Quantum BHs and Holographic complexity

Antonia Micol Frassino

ICCUB, University of Barcelona

antoniam.frassino@icc.ub.edu

XV BHW – 19th December 2022

Outline

Motivations

Set-up

- Quantum effects

Results

- Holographic complexity

Motivations

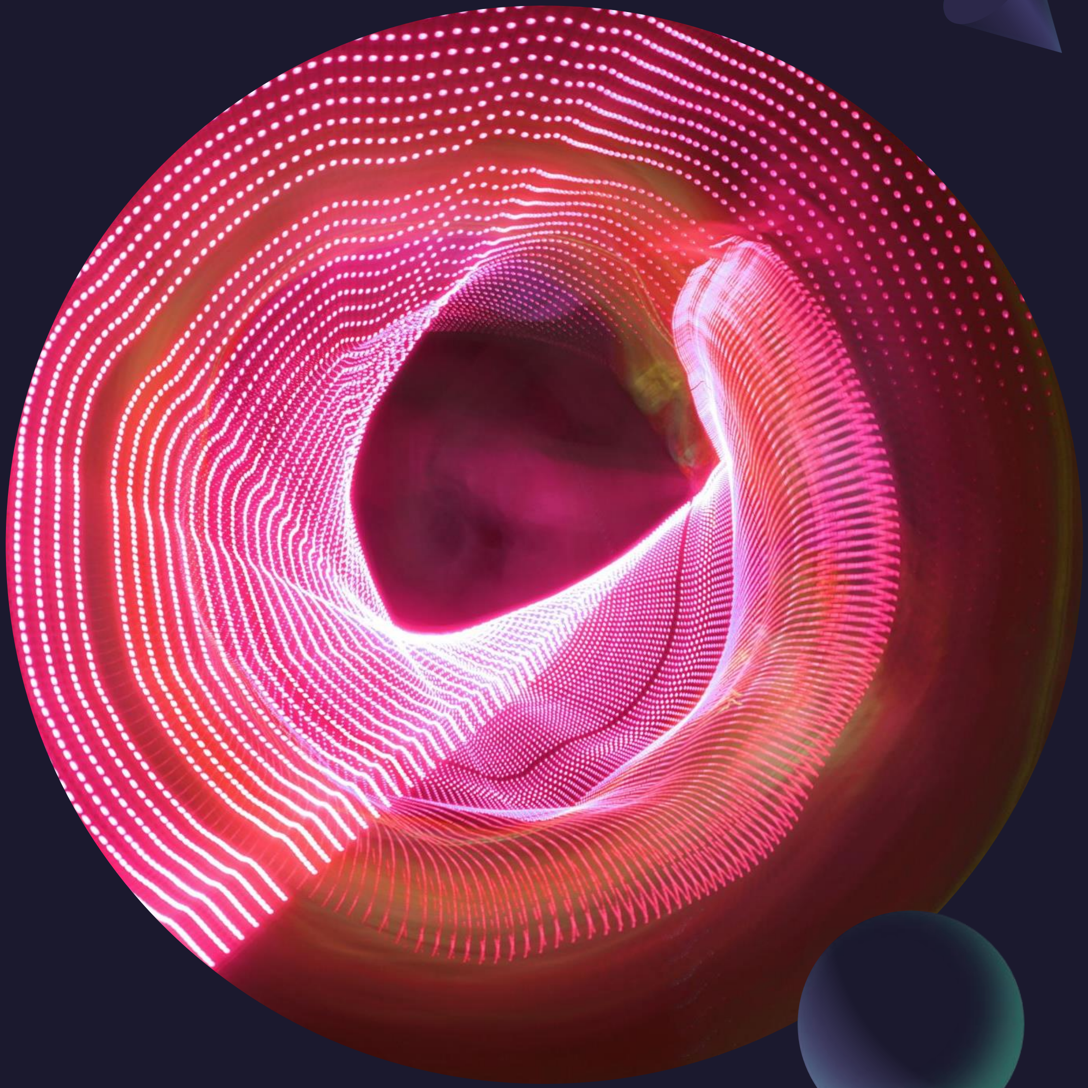
What is it that makes quantum physics
so different from classical physics?

Entanglement; the fact that you can know everything that can be known about a system and know nothing about its parts.

The second distinguishing property of quantum mechanics was pointed out by Feynman; namely, the extraordinary potential Complexity of quantum states

Entanglement

- When two qubits are entangled,
  If you perturb/look at one, it immediately affects in a ``spooky'' manner the other one



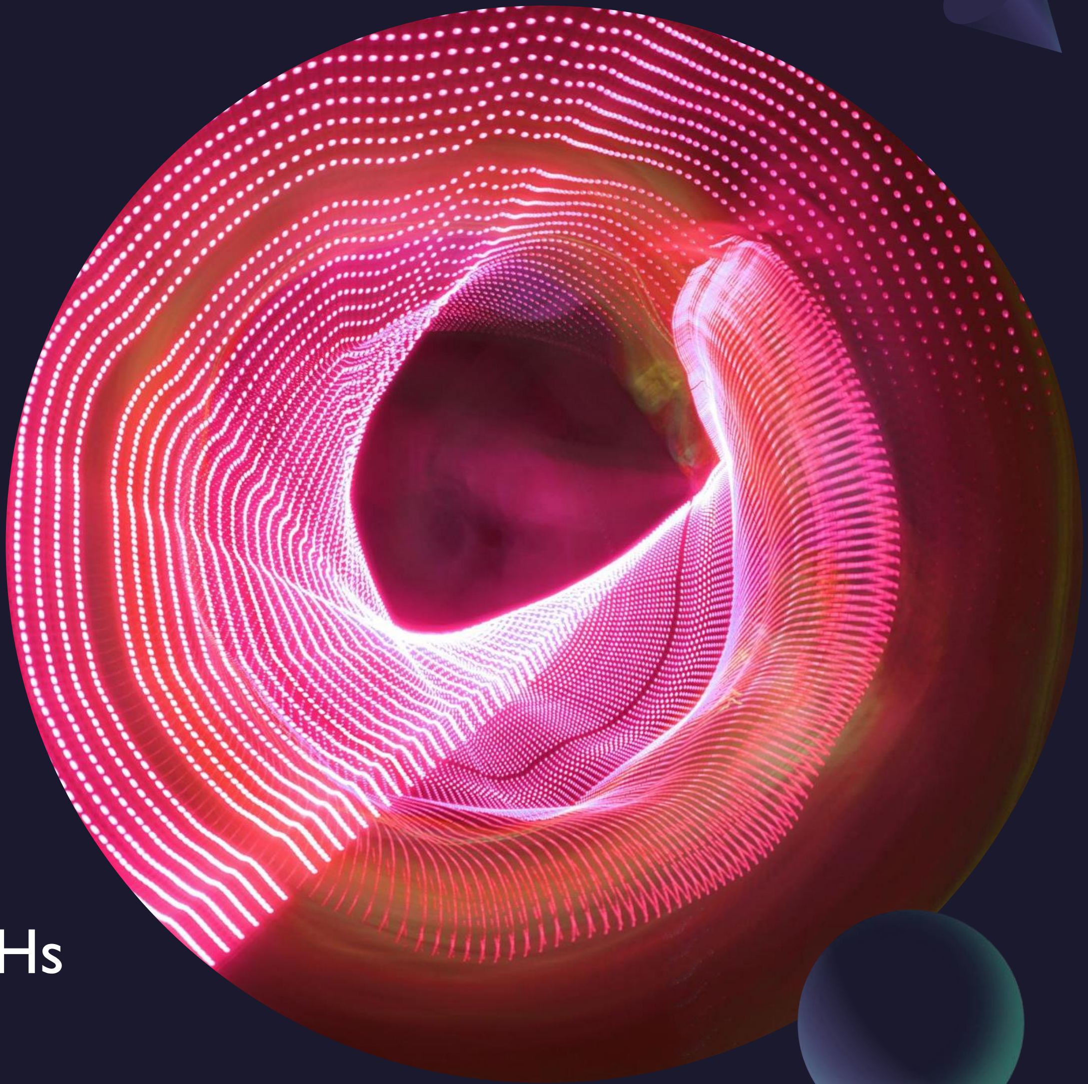
Entanglement

- What happens when you have many many particles entangled with each other?
  SYK model

- The SYK model has a scale-invariant entanglement structure: i.e., electrons are entangled at all distances

In a dual set of variables, it describes charged BHs

Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)



Complexity

Some features of complexity:

- Computational Complexity – a notion associated to the difficulty of performing some tasks
- What is the minimal amount of some simple operations required to perform a task
- Comparing timescales: max S (poly) vs max C (exp)

Holography

Explore Complexity in holography

- AdS/CFT is a holographic equivalence between:
  (quantum) gravity in asymptotically **AdS spacetime** and
  (conformal) field theories living on its 'boundary.'
- In particular, explore it in

BRANEWORLD HOLOGRAPHY

Randall, Sundrum (1999)
Karch, Randall (2001)

Braneworld holography

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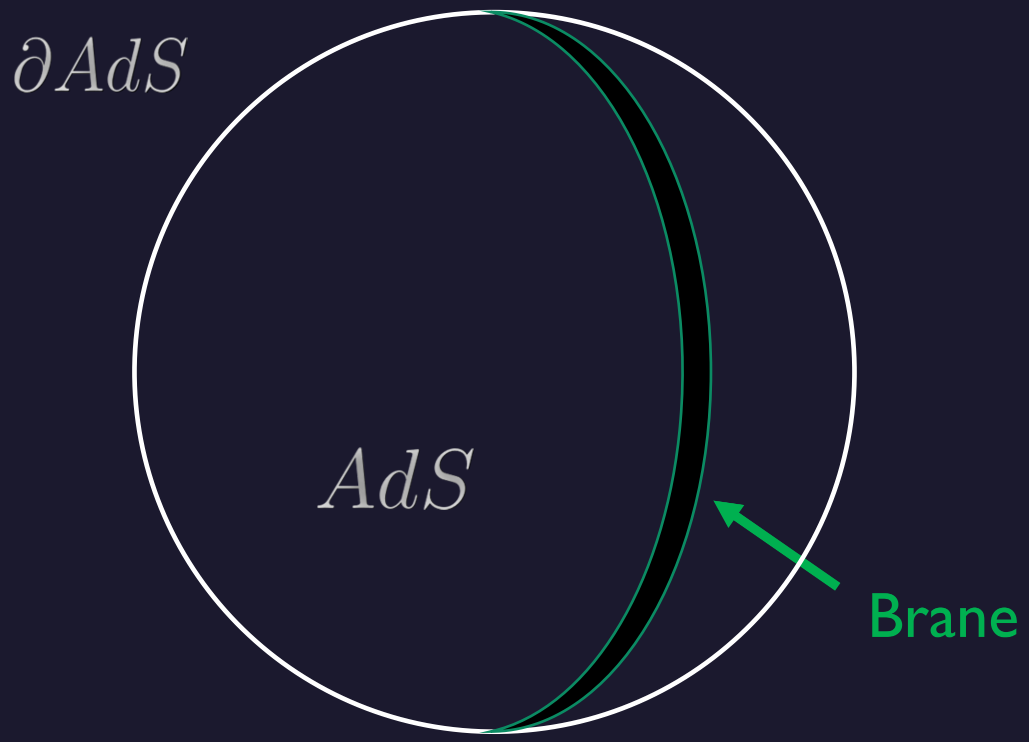
Basic idea of BRANEWORLD gravity:

Recover gravity localized on a lower dimensional surface of a higher dimensional bulk spacetime.



Cutting the bulk with a (Plank) brane

- Introduces a (D-1)-dimensional graviton massive mode localized on the brane
- The CFT is also cutoff in the UV

→ You get dynamics on the brane

Motivations to study this setup

- Add a Black hole on the Brane

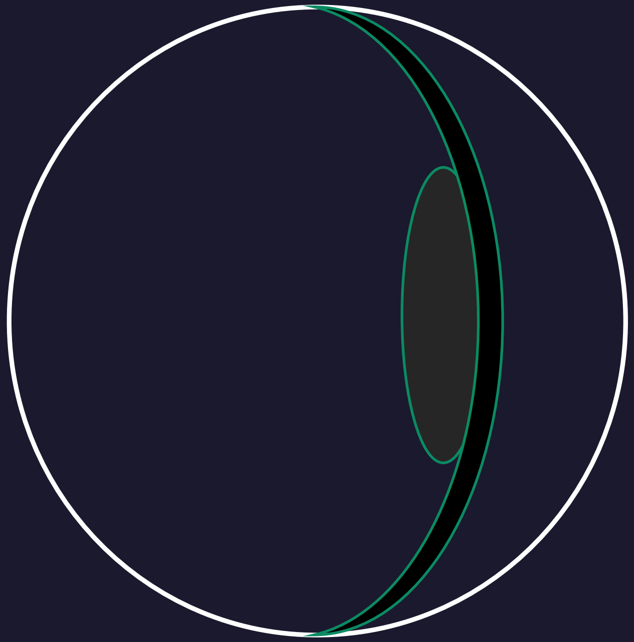
- **Double holography**

  1. CFT Perspective
  2. Brane Perspective
  3. Doubly Holographic perspective

  View the whole setup as a
  (d+2) AdS without any CFT

Motivations to study this setup

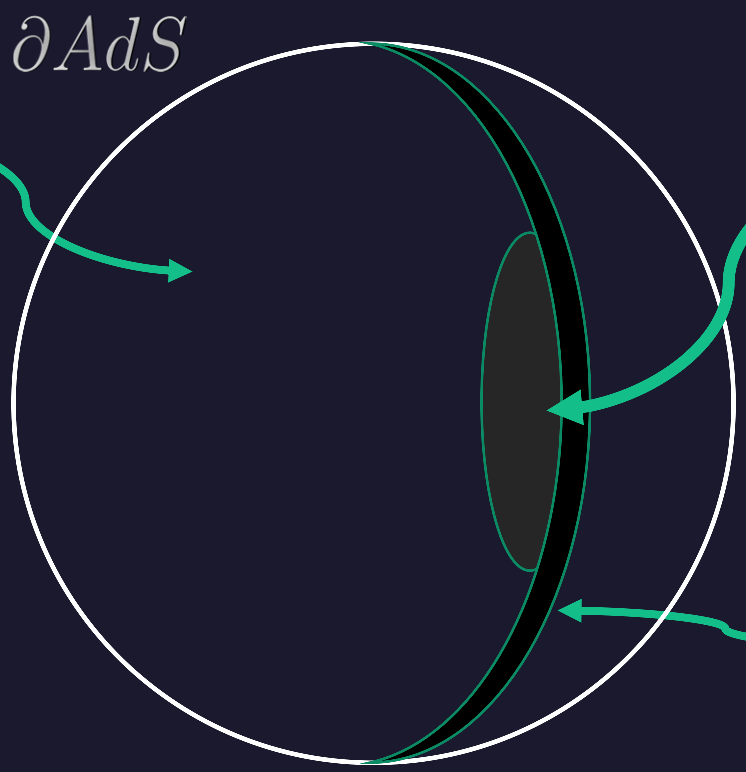
- **<u>Add a Black hole on the Brane</u>**



Black hole on the brane

Emparan, Horowitz, Myers (2000)
Emparan, Fabbri, Kaloper (2002)
Emparan, AMF, Way (2020)

The 4D theory has an asymptotic boundary where the CFT3 lives



The BH in the bulk is intersecting the brane

CFT3 on the brane

3D CC obtained from:
4D CC & brane tension

$$\frac{1}{\ell_4^2} = \frac{1}{\ell^2} + \frac{1}{\ell_3^2}$$

AMF, J. Pedraza, A. Svesko, and M. Visser (2022/23)

Quantum Backreaction

- The quantum thermal radiation does, have energy, and therefore, will affect the spacetime geometry (back-reaction)

- This *back-reaction* is governed by the semiclassical Einstein equations

$$G_{\mu\nu}(g_{\alpha\beta}) = 8\pi G_N \langle T_{\mu\nu}(g_{\alpha\beta}) \rangle$$

- Classical geometry is modified by the effects of quantum fields

- "Quantum Black Hole"

Quantum Backreaction

$$G_{\mu\nu}(g_{\alpha\beta}) = 8\pi G_N \langle T_{\mu\nu}(g_{\alpha\beta})\rangle$$

Classical Einstein tensor & metric

Quantum matter renorm stress tensor (many fields)

Difficulties in solving these equations:

- Coupled system: metric+ <QFT>
- Very hard to solve simultaneously
- Perturbative backreaction: limited insight

Exact backreaction:

- 2D models: CGHS, JT+CFT
- **Holographic reformulation**

Emparan, Fabbri, Kalopper (2002)

AdS_4 C-metric

The classical metric of this state:

$$ds^2 = \frac{\ell^2}{(\ell + xr)^2}\left[-H(r)dt^2 + \frac{dr^2}{H(r)} + r^2\left(\frac{dx^2}{G(x)} + G(x)d\phi^2\right)\right]$$

$$H(r) = \frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r} \qquad G(x) = 1 - \kappa x^2 - \mu x^3$$

ℓ_3 Brane curvature radius

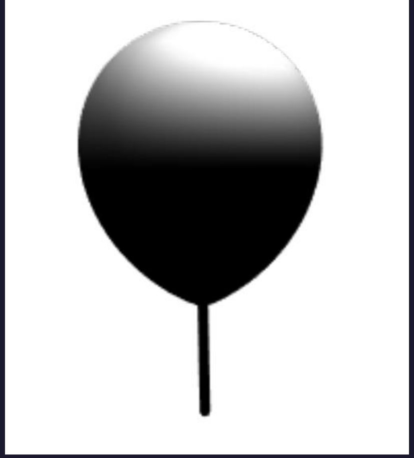
ℓ Brane position, tension^(-1)

μ Quantum corrections on the brane

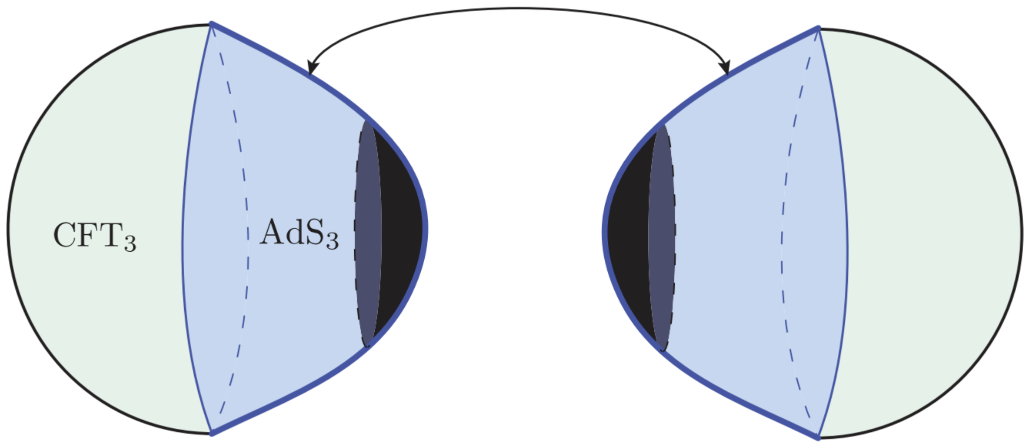
Note: the black hole is NOT in the center of AdS_4 – it's accelerating

AdS_4 C-metric

ACCELERATED BH

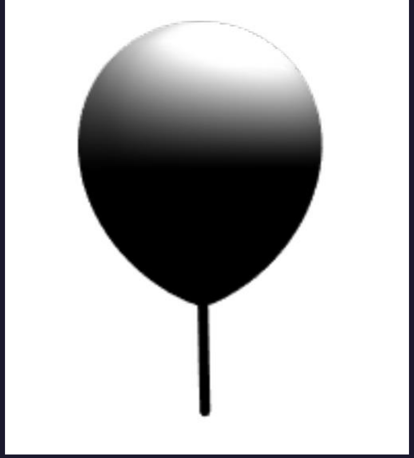


BLACK HOLE ON THE BRANE

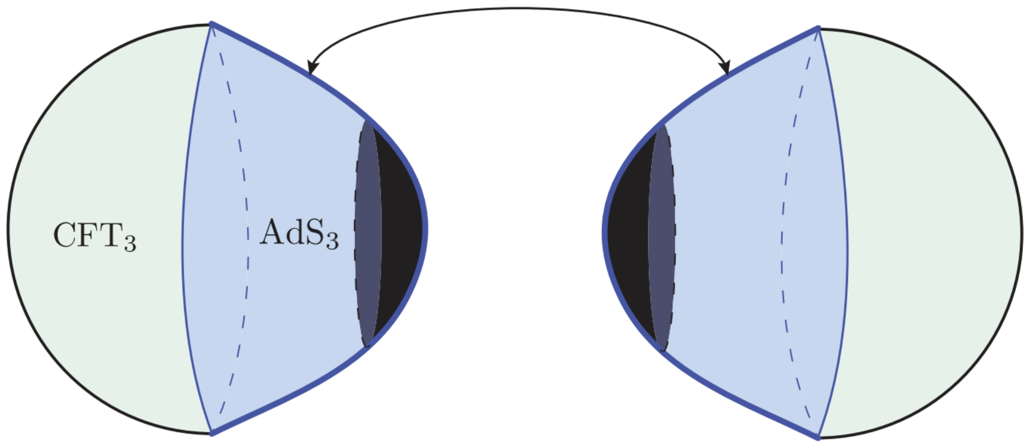


AdS_4 C-metric

ACCELERATED BH



BLACK HOLE ON THE BRANE



Metric quBTZ

The 3D metric induced on the brane at x=0

$$ds^2 = -\left(\frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r}\right)dt^2 + \frac{1}{\frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r}}dr^2 + r^2 d\phi^2$$

<u>Classical limits</u> $\mu = 0$

- $\kappa = -1$ BTZ
- $\kappa = +1$ Global or Conical AdS_3

$\mu \neq 0$ $\kappa = -1$ quBTZ, different properties of the horizon, has curvature singularity

Metric quBTZ

The 3D metric induced on the brane at x=0

$$ds^2 = -\left(\frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r}\right)dt^2 + \frac{1}{\frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r}}dr^2 + r^2 d\phi^2$$

$\mu \neq 0 \qquad \kappa = -1$

quBTZ, different properties of the horizon, has curvature singularity

Interpretation: is as a solution of a theory of 3D gravity, with higher curvature terms, coupled to a large number of quantum conformal fields, namely, the holographic CFT dual to the 4D bulk.

Holographic Complexity

Some features of **holographic complexity**

- Volume and Action – several proposals   Susskind et al (2014)

$$\mathcal{C}_V\left(|\Psi\rangle\right) = \frac{\mathrm{Vol}(\Sigma)}{G\hbar\, L} \qquad \mathcal{C}_A\left(|\Psi\rangle\right) = \frac{I(\mathcal{W})}{\pi\hbar}$$

- New developments   Belin, Myers, Ruan, Sárosi, Speranza (2021)

→ Using the C-metric we can compute quantum corrections using a classical setup

Quantum corrections

- These proposals (action/volume) have passed many tests, and they give the same results in almost all cases
- They are similar in spirit to the RT formula, where we also connected a purely geometric notion (area) to a quantum property of the state (von Neumann entropy)

However, in the case of RT, adding quantum corrections led to unexpected new results that purely classical bulk could not have given us

Engelhardt, Wall '14
Penington `19
Almheiri, Engelhardt, Marolf, Maxfield `19

Results for the Volume

Emparan, AMF, Sasieta, Tomašević (2022)

The quantum-corrected VC formula reproduces the expected computation rate for a semiclassical black hole

$$\left.\frac{\mathrm{d}\mathcal{C}_V}{\mathrm{d}\bar{t}}\right|_{t\gg\beta} = 2M\left(1+\sqrt{2}\,\frac{\mu\ell}{\ell_3}+\ldots\right)$$

Brane BH entropy +
The entropy of the CFT in the presence of the BH

$$\left.\frac{\mathrm{d}\mathcal{C}_V}{\mathrm{d}t}\right|_{t\gg\beta} \sim TS_{\mathrm{gen}} > (TS)_{BTZ}$$

up to an O(1) coefficient that depends on the mass of the black hole.

- The quBTZ computes at a faster rate than the respective BTZ

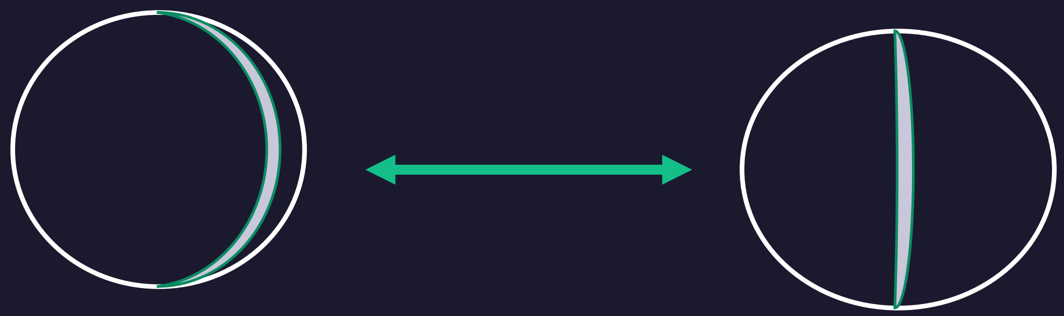
Results for the Action

Emparan, AMF, Sasieta, Tomašević (2022)

- It doesn't reproduce the complexity rate of the BTZ:

$$\frac{dC_A}{dt} \neq (TS)_{BTZ} \text{ when } \frac{c_3}{c_2}\left(\text{or} \frac{\ell}{\ell_3}\right) \to 0$$

- And, since the same parameters control the inverse tension of the brane, it doesn't distinguish between different bulk geometry:



$\ell \to 0$ $\ell \to \infty$

$$C_A(|\text{quBTZ}\rangle) \neq C_A^{\text{BTZ}} + \delta C_A$$

$\mathcal{O}(c_2)$ $\mathcal{O}(c_3)$

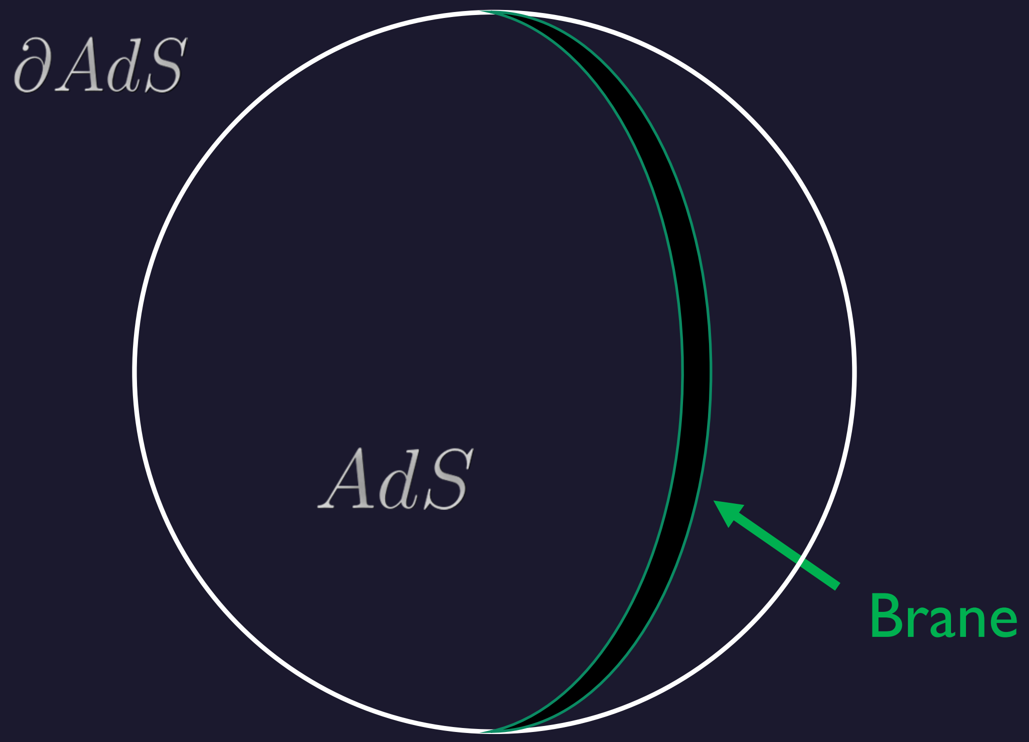
Summary

- The quantum-corrected VC formula correctly reproduces the expected computation rate for a semiclassical black hole
- AC remains still puzzling because it does not give the correct classical limit
- Sensitivity of the AC to the singularity

Braneworld holography

Randall, Sundrum (1999)
Karch, Randall (2001)

Basic idea of BRANEWORLD gravity:
recover gravity localized on a lower dimensional surface of a higher dimensional bulk spacetime.



Cutting the bulk with a (Plank) brane

- Introduces a (D-1)-dimensional graviton massive mode localized on the brane
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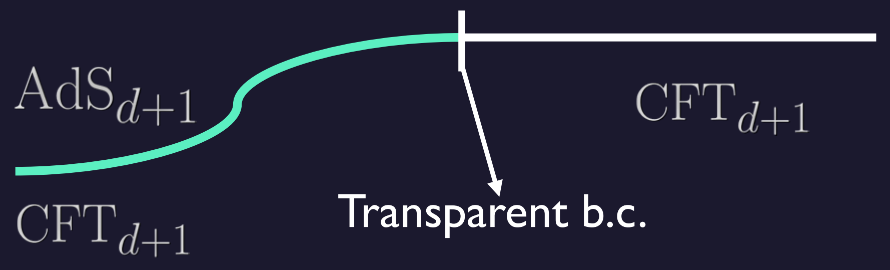
Doubly holographic interpretation

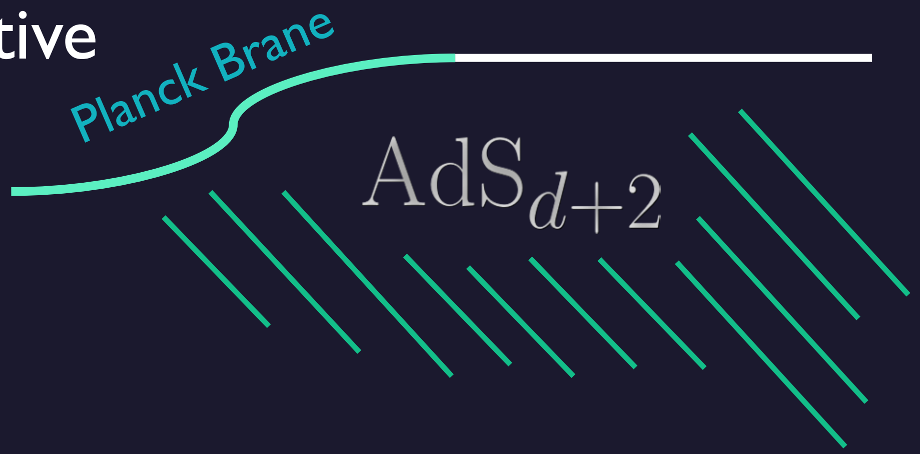
1. UV Perspective

2. Brane Perspective

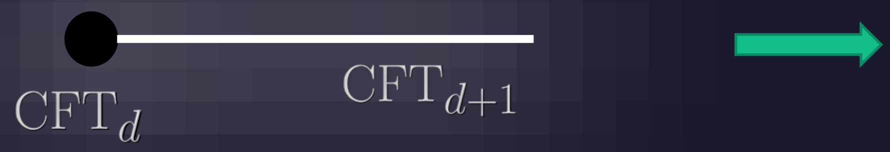


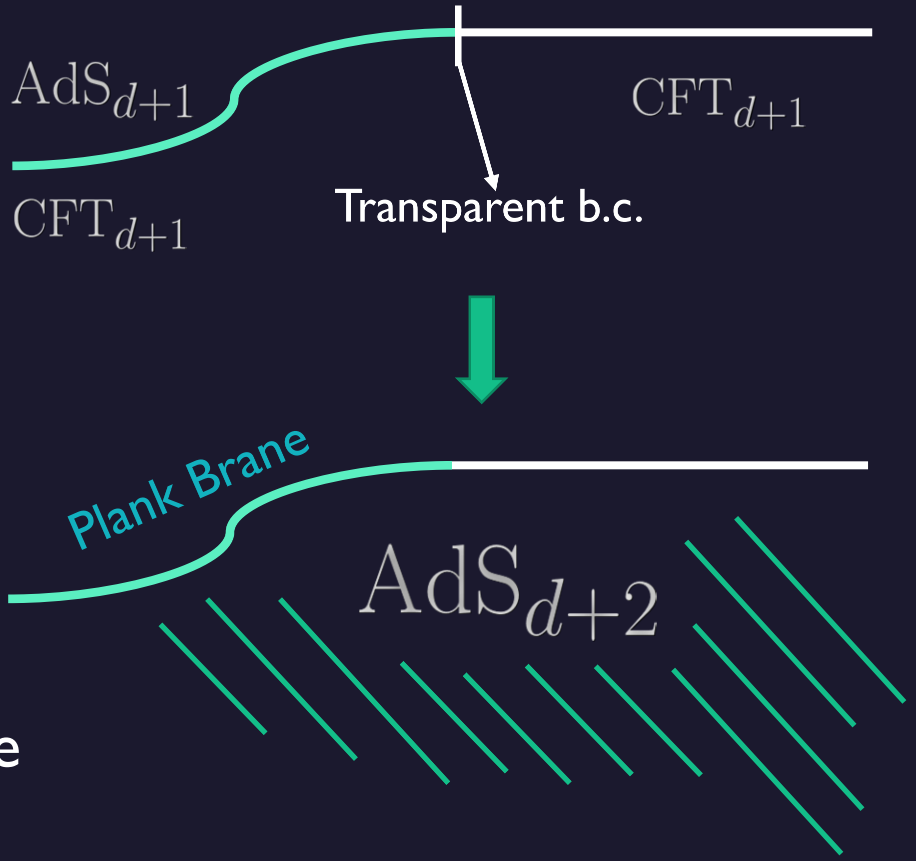
3. Doubly Holographic perspective

View the whole setup as a (d+2) AdS without any CFT



Doubly holographic interpretation





- Now this is just a dynamical (d+1) spacetime (a brane) in the (d+2) dimensional bulk.

- The EFT of gravity in the (d+1) is just a localization of this (d+2) = D gravity on the brane (like in the RS setup)

Black hole on the brane

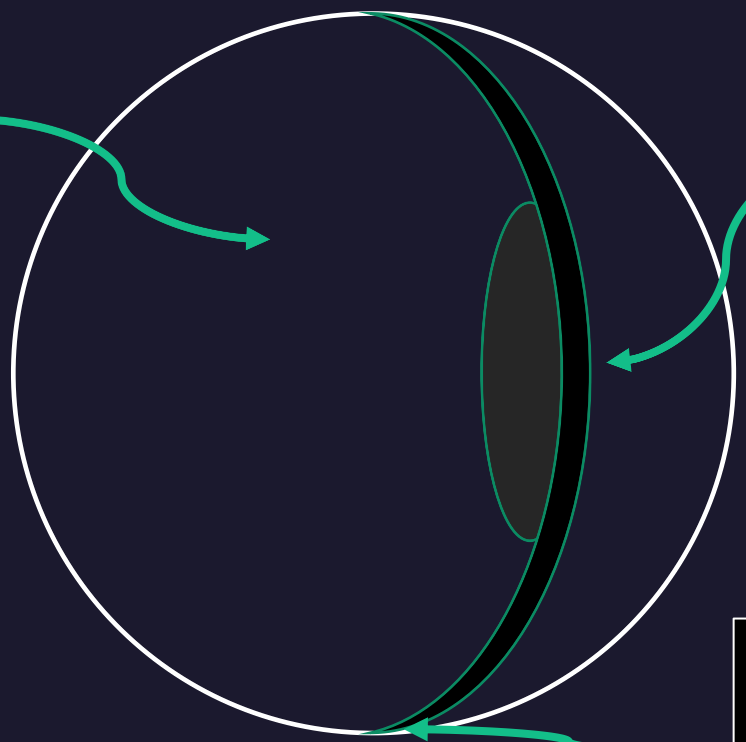
Emparan, Horowitz, Myers (2000)
Emparan, Fabbri, Kaloper (2002)
Emparan, AMF, Way (2020)

EE with negative CC in 4D

The bulk spacetime is a solution of the vacuum D

3D CC obtained from:
4D CC & brane tension

$$\frac{1}{\ell_4^2} = \frac{1}{\ell^2} + \frac{1}{\ell_3^2}$$



The BH in the bulk is intersecting the brane

Transparent boundary conditions for the CFT
btw the boundary and the brane
(we consider the dCFT)

Holographic Complexity

Some features of **holographic complexity**

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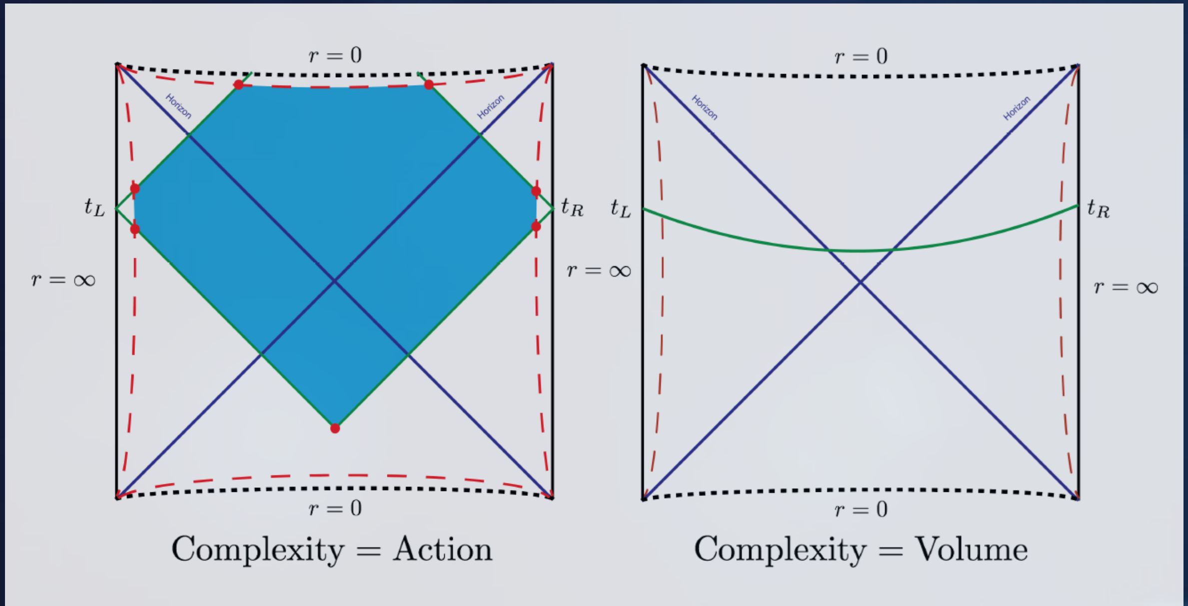

Holographic complexity

Emparan, AMF, Sasieta, Tomašević (2022)

- ❑ Our system describes a 2-sided BH
- ❑ we can compute the CV complexity of this state by the volume in 4D or the CA by the action in 4D

$$C_V(|\mathrm{quBTZ}\rangle) = \frac{\mathrm{Vol}_4(\Sigma)}{G_4 L} \qquad C_A(|\mathrm{quBTZ}\rangle) = \frac{I_4}{\hbar}$$

These quantities should compute the complexity of the quBTZ including the bulk complexity of the quantum fieldsin the large limit of the c_3 central charge.

Results

VOLUME

ACTION

$$ds^2 = \frac{\ell^2}{(\ell + xr)^2}\left[-H(r)dt^2 + \frac{dr^2}{H(r)} + r^2\left(\frac{dx^2}{G(x)} + G(x)d\phi^2\right)\right]$$

The rotating AdS C-metric:

- Rotation parameter a
- Similar but more complicated:
- Bulk structure similar to Kerr-AdS4 (inner and outer horizons, ring singularity)

↓

Interesting features

Emparan, Tomašević (2020)
Emparan, AMF, Way (2020)

Black hole on the brane

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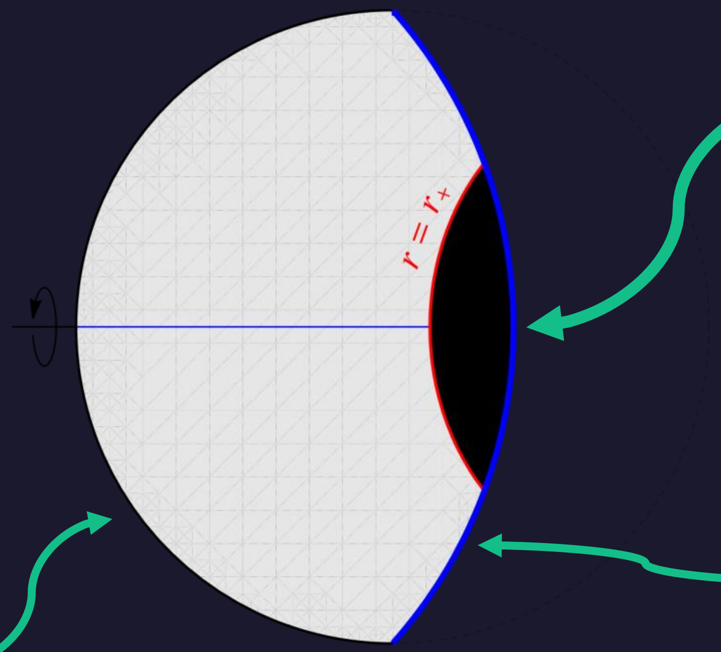
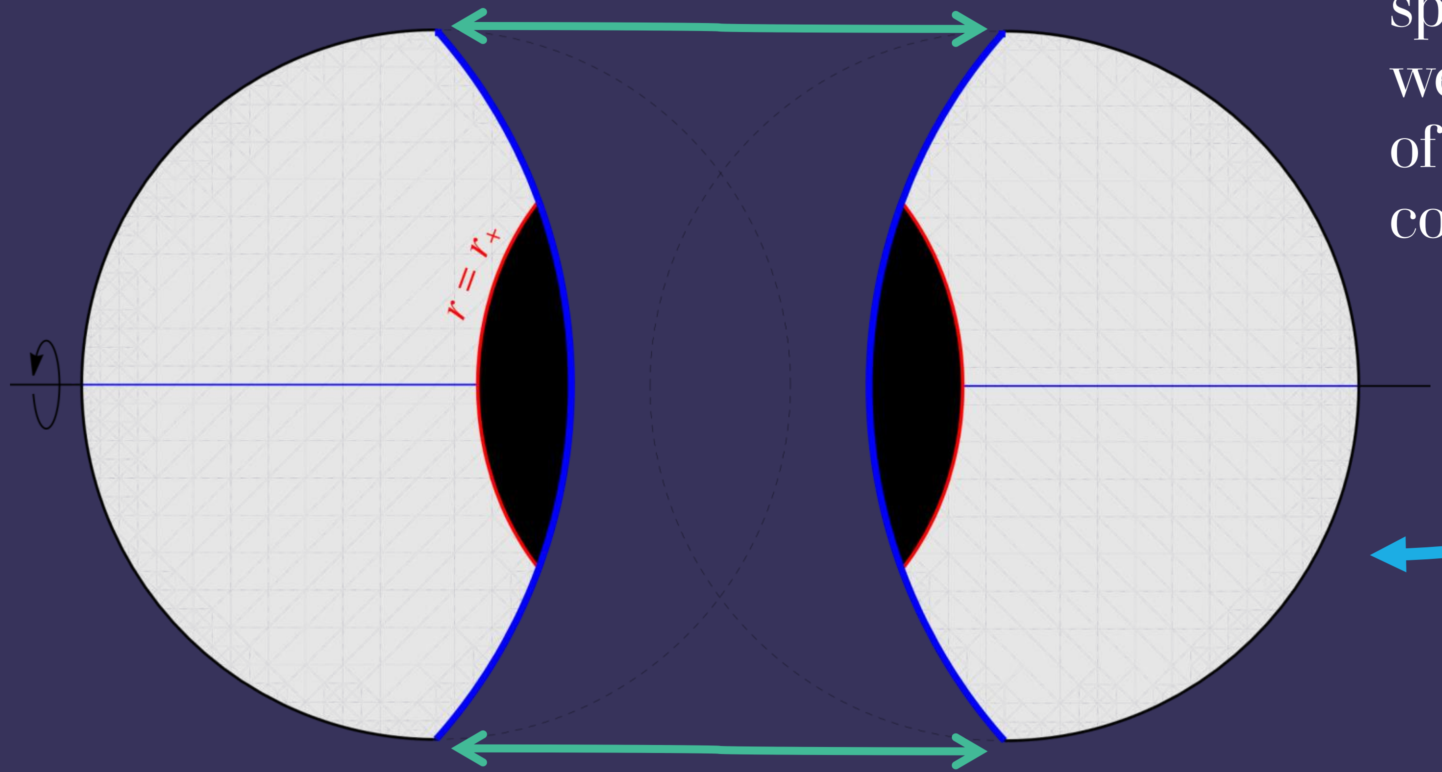


The BH in the bulk is intersecting the brane

CFT_3 on the brane

The 4D theory has an asymptotic boundary where the CFT_3 lives

When the brane is placed in the spacetime,
we erase the other part of the spacetime and consider another copy



The gluing between the two parts of the spacetime is done using Israel gluing conditions along the brane

Global aspects of the bulk

$\ell \to \infty$ | $0 < \ell < \infty$ | $\ell \to 0$

4D AdS BH (only Z2 symmetry)

BH+CFT

$\kappa = -1$ bd geometry has a BTZ

Double Wick rotation of the Schwarzschild-AdS_4

Quantum corrections

- Parameter l measures the effect of the CFT3 on the brane l/l3~geff, where l3 is the 3D curvature radius
- For example, when geff<<1, grav. Backreaction of quantum fields is small
- What is mu then? – it parametrizes the quantum state of the CFT3 which is coupled to the quBTZ black hole
- It alco cannot be arbitrary – it's set by the regularity of the bulk geometry

Add stress energy tensor

The CFT3 stress tensor, where c3 is it's central charge