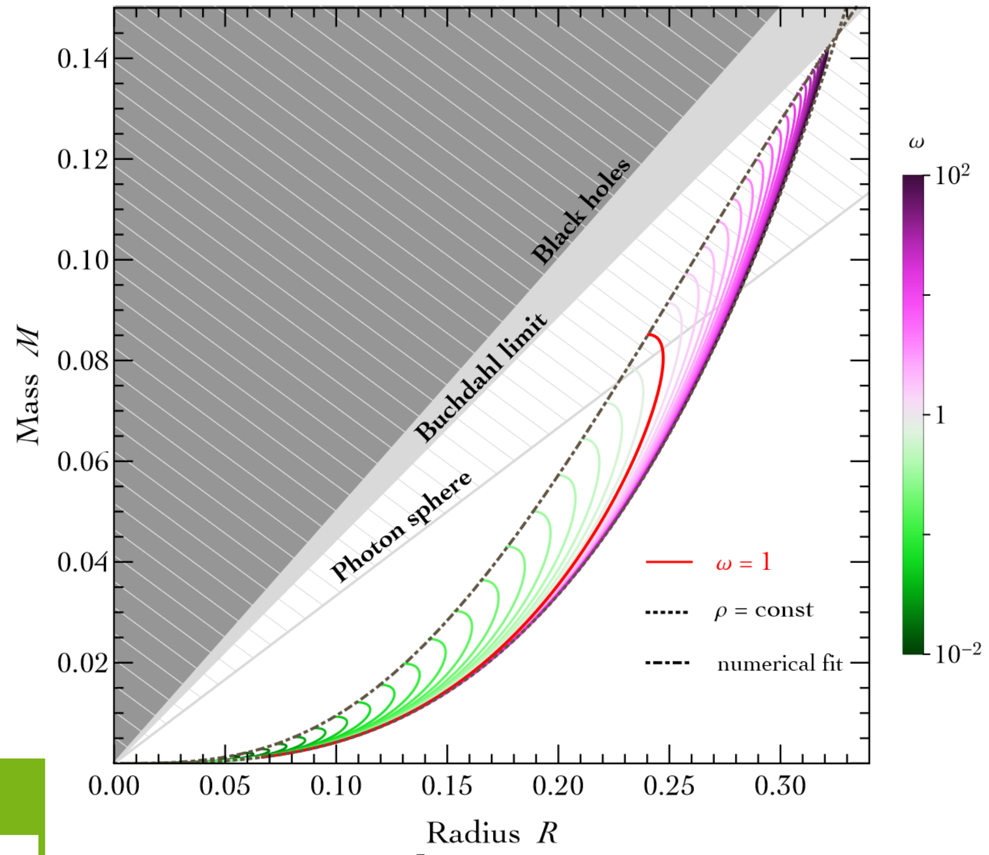
Buchdahl Bound

$$\frac{M}{R} \lesssim 4/9$$

Linear Equation of State

$$P = c_s^2(\rho - \rho_0)$$

Causal Buchdahl Bound	Causal + stable Buchdahl Bound
$\frac{M}{R} \lesssim 0.364$	$\frac{M}{R} \lesssim 0.354$



[Urbano, Veermäe, 1810.07137]

Constant Sound Speed EOS

$$\rho = \rho_0 \left[\left(\frac{1}{\gamma} - \varepsilon - 2\theta \right) (n_1 n_2 n_3)^\gamma + \varepsilon (n_1^\gamma + n_2^\gamma + n_3^\gamma) \right.$$
$$\left. + \theta \left[(n_1 n_2)^\gamma + (n_2 n_3)^\gamma + (n_3 n_1)^\gamma \right] + \frac{\gamma - 1}{\gamma} - 2\varepsilon - \theta \right]$$

Important Remarks:

- For elastic materials we cannot set all sound speeds constant.
- Ultrarigid EOS fixes sound speed of **longitudinal waves**!

$$c_{\mathrm{L}i}^2 = n_i^2 \frac{\partial^2 \rho}{\partial n_i^2} / (\rho + p_i) = \gamma - 1$$

4 parameters in the EOS

Compactness Bounds

Energy Conditions

WEC: $\rho > 0$, $\rho + p_{\rm tan} > 0$, $\rho + p_{\rm rad} > 0$

NEC: $\rho + p_{\rm tan} > 0$, $\rho + p_{\rm rad} > 0$

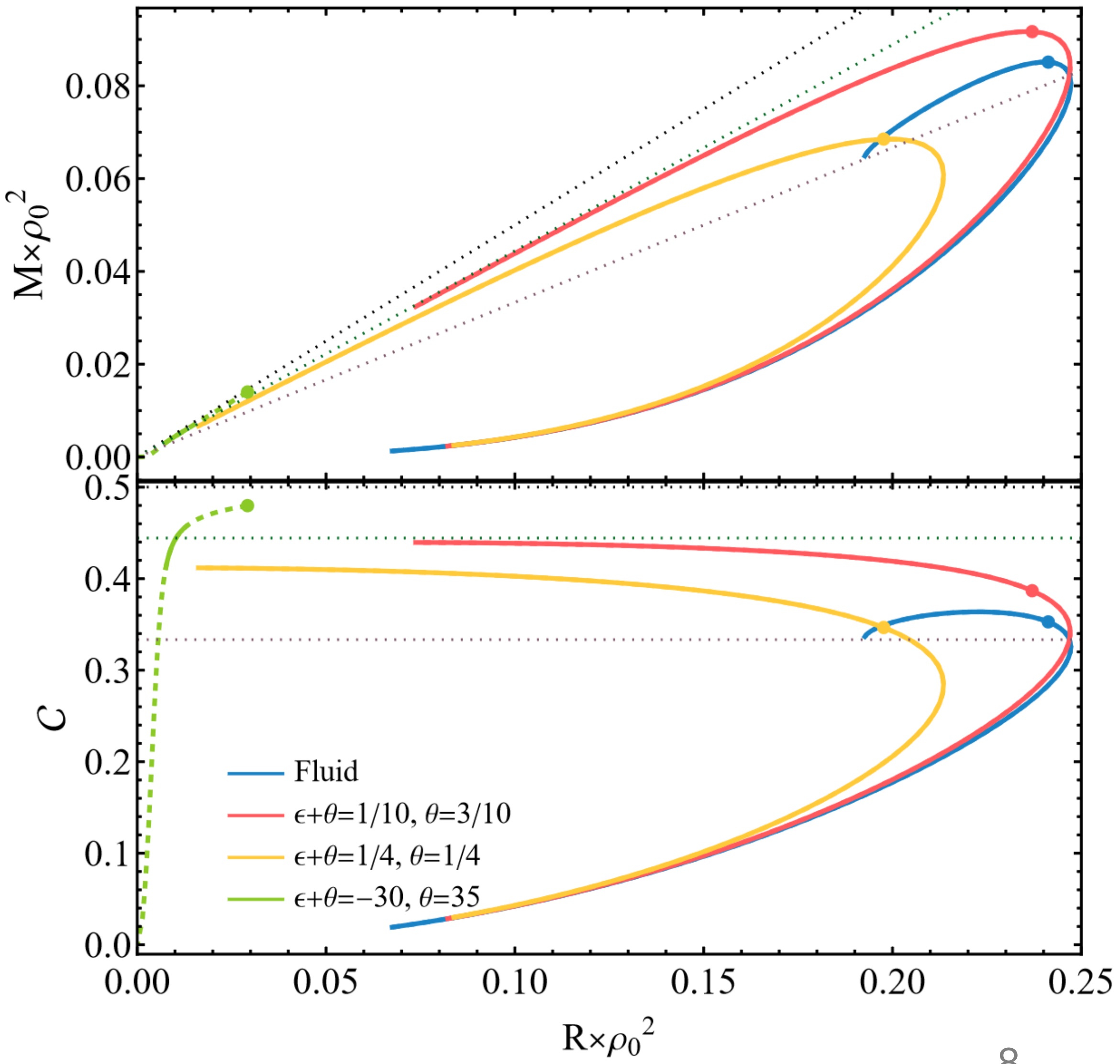
SEC: $\rho + p_{\rm tan} > 0$, $\rho + p_{\rm rad} > 0$, $\rho + p_{\rm rad} + 2p_{\rm tan} > 0$

DEC: $\rho > |p_{\rm tan}|$, $\rho > |p_{\rm rad}|$

Subluminal Sound Speeds

$$c_L, \bar{c}_L, c_T, \bar{c}_T, \bar{c}_{TT} < 1$$

Causal Bound	Causal + stable Bound
$\mathcal{C}_{\rm PA} \lesssim 0.469$	$\mathcal{C}_{\rm PAS} \lesssim 0.389$



Compactness Bounds

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$$\rho > |p_{\rm tan}|$$

$$\rho > |p_{\rm rad}|$$

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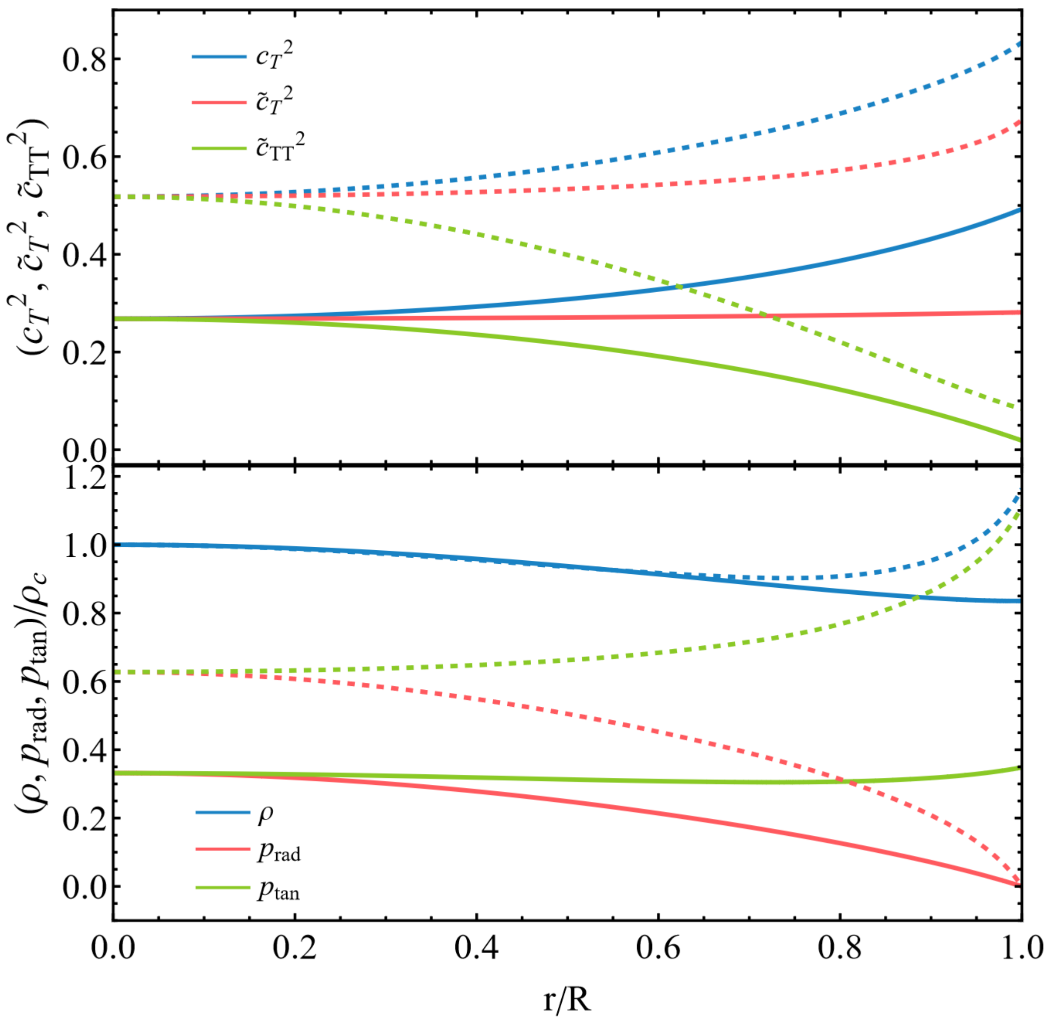

Summary and Final Remarks

Formulation:

- Introduced a simple relativistic formalism to include elastic effects in the description of compact objects;

Elastic EOS

- Generalization of the polytrope: **Quadratic elastic correction;**
- Generalization of linear (fluid) EOS: **Ultrarigid EOS**

Results:

- Elastic Stars: elasticity contributes to increase the maximum mass and compactness;
- Derive a novel set of compactness bounds that extend Buchdahl's results.

	$\mathcal{C}_{\text{Buchdahl}}$	$\mathcal{C}_{\text{PA}}$	$\mathcal{C}_{\text{PAS}}$
Fluid	$4/9$	0.365	0.354
Elastic	$1/2$	0.469	0.389

Ongoing Work

Multilayer Neutron Stars

- Setup of stars with different elastic layers and combination of fluid and elastic layers.

- Effects of the solid crust/solid core in fluid neutron stars.

Beyond Spherical symmetry

- Extend the formalism in Part I to spacetimes with less symmetry;

- Rotating Stars, Deformed Stars, Tidal deformations

1+1 NR evolution

Thank you

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Part III

Extra Slides

Relativistic Elasticity

[Carter&Quintana; Kijowski&Magli; **Beig&Schmidt,2018**]

Concrete example: Spherically Symmetric, Static Spacetime

- **Step 1:** Set the **geometry** in the two physical spacetime and the material spacetime

$$ds^2 = -e^{2\alpha}dt^2 + e^{2\beta}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \qquad \gamma = e^{2\beta_0(R)}dR^2 + R^2(d\Theta^2 + \sin^2\Theta d\Phi^2).$$

- **Step 2:** Assign the ***mapping*** between both and the ***configuration gradient*** $f^A_\mu = \partial X^A / \partial x^\mu$

$$R = R(t, r), \qquad \Theta = \theta, \qquad \Phi = \varphi,$$

- **Step 3:** Compute the pushforward metric H^{AB} and orthogonal metric to 4-velocity;

$$H^{RR} = e^{-2\beta}(R')^2, \qquad H^{\Theta\Theta} = \frac{1}{r^2}, \qquad H^{\Phi\Phi} = \frac{1}{r^2\sin^2\theta}$$

$$h_{rr} = e^{2\beta}, \quad h_{\theta\theta} = r^2, \quad h_{\varphi\varphi} = r^2\sin^2\theta,$$

Relativistic Elasticity

Concrete example: Spherically Symmetric, Static Spacetime

- **Step 4:** Dynamics and matter:

$$S[\mathbf{\Psi}] = \int_{\mathcal{M}} \rho(\mathbf{\Psi}, d\mathbf{\Psi}) \sqrt{-\det(g)} d^4x$$

[Natário, 2019]

$$T_{\mu\nu} = 2\frac{\partial \rho}{\partial g^{\mu\nu}} - \rho g_{\mu\nu}, \qquad T_{\mu\nu} = \rho u_\mu u_\nu + \sigma_{\mu\nu}, \qquad \sigma_{\mu\nu}u^\nu = 0,$$

- **Step 5:** Homogeneous and isotropic materials;

$$\rho = \hat{\rho}(\underbrace{i_1(\boldsymbol{\mathcal{H}}), i_2(\boldsymbol{\mathcal{H}}), i_3(\boldsymbol{\mathcal{H}})}_{\text{Principal invariants}}),$$

$$\sigma_{\mu\nu} = \sum_{i=1}^{3} (\underbrace{n_i \frac{\partial \rho}{\partial n_i} - \rho}_{\text{Principal pressure in the i'th direction}}) e_{(i)\mu} e_{(i)\nu}, \quad \sigma^\mu_\nu e^\nu_{(i)} = p_i e^\mu_{(i)},$$

Relativistic Elasticity

Concrete example: Spherically Symmetric, Static Spacetime

First Result: Pressures can be computed from the energy density.

$$P_i = (n_i \frac{\partial \rho}{\partial n_i} - \rho)$$

First law of thermodynamics for perfect-fluids:

$$P = (\rho_b \frac{\partial \rho}{\partial \rho_b} - \rho)$$

Not enough to close the system of equations.

- **Step 6:** Principle Linear Densities

$$n_1(t,r) = e^{-\beta} R'(r), \qquad n_2(t,r) = n_3(t,r) = \frac{R(r)}{r}.$$

Final Equation to close the system

Relativistic Elasticity - Summary

Anisotropic TOV Equations

$$m' = 4\pi r^2 \rho \qquad \alpha' = \frac{e^{2\beta}}{r}\left(\frac{m}{r} + 4\pi r^2 P_r\right) \qquad P_r' = \frac{2}{r}(P_t - P_r) - (P_r + \rho)\alpha'$$

Equation of State

$$\rho = \hat{\rho}(\delta, \eta), \qquad P_r = \delta \partial_\delta \hat{\rho} - \rho, \qquad P_t = P_r + 3/2\eta\partial_\eta \rho,$$

Elastic Equation

$$\eta' = -\frac{3}{r}(\eta - e^{\beta}\delta)$$

Part II:

Elastic Stars

arXiv: 2107.12272

Polytropic Elastic Stars

Polytropic (Fluid) Equation of State:

$$\rho = \rho_b + n\mathcal{K}\rho_b^{1+1/n} \qquad P = \mathcal{K}\rho_b^{1+1/n}$$

In the elastic notation:

$$\widehat{\rho}(\delta,\eta) = \left(\rho_0 - \frac{n^2 K}{1+n}\right)\delta + \frac{n^2 K}{1+n}\delta^{1+1/n}$$

Simplest Elastic generalization:

$$\widehat{\rho}(\delta,\eta) = \underbrace{\left(\rho_0 - \frac{n^2 K}{1+n}\right)\delta + \frac{n^2 K}{1+n}\delta^{1+1/n}}_{\text{Perfect-fluid Polytrope}} + \underbrace{\frac{2\mu}{3}\eta^2\left(1-\frac{\delta}{\eta}\right)^2}_{\text{Quadratic Correction}}$$

Polytropic Elastic Stars

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Reference-frame invariance

$$\widehat{\rho}(\delta,\eta) = \left(\rho_0 - \frac{n^2 K}{1+n}\right)\delta + \frac{n^2 K}{1+n}\delta^{1+\frac{1}{n}} + \frac{2\mu}{3}\eta^2\left(1-\frac{\delta}{\eta}\right)^2$$

- Why does the reference state do not appear when we study polytropes?
- Consider a new reference state compressed by a volume factor:

$$\delta = f\tilde{\delta} \qquad \eta = f\tilde{\eta}$$

$$\hat{\rho}(\tilde{\delta},\tilde{\eta}) = \left(\rho_0 - \frac{n^2 K}{1+n}\right) f\tilde{\delta} + \frac{n^2 K}{1+n} f^{1+\frac{1}{n}}\tilde{\delta}^{1+\frac{1}{n}} + \frac{2\mu}{3} f^2(\tilde{\delta}-\tilde{\eta})^2$$

- However, by appropriate parameter redefinition:

$$\hat{\rho}(\tilde{\delta},\tilde{\eta}) = \left(\tilde{\rho}_0 - \frac{n^2 \tilde{K}}{1+n}\right)\tilde{\delta} + \frac{n^2 \tilde{K}}{1+n}\tilde{\delta}^{1+\frac{1}{n}} + \frac{2\tilde{\mu}}{3}(\tilde{\delta}-\tilde{\eta})^2$$

Reference-frame choice is akin to a gauge choice

Reference-frame invariance

Quantities invariant under renormalization of reference state:

Baryonic Density ρ_b

Polytropic factor $\mathcal{K}$

Dimensionless Shear Parameter $\mathcal{E}$

"Anisotropic" Density ρ_{ani}

Elastic Polytropic EOS

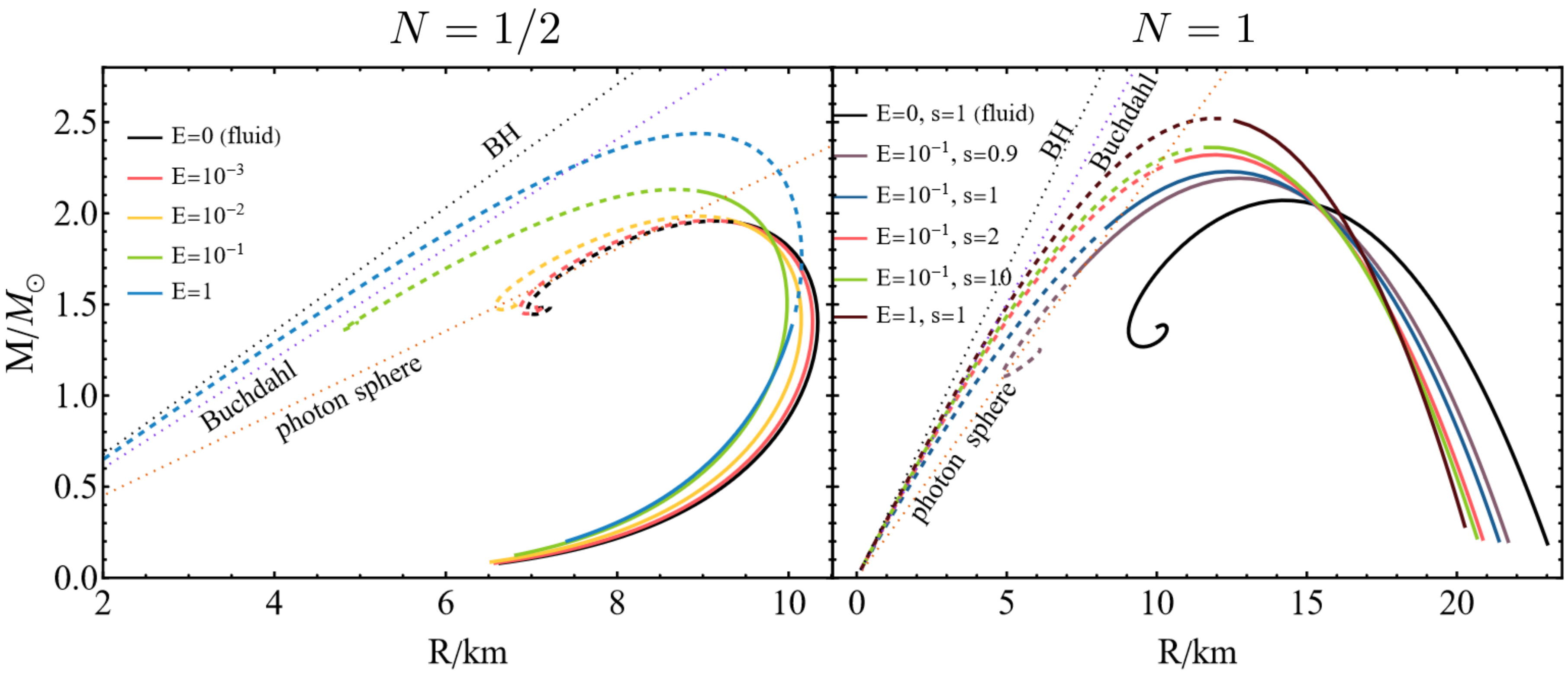
$$\rho = \rho_b + n\mathcal{K}\rho_b^{1+1/n} + \mathcal{E}\mathcal{K}^n(\rho_b - \rho_{\text{ani}})^2$$

Reference-frame dependent quantities do not appear in the final equations

Equilibrium Configurations

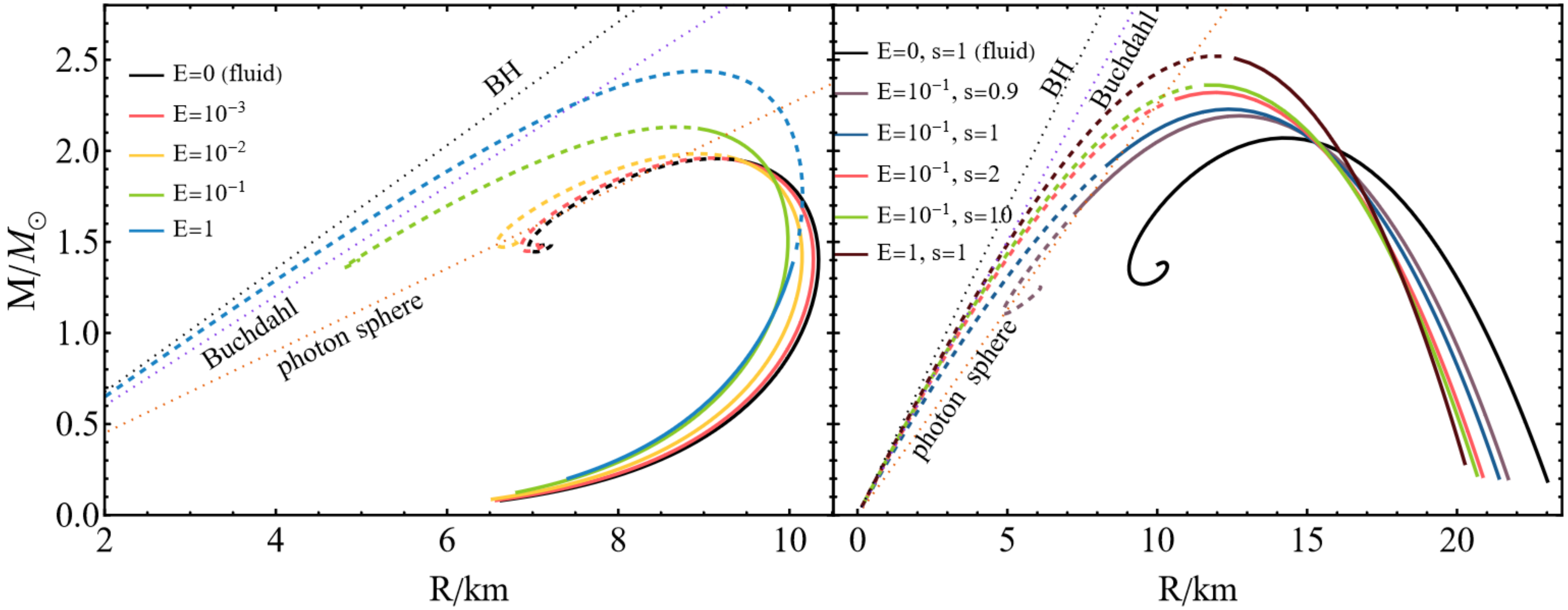
$$\rho = \rho_b + n\mathcal{K}\rho_b^{1+1/n} + \mathcal{E}\mathcal{K}^n(\rho_b - \rho_{\rm ani})^2 \qquad P_r = \rho_b \frac{\partial \rho}{\partial \rho_b} - \rho \qquad P_t - P_r = \frac{3}{2}\rho_{\rm ani}\frac{\partial \rho}{\partial \rho_{\rm ani}}$$

For each $\mathcal{E}$ there is a one parameter family of solutions depending on central density.



Equilibrium Configuration

Properties of Equilibrium Configurations:



1. Elasticity increases the maximum mass and compactness of star;
2. Ultracompact and beyond-Buchdahl configurations;

Viability Conditions

Energy Conditions

WEC

$$\rho > 0$$

$$\rho + p_{\tan} > 0$$

$$\rho + p_{\rm rad} > 0$$

NEC

SEC

$$\rho + p_{\rm rad} + 2p_{\tan} > 0$$

DEC

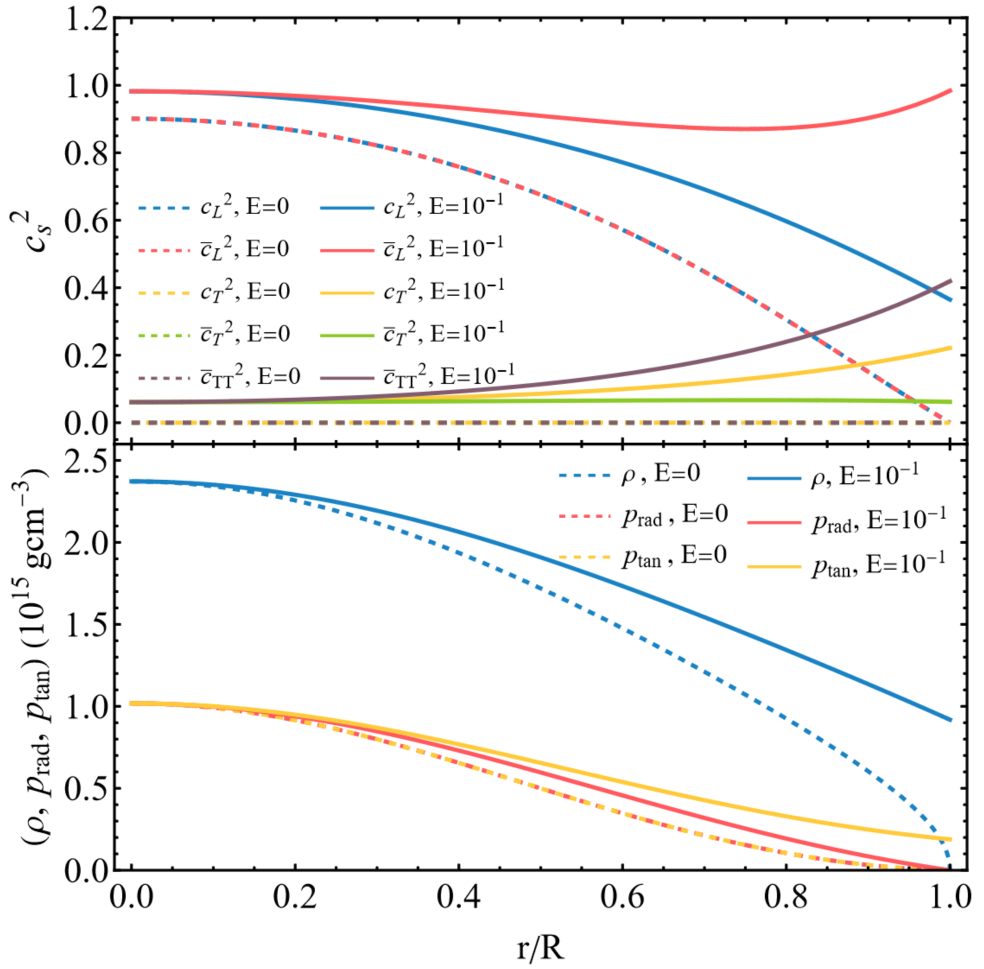
$$\rho > |p_{\tan}|$$

$$\rho > |p_{\rm rad}|$$

Subluminal Sound Speeds

Missed in most of anisotropic studies!

- c_L Longitudinal Waves in the Radial direction
- $\bar{c}_L$ Longitudinal Waves in the Tangential direction
- c_T Transverse Waves in the Radial direction
- $\bar{c}_T$ Transverse Waves in the Tangential direction



Viability Conditions

Stability:

Linear Radial Stability

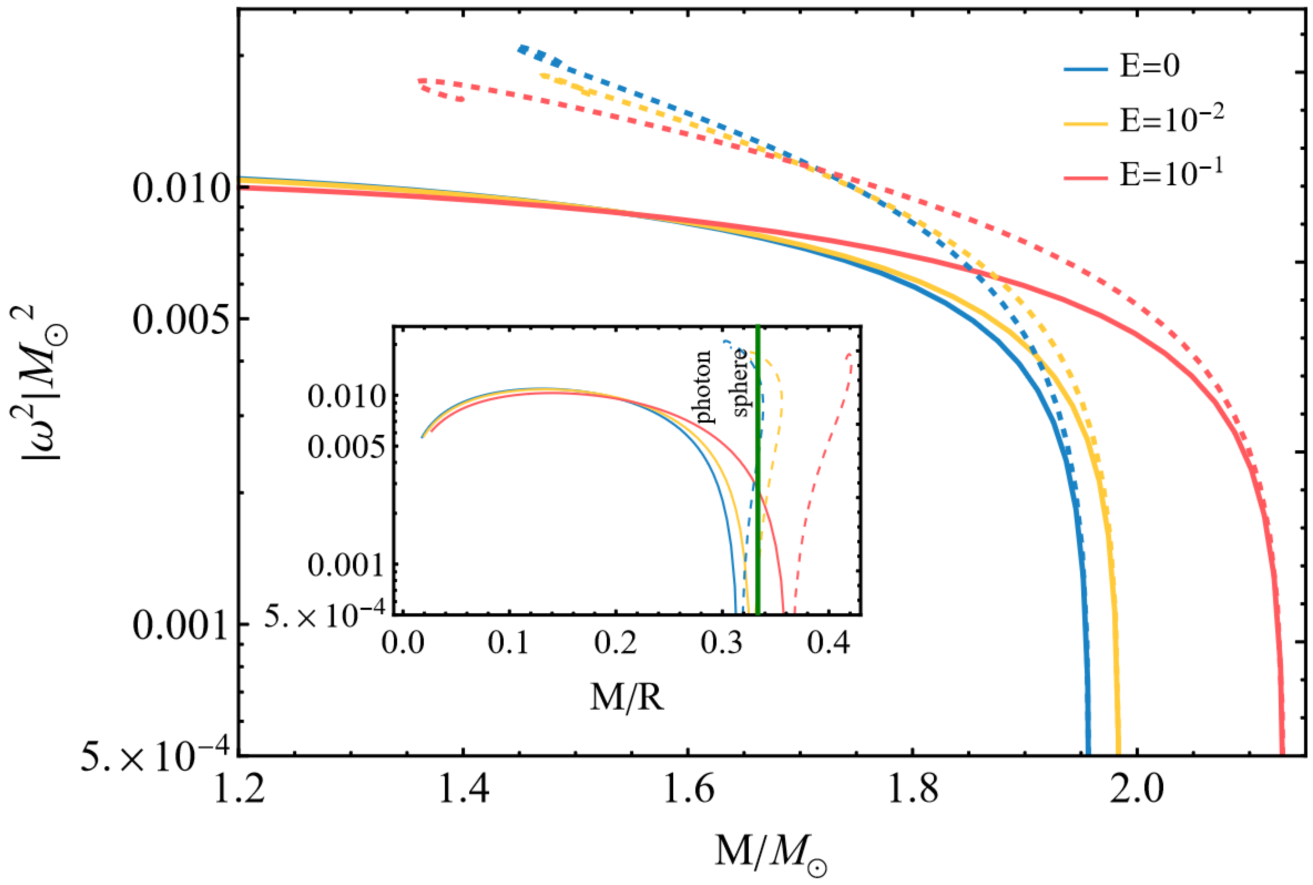
$$\xi(t,r) = \xi(r)e^{i\omega t}$$

$$\chi(t,r) = \chi(r)e^{i\omega t}$$

Unstable	Stable
$\omega^2 < 0$	$\omega^2 > 0$

Same as perfect-fluids

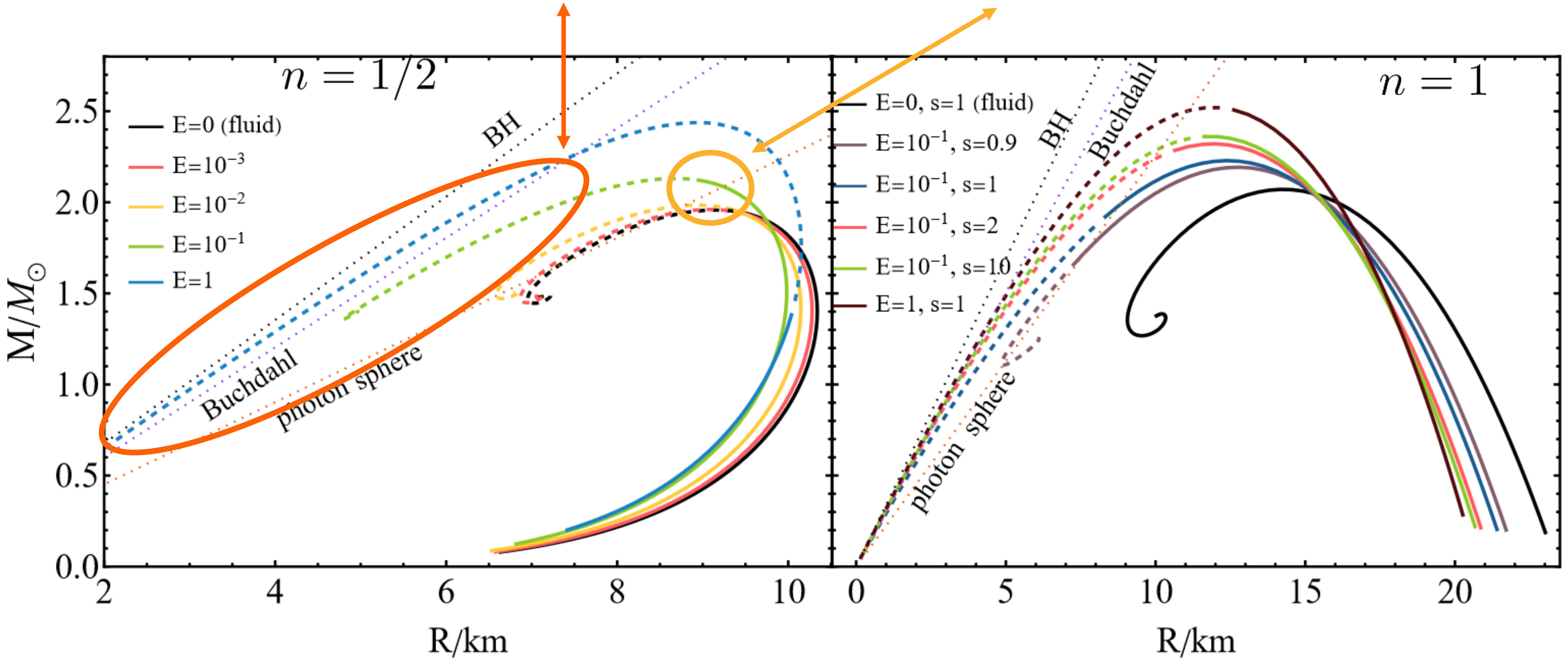
Radial stability threshold is at the maximum mass!



Part I - Final Remarks

Objects beyond Buchdahl limit are **unstable** and **superluminal sound speeds** and **break dominant energy conditions**

Ultracompact stars with LR, while stable and causal wave propagation! $R < 3M$



$M_{\max}$ can increase up to $\approx 22\%$

For fluids:

$$M_{\max} \sim 2M_\odot$$

For elastic:

$$M_{\max} \sim 2.5M_\odot$$

Compatible with GW190814 ?

$$\mu \approx 7 \times 10^{26} \left(\frac{\rho_0}{10^{11}\ \mathrm{g/cm^3}} \right)^2 \sqrt{\frac{K}{10^5 M_\odot^4}} \left(\frac{E}{0.1} \right) \mathrm{erg/cm^3}$$

Part III

Compactness Bounds in GR

arXiv: 2202.00043

Buchdahl Bound

How compact can a "non-exotic" compact object be?

TOV + Incompressible Fluid EOS

$$m' = 4\pi r^2 \rho$$

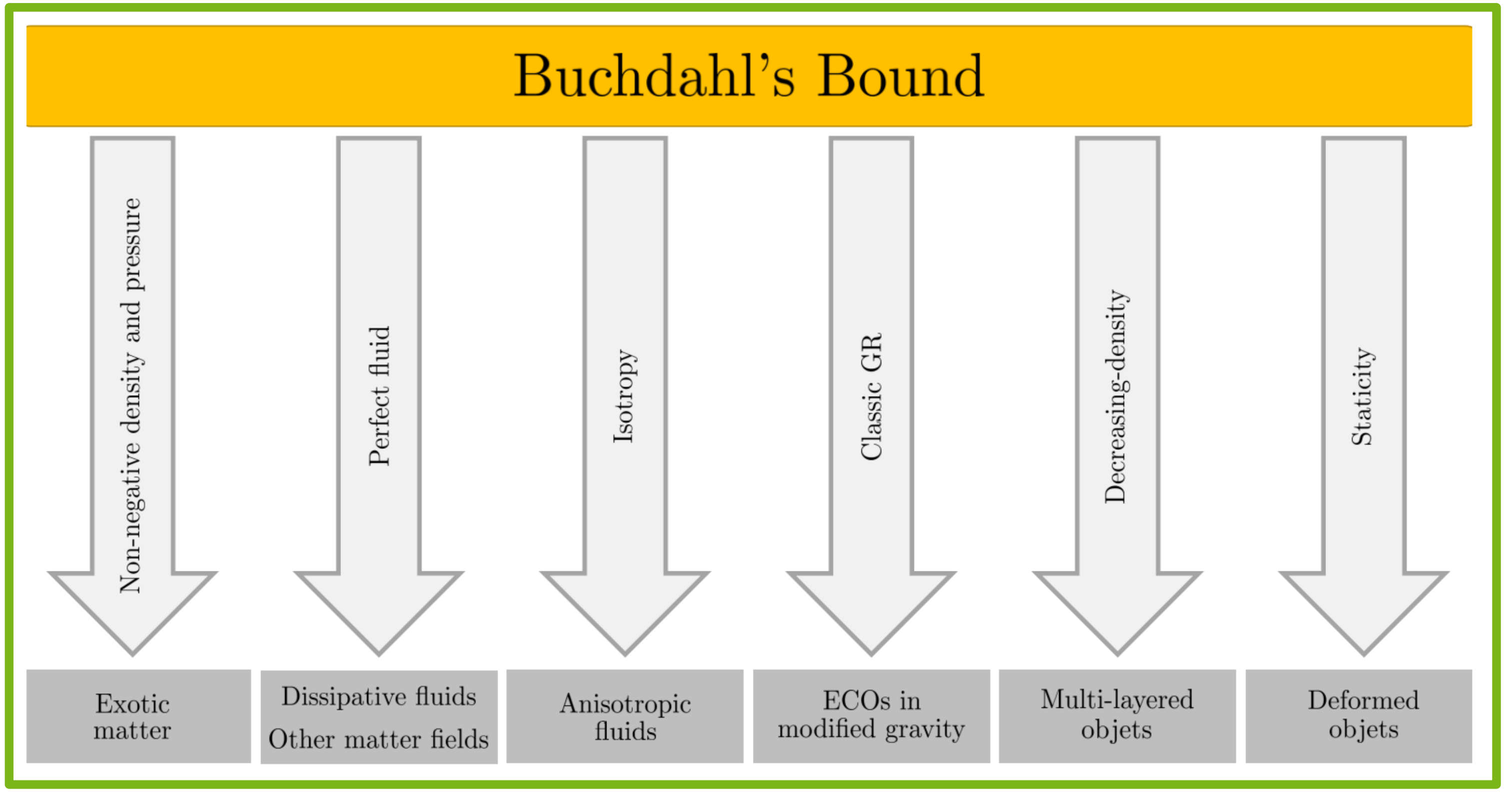
$$\alpha' = \frac{e^{2\beta}}{r}\left(\frac{m}{r} + 4\pi r^2 P_r\right)$$

$$P' = -(P+\rho)\alpha'$$

$$\rho(r) = \rho_c$$

Can be solved analytically:

$$\frac{M}{R} = \frac{2(p_c\rho_c + 2p_c^2)}{(3p_c+\rho_c)^2} < \frac{4}{9}$$



Causal Buchdahl Bound

*How compact can a "non-exotic" **and casual** compact object be?*

Causality Condition:

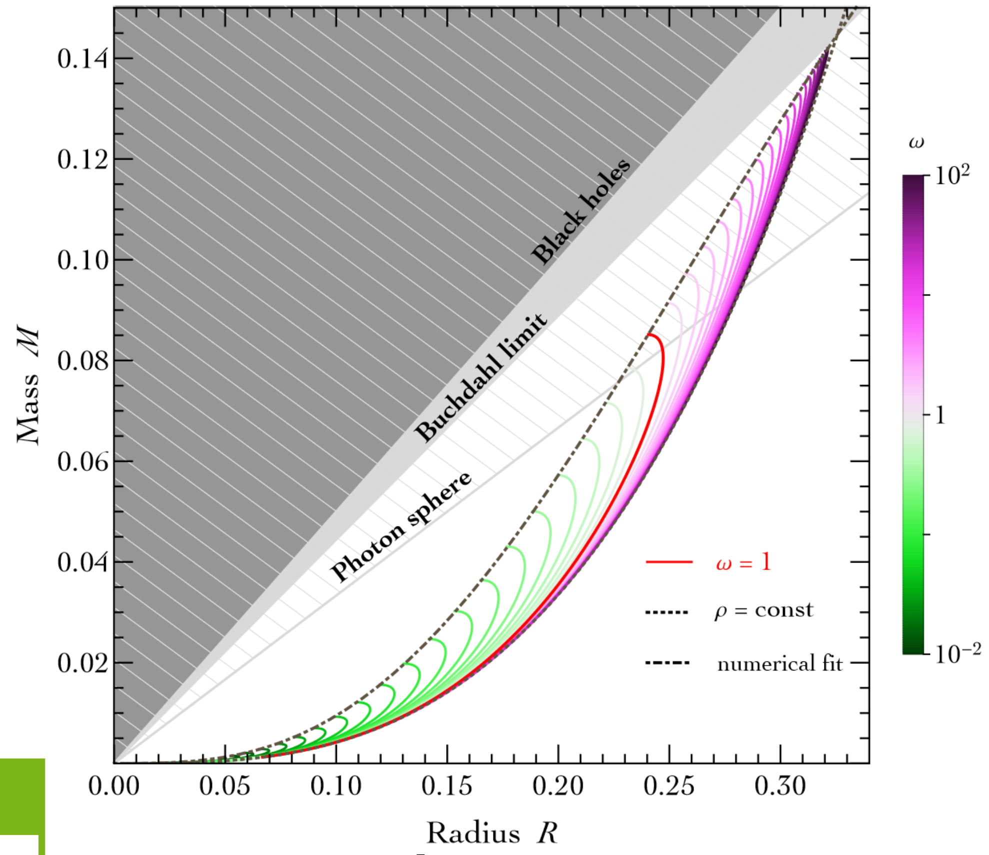
$$0 \leq c_s^2 \equiv dP/d\rho \leq 1$$

Linear Equation of State

$$P = c_s^2(\rho - \rho_0)$$

No analytical solution: Solve numerically TOV

Causal Buchdahl Bound	Causal + stable Buchdahl Bound
$\frac{M}{R} \lesssim 0.364$	$\frac{M}{R} \lesssim 0.354$



[Urbano, Veermäe, 1810.07137]

Constant Sound Speed EOS

Generalization of Linear Equation of State

$$P = c_s^2(\rho - \rho_0)$$

Important Remarks:

- For elastic materials we cannot set all sound speeds constant.
- At the center: Longitudinal waves must propagate faster than transverse waves;
- Best choice: $c_{\mathrm{L}i}^2 = n_i^2 \frac{\partial^2 \rho}{\partial n_i^2} / (\rho + p_i) = \gamma - 1$

Ultrarigid EOS

$$\begin{aligned}\rho =& \rho_0 \left[\left(\frac{1}{\gamma} - \varepsilon - 2\theta \right) (n_1 n_2 n_3)^\gamma + \varepsilon (n_1^\gamma + n_2^\gamma + n_3^\gamma) \right. \\ & \left. + \theta \left[(n_1 n_2)^\gamma + (n_2 n_3)^\gamma + (n_3 n_1)^\gamma \right] + \frac{\gamma - 1}{\gamma} - 2\varepsilon - \theta \right]\end{aligned}$$

Stress-Free Material: Reference frame matters.

Constant Sound Speed EOS

4 EOS parameters

1 Dimensionful
ρ_0
• Set the scale of the mass and radius;
• Absorbed in the choice of units;
• Does not affect the compactness

3 Dimensionless		
c_S^2	ϵ	θ
• Characterize the profile of the solutions;		
• Fix the sound speed (=1, saturate causality);		
• 2 parameters remaining;		

Particular cases:

$(\epsilon = \theta = 0)$

Christodoulou's hard phase material;

$(\epsilon = 0, \theta = 1/4)$

Karlovini-Samuelsson SUREOS

$(\epsilon = \theta = 1/8)$

Brotas Rigid Solid

Generalized Buchdahl's Bound

For each set of parameters there is a one parameter family of solutions depending on central density.

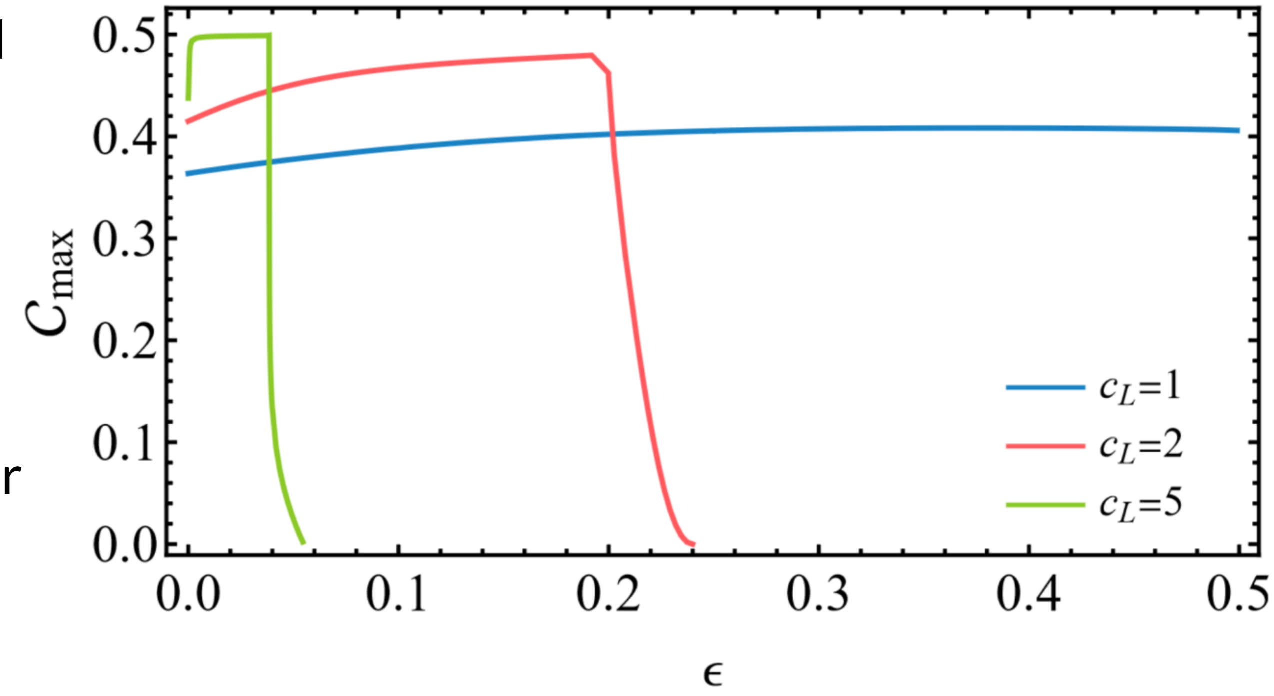
- The maximum compactness grows with the sound speed;

- In the **fluid limit** we recover **Buchdahl's bound**.

$$\mathcal{C}_{\max}(\mu = 0) \to 4/9$$

- **Buchdahl bound** abruptly tends to the **BH limit** for elastic materials (even for small parameters).

$$\mathcal{C}_{\max}(\mu \neq 0) \to 1/2$$



However:
Buchdahl's bound, are obtained in **unphysical** configurations!

Bounds on the parameter space

- Positivity of the bulk modulus;

$$\epsilon + \theta < \frac{3}{8}$$

- Real transverse sound speeds in the **reference state**:

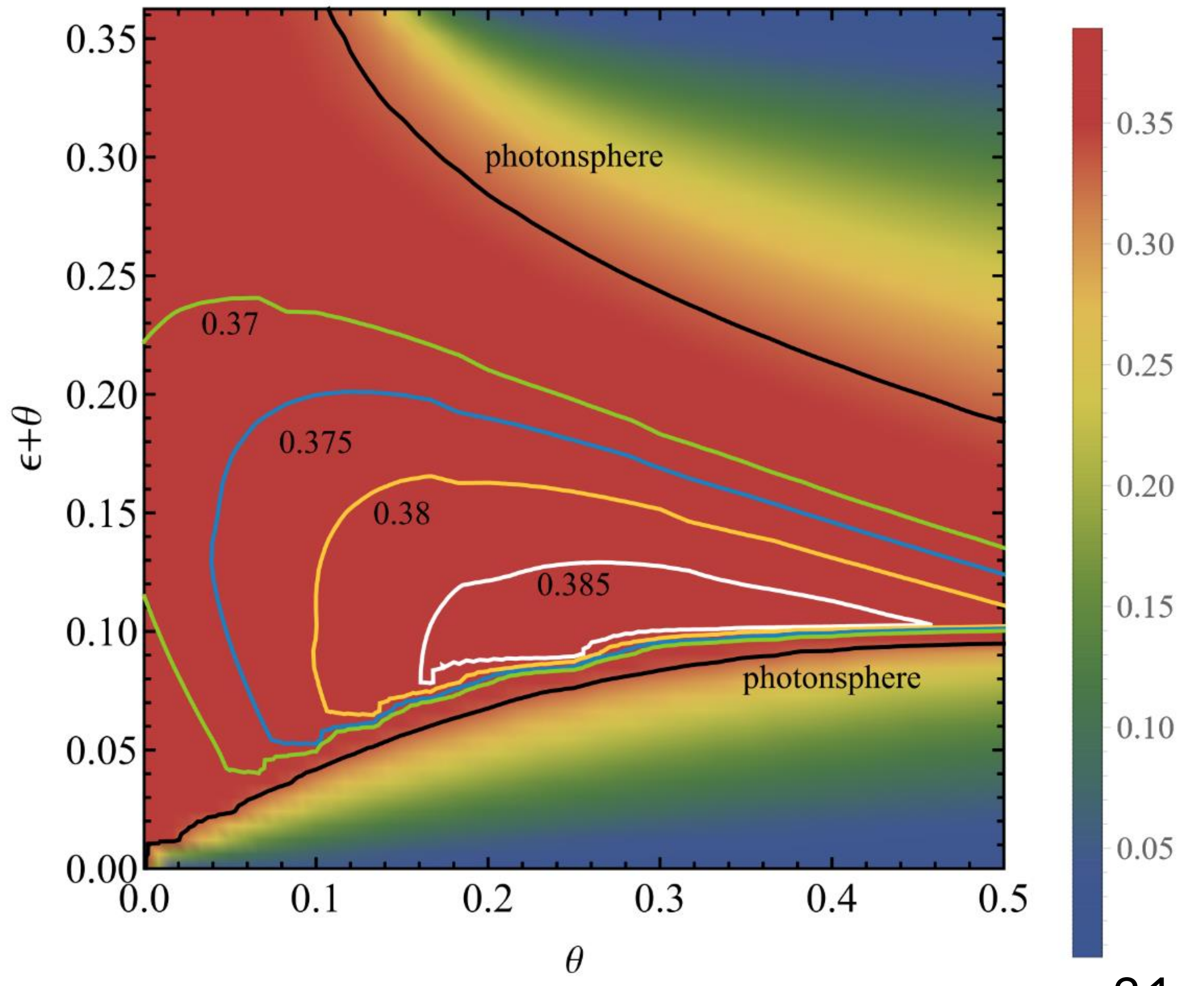
$$\epsilon + \theta > 0$$

- Positivity of bulk modulus, pressure and density for **large densities**

$$\epsilon + 2\theta < \frac{1}{2}$$

- Reality of the transverse speed in the isotropic state for **large densities**

$$\theta > 0$$



Physically Admissible

Energy Conditions

WEC: $\rho > 0$, $\rho + p_{\rm tan} > 0$, $\rho + p_{\rm rad} > 0$

NEC: $\rho + p_{\rm tan} > 0$, $\rho + p_{\rm rad} > 0$

SEC: $\rho + p_{\rm tan} > 0$, $\rho + p_{\rm rad} > 0$, $\rho + p_{\rm rad} + 2p_{\rm tan} > 0$

DEC

$$\rho > |p_{\rm tan}|$$

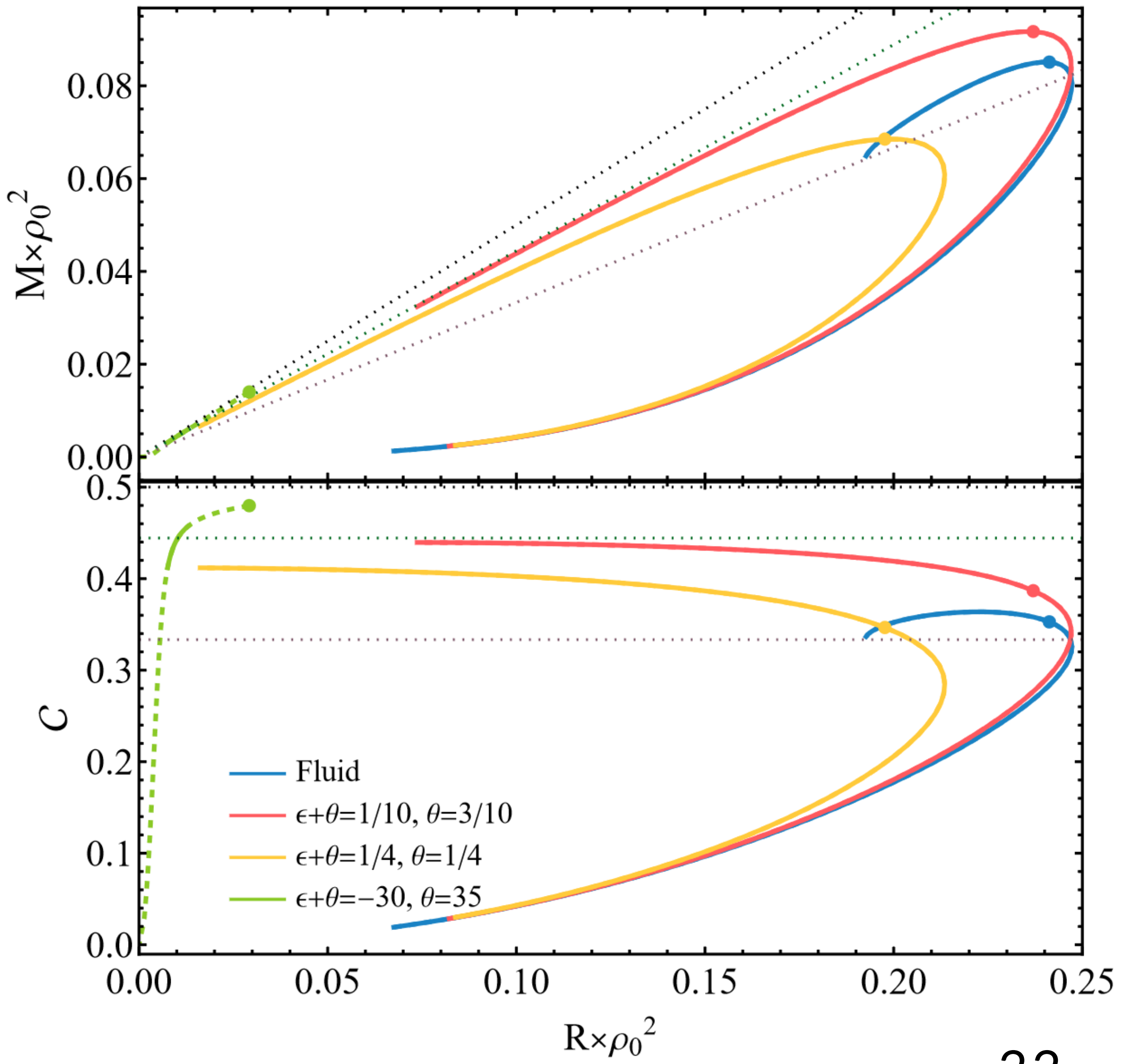
$$\rho > |p_{\rm rad}|$$

Subluminal Sound Speeds

$$c_L, \bar{c}_L, c_T, \bar{c}_T, \bar{c}_{TT} < 1$$

Maximum Compactness of a Physically Admissible Star

$$\mathcal{C}_{\rm PA} \lesssim 0.469$$



Physically Admissible and Stable

Energy Conditions

WEC: $\rho > 0$, $\rho + p_{\rm tan} > 0$, $\rho + p_{\rm rad} > 0$

NEC: $\rho + p_{\rm tan} > 0$, $\rho + p_{\rm rad} > 0$

SEC: $\rho + p_{\rm tan} > 0$, $\rho + p_{\rm rad} > 0$, $\rho + p_{\rm rad} + 2p_{\rm tan} > 0$

DEC

$$\rho > |p_{\rm tan}|$$

$$\rho > |p_{\rm rad}|$$

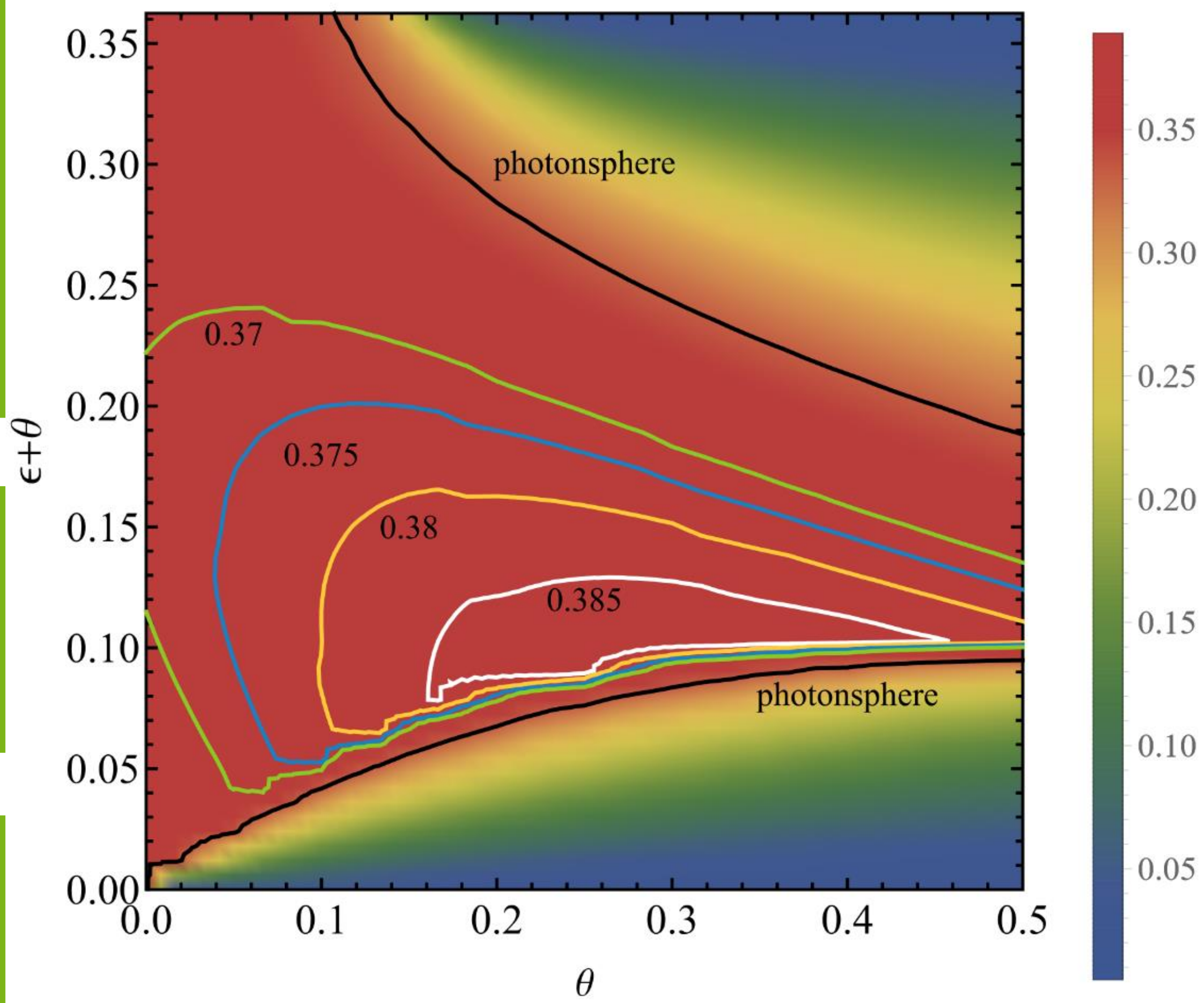
Subluminal Sound Speeds

$$c_L, \bar{c}_L, c_T, \bar{c}_T, \bar{c}_{TT} < 1$$

Radial Stability

$$\rho_c < \rho_c(M_{\rm max})$$

"On the right of the maximum mass in the mass-radius diagram"



$$\mathcal{C}_{\rm PAS} \lesssim 0.389$$

Physically Admissible and Stable

Energy Conditions

WEC:
$$\rho > 0$$
$$\rho + p_{\rm tan} > 0$$
$$\rho + p_{\rm rad} > 0$$

NEC: $\rho + p_{\rm tan} > 0$, $\rho + p_{\rm rad} > 0$

SEC: $\rho + p_{\rm tan} > 0$, $\rho + p_{\rm rad} > 0$, $\rho + p_{\rm rad} + 2p_{\rm tan} > 0$

DEC
$$\rho > |p_{\rm tan}|$$
$$\rho > |p_{\rm rad}|$$

Subluminal Sound Speeds

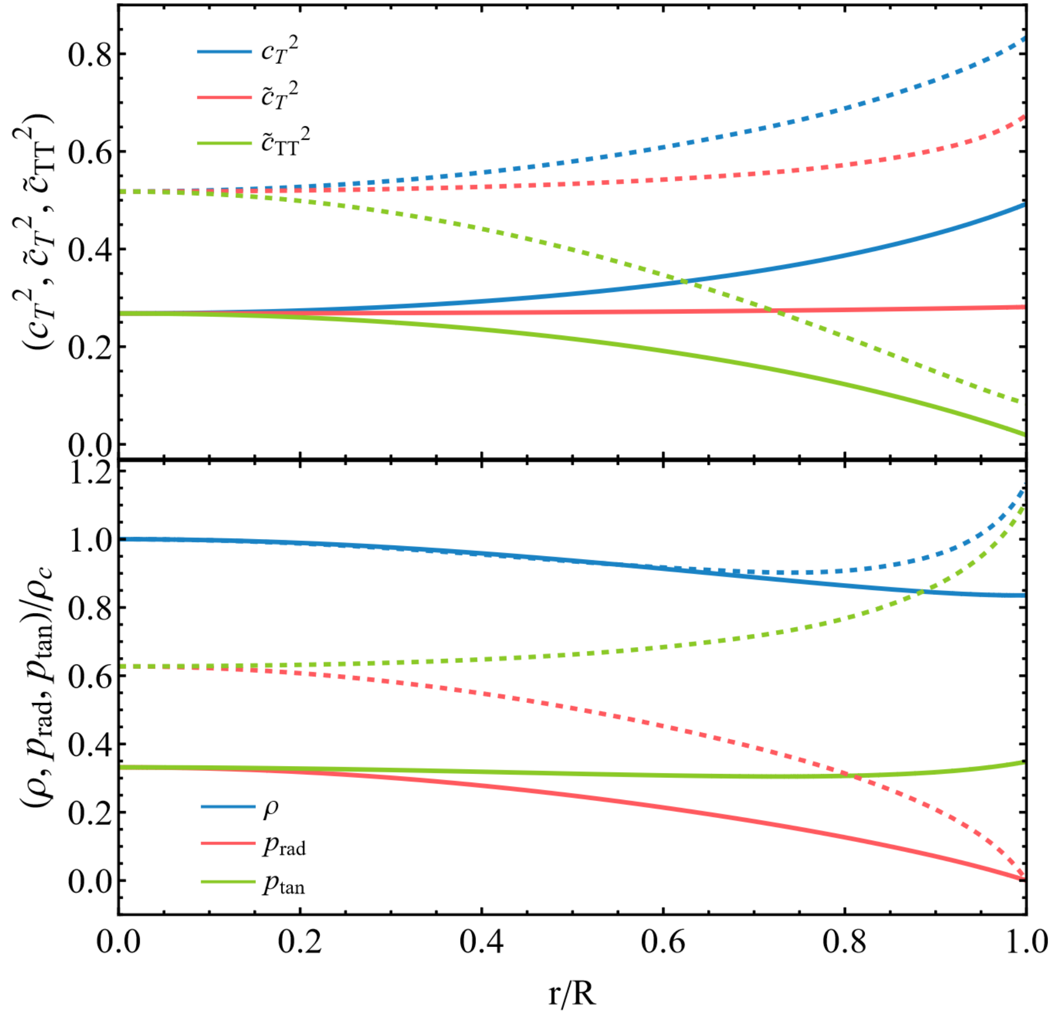
$$c_L, \bar{c}_L, c_T, \bar{c}_T, \bar{c}_{TT} < 1$$

Radial Stability

$$\rho_c < \rho_c(M_{\rm max})$$

"On the right of the maximum mass in the mass-radius diagram"

Dashed: Saturating PA bound;
Solid: Solution at PAS bound



Summary and Final Remarks

Part I:

- Introduced a simple relativistic formalism to include elastic effects in the description of compact objects;

Part II: Elastic Stars

- Introduce the simplest generalization to polytropic fluids;
- We construct elastic stars: elasticity increases the mass and compactness of the stars.
- Elasticity can be used to construct ECOs and also more accurately model NS.

Part III: Generalized Compactness Bounds

- Introduce the most general equation of state for a rigid body;
- Derive a novel set of compactness bounds that extend Buchdahl's results.

	$\mathcal{C}_{\text{Buchdahl}}$	$\mathcal{C}_{\text{PA}}$	$\mathcal{C}_{\text{PAS}}$
Fluid	$4/9$	0.365	0.354
Elastic	$1/2$	0.469	0.389

Ongoing Work

Multilayer Neutron Stars

- Setup of stars with different elastic layers and combination of fluid and elastic layers.
- Effects of the solid crust/solid core in fluid neutron stars.

Beyond Spherical symmetry

- Extend the formalism in Part I to spacetimes with less symmetry;
- Rotating Stars, Deformed Stars, Tidal deformations

1+1 NR evolution

Thank you