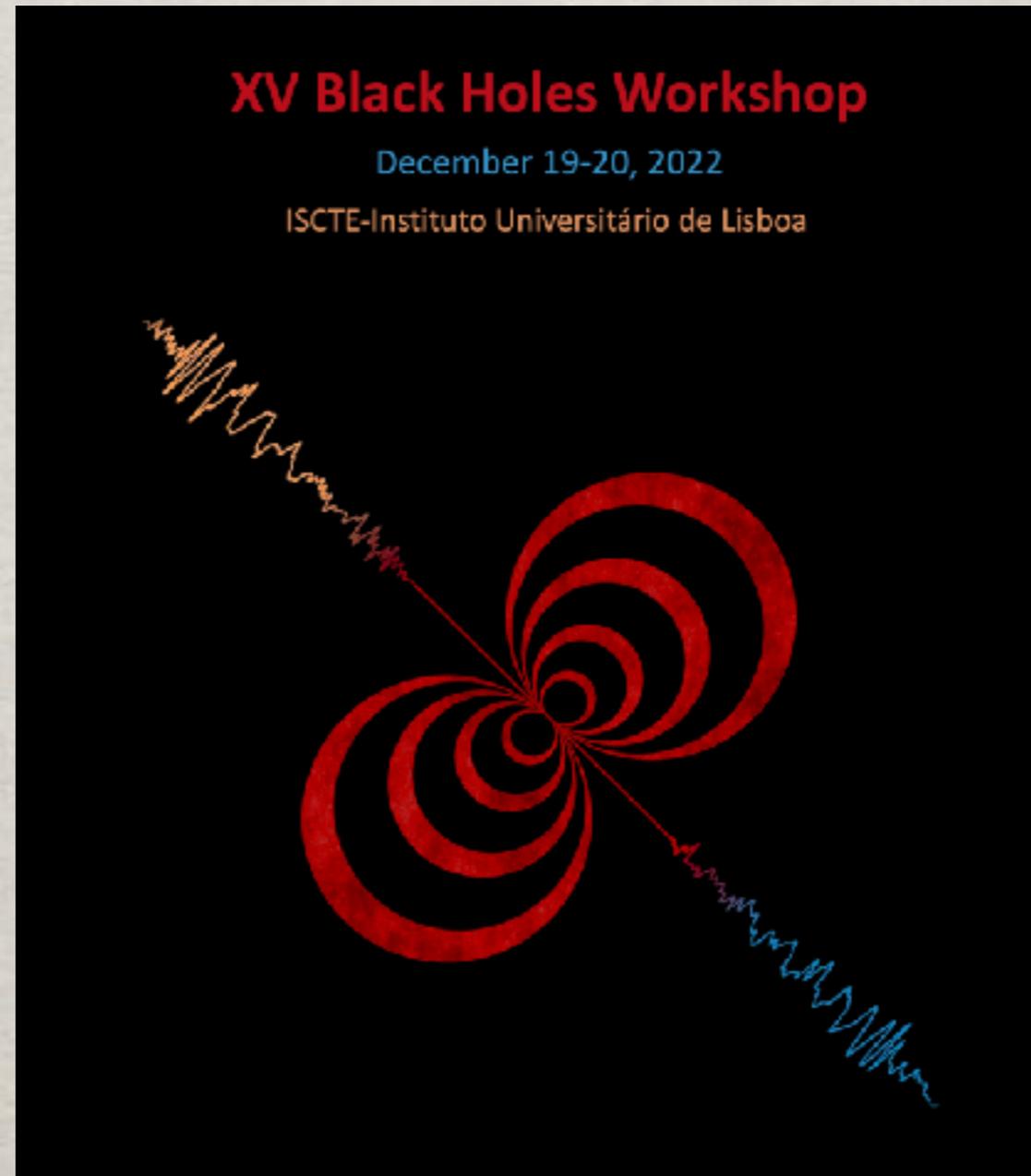


On the fate of the Light Ring instability

C. Herdeiro

Departamento de Matemática e CIDMA, Universidade de Aveiro, Portugal
XV Black Holes Workshop, ISCTE, Lisbon, Dec 19th 2022



Based on
2207.13713
with

P. Cunha, E. Radu, N. Sanchis-Gual
(to appear in Phys. Rev. Lett.)



3 statements and 3 questions

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In my talk I will give partial, but hopefully informative, answers to these three questions.

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PHYSICAL REVIEW LETTERS **124**, 181101 (2020)

Editors' Suggestion

Stationary Black Holes and Light Rings

Pedro V. P. Cunha¹ and Carlos A. R. Herdeiro²

¹*Max Planck Institute for Gravitational Physics—Albert Einstein Institute, Am Mühlenberg 1, Potsdam 14476, Germany*

²*Departamento de Matemática da Universidade de Aveiro and CIDMA, Campus de Santiago, 3810-183 Aveiro, Portugal*

 (Received 19 March 2020; accepted 15 April 2020; published 8 May 2020)

The ringdown and shadow of the astrophysically significant Kerr black hole (BH) are both intimately connected to a special set of bound null orbits known as light rings (LRs). Does it hold that a *generic* equilibrium BH *must* possess such orbits? In this Letter we prove the following theorem. A stationary, axisymmetric, asymptotically flat black hole spacetime in $1+3$ dimensions, with a nonextremal, topologically spherical, Killing horizon admits, at least, one standard LR outside the horizon for each rotation sense. The proof relies on a topological argument and assumes C^2 smoothness and circularity, but makes no use of the field equations. The argument is also adapted to recover a previous theorem establishing that a horizonless ultracompact object must admit an even number of nondegenerate LRs, one of which is stable.

DOI: [10.1103/PhysRevLett.124.181101](https://doi.org/10.1103/PhysRevLett.124.181101)

Q1: Do all theoretical black hole solutions have Kerr-like LRs?

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Central idea:

LRs are *critical points* of two potentials

$$\nabla H_{\pm} \Big|_{\text{LR}} = 0$$

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Central idea:

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These potentials define vector fields as their gradients:

$$\mathbf{V}_{\pm} = \nabla H_{\pm}$$

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$$\oint_C d\Omega = 2\pi w$$

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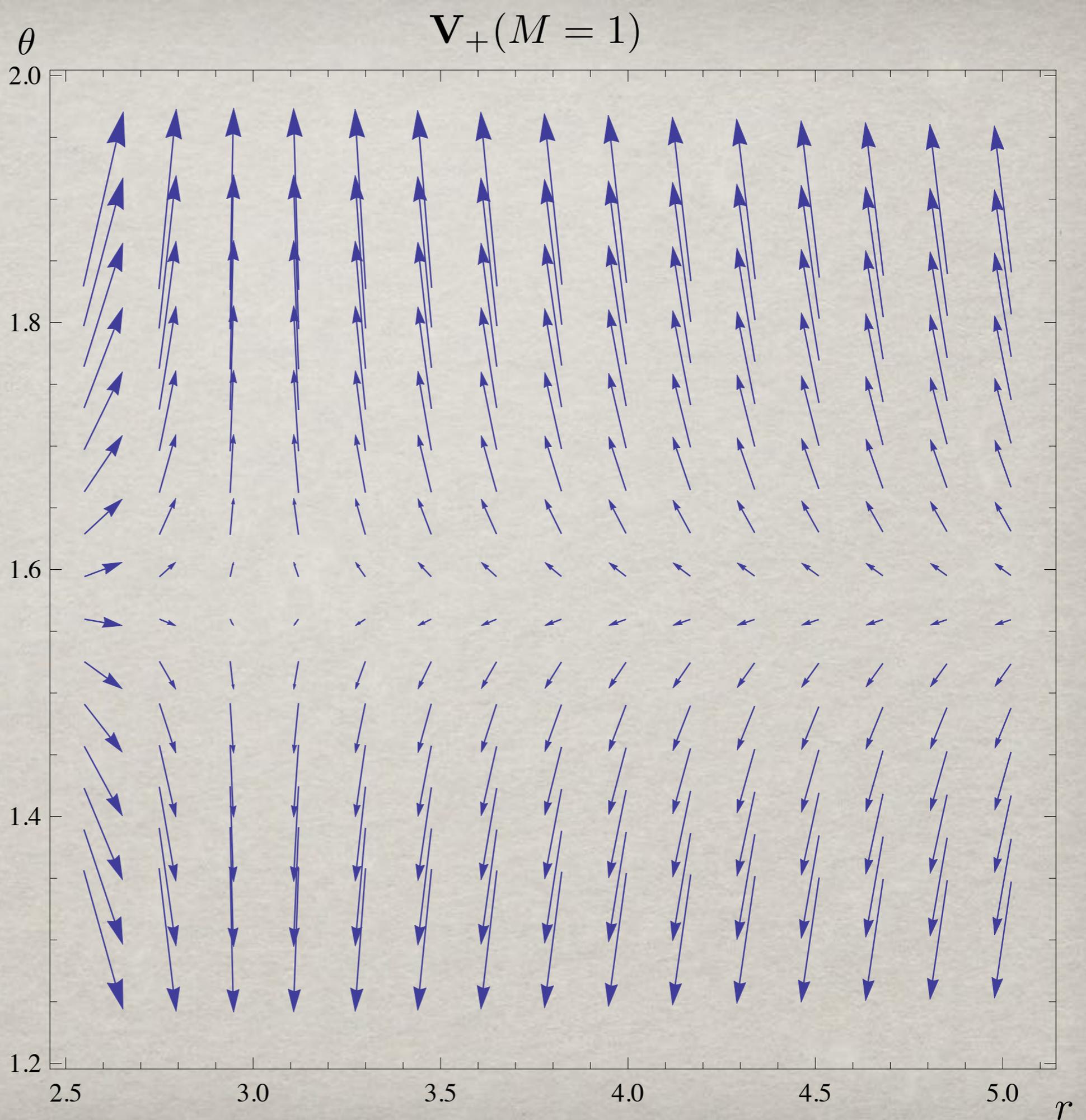
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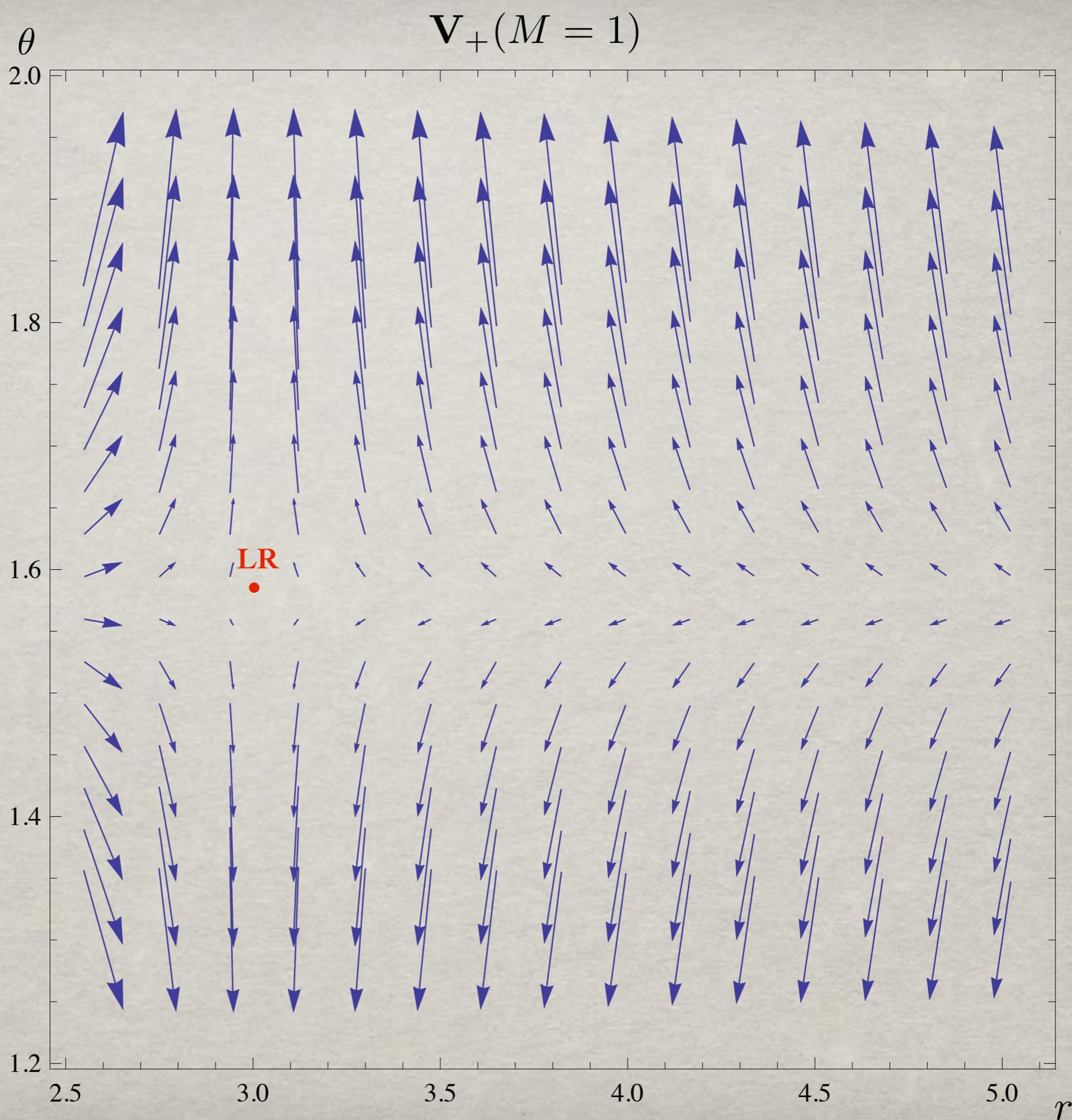
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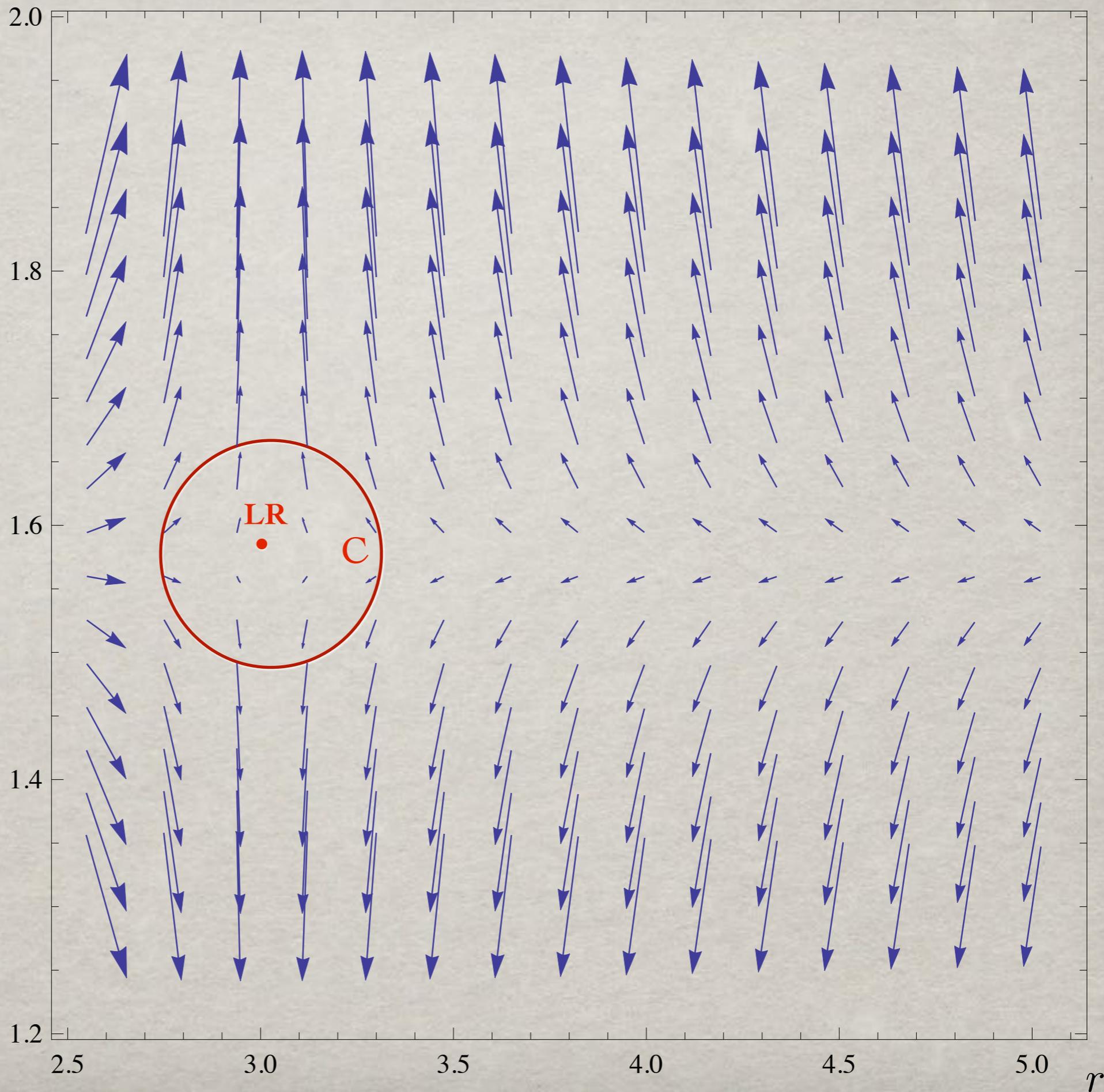


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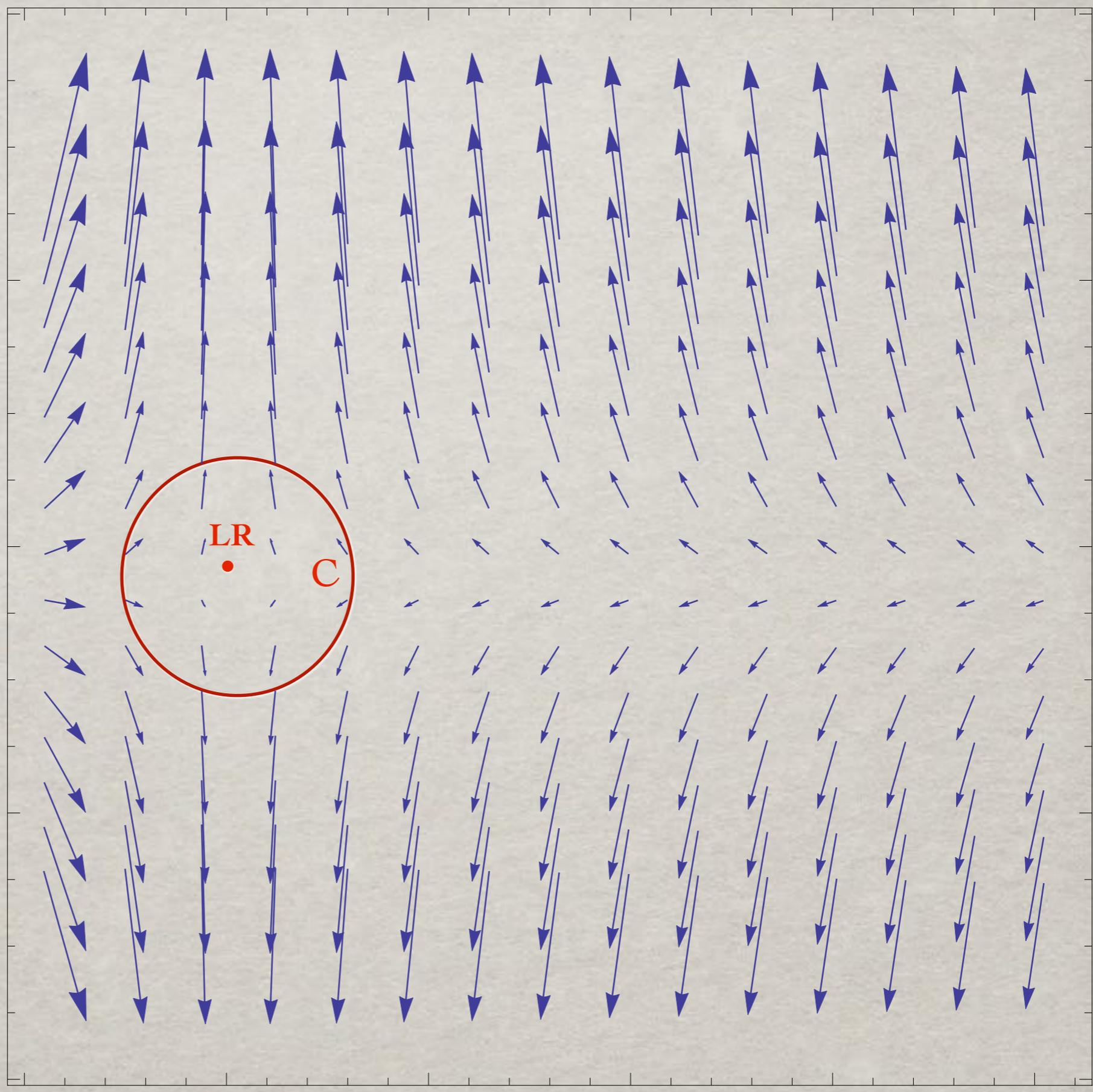
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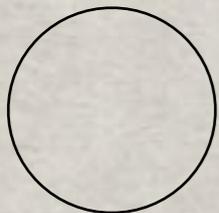


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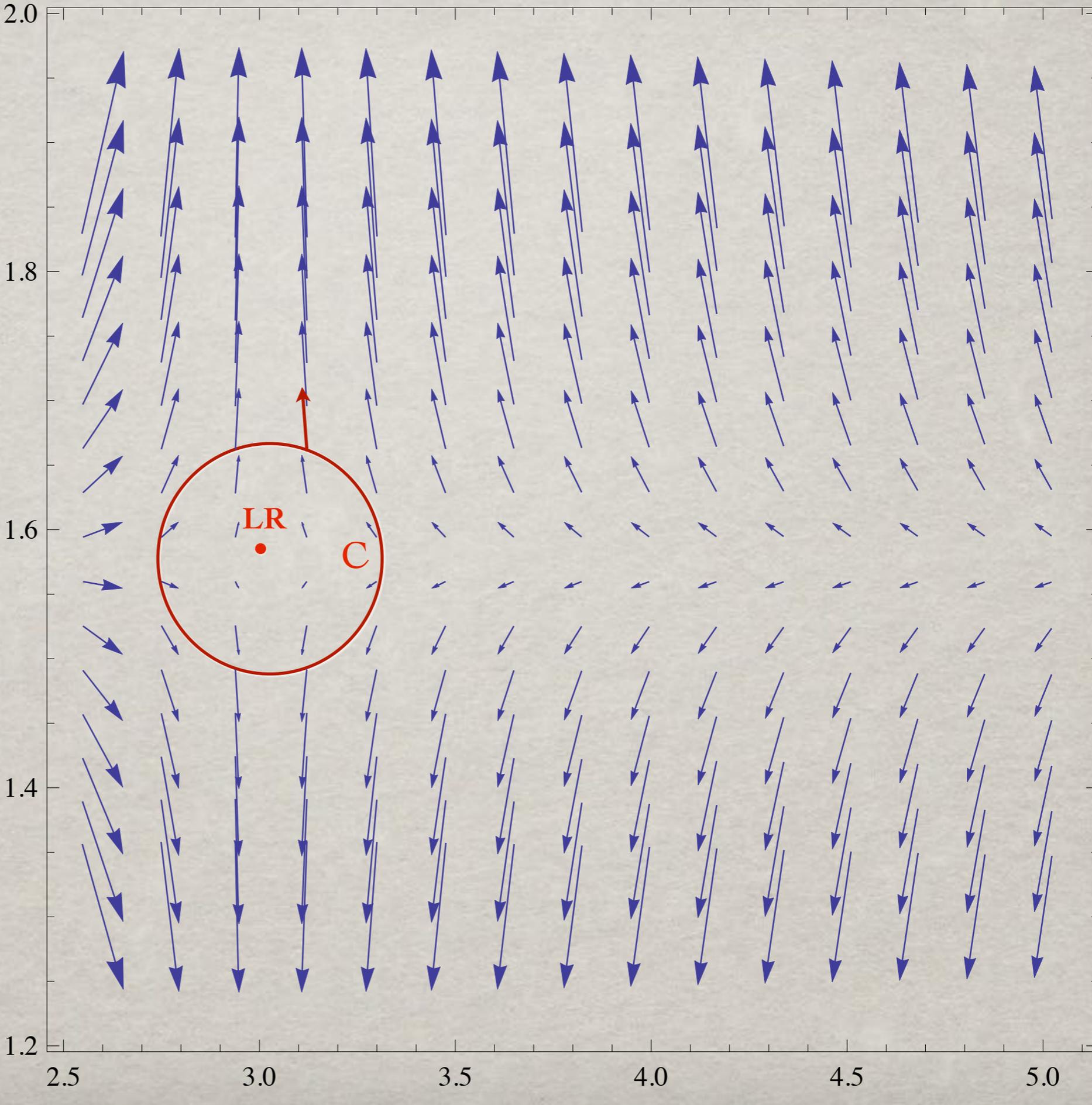
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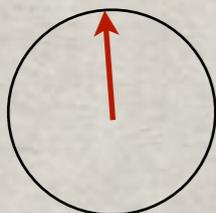
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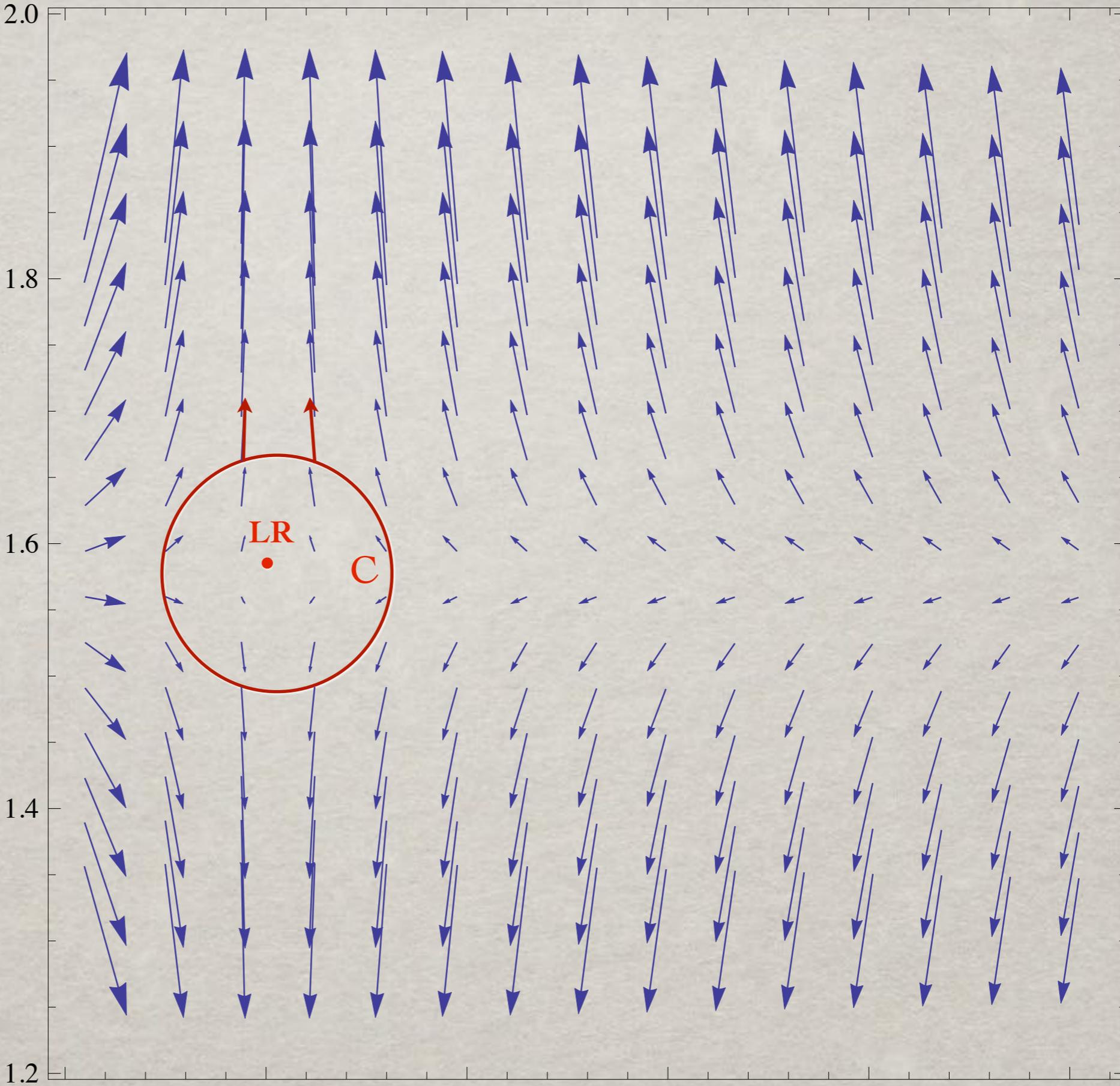
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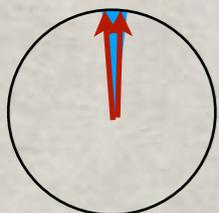
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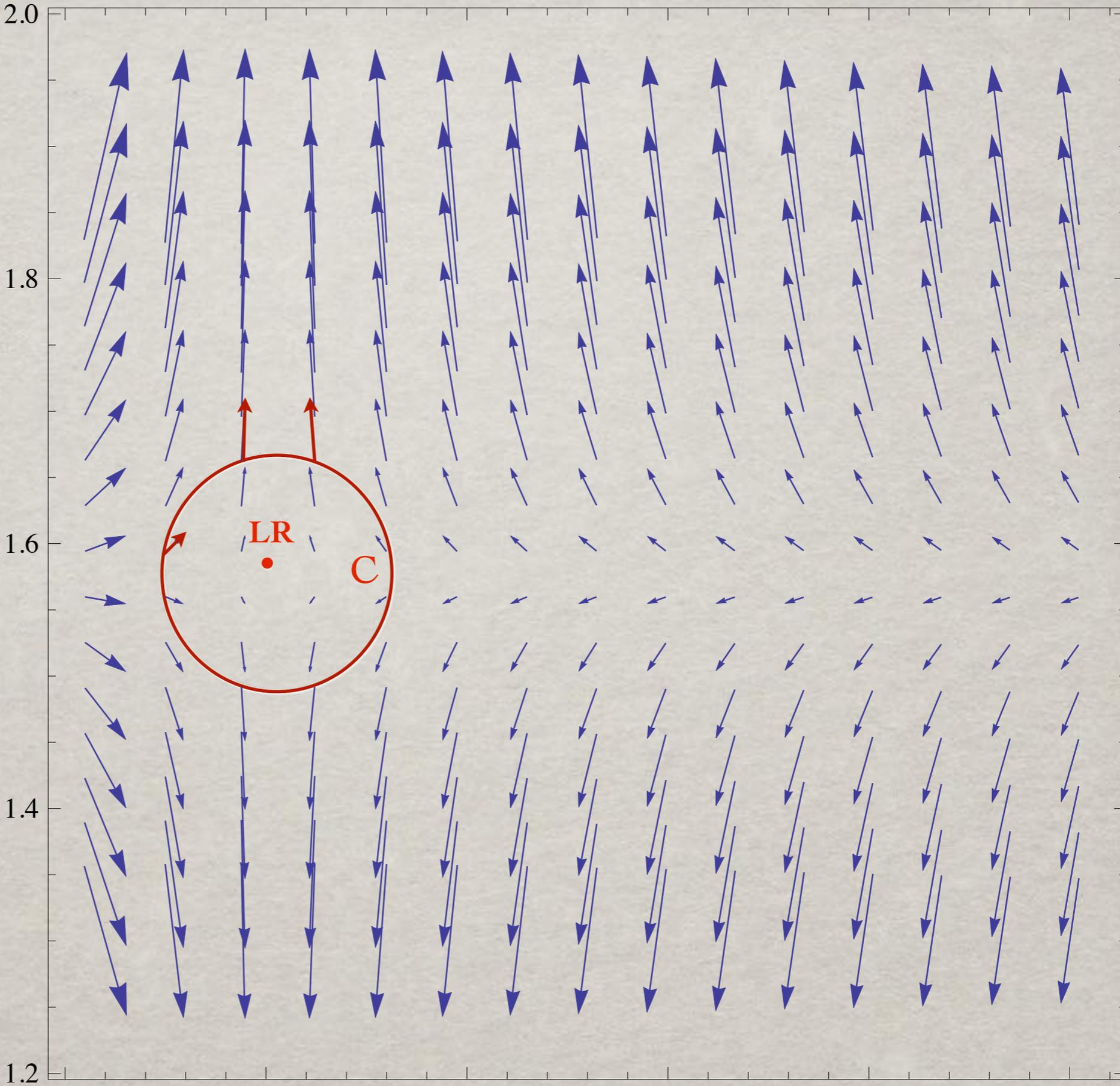
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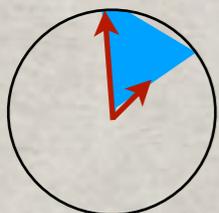
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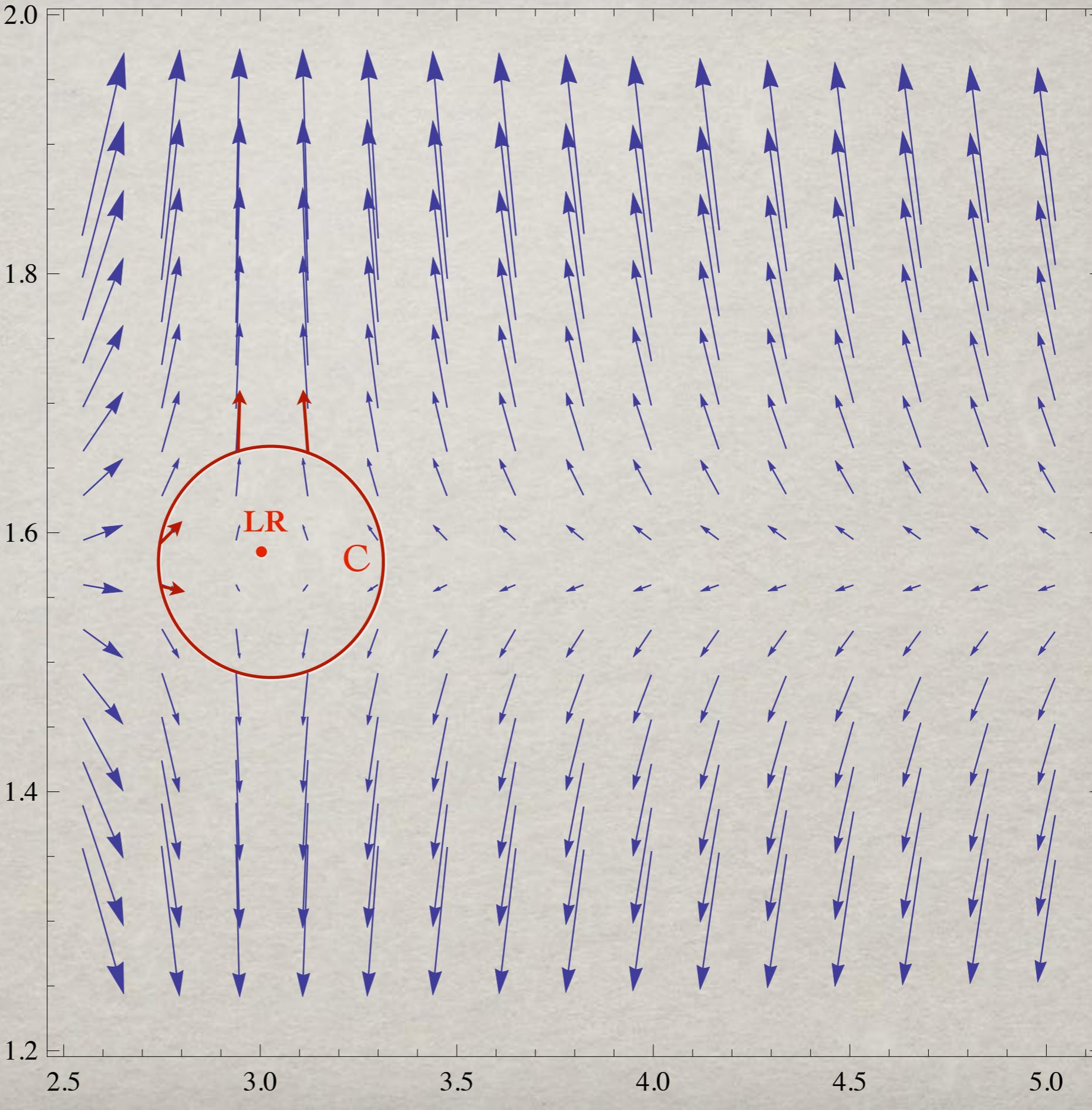
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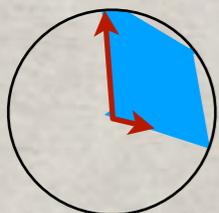
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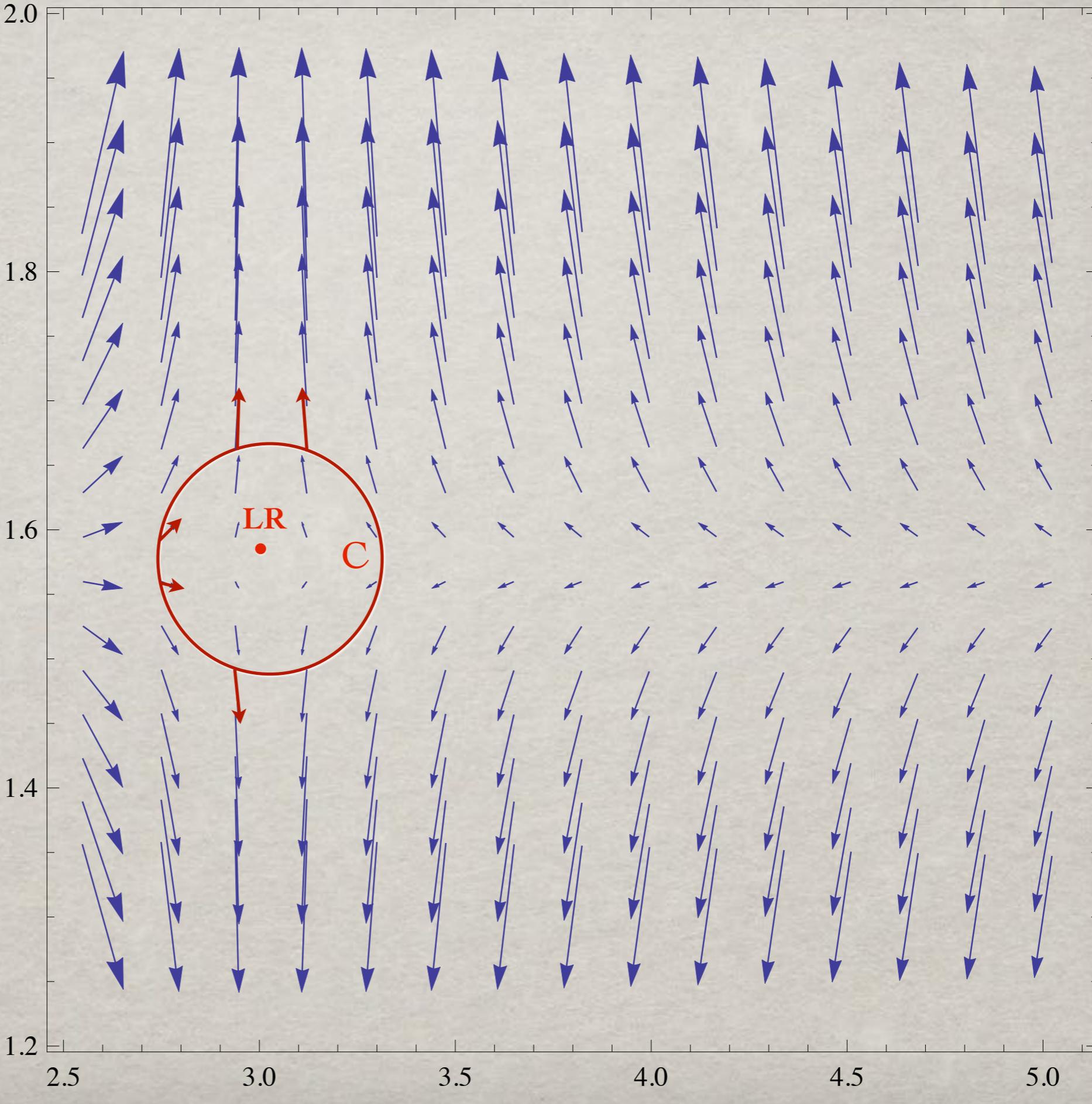
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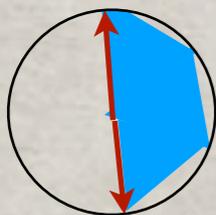
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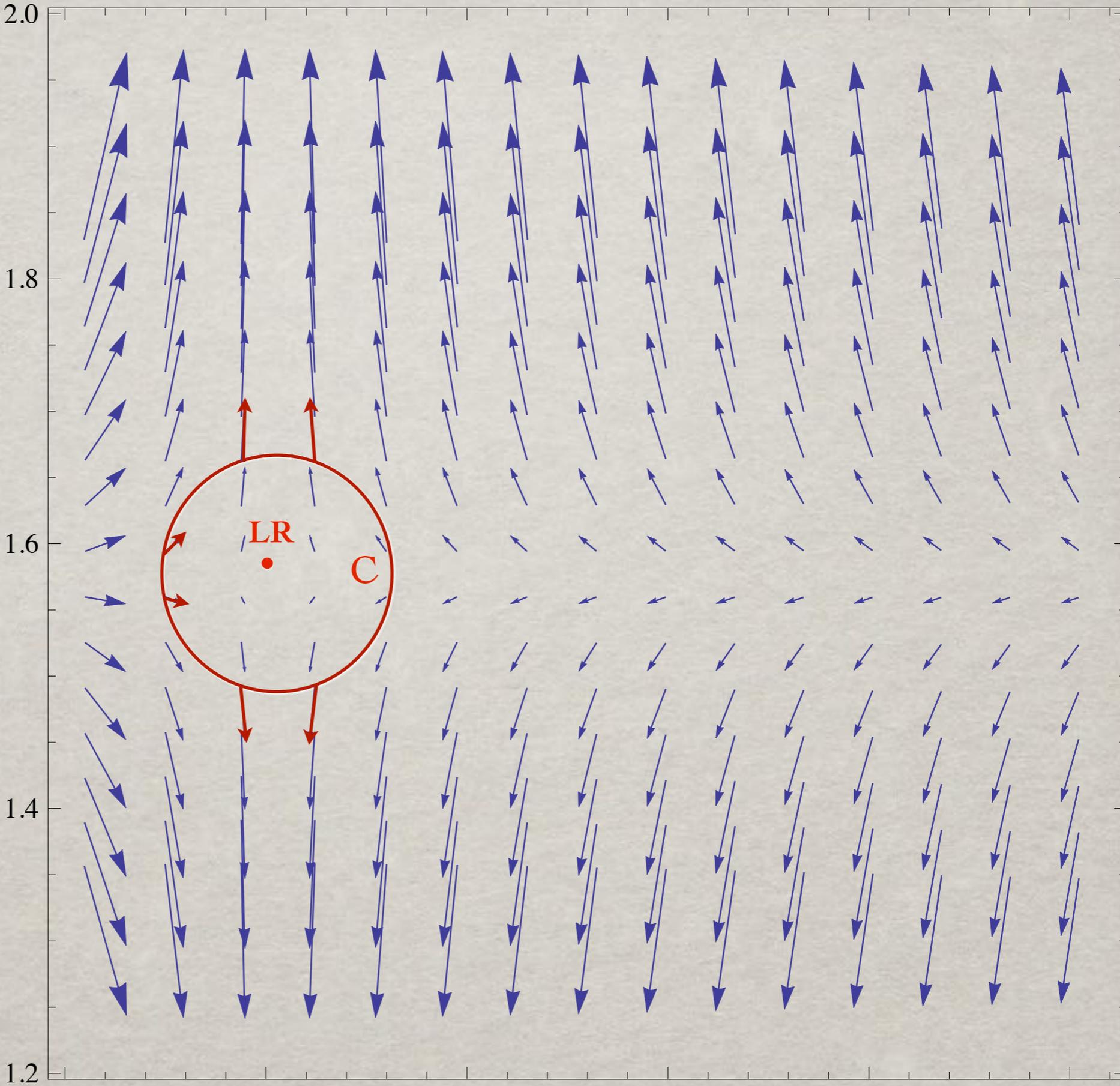
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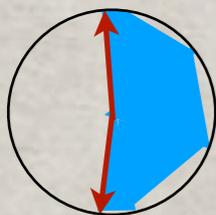
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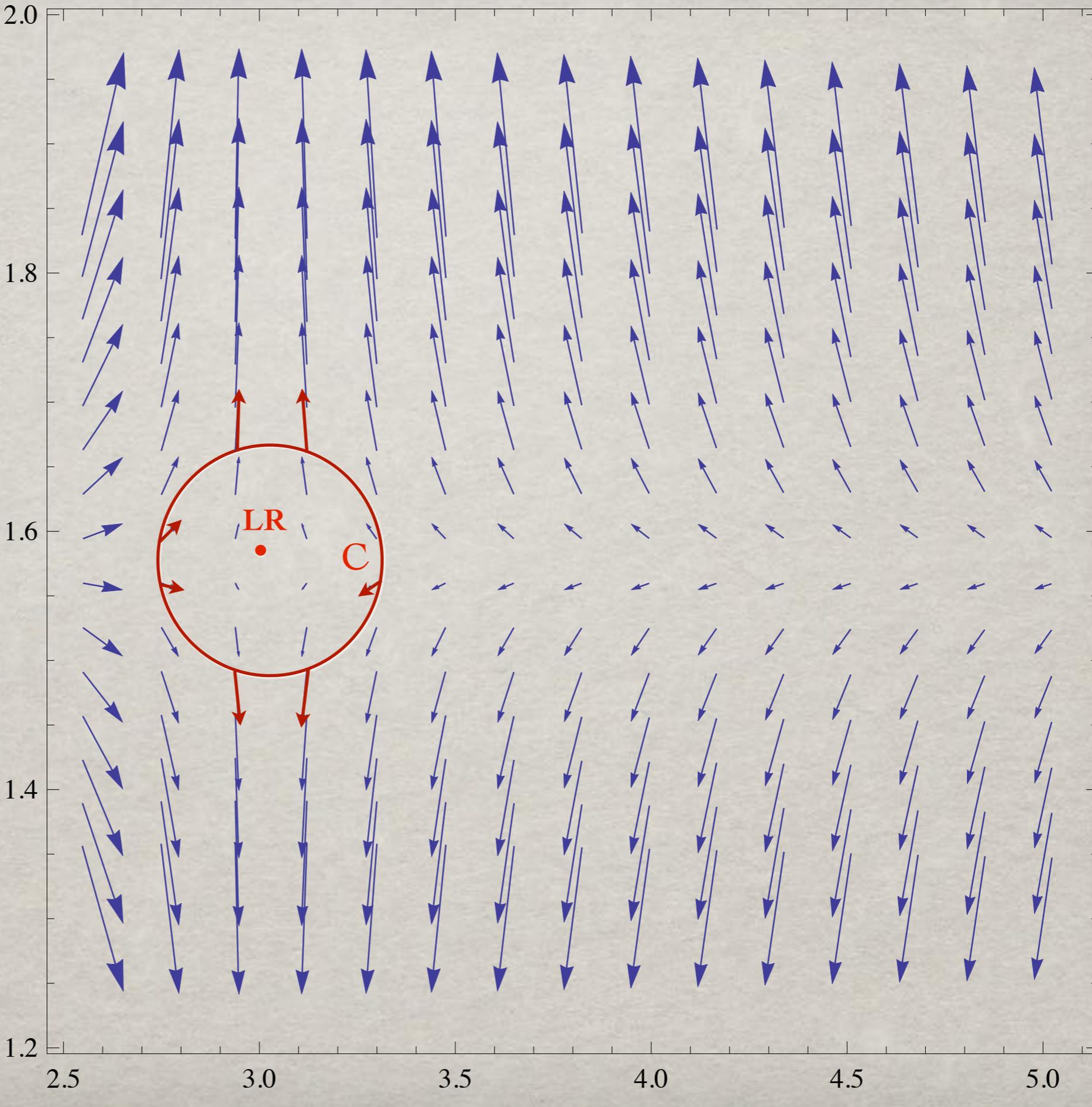
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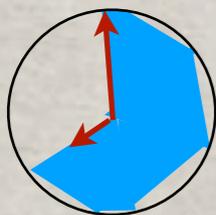
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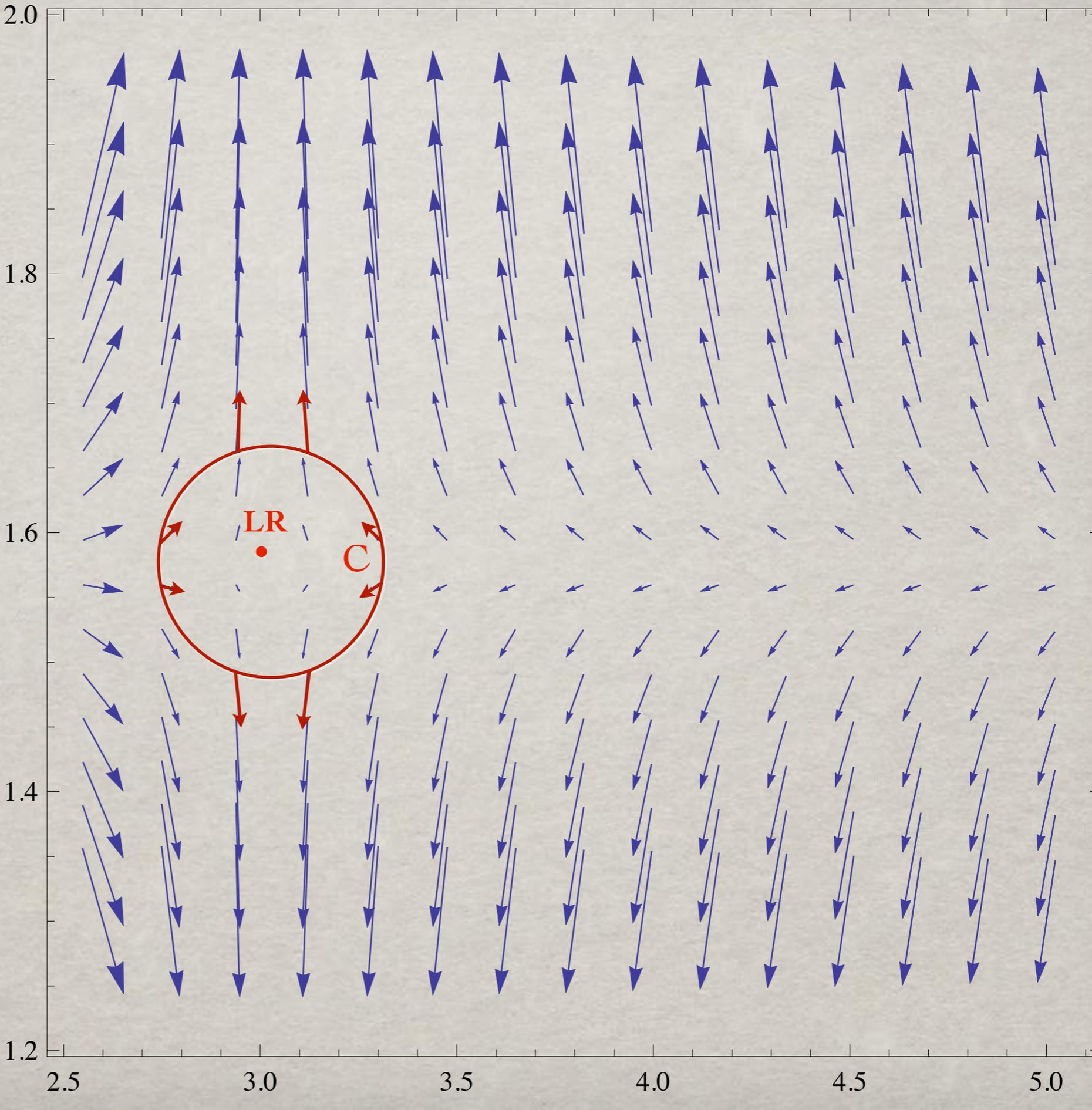
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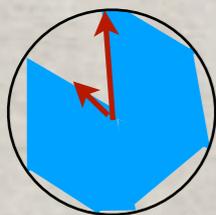
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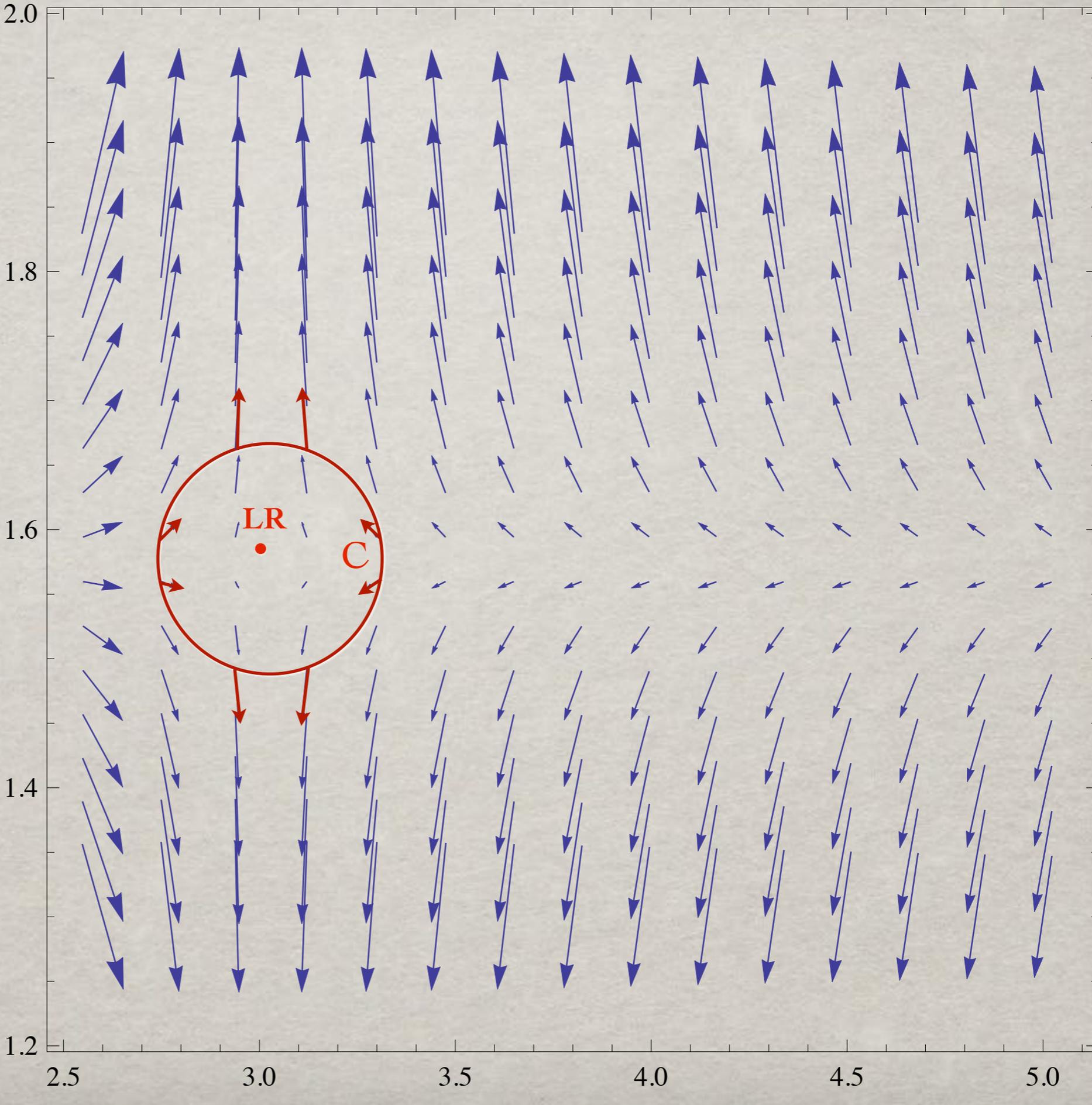
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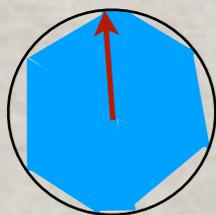
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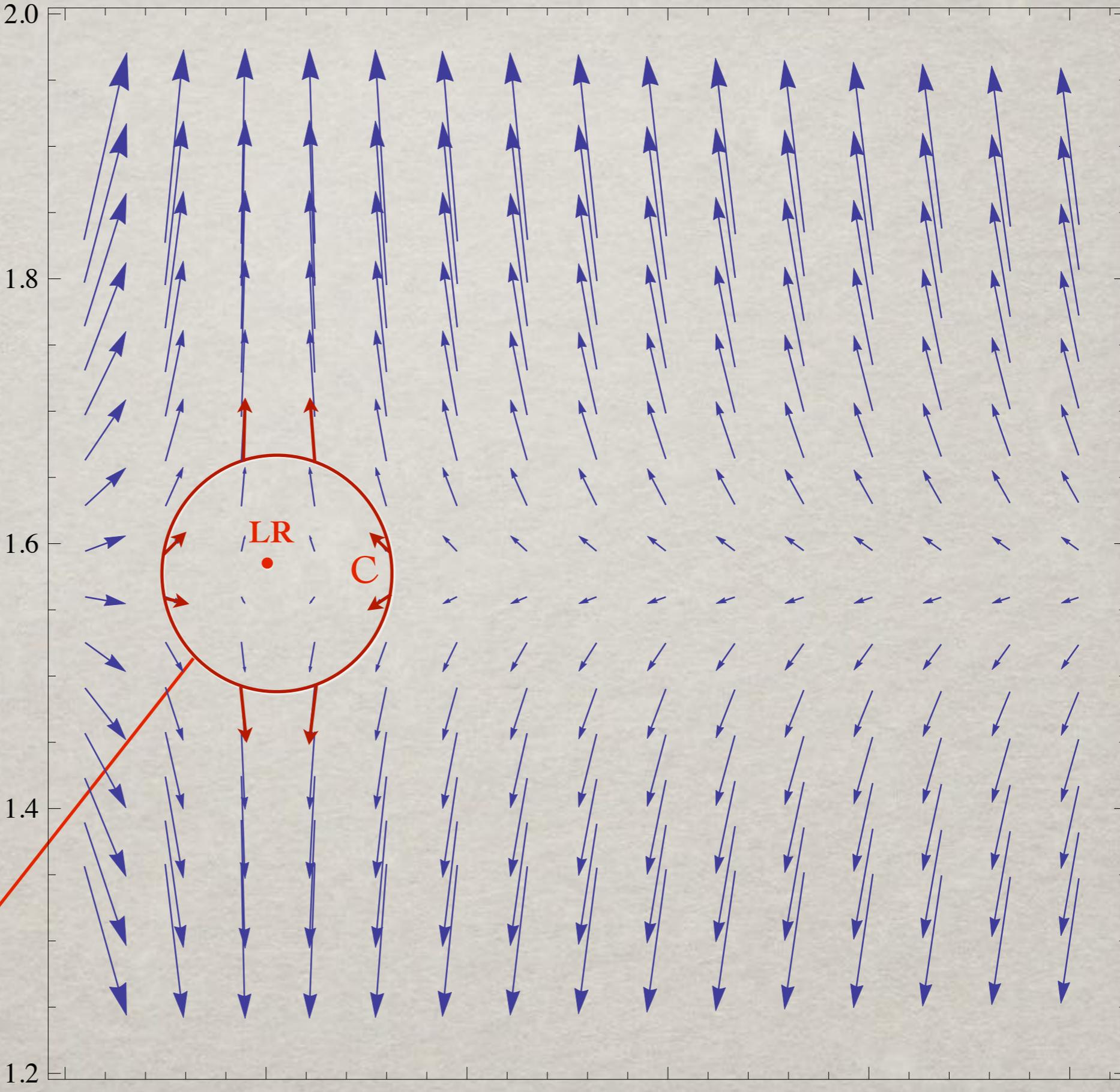
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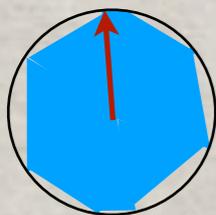
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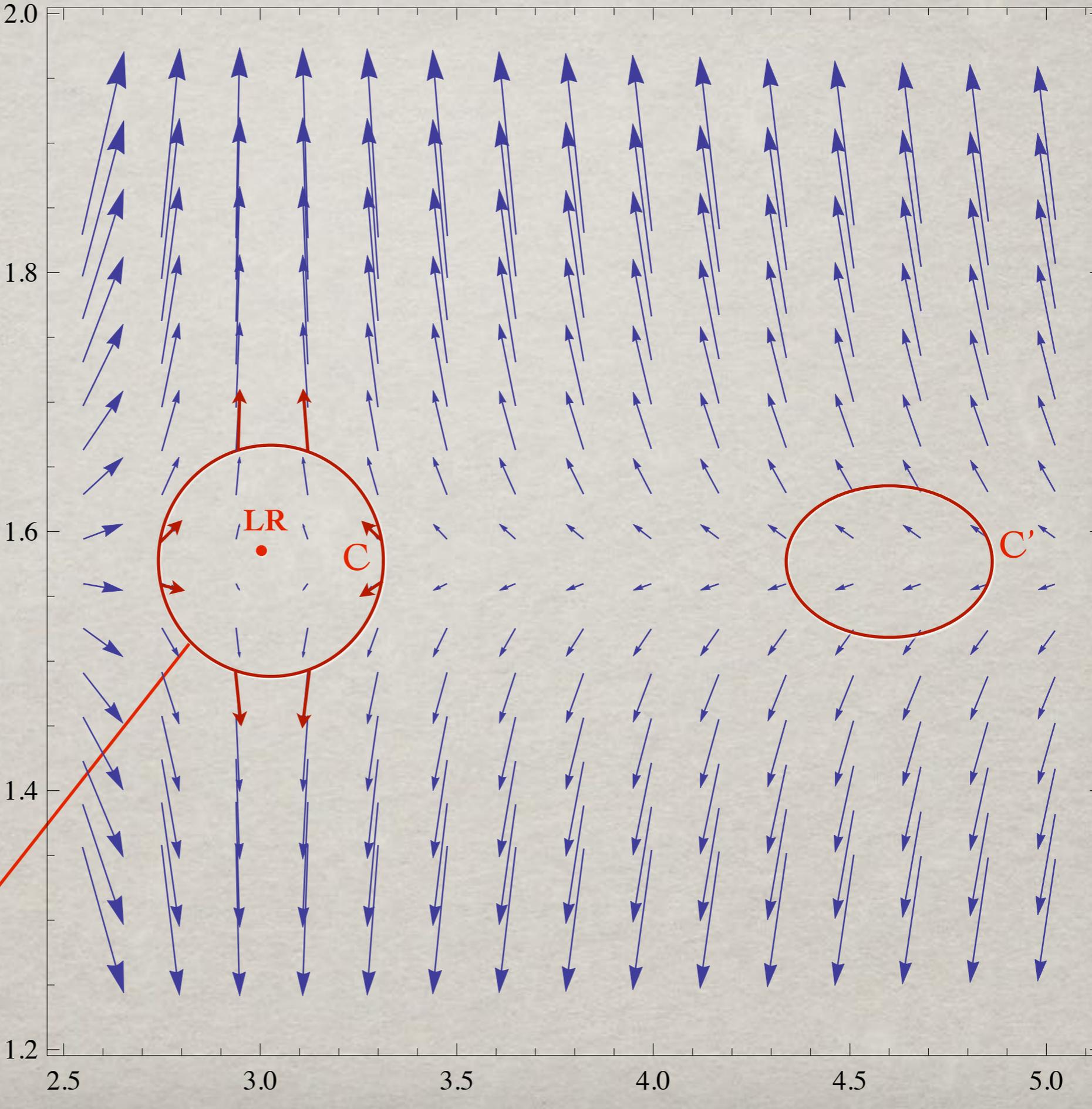
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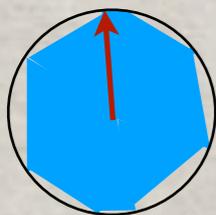
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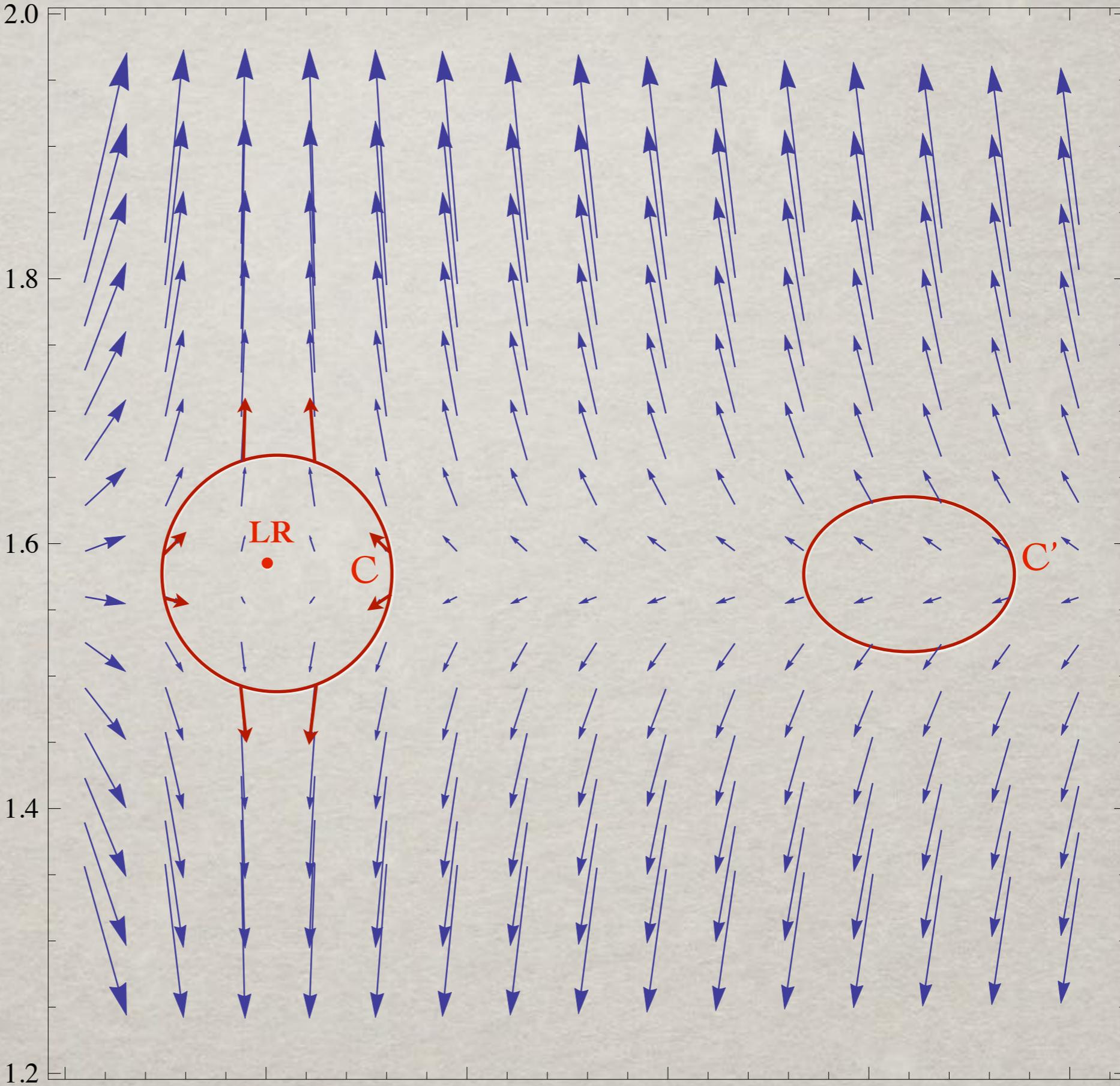
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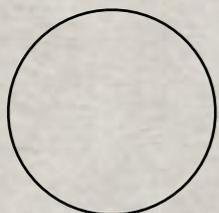
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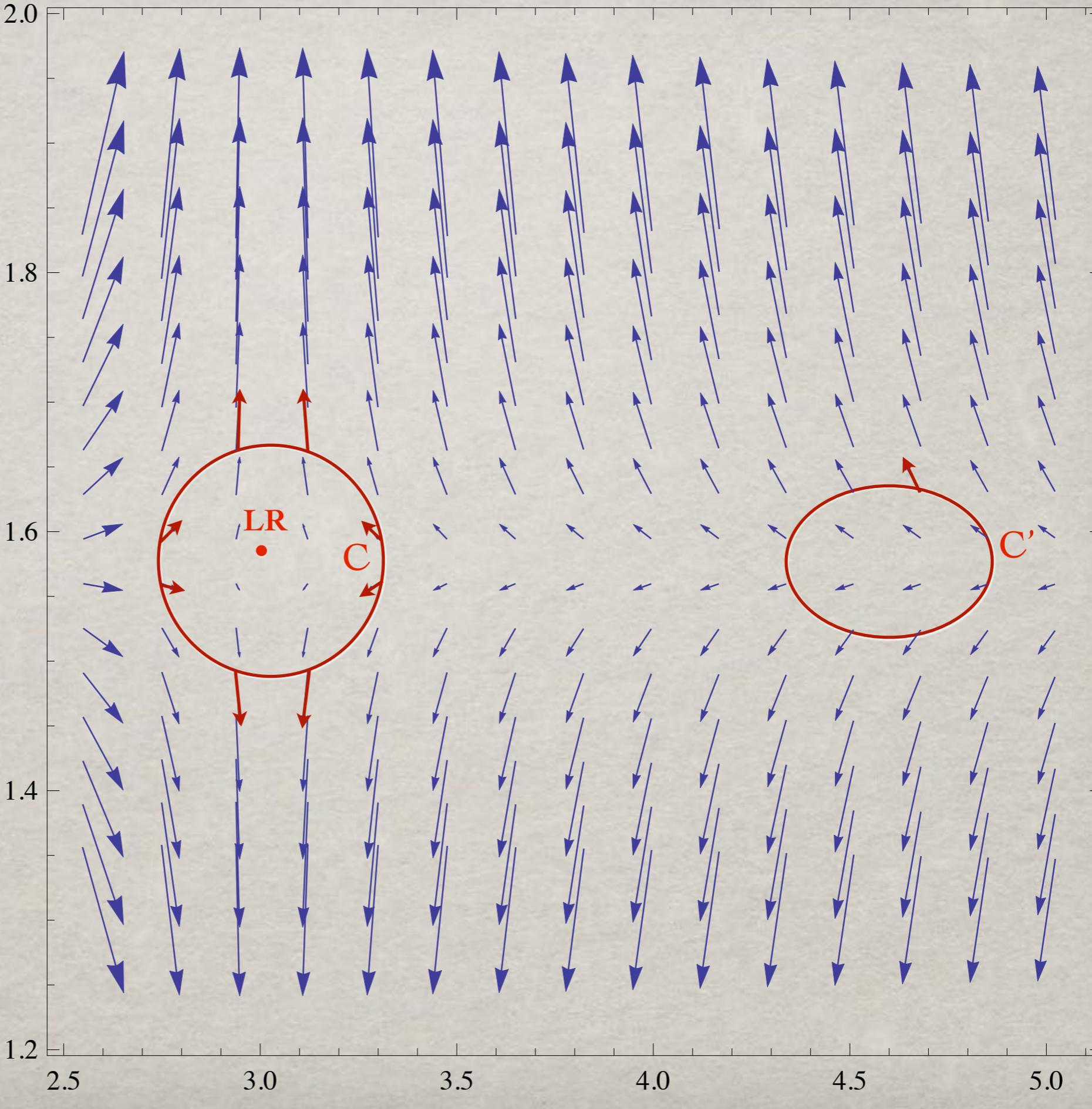
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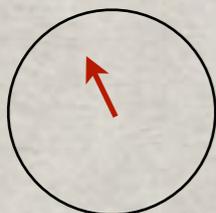
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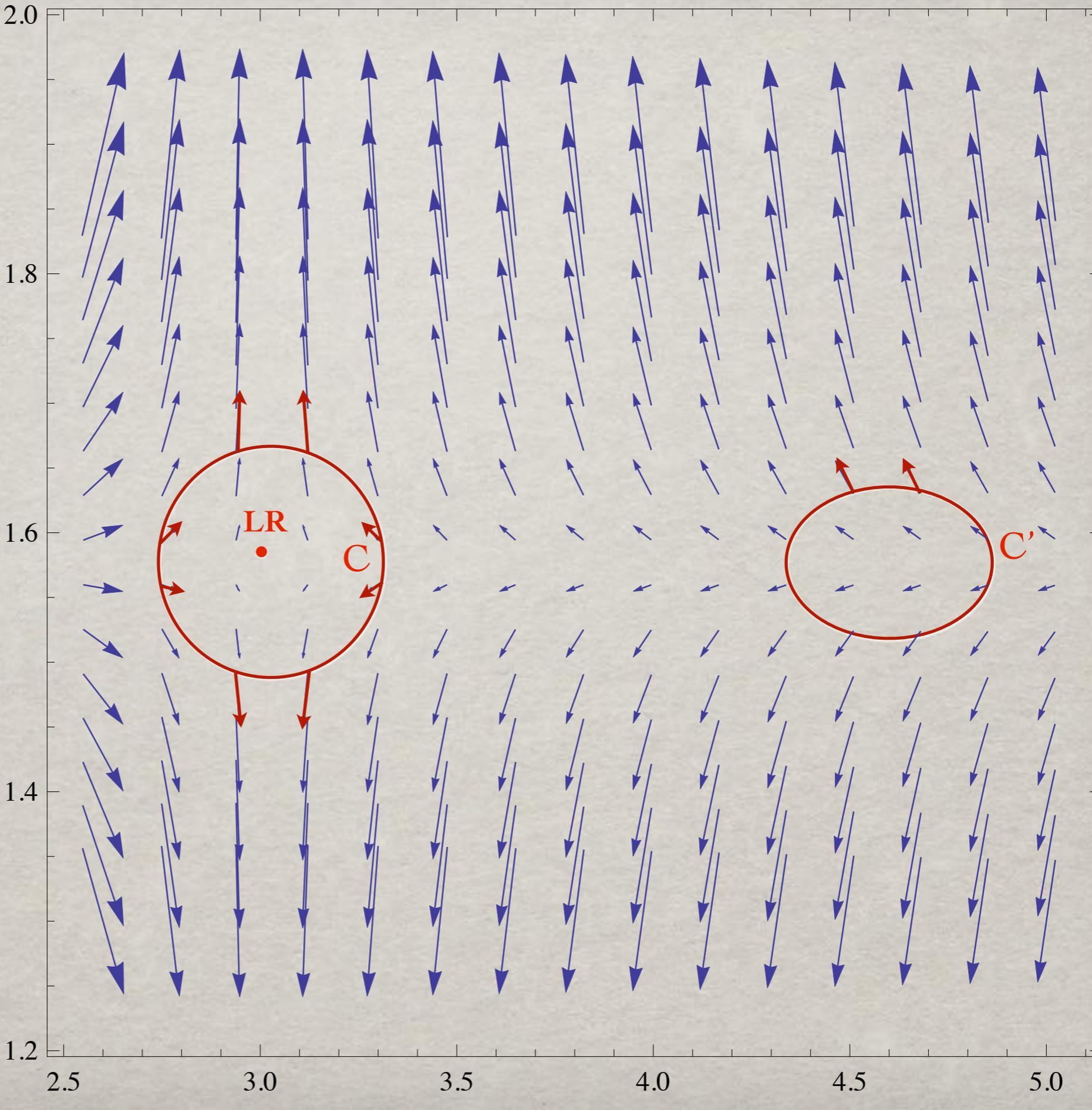
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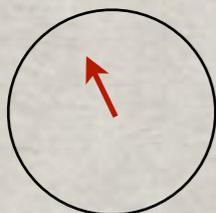
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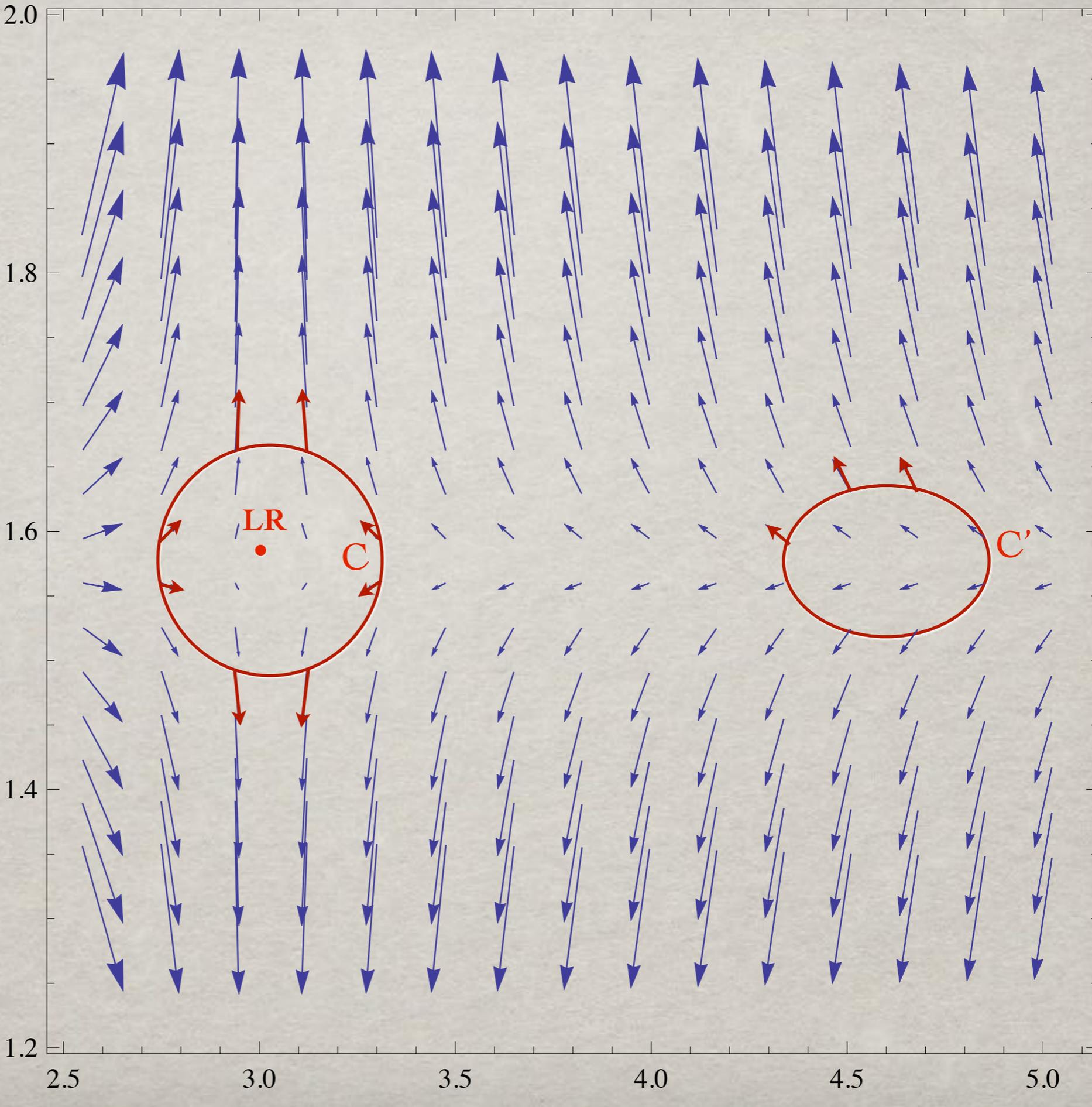
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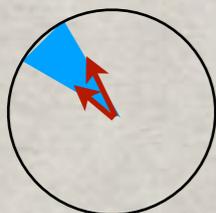
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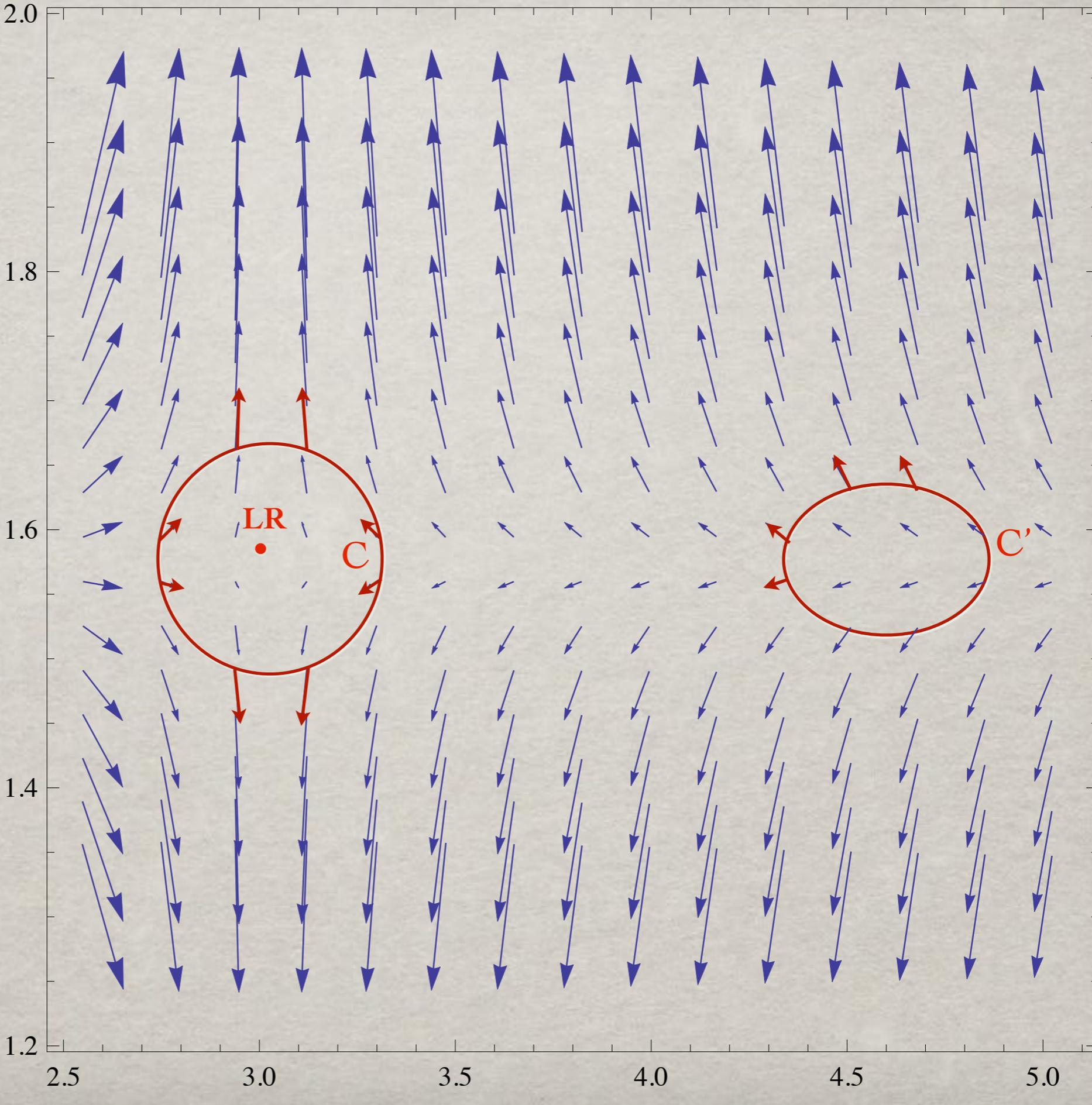
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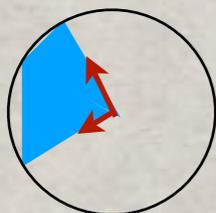
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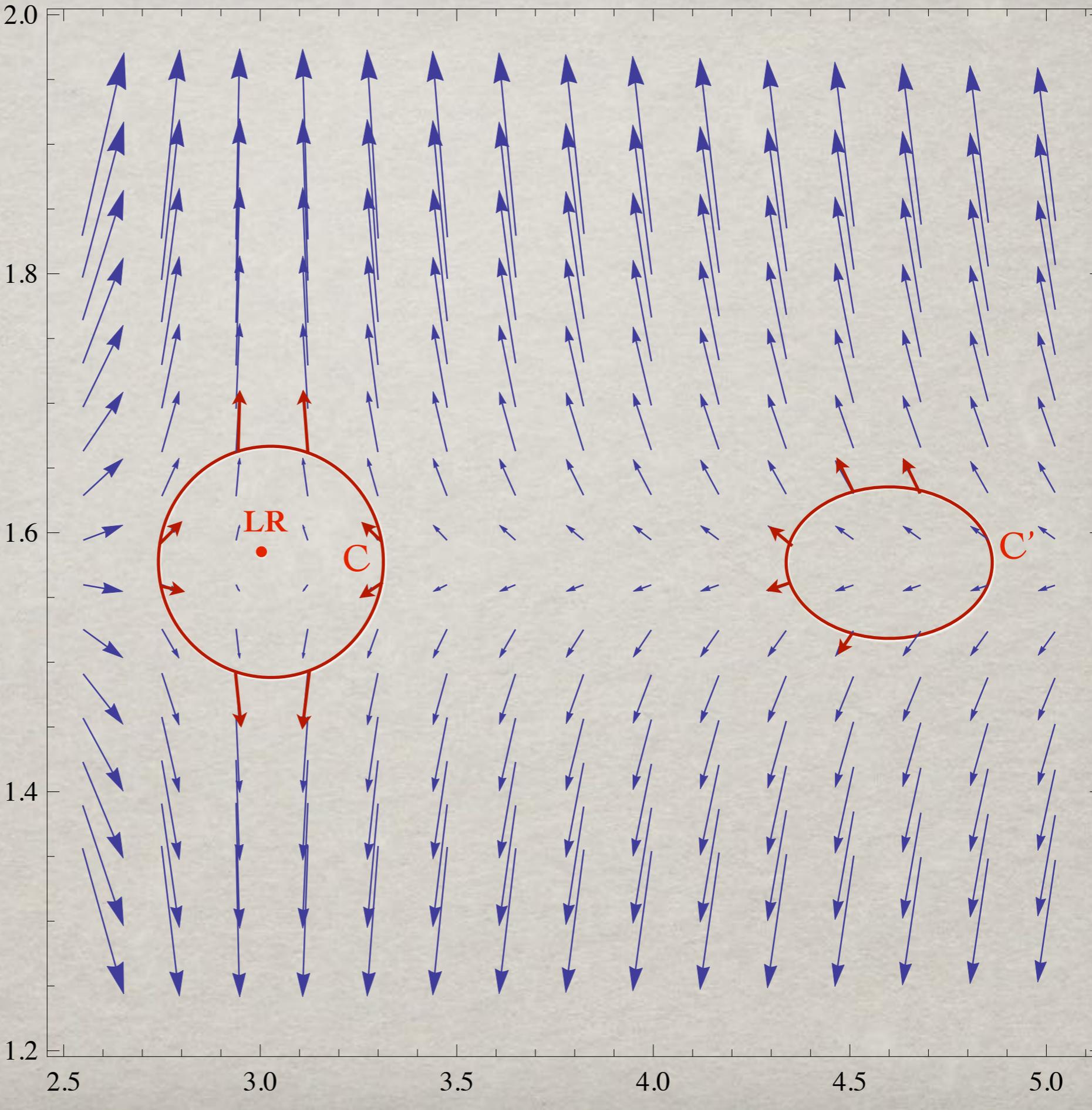
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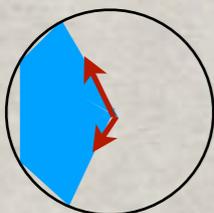
5.0

 r 

The circulation of these
vector fields around
closed contours defines
an integer
topological charge w :

$$\oint_C d\Omega = 2\pi w$$

Winding of
the vector field



$$\mathbf{V}_+(M=1)$$

 θ

2.0

1.8

1.6

1.4

1.2

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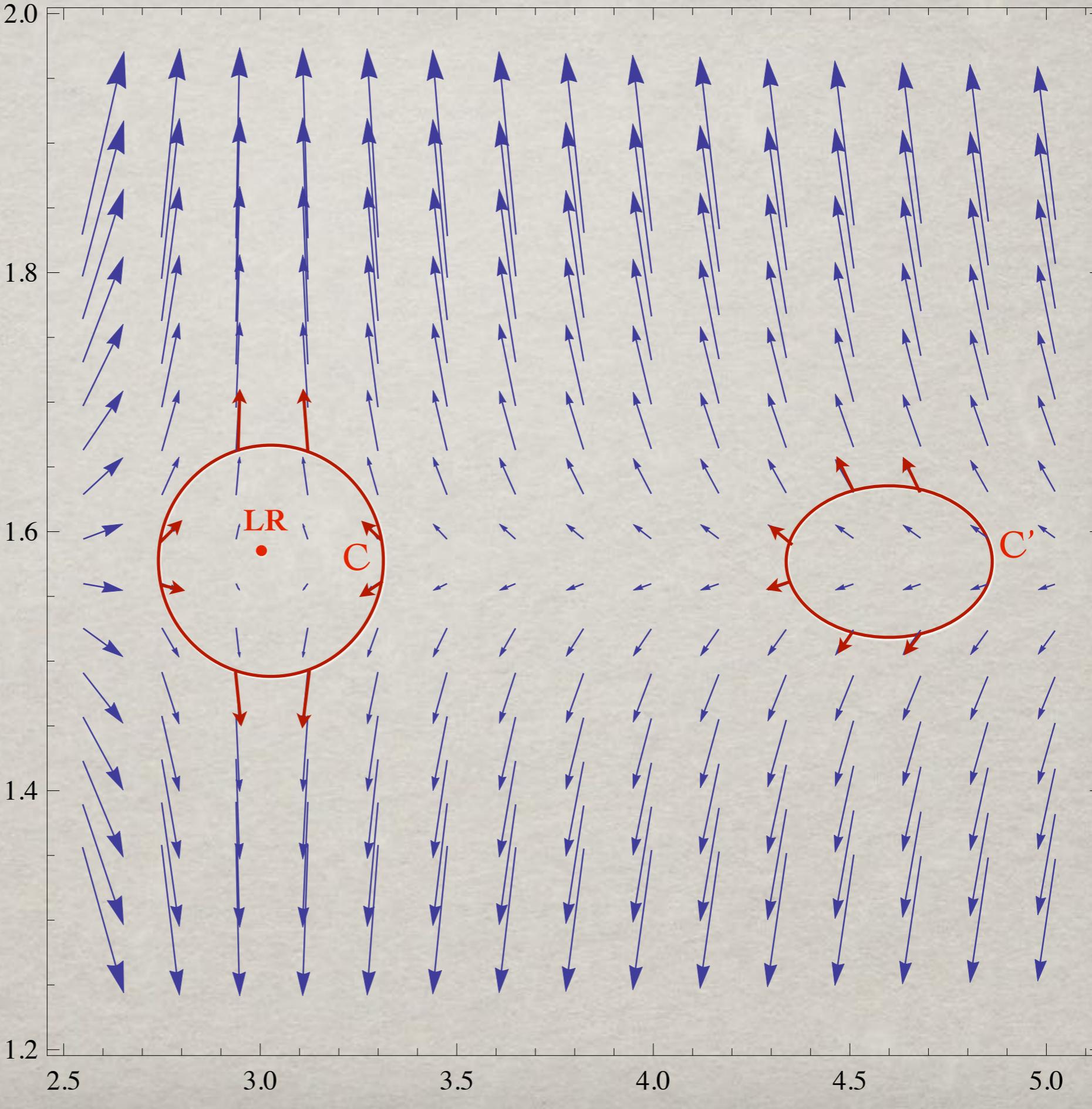
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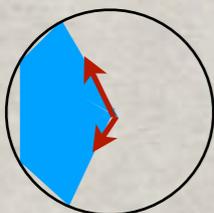
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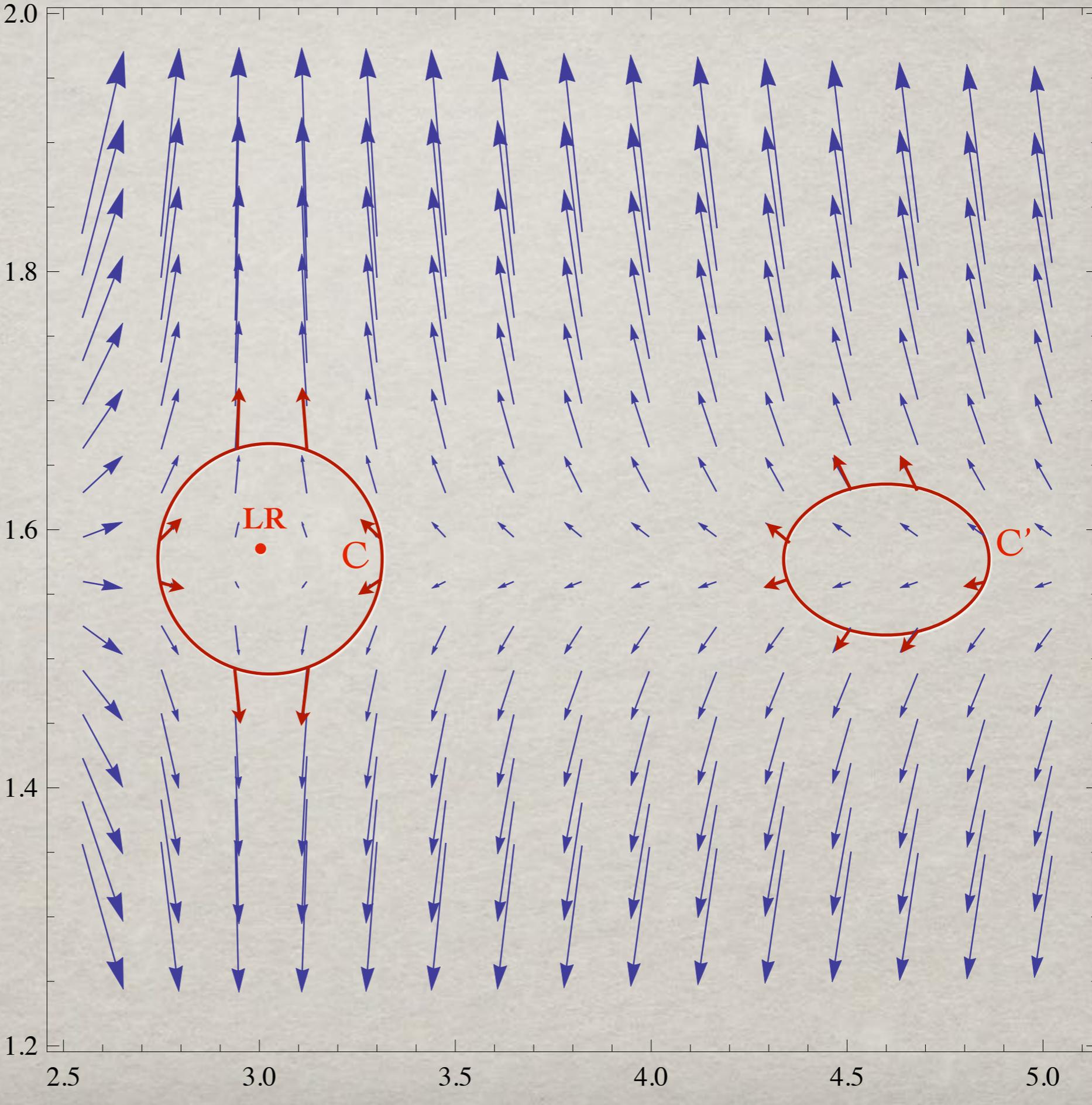
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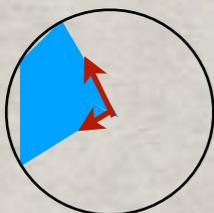
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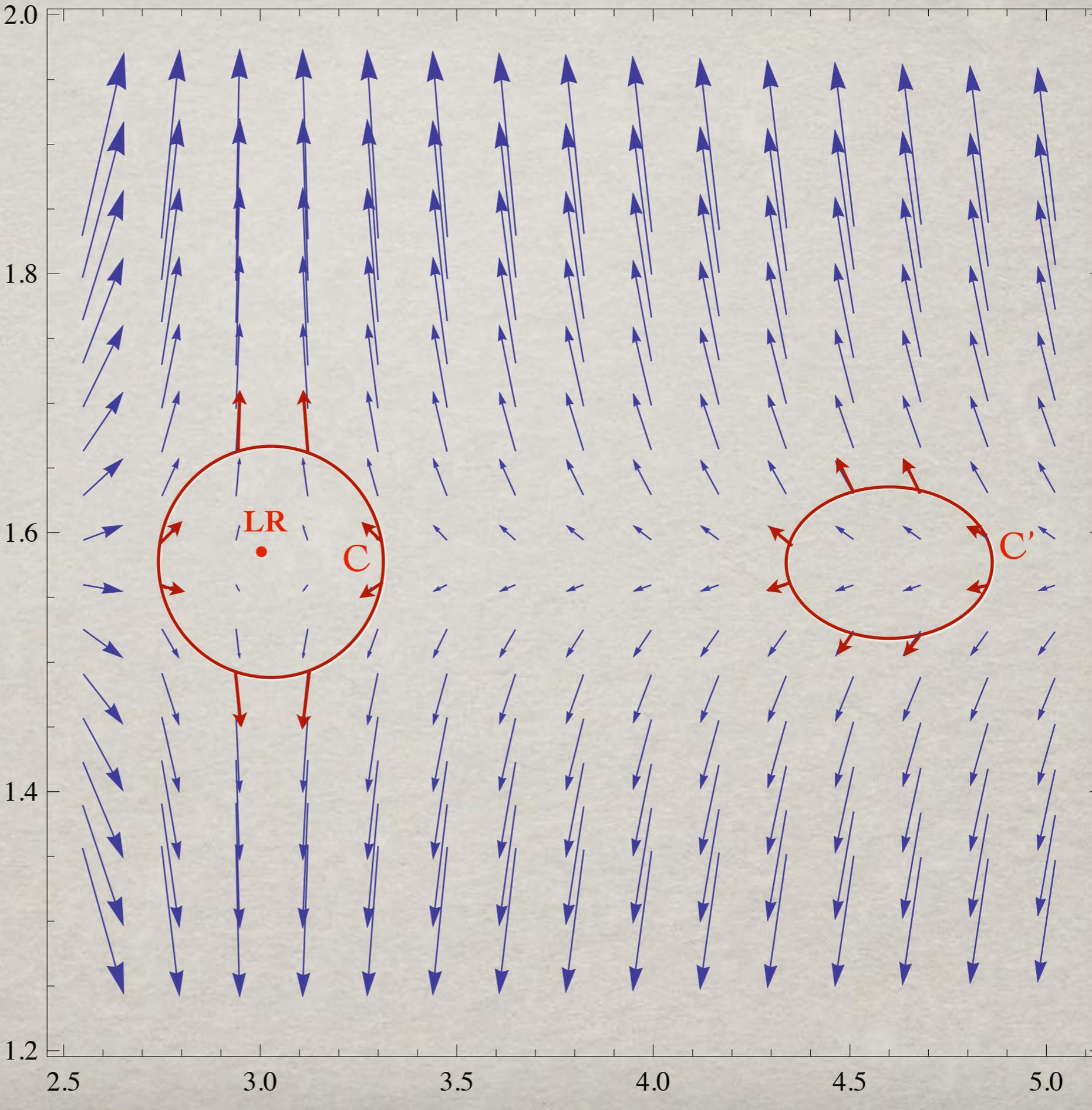
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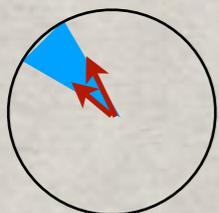
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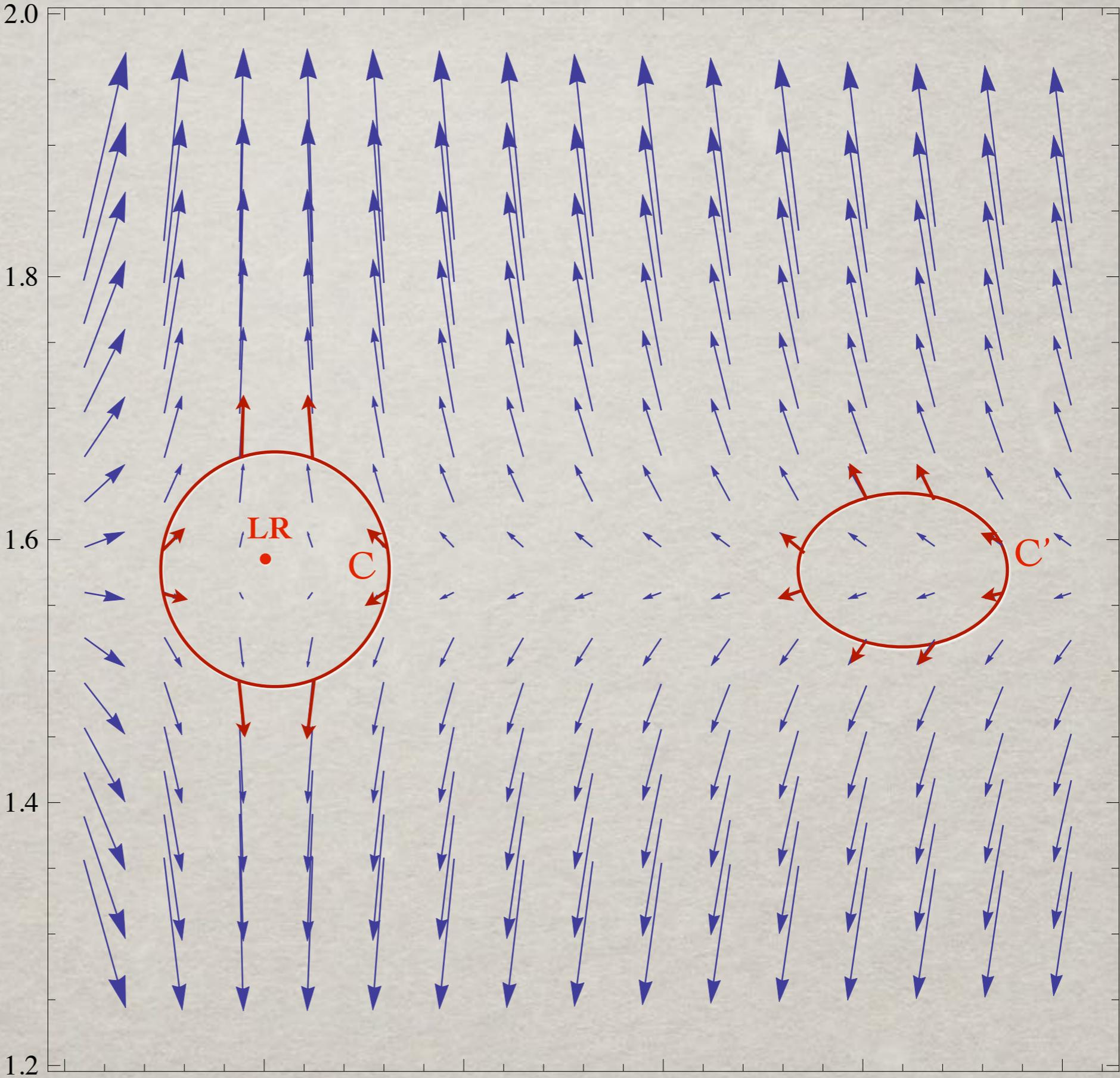
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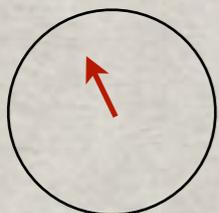
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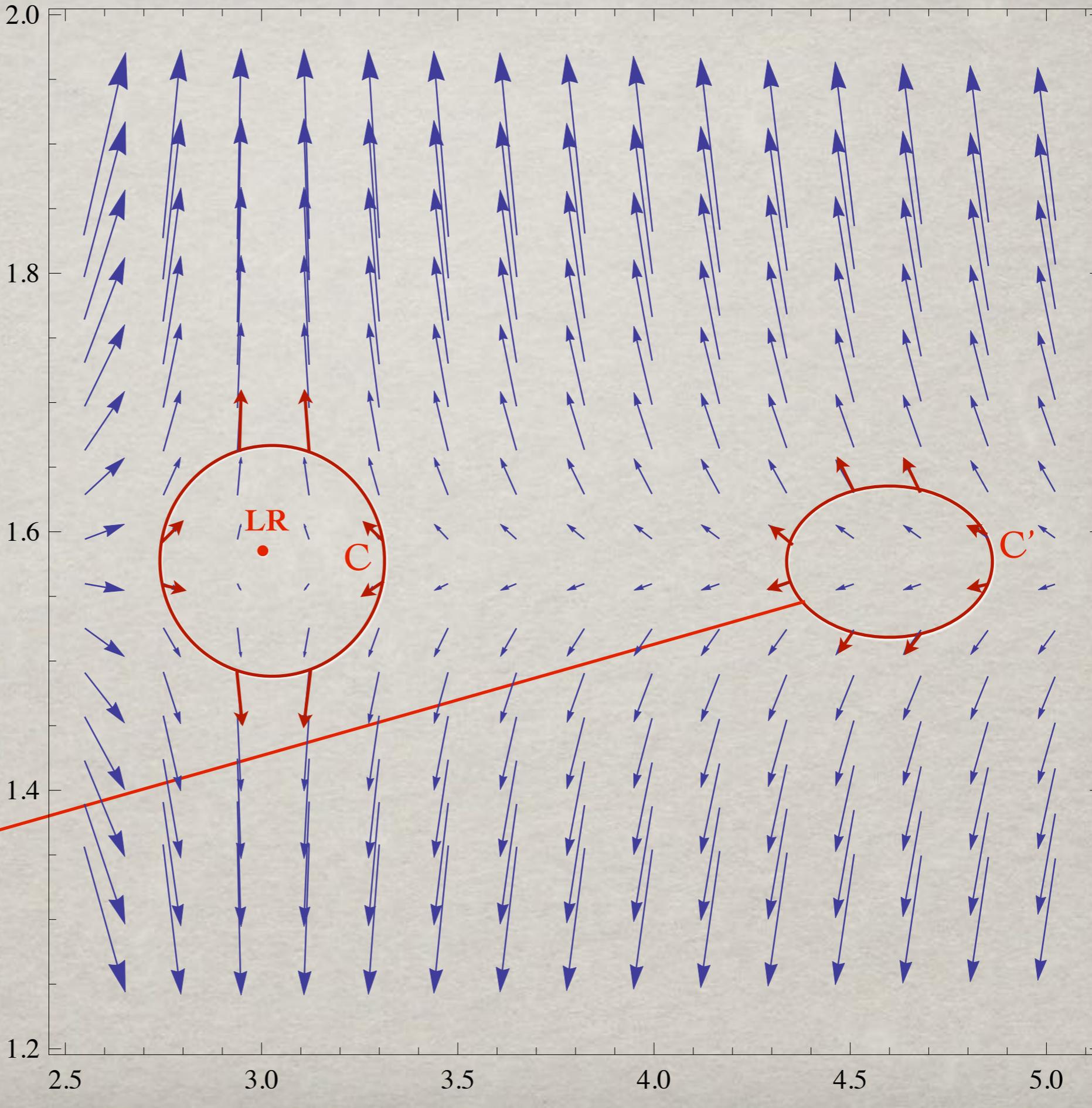
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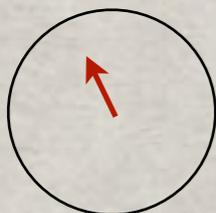
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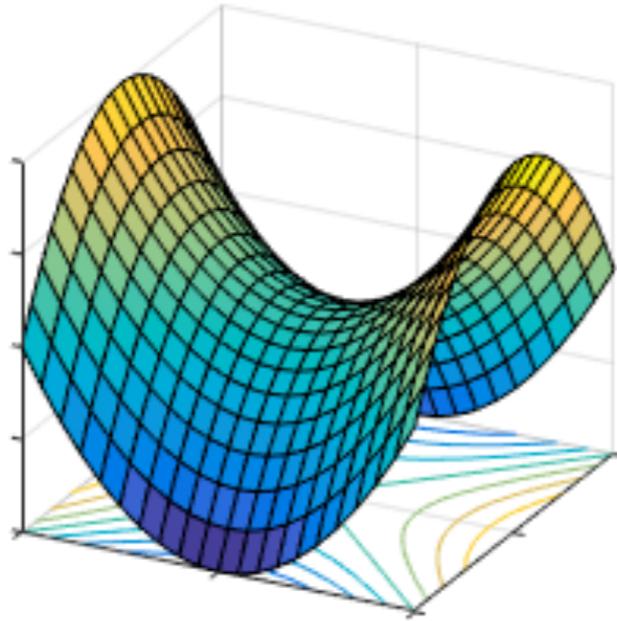
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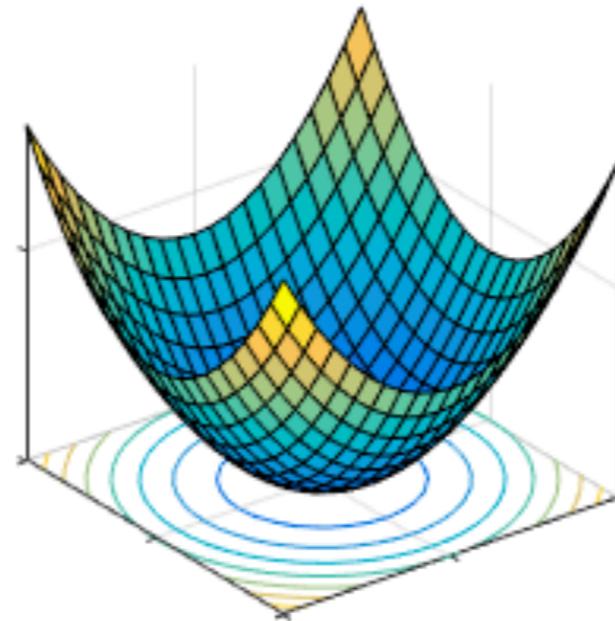
The winding # of \mathbf{V}
circulating around is
 $w=0$

But not all possible LR's have $w=-1$:

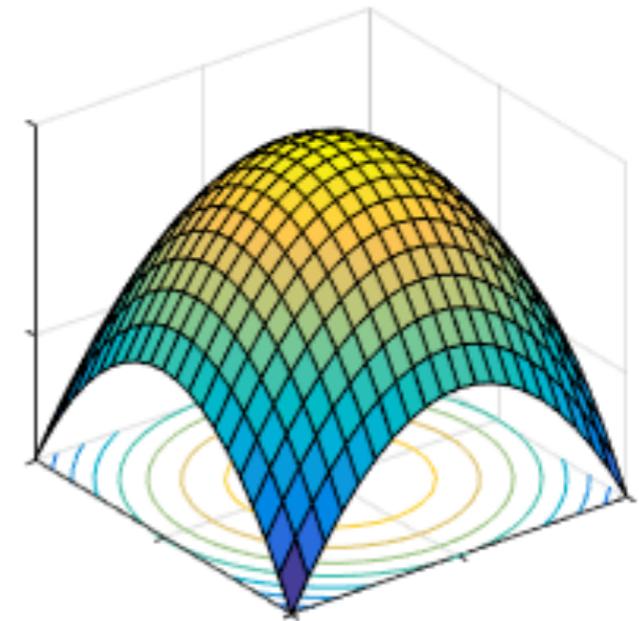
saddle point



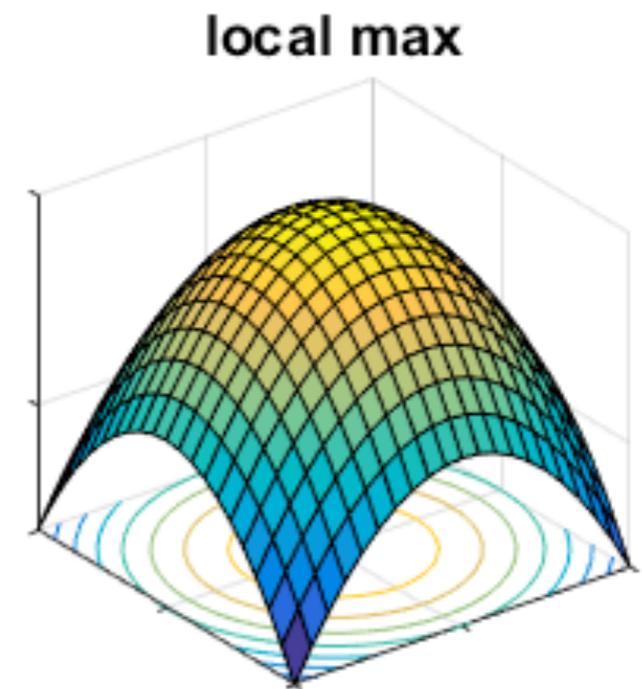
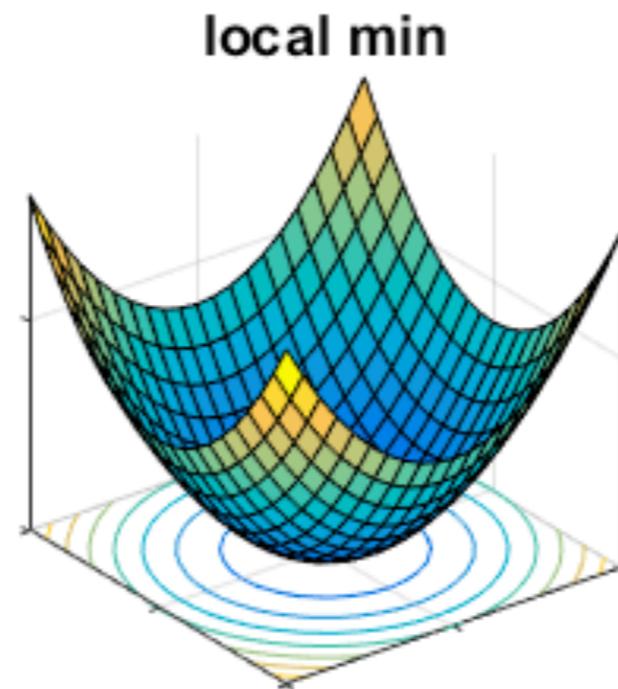
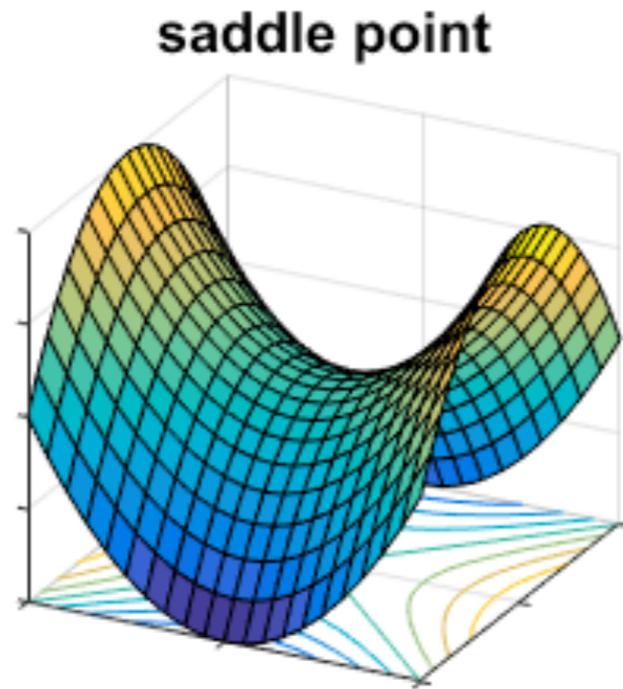
local min



local max



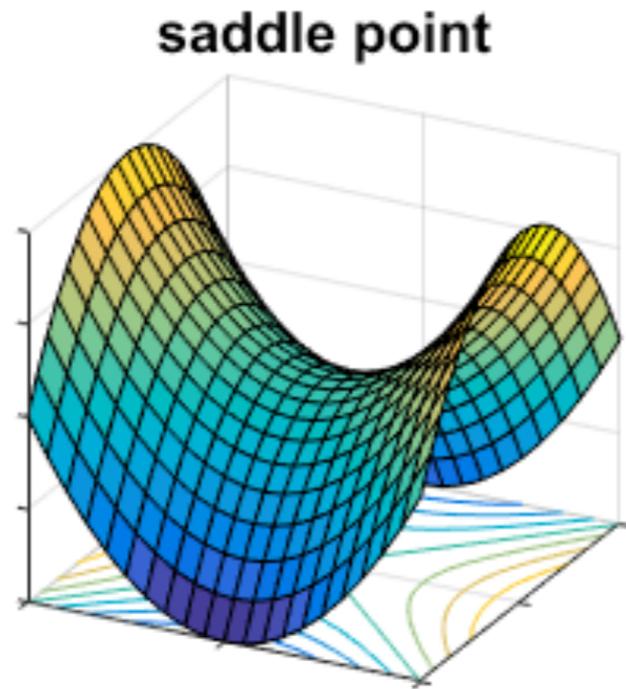
But not all possible LRs have $w = -1$:



$$w = -1$$

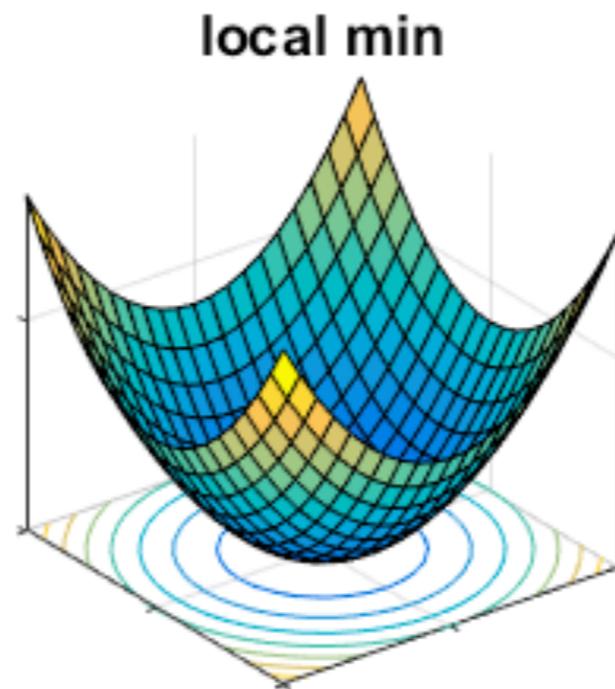
Standard LRs
(Kerr-like)

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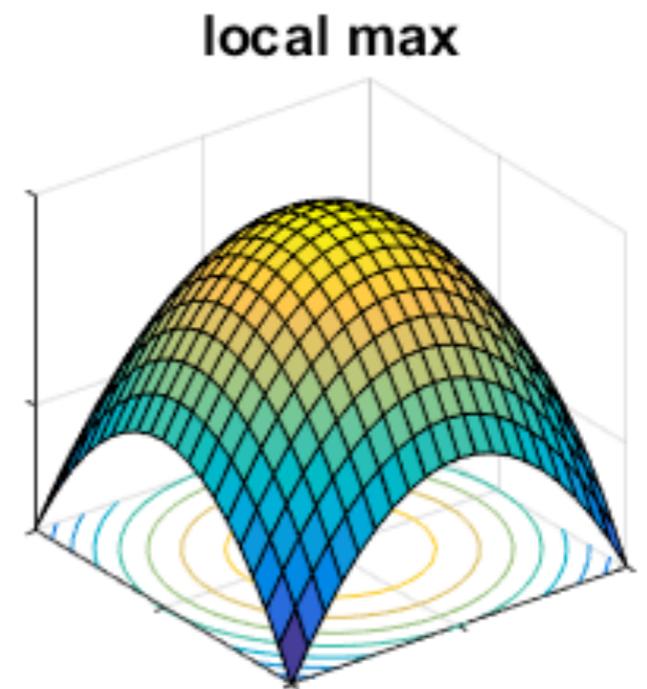
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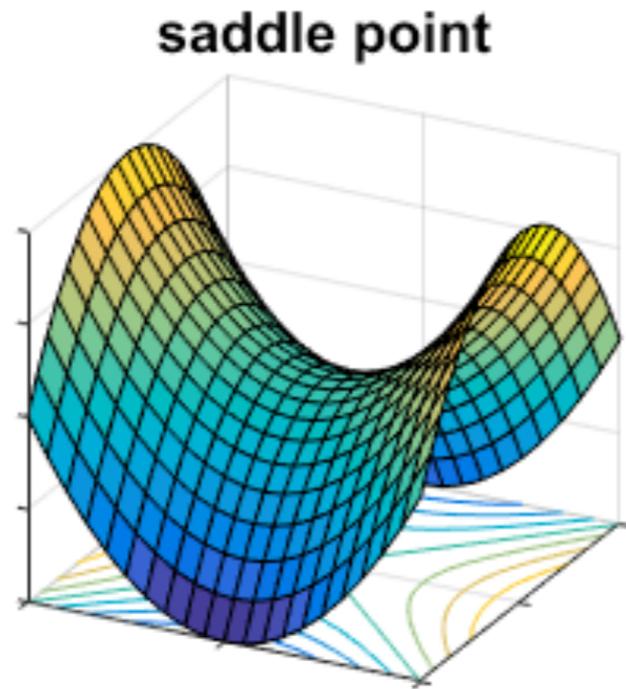
$$w = +1$$

Exotic LRs
(Kerr-unlike)



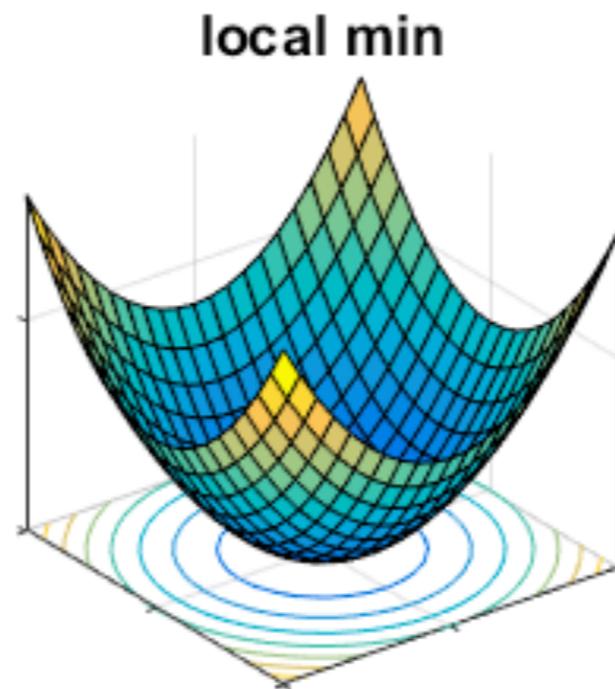
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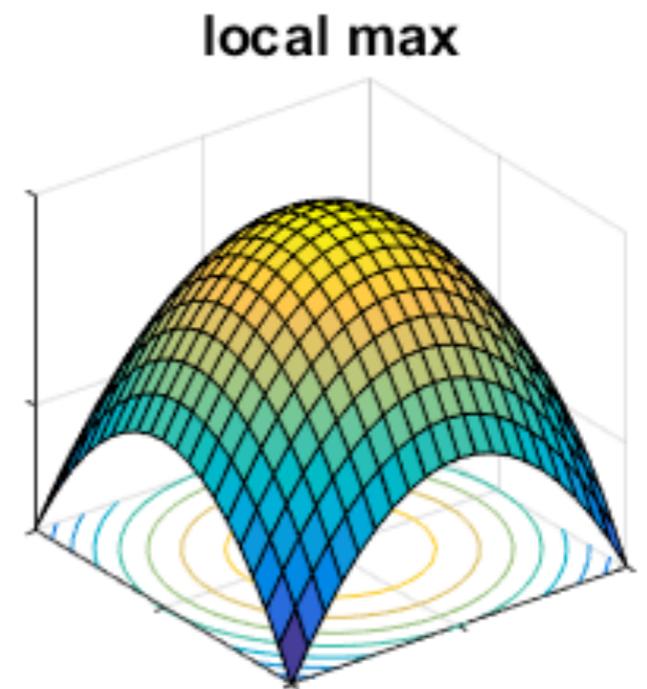
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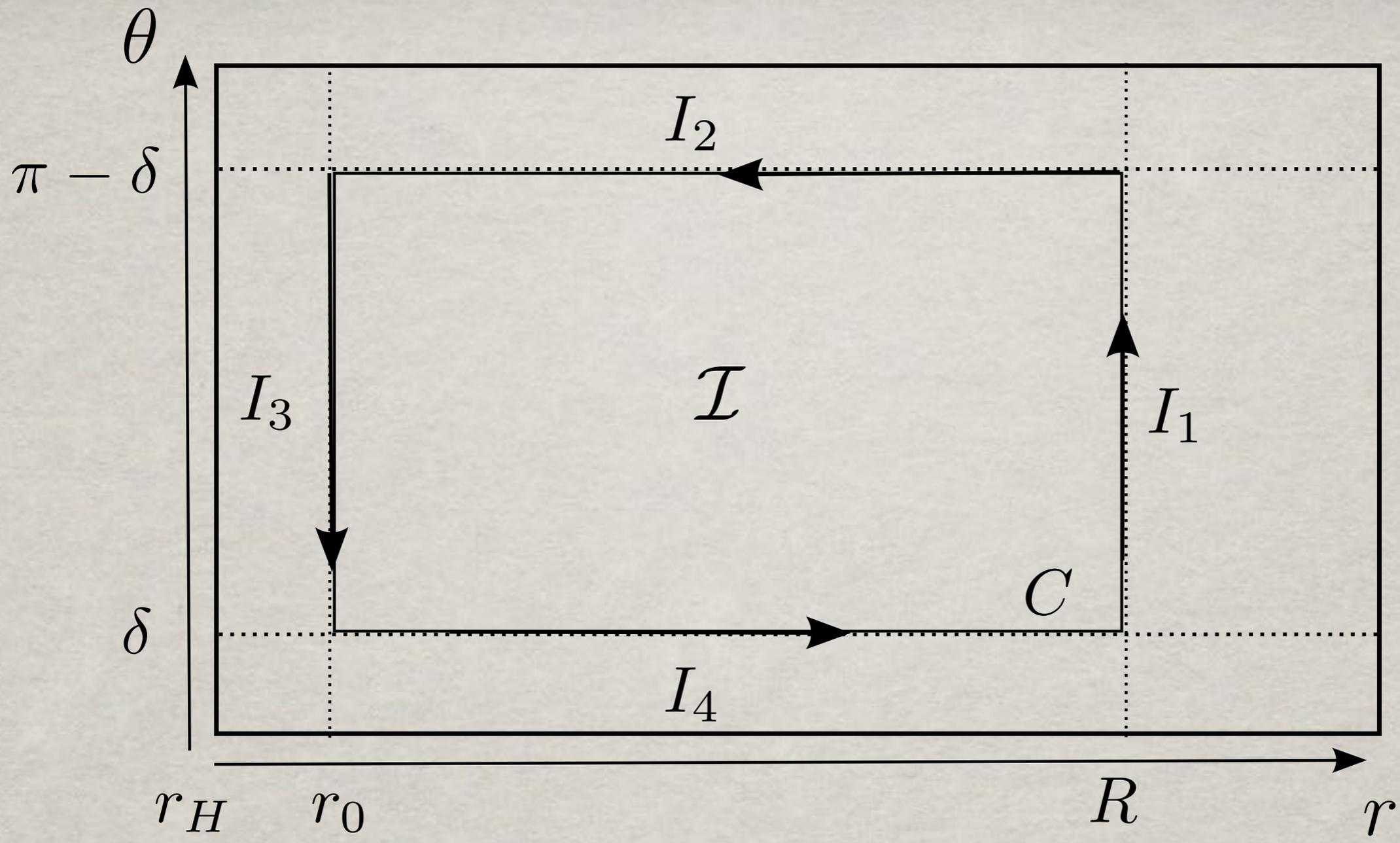
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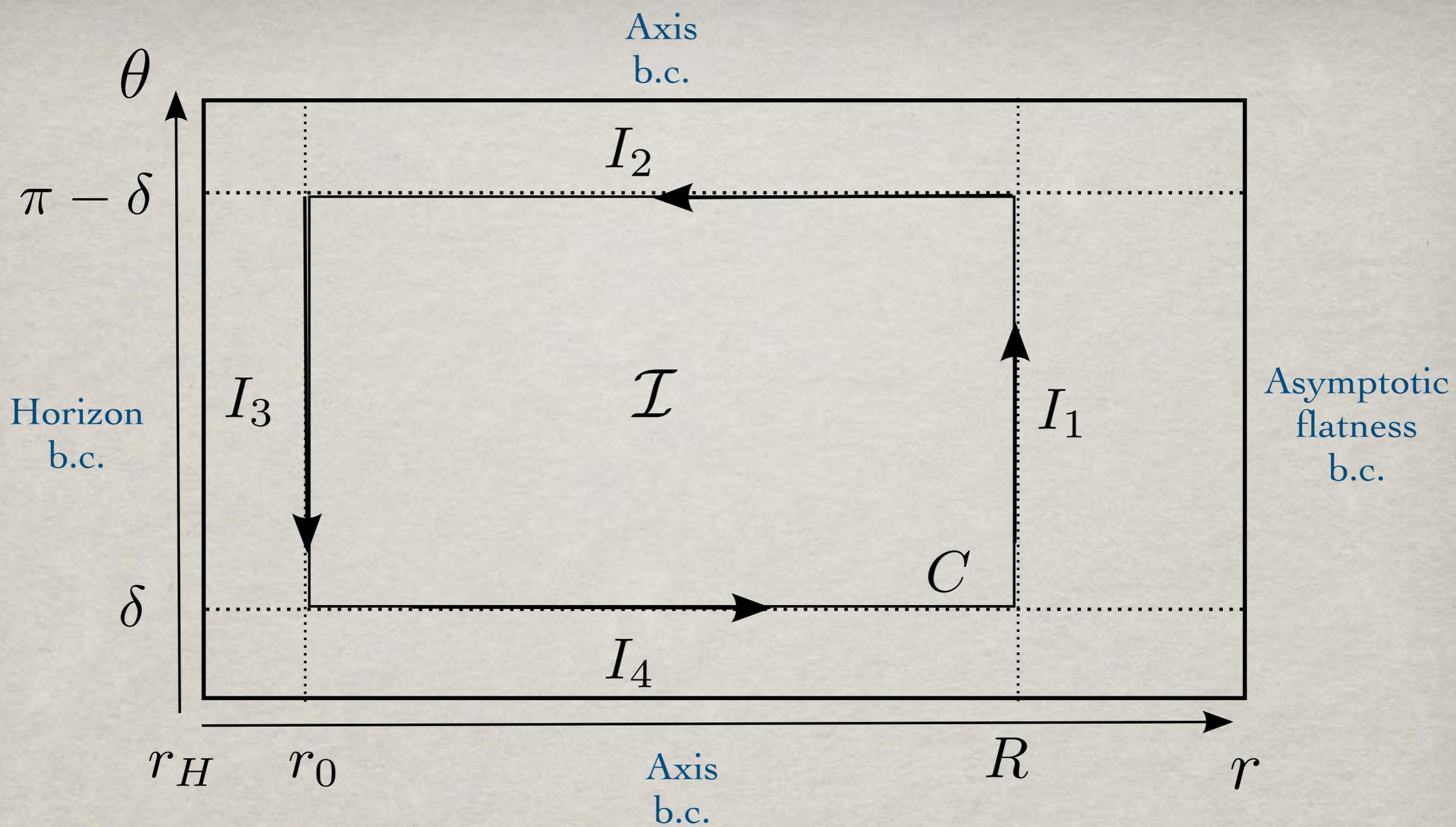


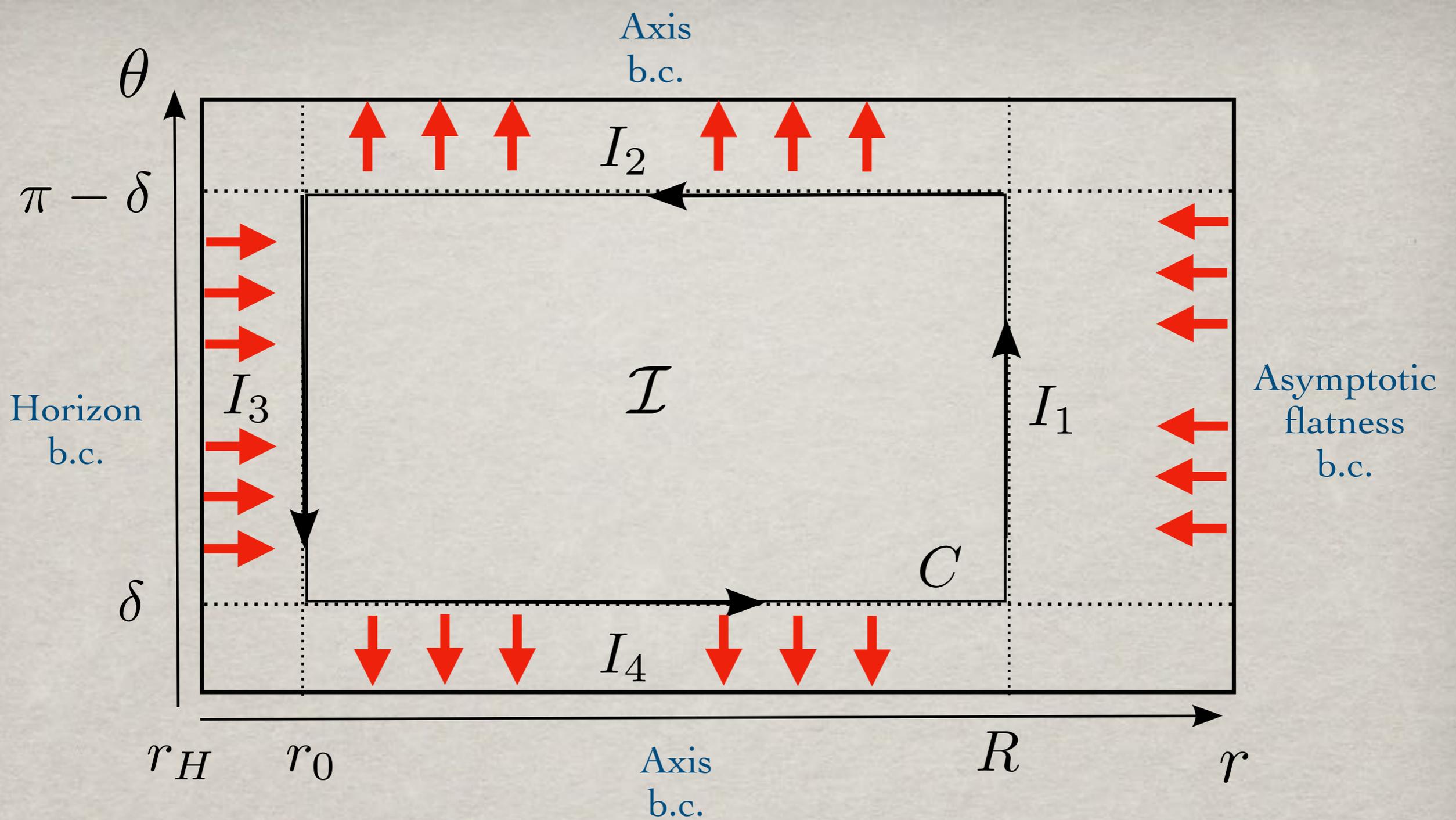
$$w = +1$$

The topological charge is additive:

$$\oint_C d\Omega = 2\pi \sum_i w_i, \quad w_i = -1, 1.$$







$$w = \lim_{R \rightarrow +\infty} \lim_{r_0 \rightarrow r_H} \left(\lim_{\delta \rightarrow 0} \oint_C d\Omega \right) = -1$$

Q1: Do all theoretical black hole solutions have Kerr-like LRs?

(Partial) R1:

Yes,

under the stated conditions of the theorem (and possibly even more LRs).

But,

can be circumvented

(e.g.) by changing the boundary conditions.

Example of a BH without LRs (asymptotically Melvin):

Júnior, Cunha, CH, Crispino, *Phys. Rev. D* 104 (2021) 044018

Q2: Can theoretical horizonless exotic ECOs have Kerr-like LRs?

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As BH mimickers, ECOs have the appeal that they could solve the singularity problem of BHs and perhaps connect to the dark matter issue.

Many models of horizonless ECOs have been proposed throughout the years, with different motivations, not always to replace black holes. Some co-exist with black holes.

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- c) gravastars (Mazur and Mottola, gr-qc/0109035)
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Generic statements?

A theorem for ultracompact ECOs that form from incomplete gravitational collapse

PRL **119**, 251102 (2017)

PHYSICAL REVIEW LETTERS

week ending
22 DECEMBER 2017

Light-Ring Stability for Ultracompact Objects

Pedro V. P. Cunha,^{1,2} Emanuele Berti,^{3,2} and Carlos A. R. Herdeiro¹

¹*Departamento de Física da Universidade de Aveiro and CIDMA, Campus de Santiago, 3810-183 Aveiro, Portugal*

²*CENTRA, Departamento de Física, Instituto Superior Técnico, Universidade de Lisboa, Avenida Rovisco Pais 1, 1049 Lisboa, Portugal*

³*Department of Physics and Astronomy, The University of Mississippi, University, Mississippi 38677, USA*

(Received 3 August 2017; revised manuscript received 18 October 2017; published 18 December 2017)

We prove the following theorem: axisymmetric, stationary solutions of the Einstein field equations formed from classical gravitational collapse of matter obeying the null energy condition, that are everywhere smooth and ultracompact (i.e., they have a light ring) must have at least *two* light rings, and one of them is *stable*. It has been argued that stable light rings generally lead to nonlinear spacetime instabilities. Our result implies that smooth, physically and dynamically reasonable ultracompact objects are not viable as observational alternatives to black holes whenever these instabilities occur on astrophysically short time scales. The proof of the theorem has two parts: (i) We show that light rings always come in pairs, one being a saddle point and the other a local extremum of an effective potential. This result follows from a topological argument based on the Brouwer degree of a continuous map, with no assumptions on the spacetime dynamics, and, hence, it is applicable to any metric gravity theory where photons follow null geodesics. (ii) Assuming Einstein's equations, we show that the extremum is a local minimum of the potential (i.e., a stable light ring) if the energy-momentum tensor satisfies the null energy condition.

DOI: [10.1103/PhysRevLett.119.251102](https://doi.org/10.1103/PhysRevLett.119.251102)

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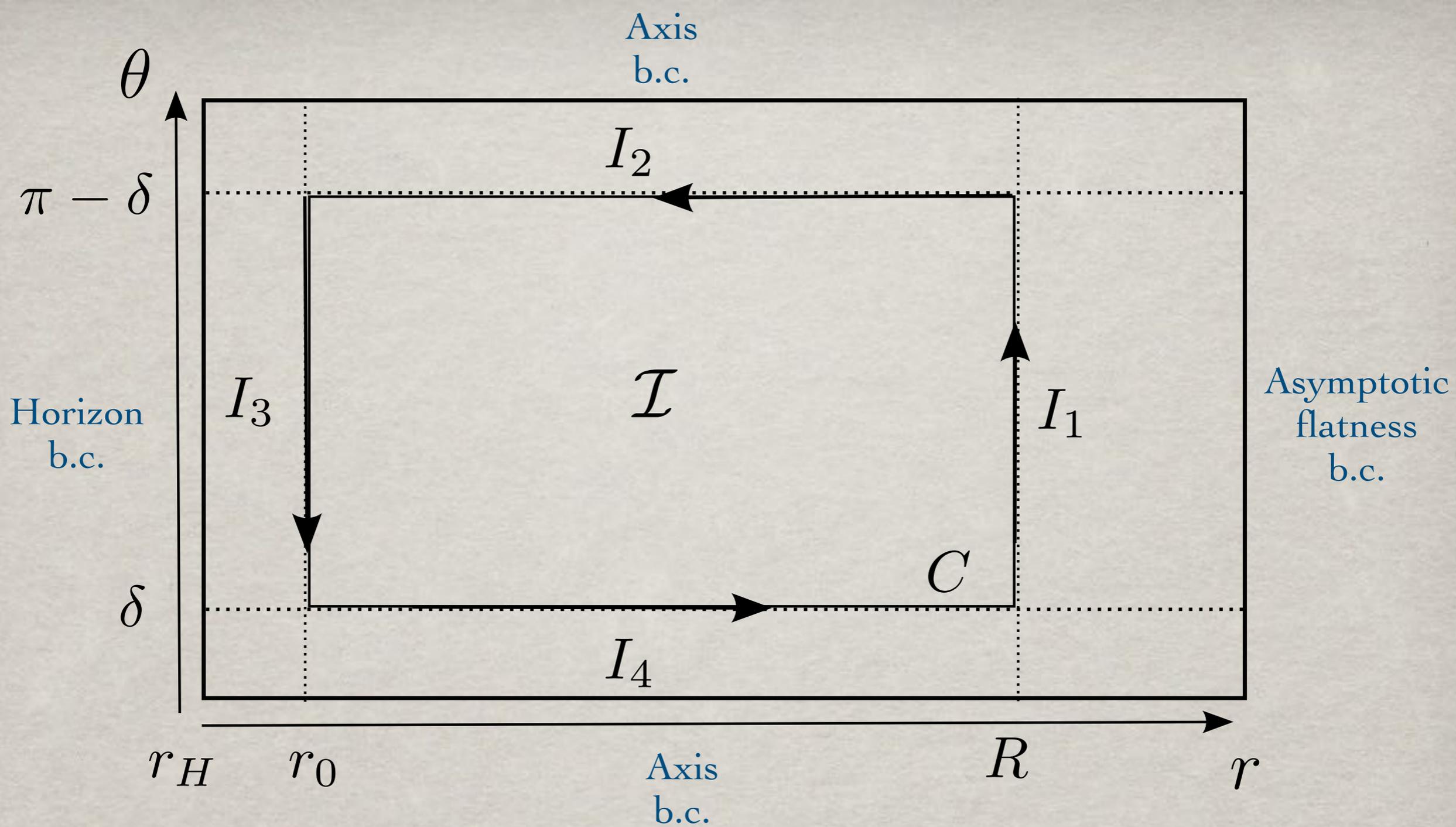
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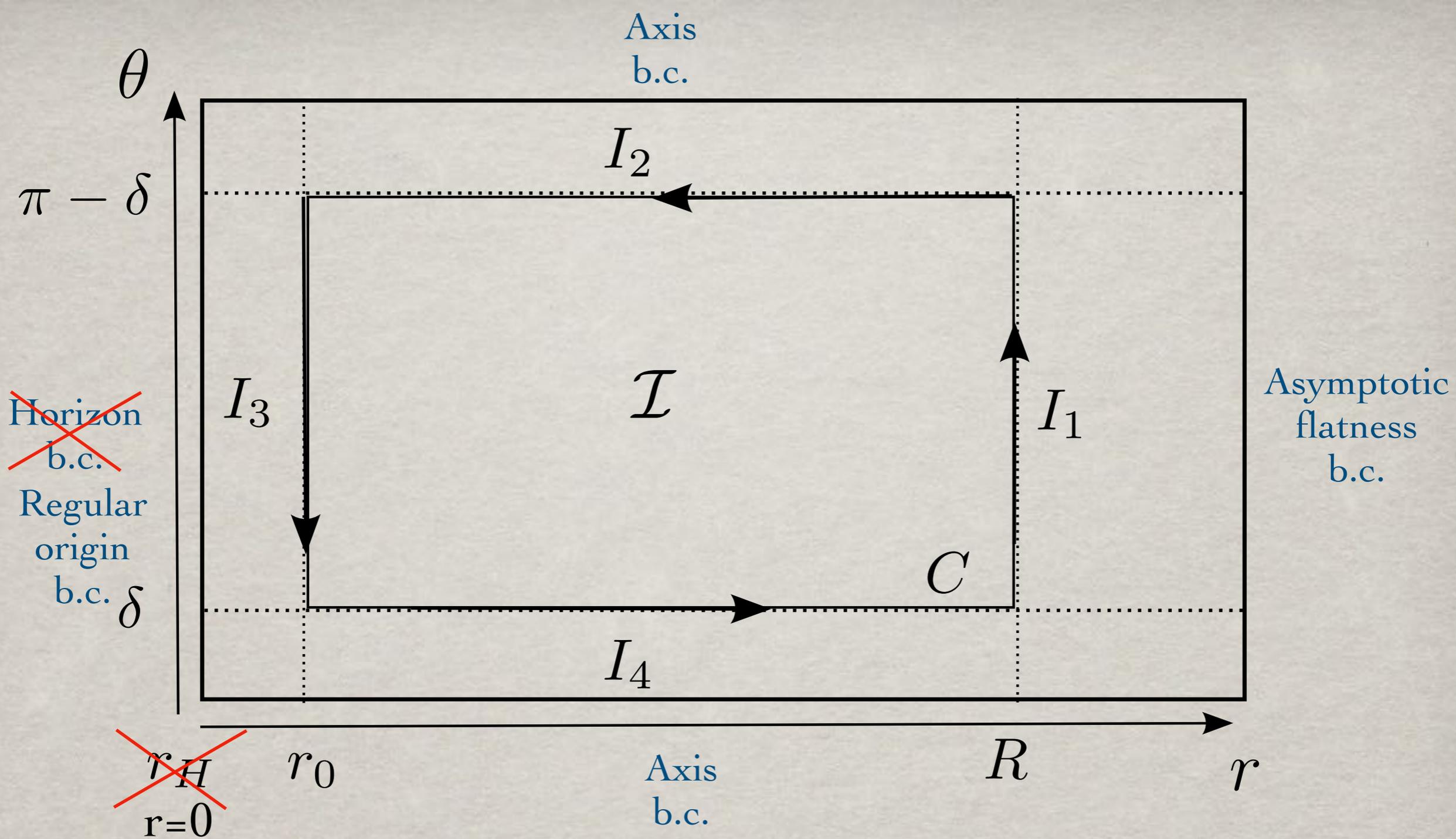
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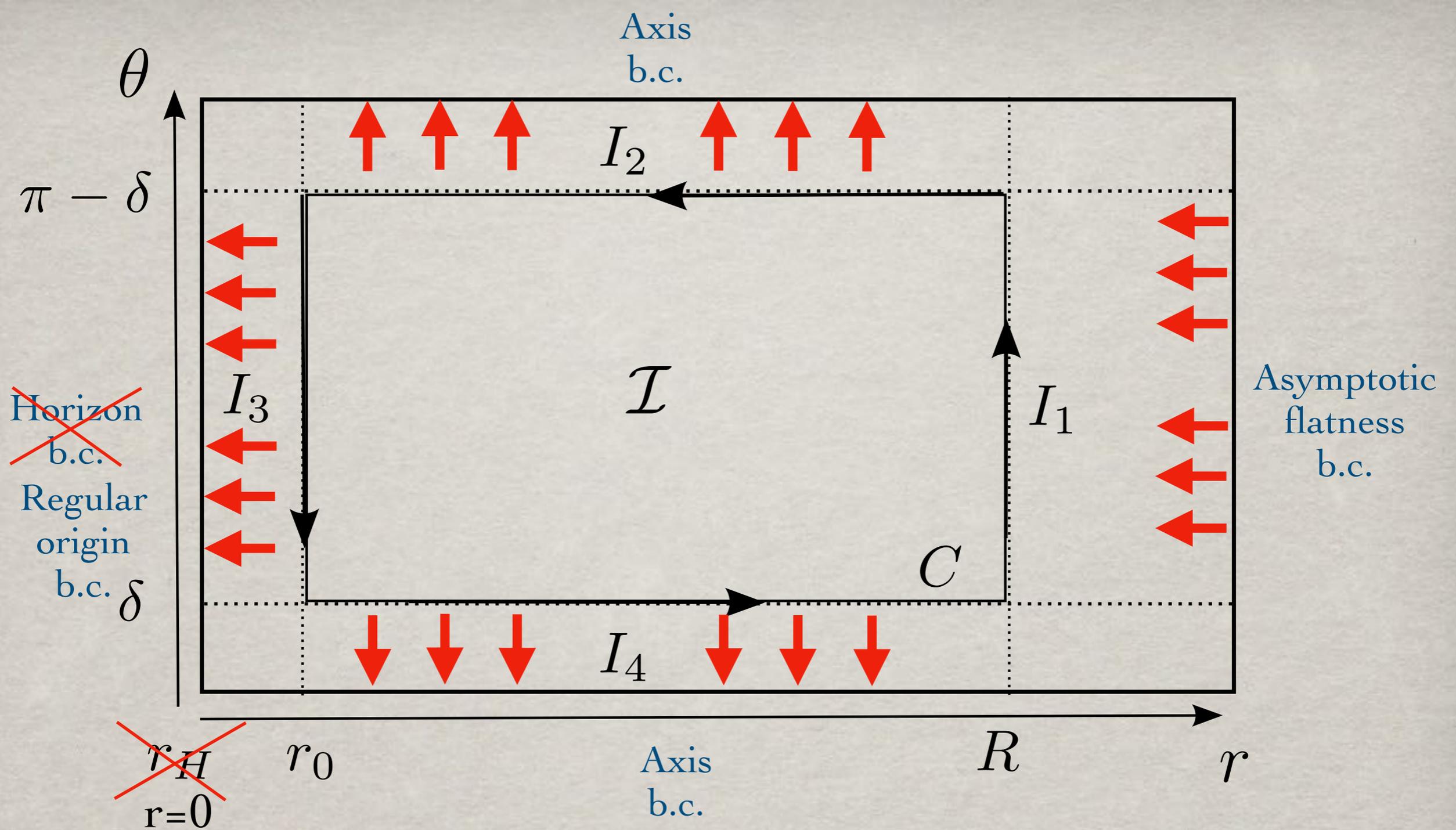
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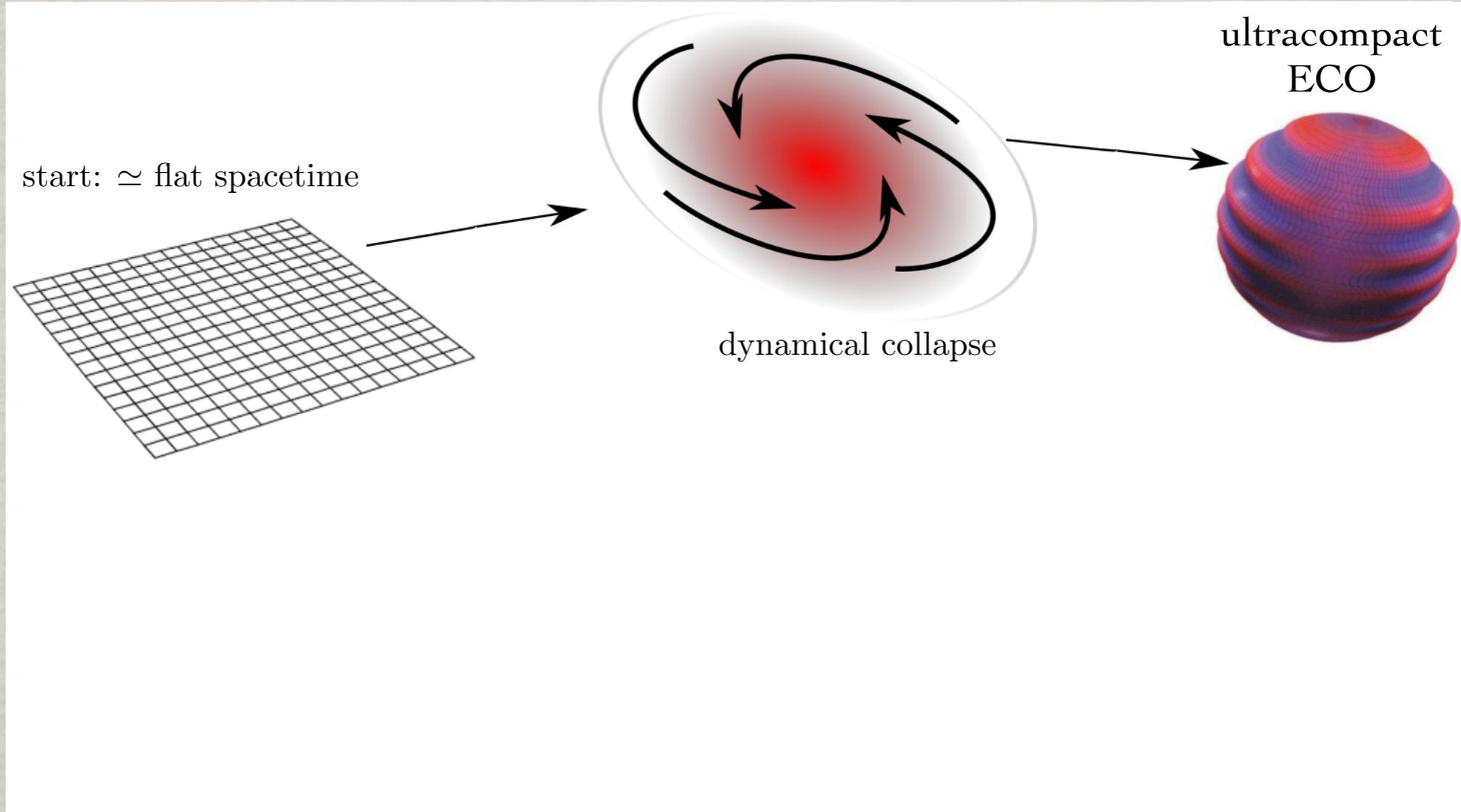




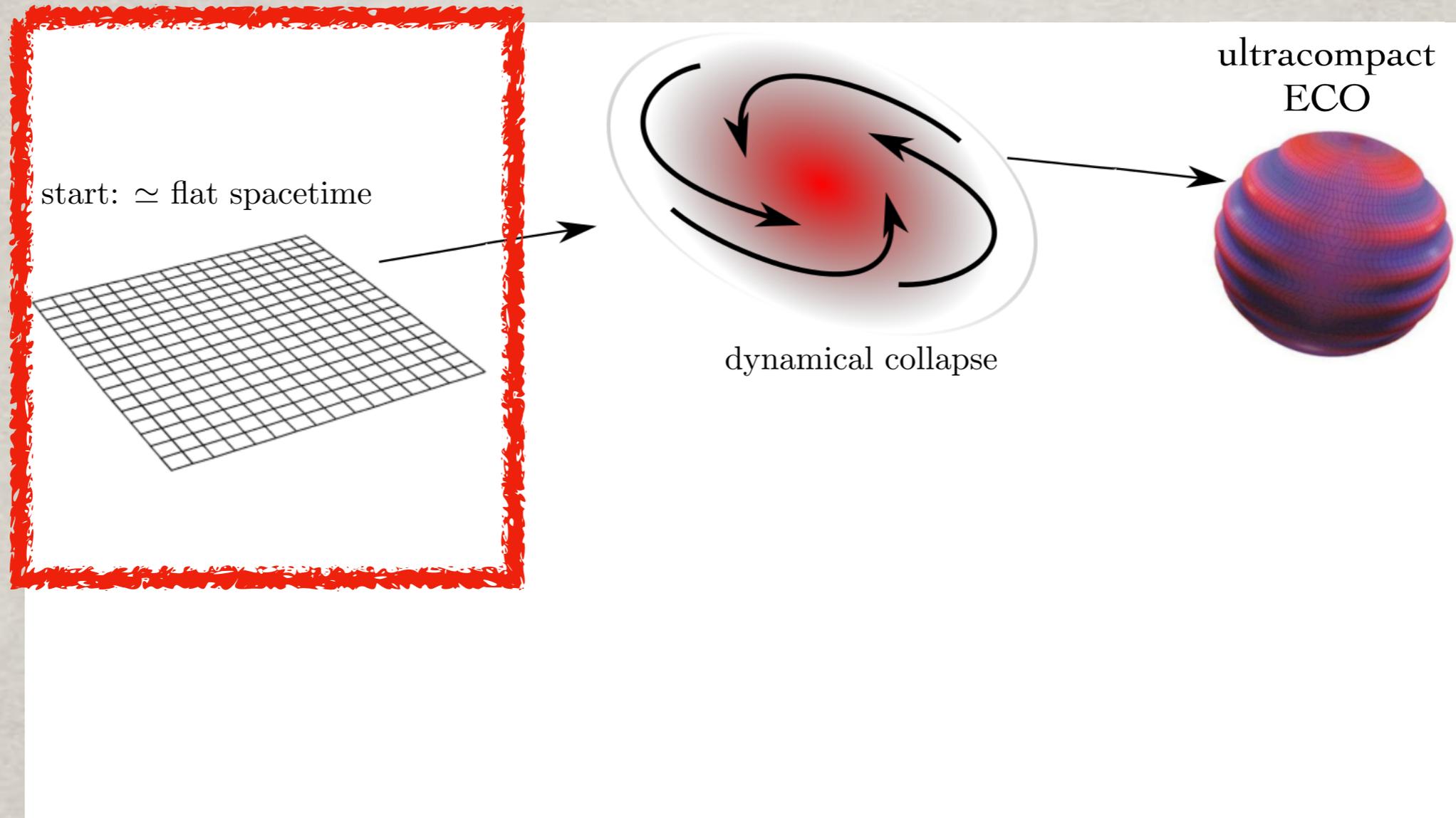


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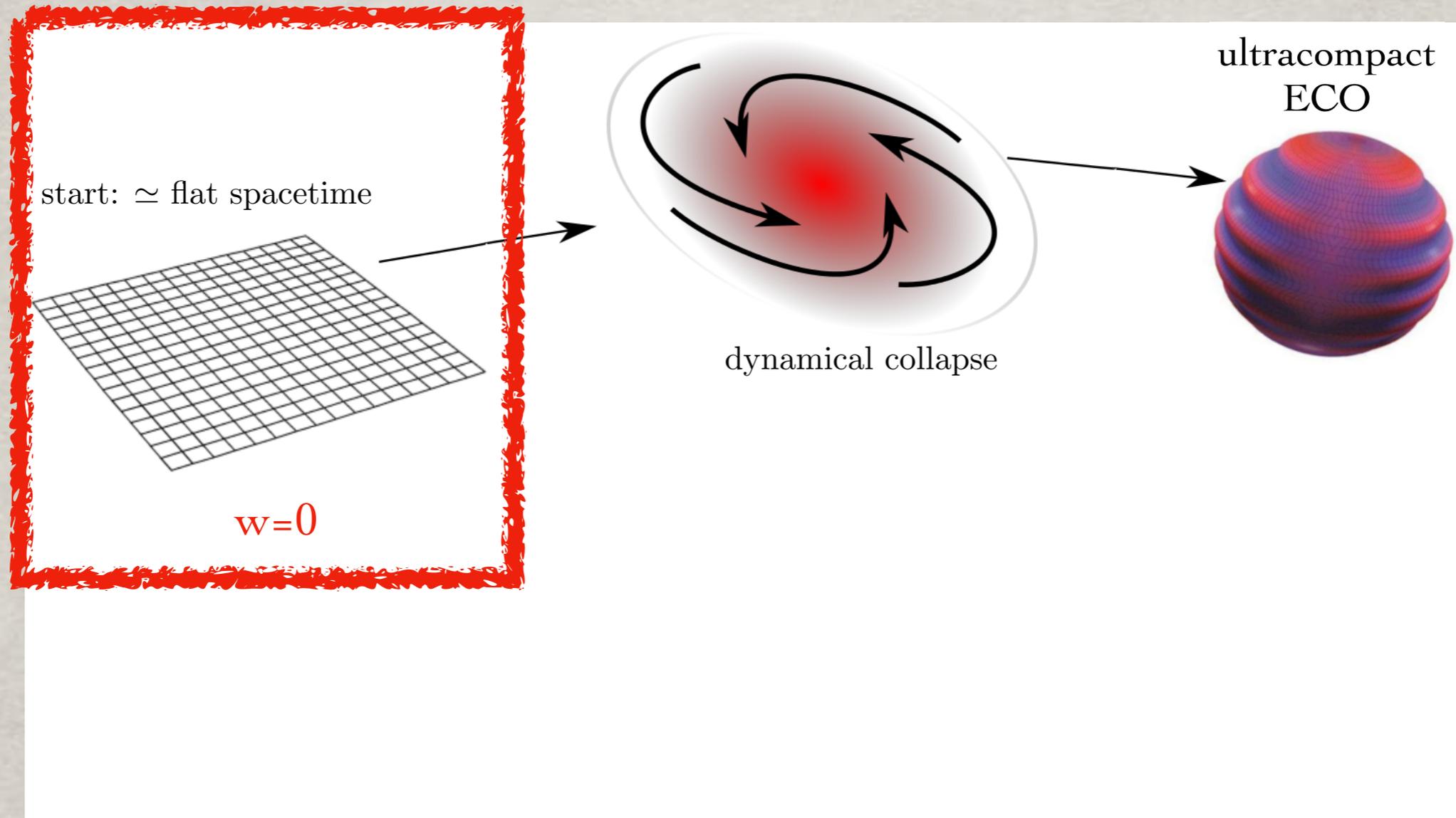
A generic dynamical picture



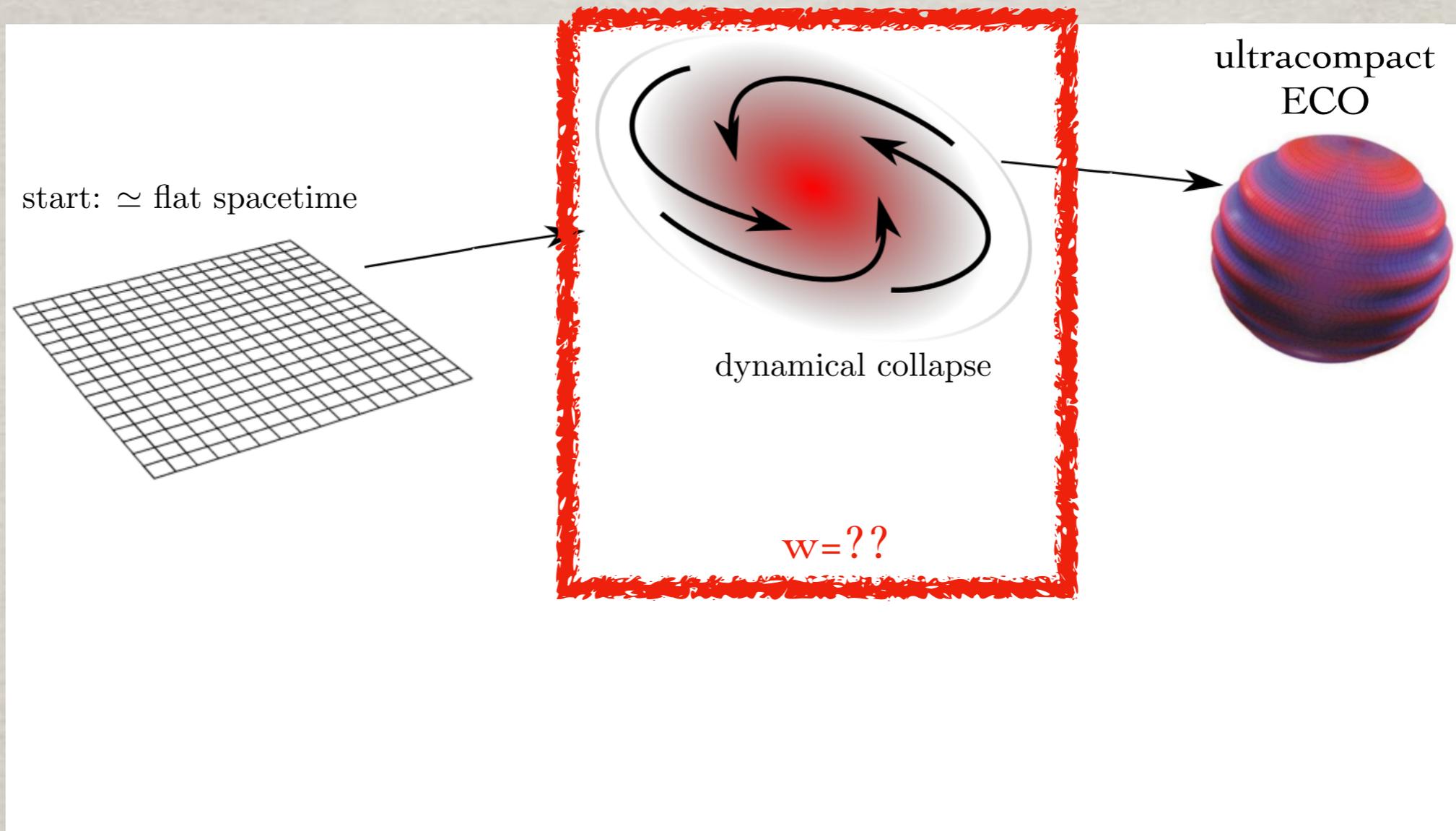
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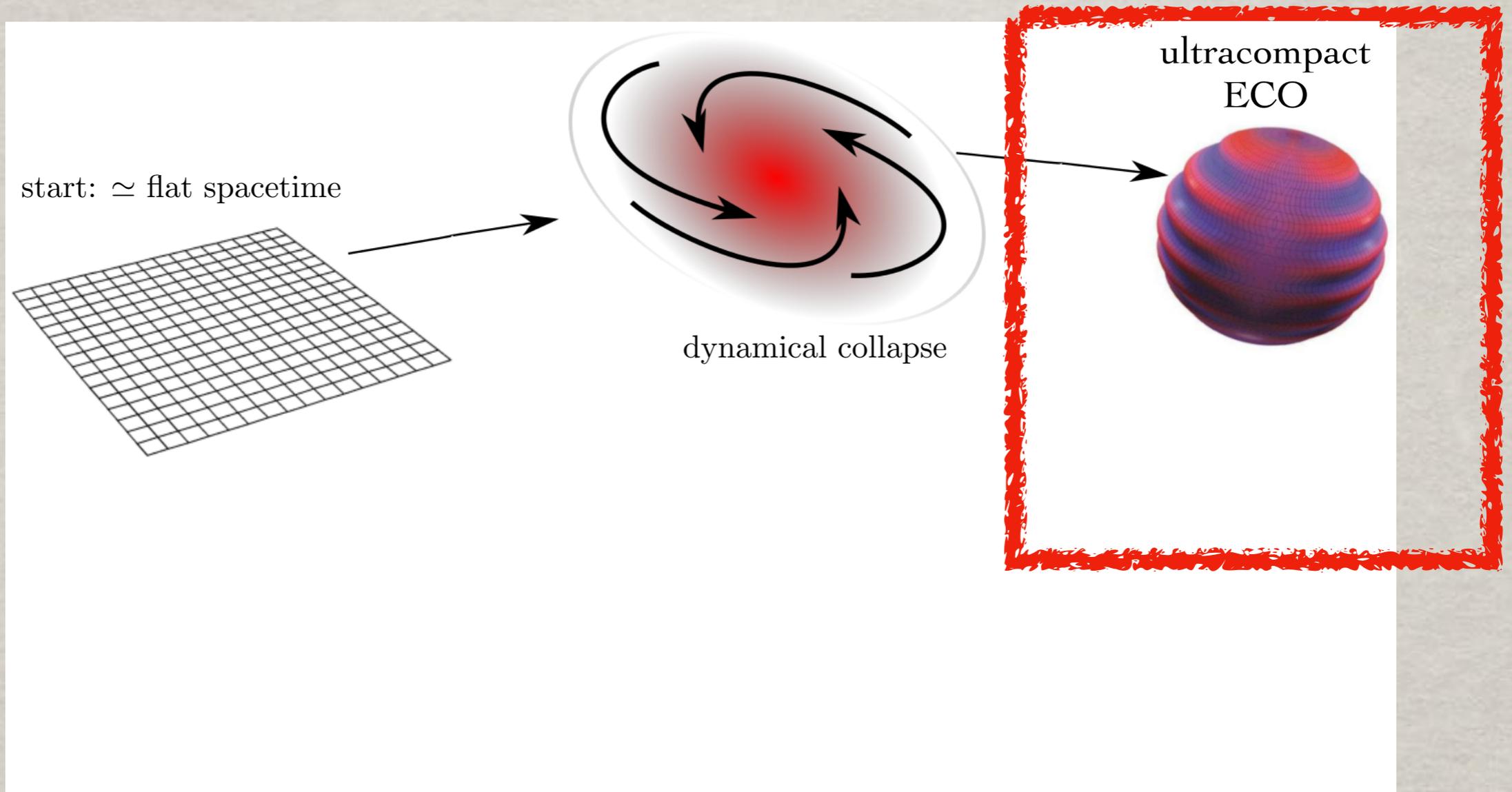
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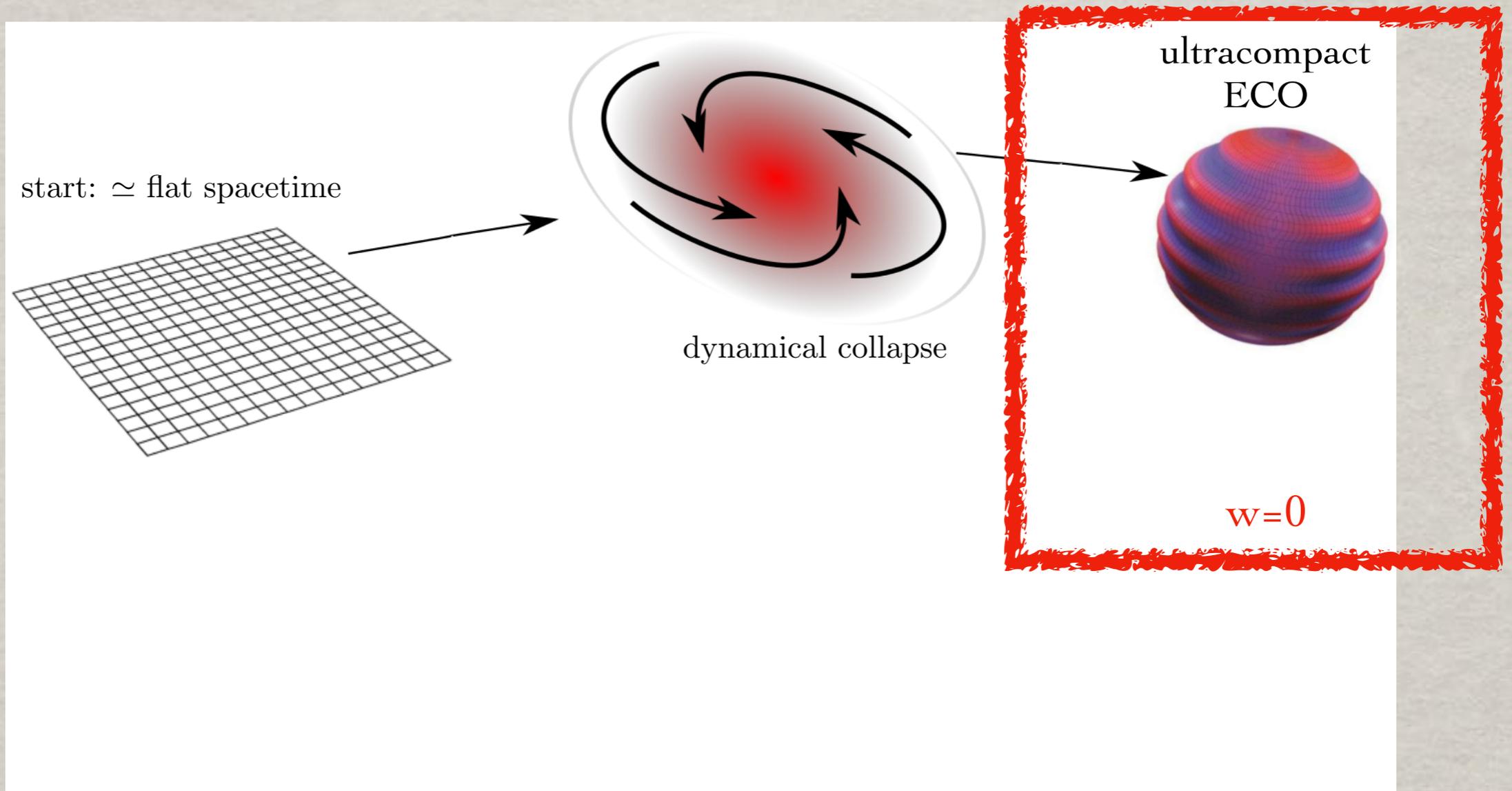
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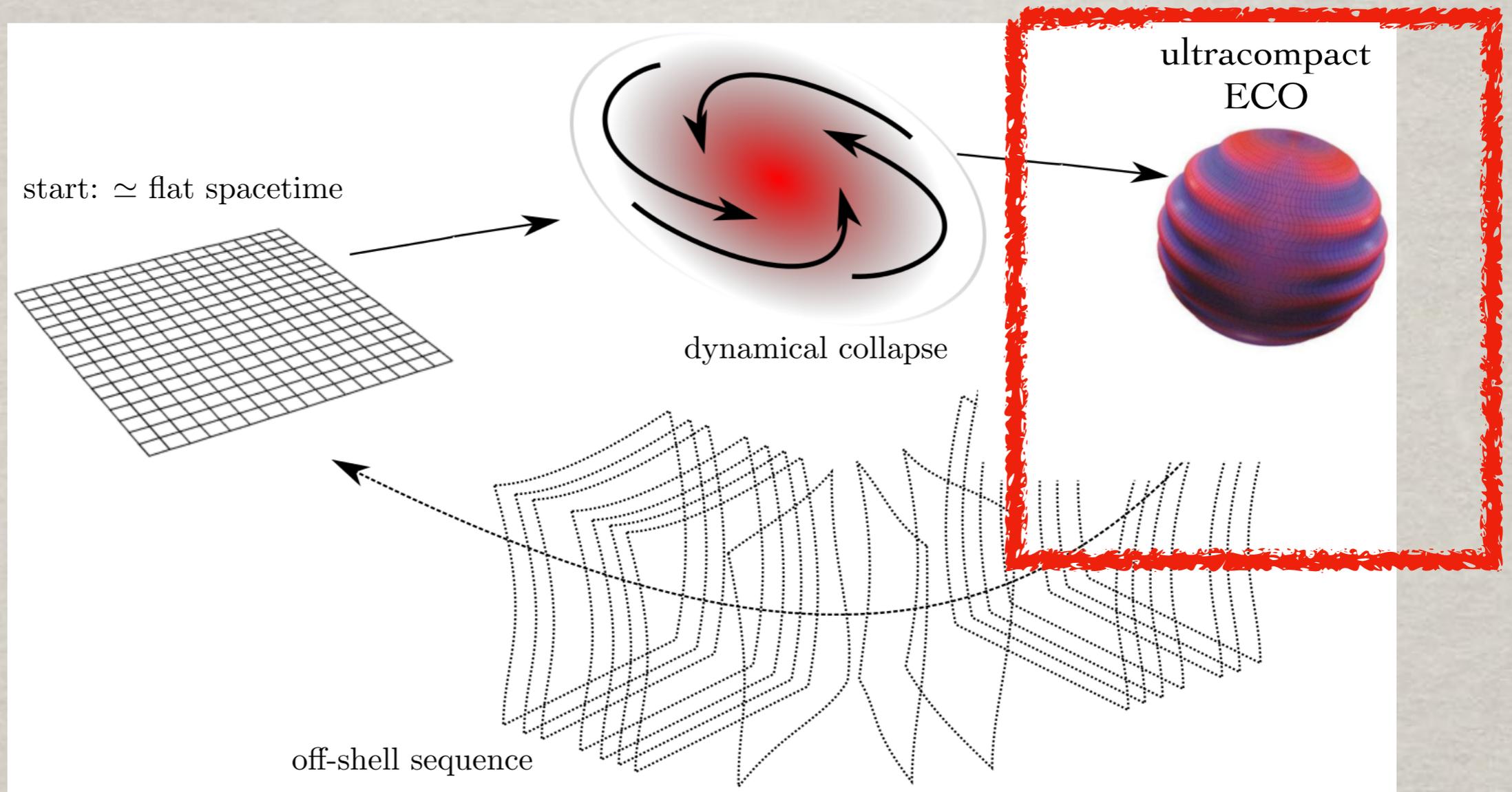
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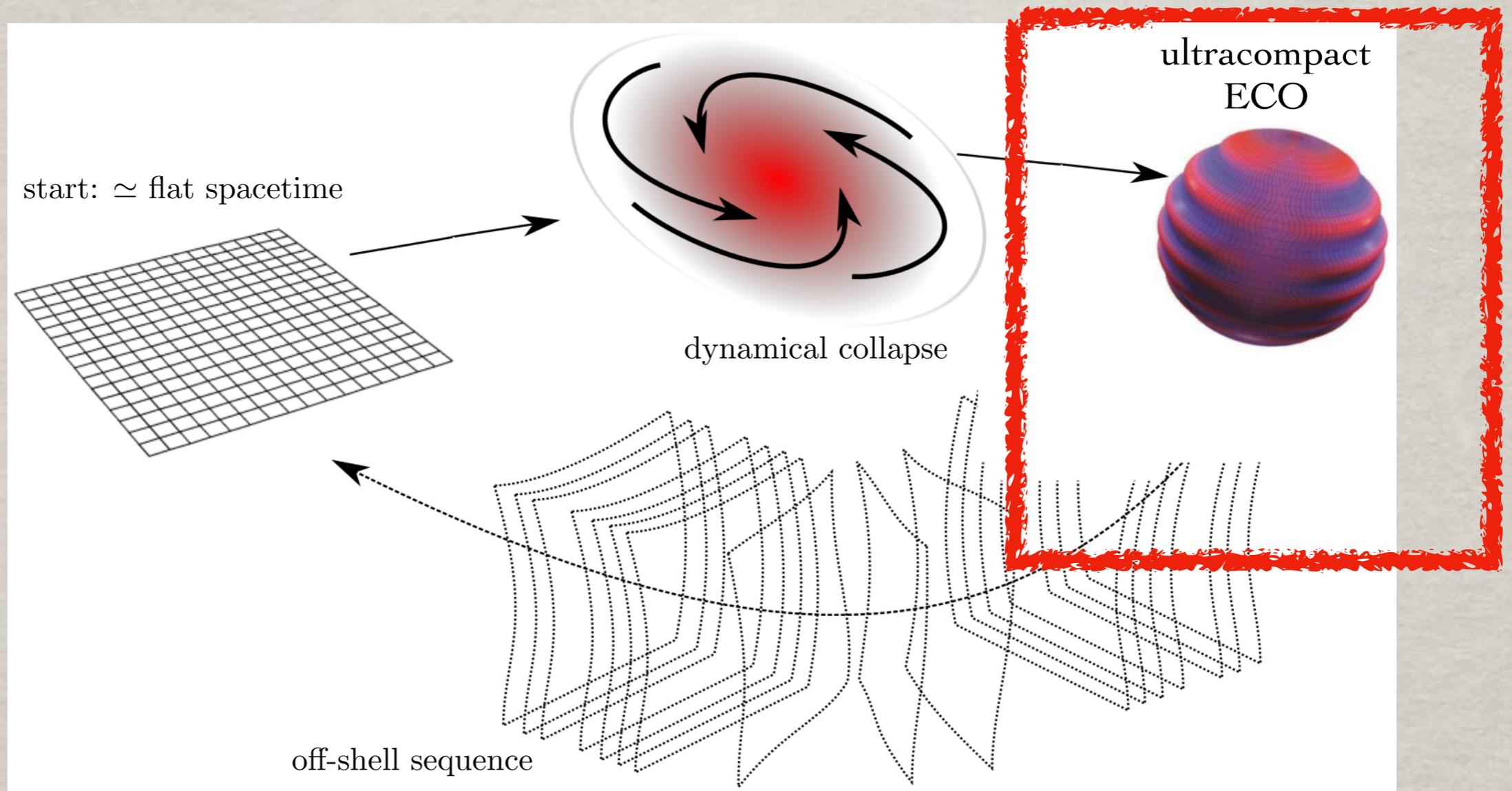
A generic dynamical picture



A generic dynamical picture



A generic dynamical picture



Punch line:

any (stationary, axi-symmetric, circular, topologically trivial) ECO that forms from an incomplete gravitational collapse which has a standard LR, must have an exotic one as well.

The exotic LR must be stable, if the Null Energy Condition (NEC) is obeyed.

Q2: Can theoretical horizonless exotic compact objects (ECOs) have Kerr-like LRs?

(Partial) R2:

Yes,

but under the stated conditions of the theorem
necessarily with extra baggage: there is an extra LR, which is stable assuming the NEC.

But,

can be circumvented (e.g.):
by non-trivial topology (e.g. wormholes),
by non-smoothness (e.g. gravastars),
by *ad hoc* boundary conditions (e.g. truncations of Kerr).

Q3: If so, could such ECOs be astrophysically viable?

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Some viability conditions:

- 1) Appear in a well motivated and consistent physical model;
- 2) Have a dynamical formation mechanism;
- 3) Be (sufficiently) stable.

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Then,

there is a possible generic viability issue for ultracompact ECOs with a stable LR:

stable LRs may lead to a trapping instability.

J. Keir, Class.Quant.Grav. 33 (2016) no.13, 135009; Benomio, arXiv:1809.07795

- Non-linear;
- Time scale?

Exotic Compact Objects and the Fate of the Light-Ring Instability

Pedro V. P. Cunha¹, Carlos Herdeiro¹, Eugen Radu¹, and Nicolas Sanchis-Gual^{2,1}

¹*Departamento de Matemática da Universidade de Aveiro and Centre for Research and Development in Mathematics and Applications (CIDMA), Campus de Santiago, 3810-183 Aveiro, Portugal*

²*Departamento de Astronomía y Astrofísica, Universitat de València, Dr. Moliner 50, 46100 Burjassot (València), Spain*

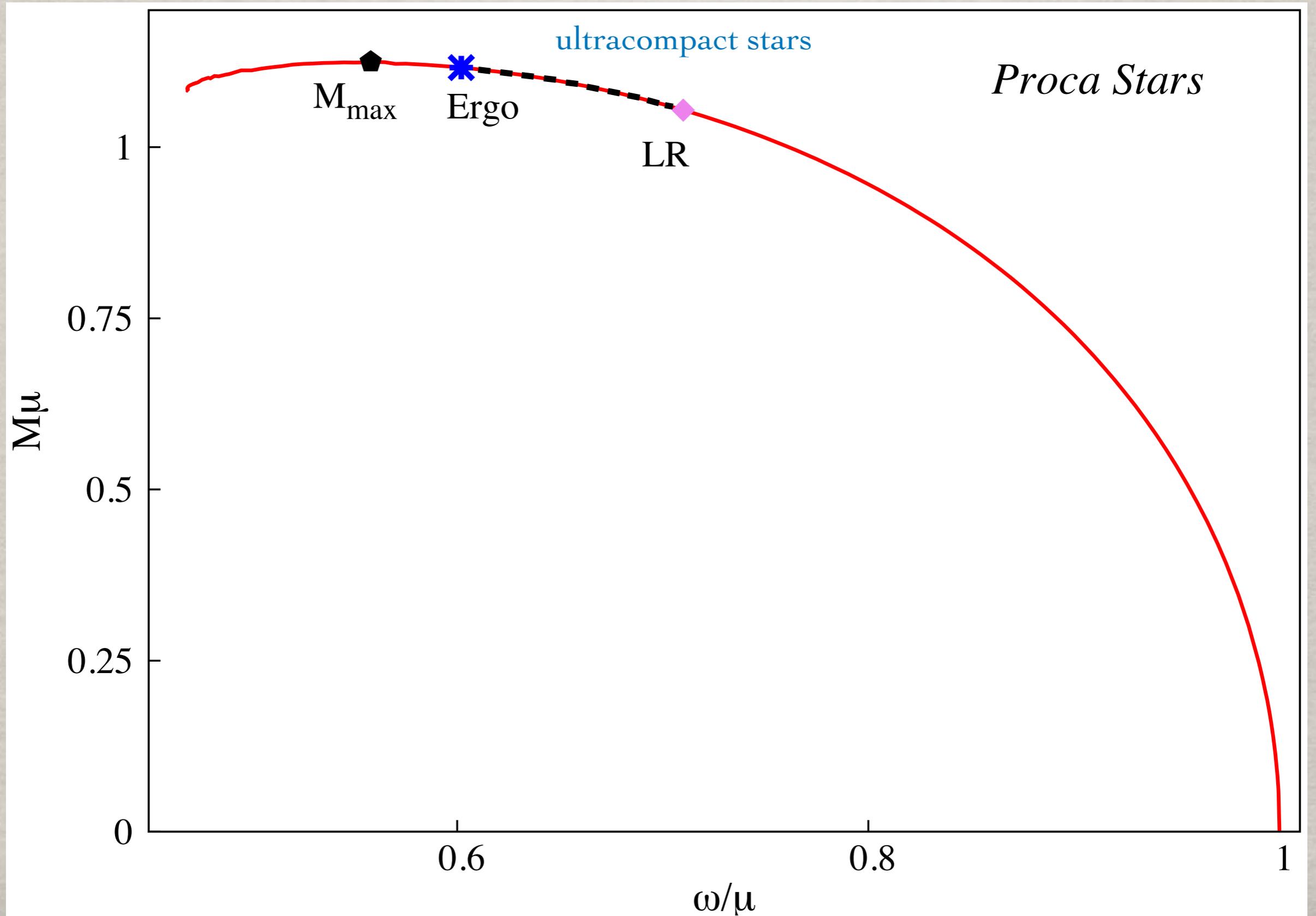
(Received 12 August 2022; accepted 6 December 2022)

Ultracompact objects with light rings (LRs) but without an event horizon could mimic black holes (BHs) in their strong gravity phenomenology. But are such objects dynamically viable? Stationary and axisymmetric ultracompact objects that can form from smooth, quasi-Minkowski initial data must have at least one *stable* LR, which has been argued to trigger a spacetime *instability*; but its development and fate have been unknown. Using fully nonlinear numerical evolutions of ultracompact bosonic stars free of any other known instabilities and introducing a novel adiabatic effective potential technique, we confirm the LRs triggered instability, identifying two possible fates: migration to nonultracompact configurations or collapse to BHs. In concrete examples we show that typical migration (collapse) timescales are not larger than $\sim 10^3$ light-crossing times, unless the stable LR potential well is very shallow. Our results show that the LR instability is effective in destroying horizonless ultracompact objects that could be plausible BH imitators.

$$\mathcal{L} = \frac{R}{16\pi G} + \mathcal{L}_m$$

$$\mathcal{L}_m = -\frac{1}{4}\mathcal{F}_{\alpha\beta}\bar{\mathcal{F}}^{\alpha\beta} - \frac{1}{2}\mu^2\mathcal{A}_\alpha\bar{\mathcal{A}}^\alpha$$

Spinning (mini)-Proca stars



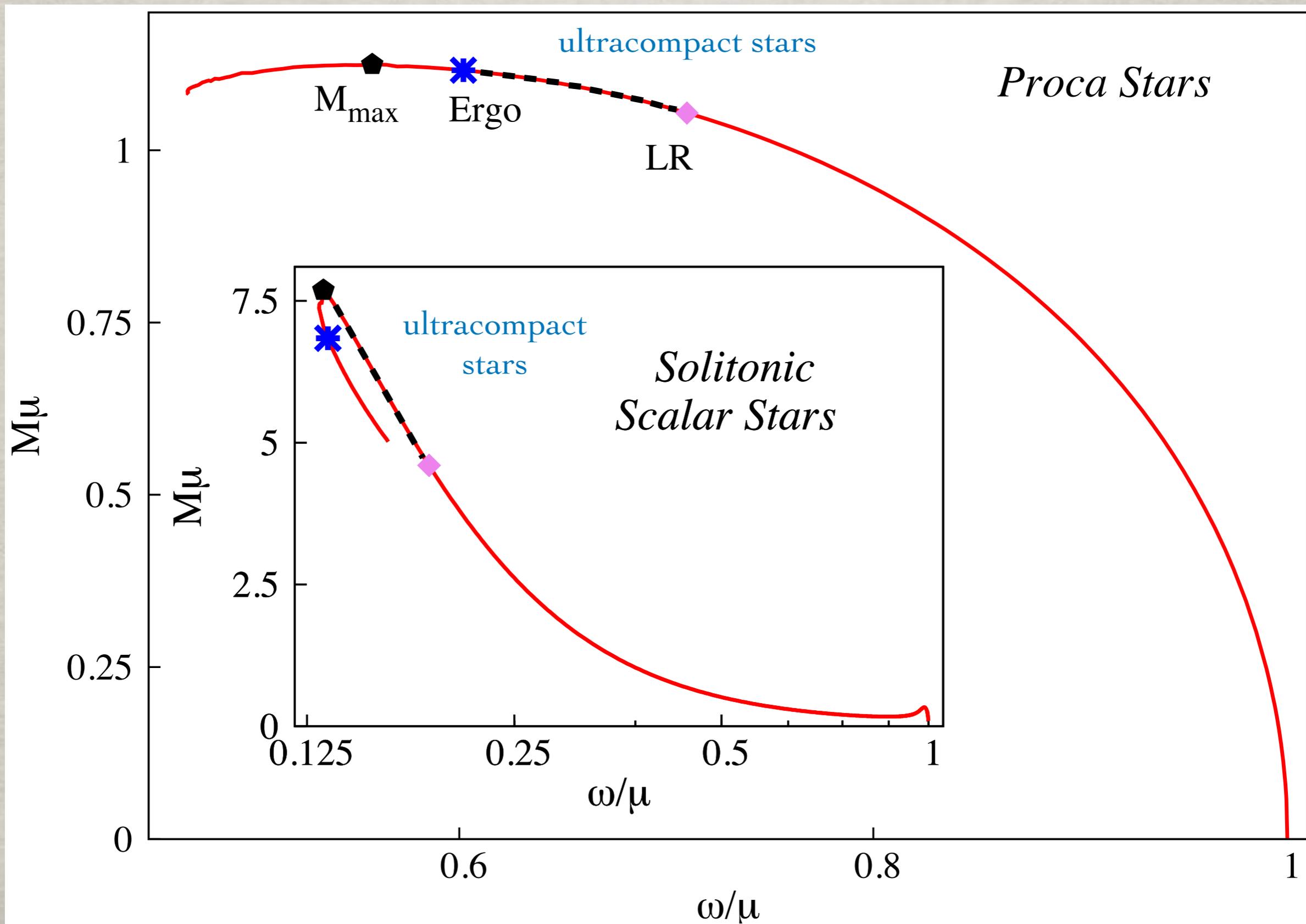
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Spinning (mini)-Proca stars

$$\mathcal{L}_m = -\partial_\alpha\Phi\partial^\alpha\bar{\Phi} - \mu^2|\Phi|^2\left[1 - \frac{2|\Phi|^2}{\sigma_0^2}\right]^2$$

Spinning solitonic scalar stars



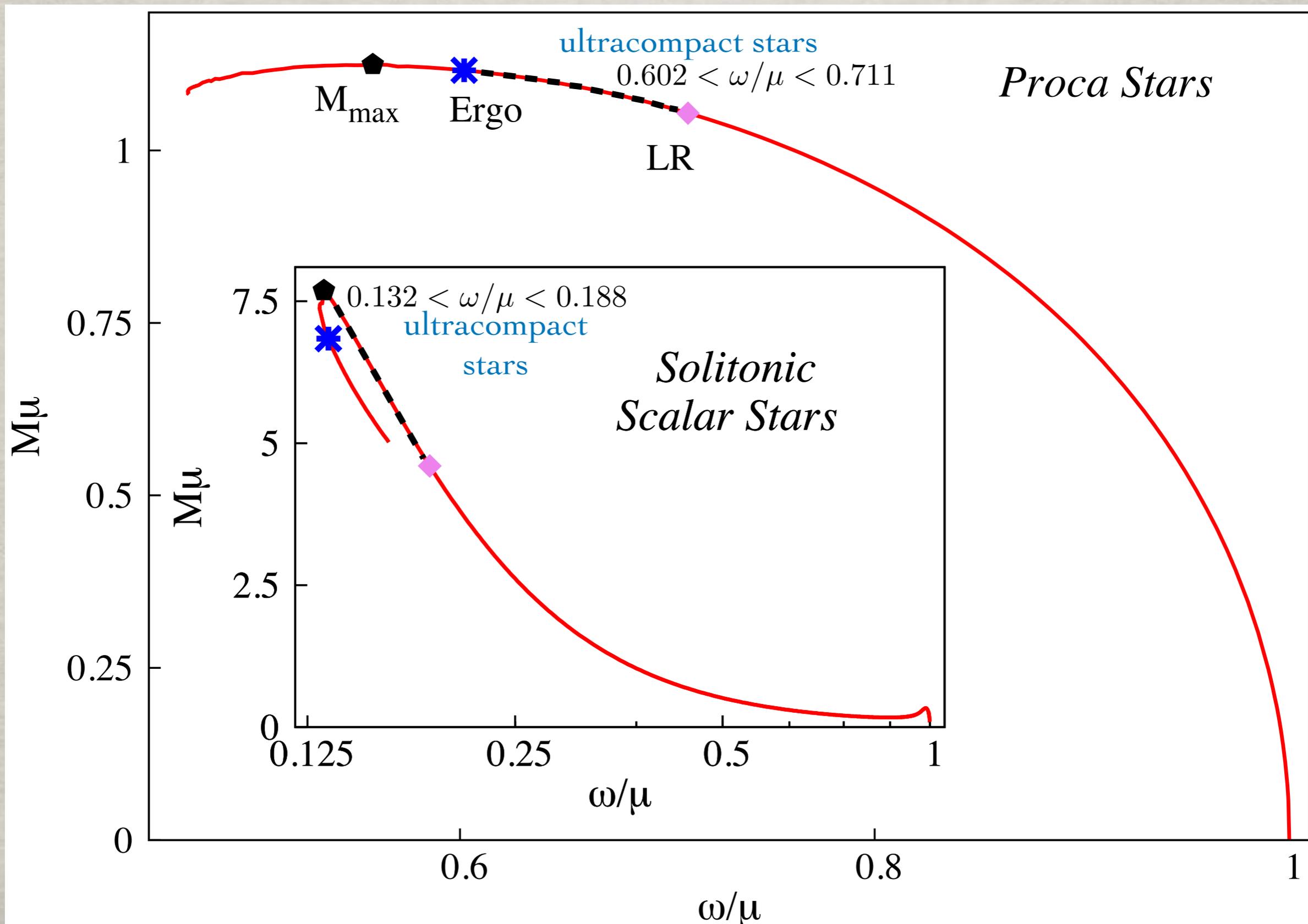
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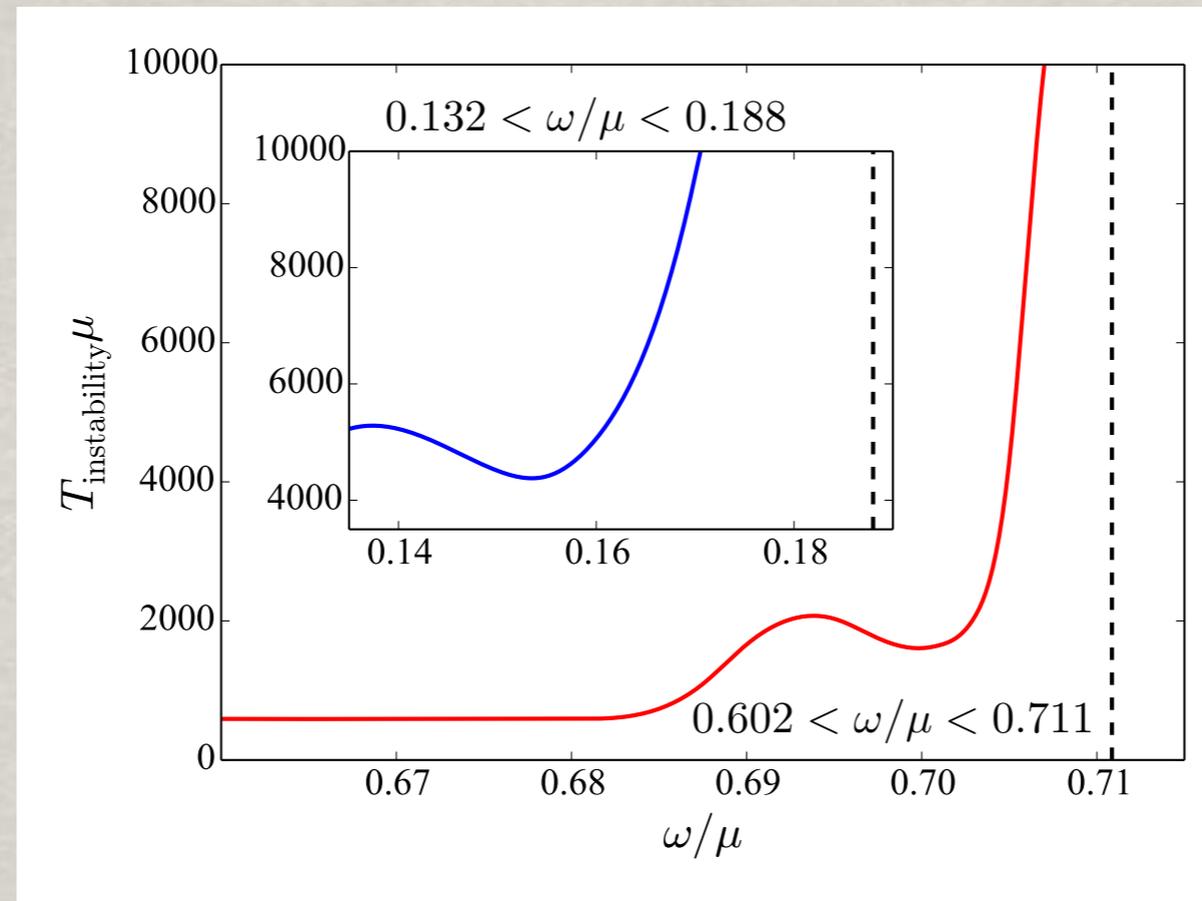
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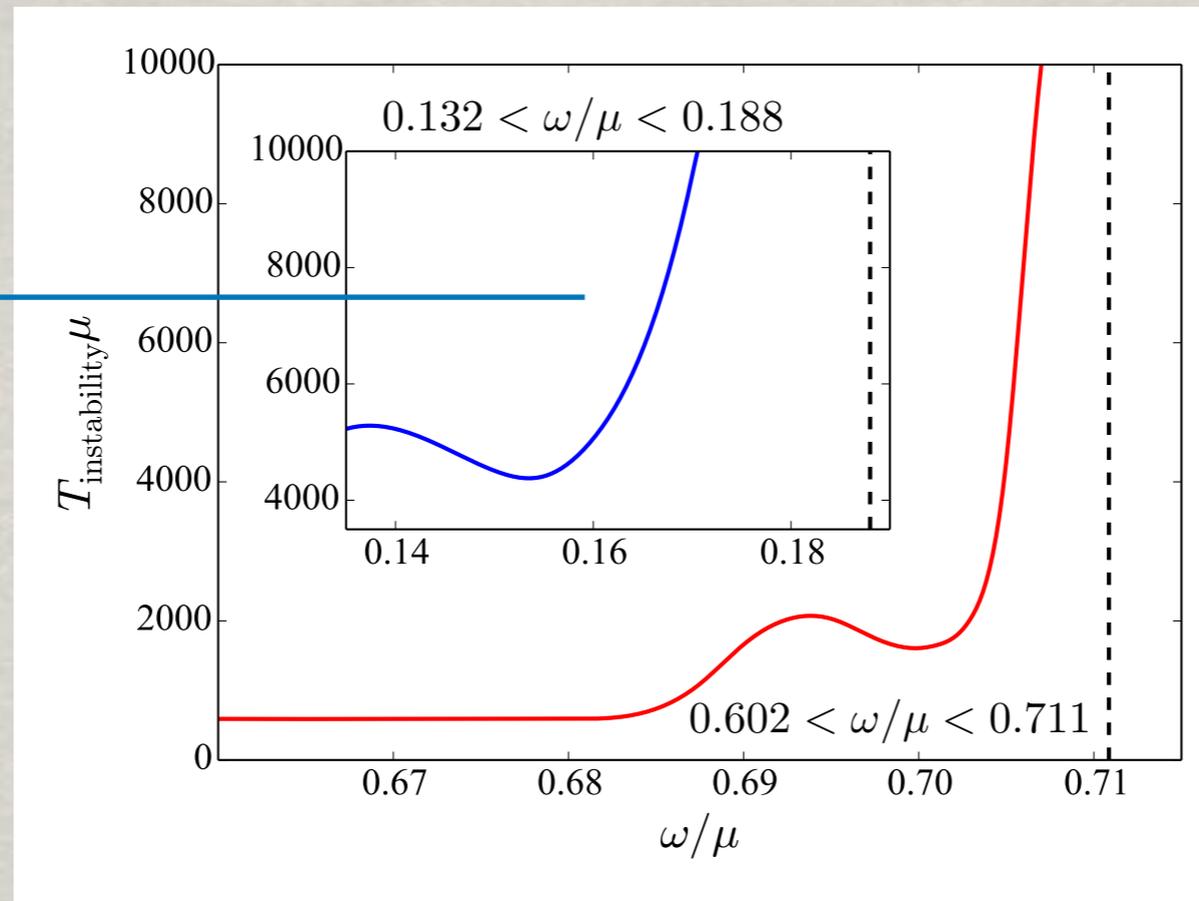
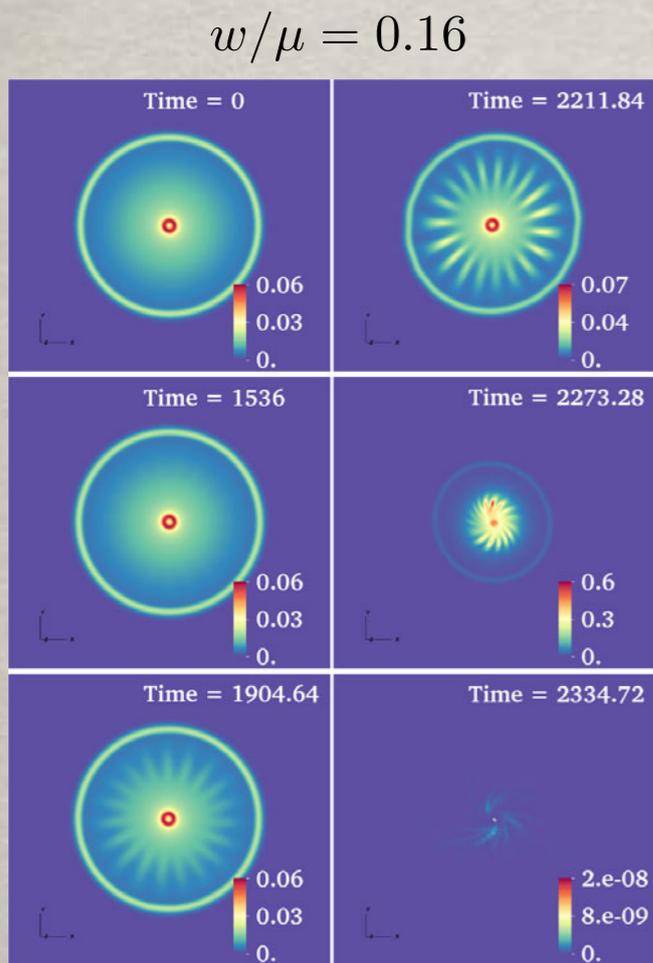
Spinning solitonic scalar stars



There is an instability and there is a transition:

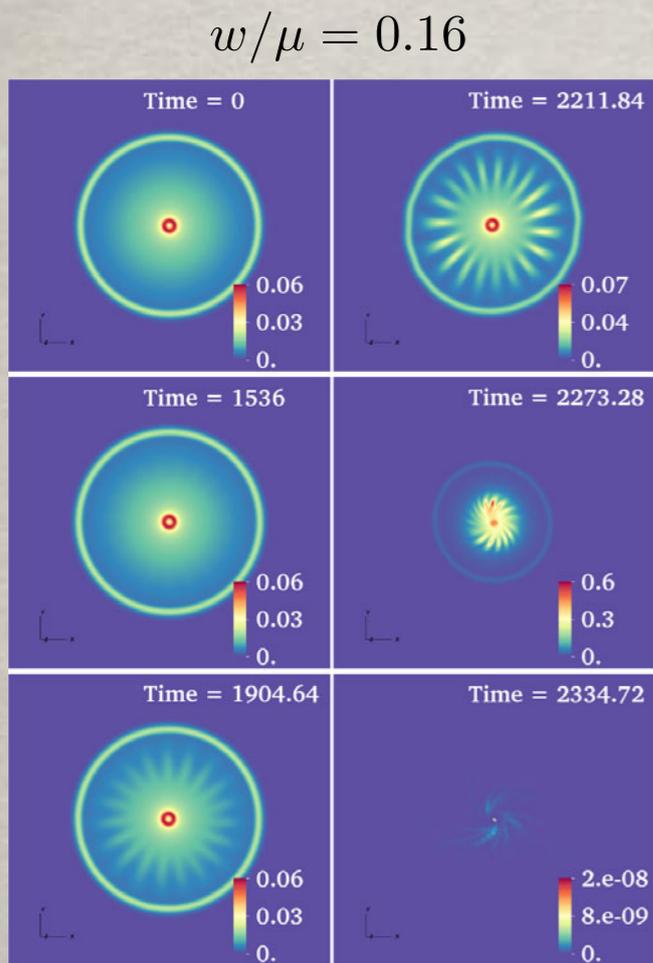


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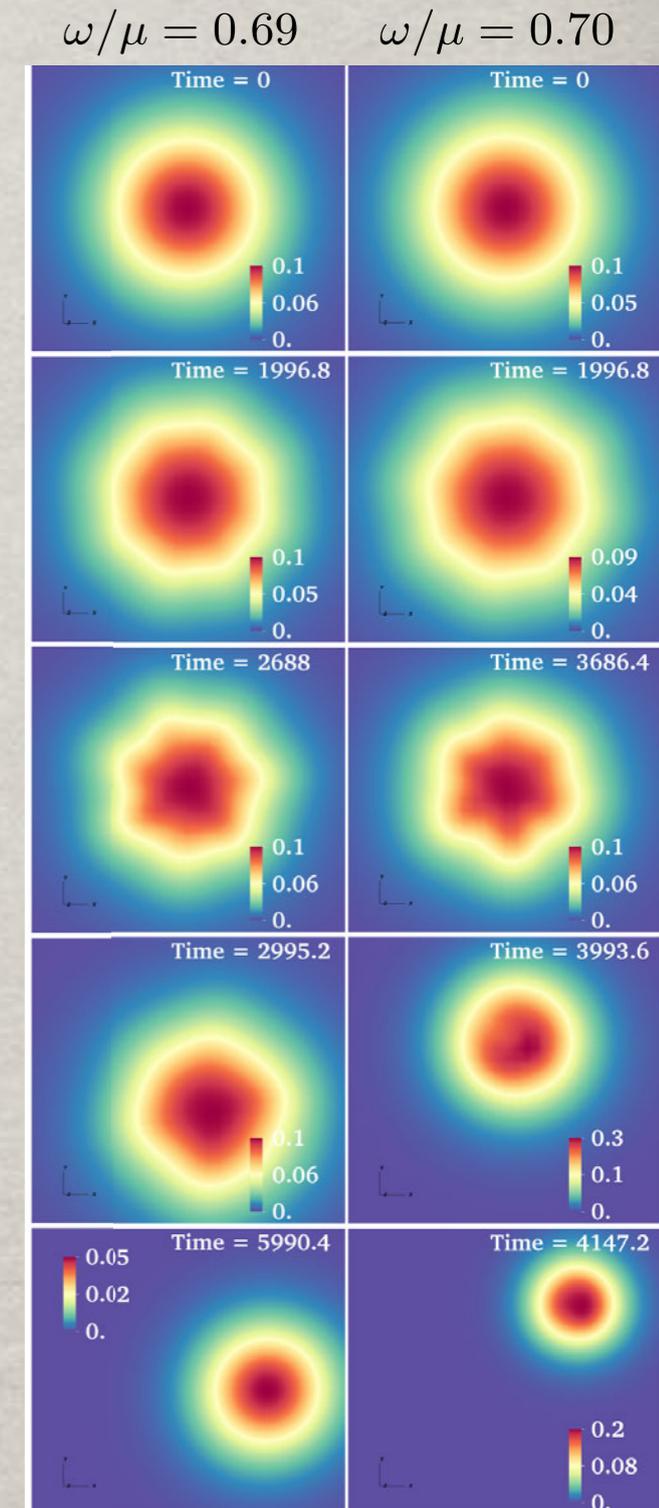
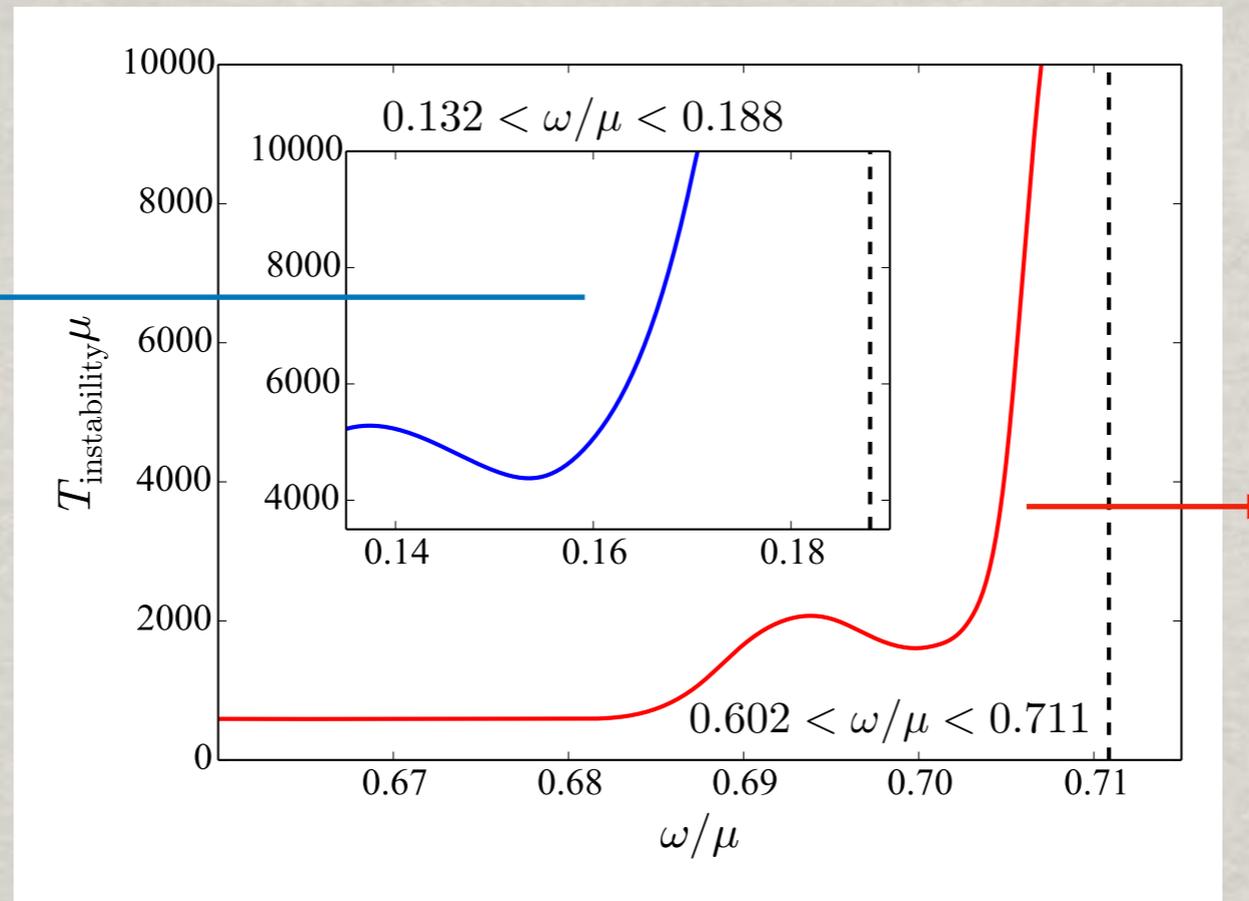


Collapse
into BH

There is an instability and there is a transition:

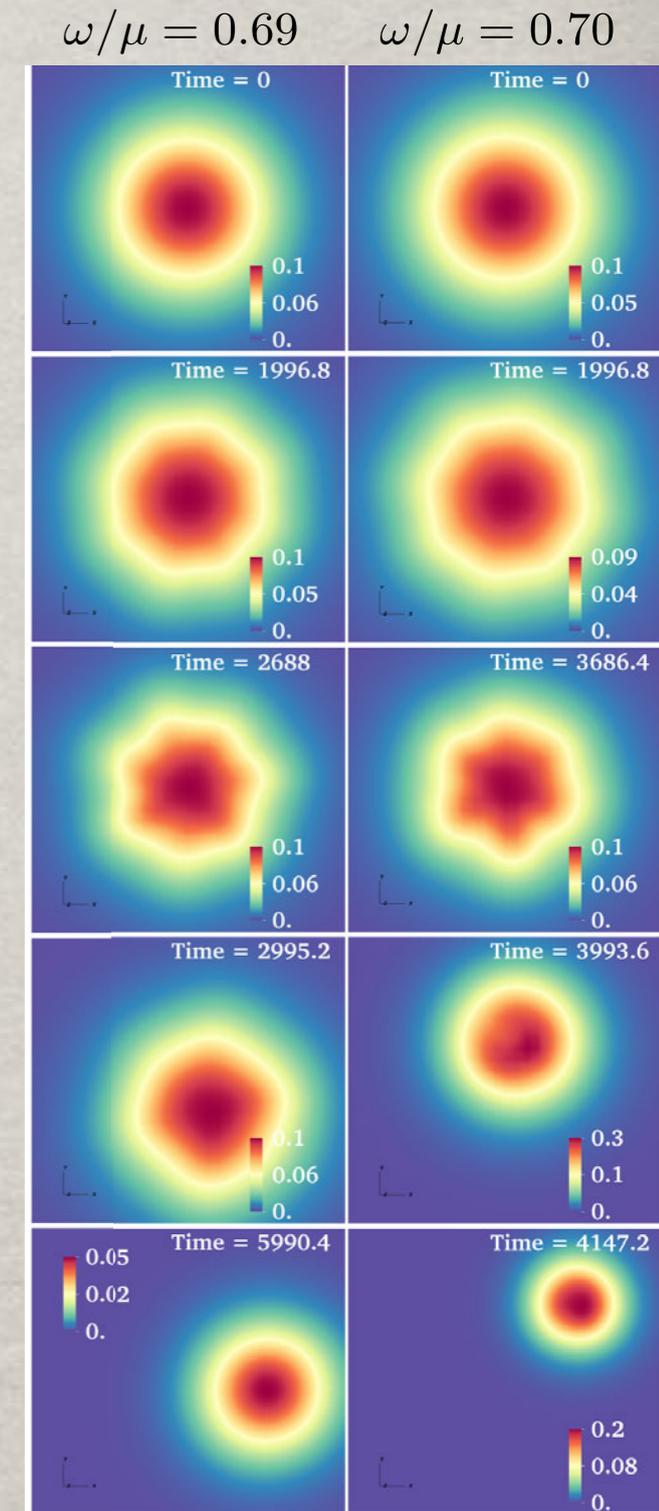
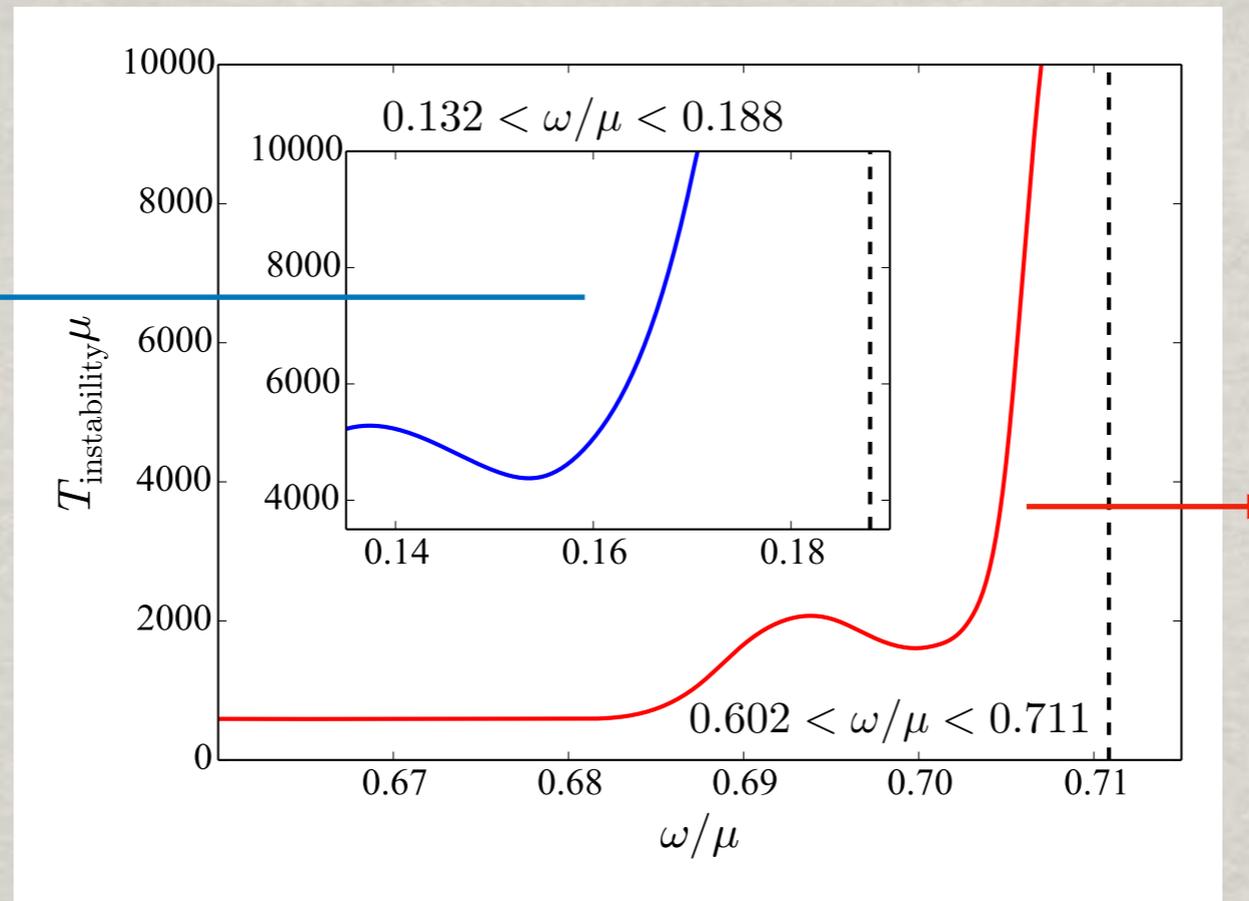
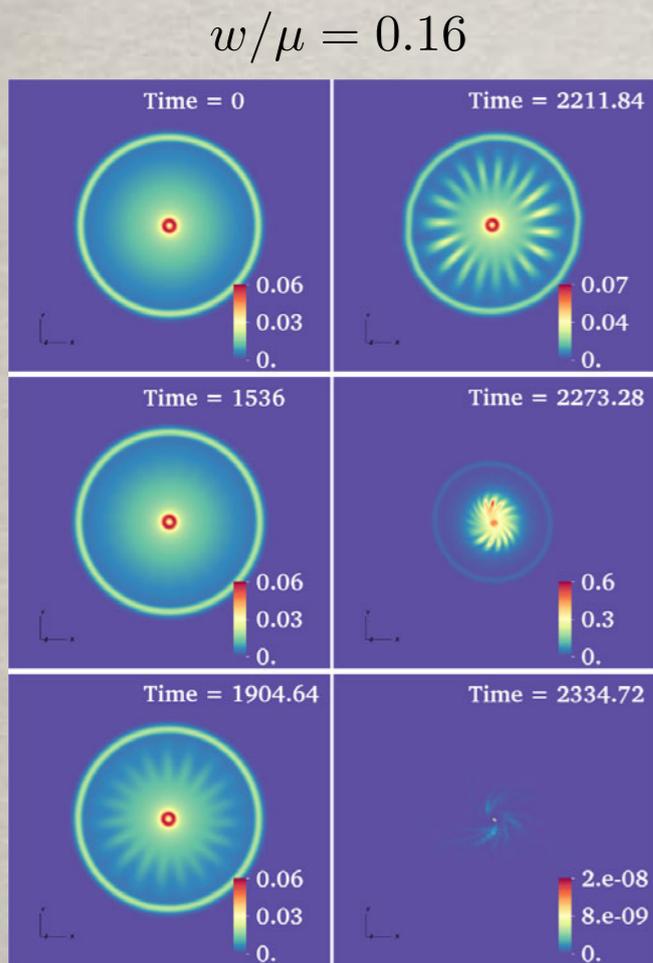


Collapse
into BH



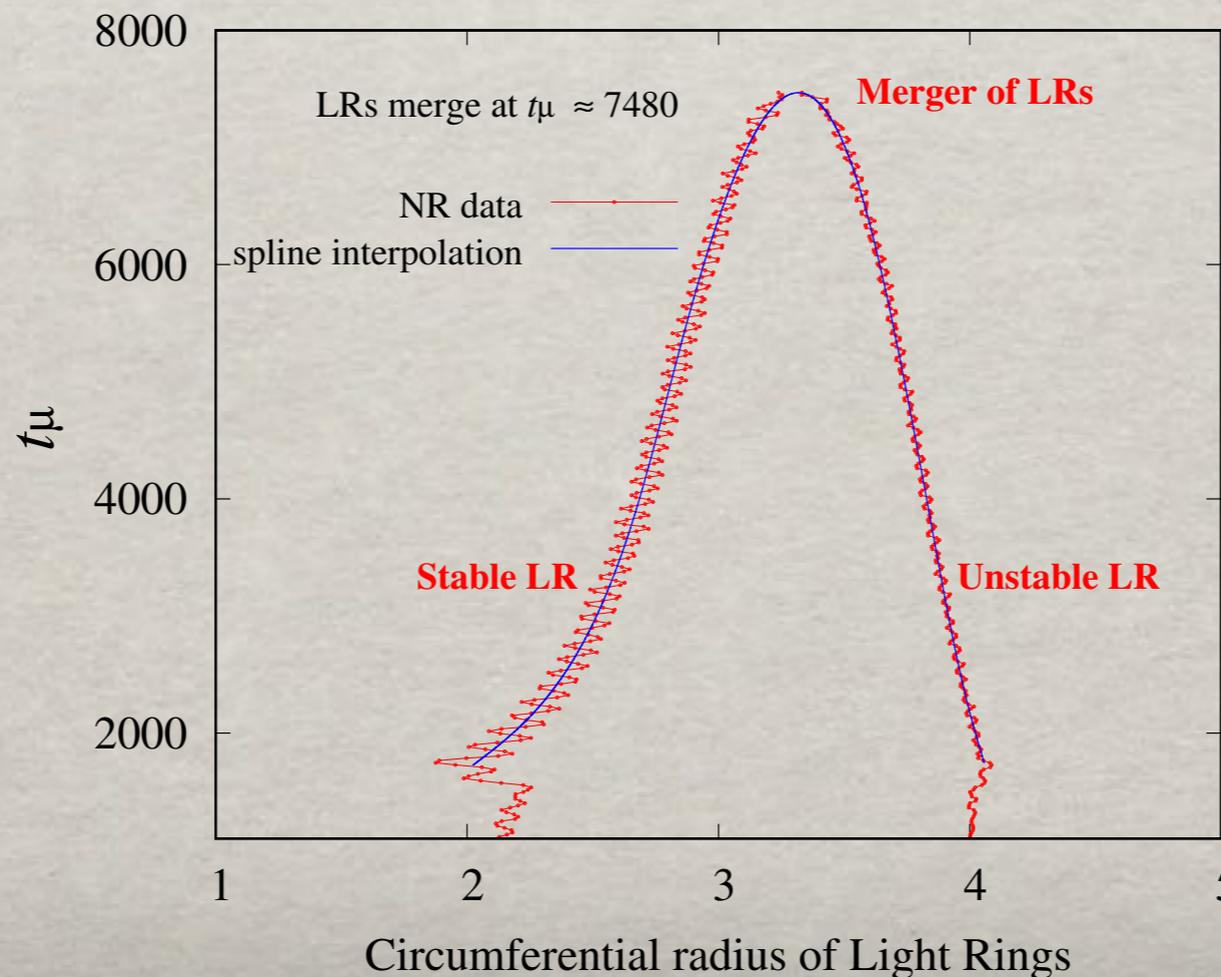
Migration into
non-ultracompact

There is an instability and there is a transition:



Collapse
into BH

Adiabatic
effective potential
allows following the LRs
during the evolution.



Migration into
non-ultracompact

Q3: If so, could such ECOs be astrophysically viable?

(Partial) R3:

The trapping instability associated to stable LRs is real in the concrete studied models
and
it needs not be too long lived, except near the critical solution,
leading to collapse or migration.

This questions the viability of ultracompact ECOs,
that have a plausible formation mechanism.

But,

only two families of examples; generality?

there are important open questions
(non-monotonic instability time scale, loss of axi-symmetry,
non-linear character of the instability, spatial correlation with stable LR,...).

On the fate of the Light Ring instability

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XV Black Holes Workshop, ISCTE, Lisbon, Dec 19th 2022



Thank you for your attention
Muito obrigado pela vossa atenção

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2207.13713
with
P. Cunha, E. Radu, N. Sanchis-Gual
(to appear in Phys. Rev. Lett.)

*“There is a crack in everything,
that is how the light gets in”*

L. Cohen