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Spinning black holes in modified gravity

A new approach and code

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Pedro G. S. Fernandes, University of Nottingham

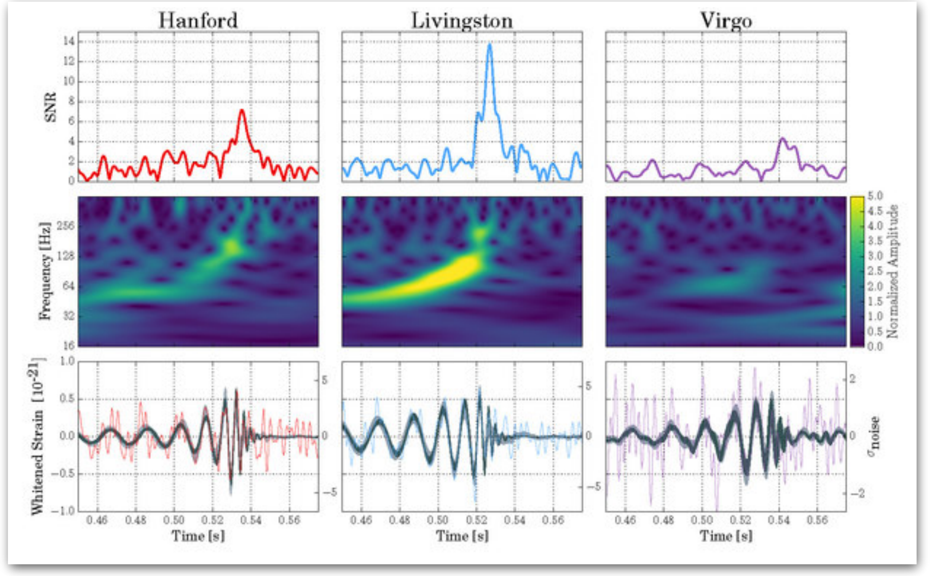
in collaboration with David J. Mulryne

Motivation

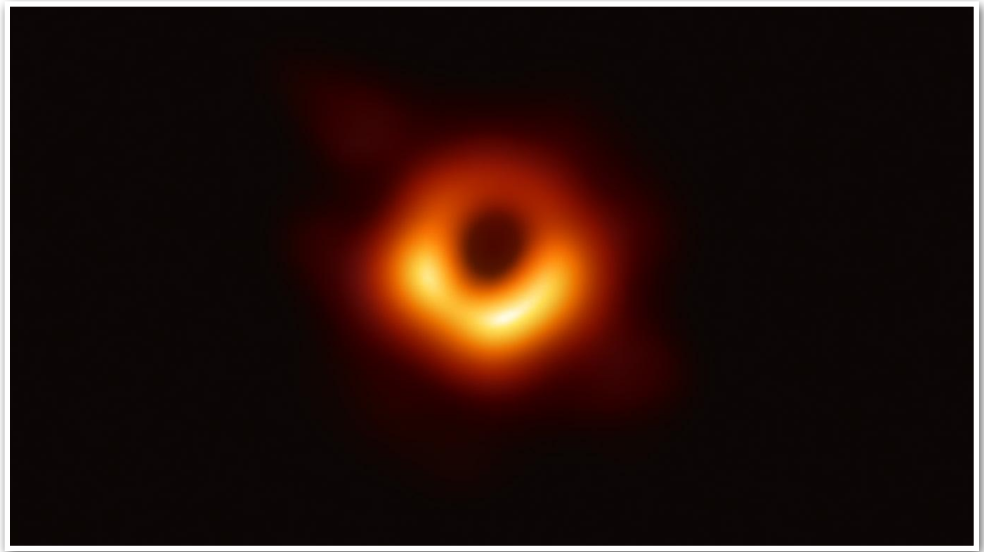
Spinning black holes in modified gravity

Motivation

- Problems with General Relativity
  - Dark matter, dark energy and other tensions in cosmology
  - Curvature singularities inside black holes, quantisation/renormalisation…
- GR has only started being tested in the strong field regime recently
  - Gravitational Waves
  - Black Hole Shadows



[LIGO collaboration, 2016]



[Event Horizon Telescope, 2019]

Spinning black holes in modified gravity

Motivation

- In GR the gravitational field of stationary black holes is described uniquely by the Kerr metric
- Any eventual deviation from Kerr would provide a smoking-gun for new physics
- To study what kind of deviations might occur in other theories we need solutions
- In modified theories of gravity the field equations are dramatically more complex, such that finding closed-form solutions is seemingly impossible. Numerical methods are needed

Spinning black holes in modified gravity

Motivation

- However, solving the field equations numerically is a challenging task as it requires solving a system of highly non-linear 2D elliptic PDE's

- Codes capable of doing so are scarce:

    - FIDISOL/CADSOL Package, used in many works. Unfortunately, it is not publicly available. Uses finite-difference methods. Estimated errors on solutions is of $\mathcal{O}\left(10^{-3}\right)$

    - XPDES code [Sullivan, Yunes and Sotiriou, 2020]. Publicly available. Uses finite-difference methods. Written in *C* language. Reports a typical maximum error of $\mathcal{O}\left(10^{-6}\right)$. Takes a few minutes to run and obtain a solution

Spinning black holes in modified gravity

Motivation

In this work we have developed another solver. We have several motivations to do so:

- We show that spectral methods are ideally suited to solving the type of equations at hand
- Our method and implementation is far more accurate
- Transparent and easy to adapt to new settings, being written in the modern programming language Julia
- Fast, converging on a solution in a matter of seconds even in highly complex settings
- Built-in routines to explore properties of these solutions

Physical Setup and Numerical Method

Physical Setup

Metric ansatz

- We will consider a stationary and axisymmetric metric ansatz written in quasi-isotropic coordinates

$$ds^2 = -f\mathcal{N}^2dt^2 + \frac{g}{f}\left[h\left(dr^2 + r^2d\theta^2\right) + r^2\sin^2\theta\left(d\varphi - \frac{W}{r}\left(1-\mathcal{N}\right)dt\right)^2\right],$$

$$\mathcal{N} \equiv \mathcal{N}(r) = 1 - \frac{r_H}{r}$$

- Contains four functions of r and θ: f, g, h, W

Numerical approach

Spectral expansion

- To solve the system of PDE's coming from the field equations we adopt a new radial coordinate

$$x = 1 - \frac{2r_H}{r}$$

mapping $r \in [r_H, \infty[\to x \in [-1,1]$.

- Use the following spectral expansion for the metric functions $\mathcal{F} = \{f, g, h, W\}$

$$\mathcal{F}^{(k)} = \sum_{i=0}^{N_x-1} \sum_{j=0}^{N_\theta-1} \alpha_{ij}^{(k)} T_i(x) \cos\left(2j\theta\right)$$

$T_i(x) \equiv i^{th}$ Chebyshev Polynomial

Numerical approach

Boundary conditions

- Axial symmetry and regularity on the axis, together with equatorial symmetry implies:

$$\partial_\theta f = \partial_\theta g = \partial_\theta h = \partial_\theta W = 0, \qquad \text{for} \quad \theta = 0, \pi/2$$

Note that with our spectral expansion these boundary conditions are automatically satisfied

- We also implement suitable boundary conditions at the horizon and at infinity

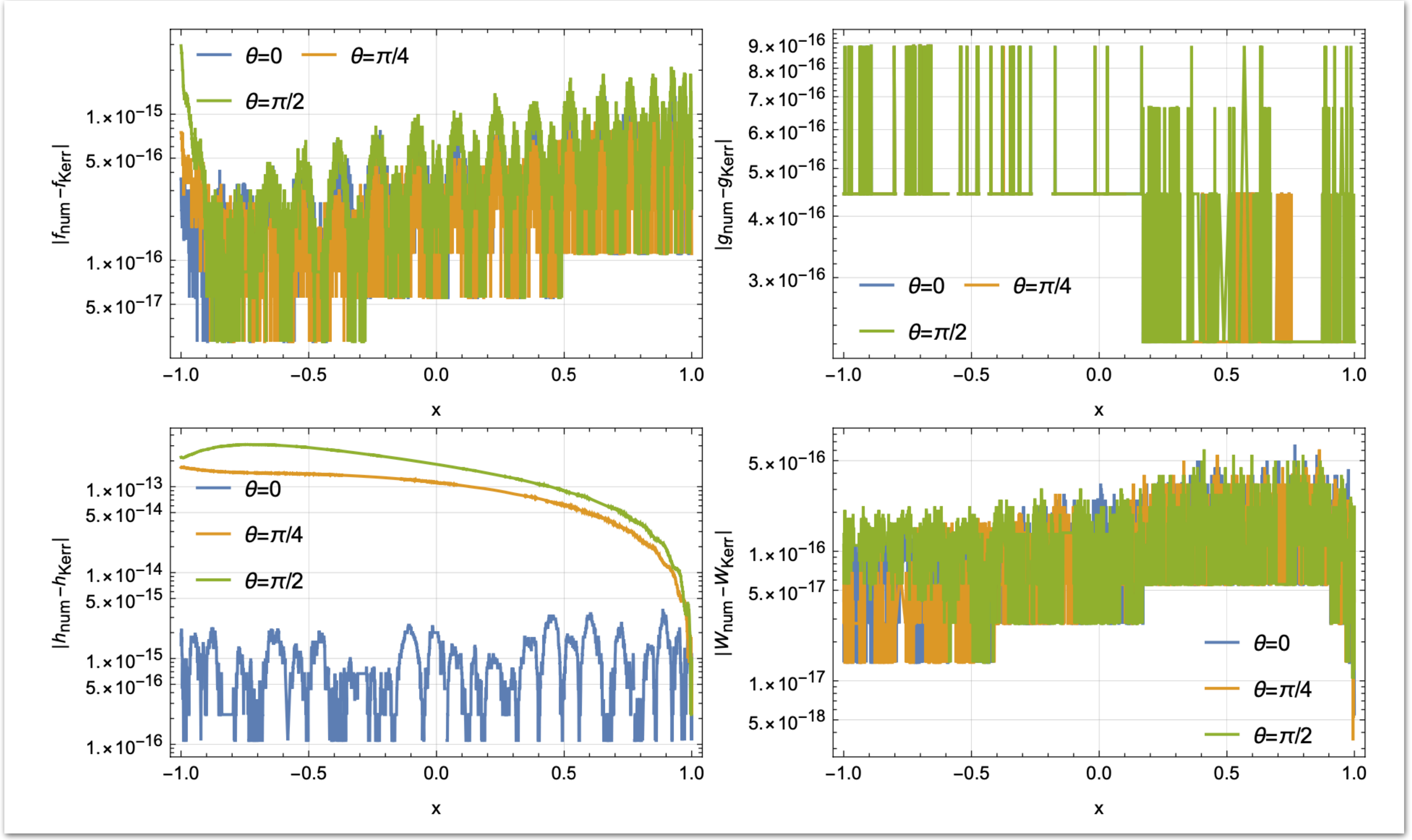
- The system is solved with the Newton-Raphson root-finding method

General Relativity

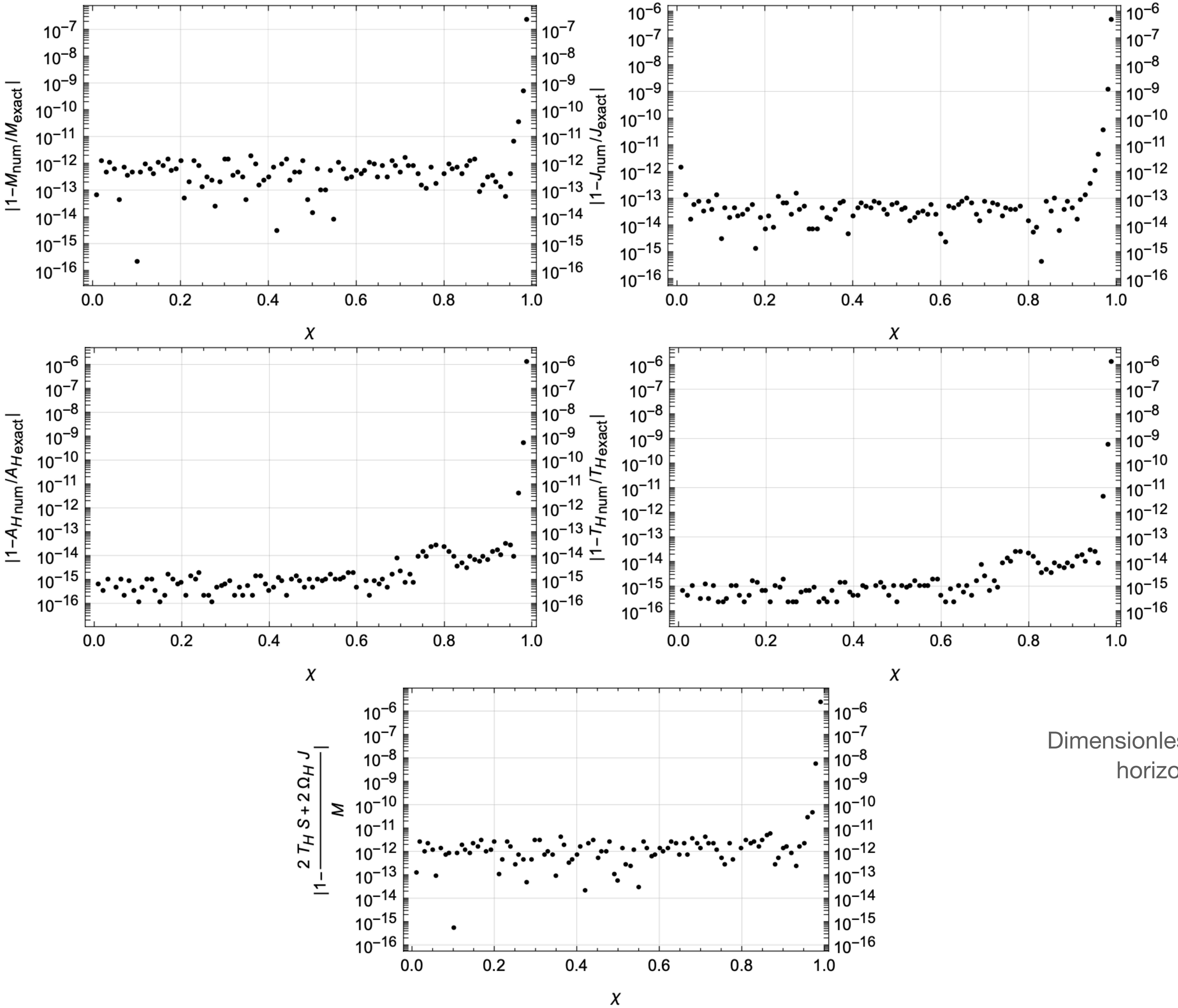
General Relativity

The Kerr Black Hole

To benchmark our code we solve the field equations of GR, $G_{\mu\nu} = 0$, and compare with the analytically known Kerr solution



$$N_x = 42, \quad N_\theta = 8, \quad \chi \equiv a/M = 0.6$$



Dimensionless spin χ on the horizontal axis!

Einstein-scalar-Gauss-Bonnet Gravity

Einstein-scalar-Gauss-Bonnet Gravity

The theory

$$\mathcal{S} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - (\nabla\phi)^2 + \frac{\alpha}{4}\xi(\phi)\,\mathcal{G}\right)$$

- We use the same spectral expansion for the scalar field, with suitable boundary conditions
- No analytical solutions are known. To test the accuracy of our solutions we use analytical relations
- For the coupling $\xi(\phi) = e^{\gamma\phi}$, solutions obey the Smarr-type relation

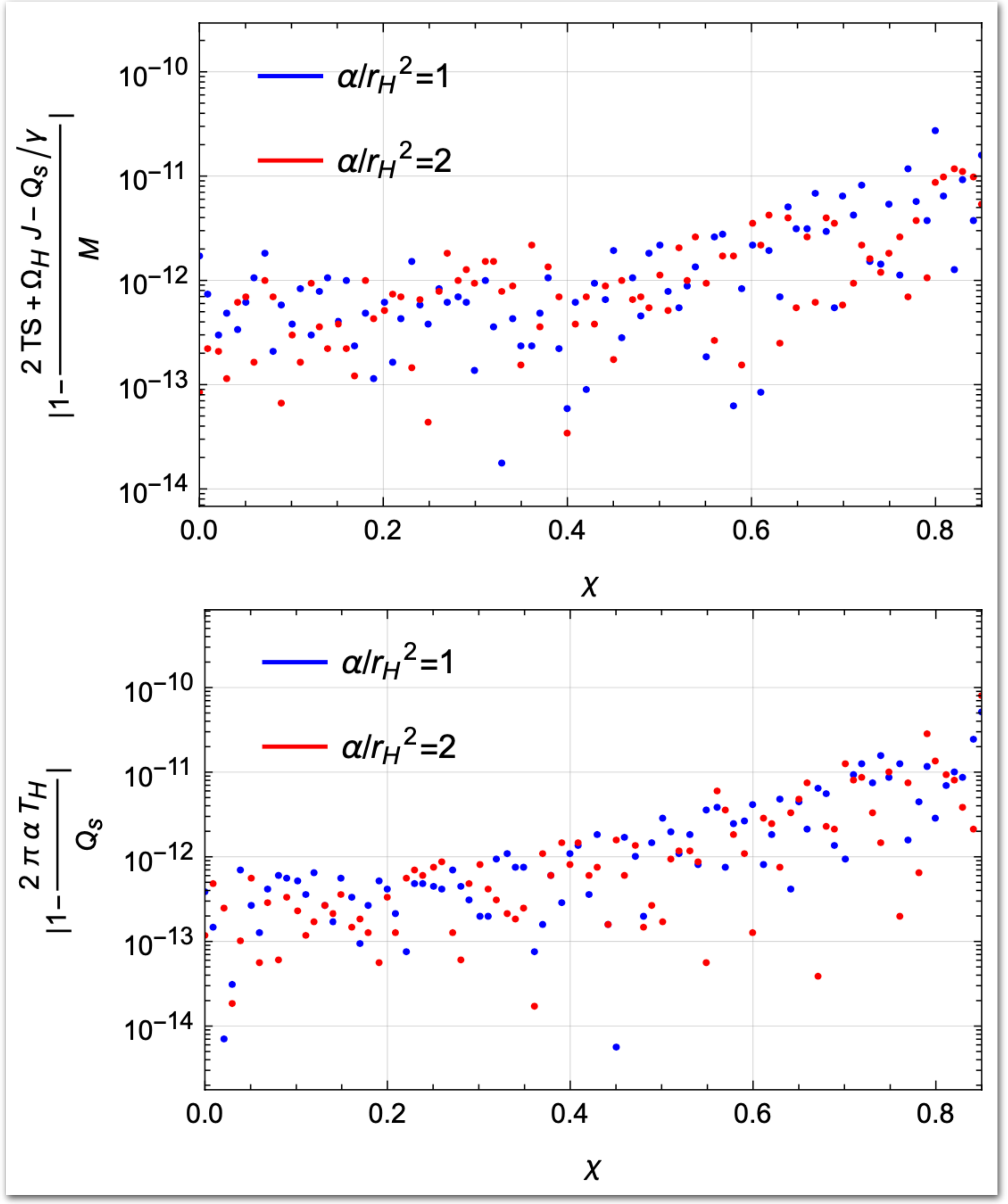
$$M + Q_s/\gamma = 2T_H S + 2\Omega_H J,$$

- For linear coupling $\xi(\phi) = \phi$, solutions obey the relation [Prabhu and Stein, 2018]

$$Q_s = 2\pi\alpha T_H$$

Einstein-scalar-Gauss-Bonnet Gravity

Accuracy of numerical solutions



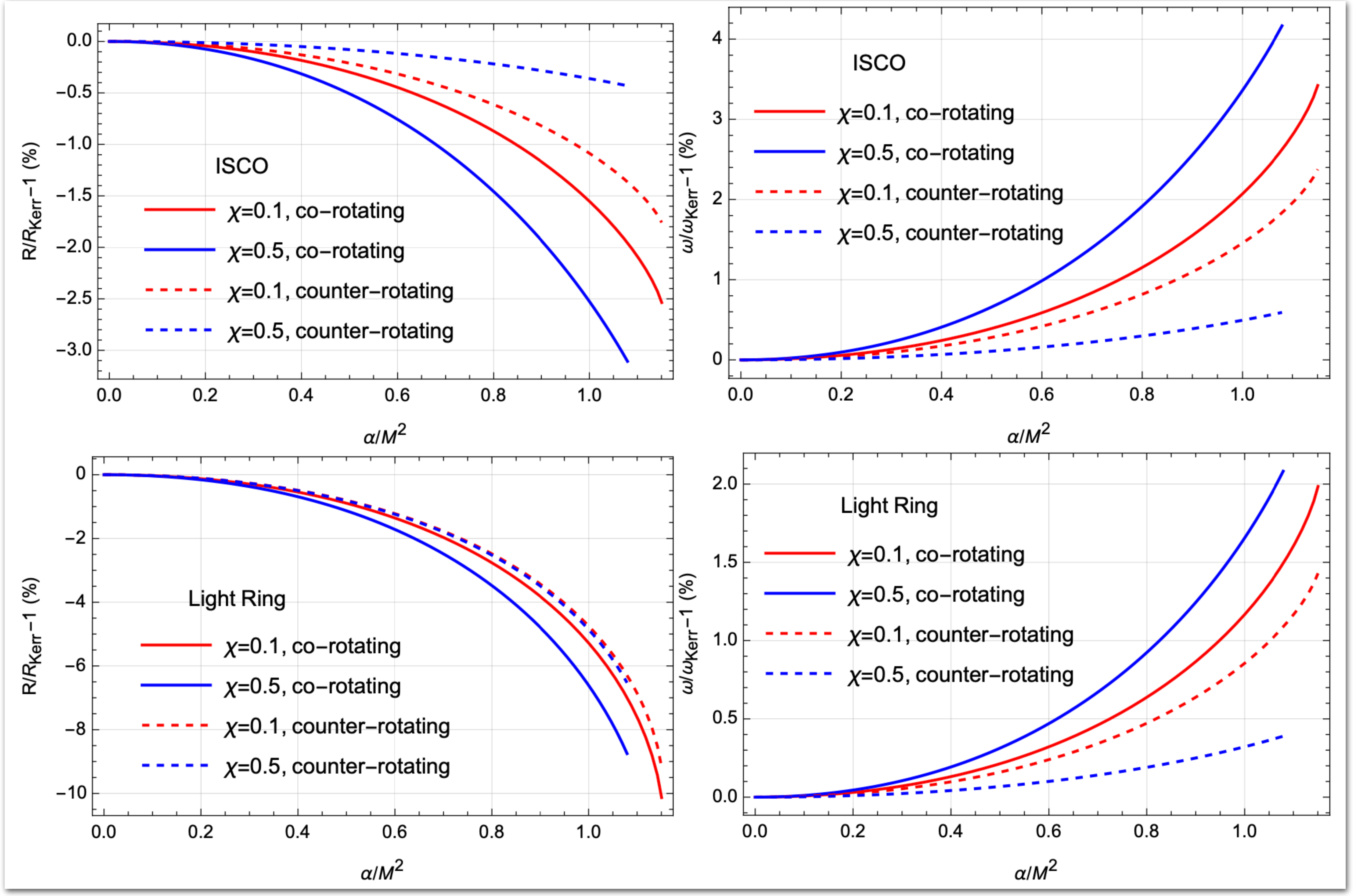

```
julia> include("Solver.jl")

Solver initated with:
α/rh^2=1.0
χ=0.6

Iter     f(x) inf-norm    Step 2-norm
------   --------------   --------------
     0     3.929702e+00              NaN
     1     7.231364e-03     4.183521e-04
     2     1.184027e-06     9.296671e-09
     3     3.623768e-13     3.904442e-16
     4     3.481659e-13     3.279524e-24

Success! Solution converged!
χ=0.6000000000000046
α/M^2=0.1597779841955325
Qs/M=0.0363666087161596
Smarr = 1.624256285026604e-13
```

About the code

The code

Built-in routines

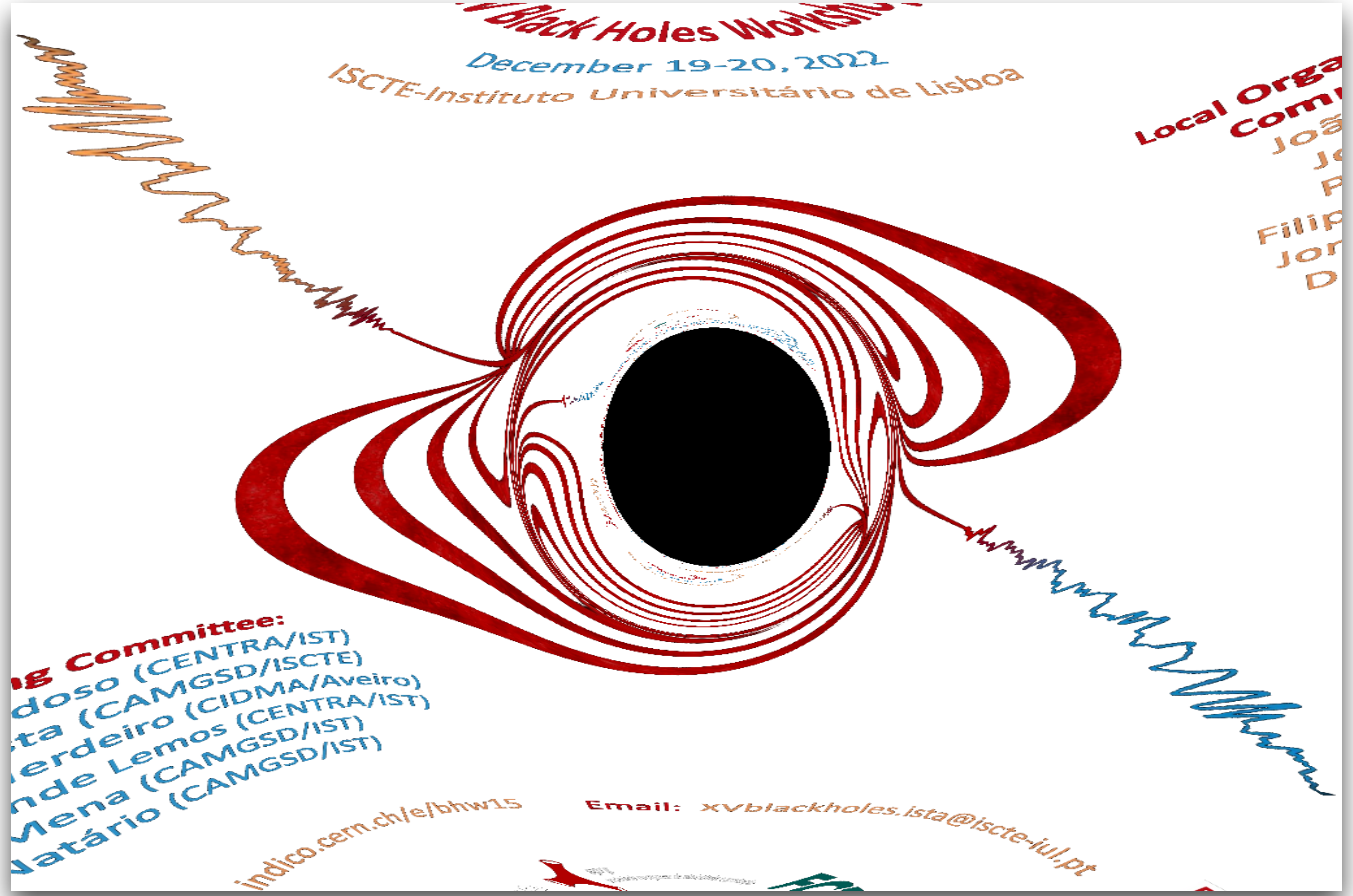
- Ergosphere
- Light rings
- ISCO
- Geodesic frequency at the ISCO
- Petrov type
- …



The code

Other studies

- Shadows
- Quasinormal modes
- Use as seeds for numerical evolutions



Shadow and gravitational lensing by a scalar-Gauss-Bonnet black hole with a dimensionless spin $\chi = 0.8$

The code

GitHub repository: https://github.com/pgsfernandes/SpinningBlackHoles.jl

SpinningBlackHoles.jl

SpinningBlackHoles is a pseudospectral solver tailor-made to obtain stationary and axisymmetric black hole solutions in modified theories of gravity.

If you use SpinningBlackHoles you are kindly asked to cite the following paper in any resulting works:

```
@article{SpinningBlackHoles,
    author = "Fernandes, Pedro G. S. and Mulryne, David J.",
    title = "{A new approach and code for spinning black holes in modified gravity}",
    eprint = "2212.07293",
    archivePrefix = "arXiv",
    primaryClass = "gr-qc",
    month = "12",
    year = "2022"
}
```

The user is also referred to this reference for details concerning the physics and mathematical setup involved.

Installation

The first step is to install Julia on your machine, if not already installed. This can be done by following the instructions on the official website.

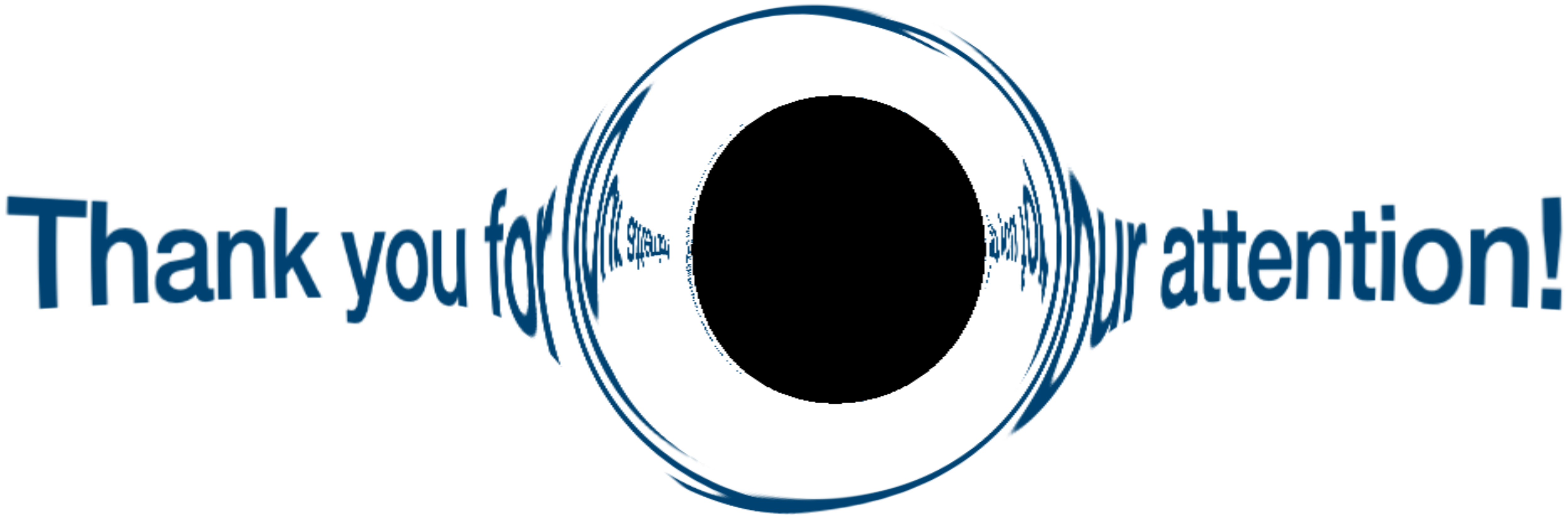
This package is not in the official registry. You can install it (and its dependencies) by first opening a Julia REPL

```
julia
```

and pressing the] key to enter the Pkg REPL. Afterwards, you must specify the URL to the repository:

```
pkg> add https://github.com/pgsfernandes/SpinningBlackHoles.jl
```

and that is it, the package is installed. To exit the Pkg REPL simply press backspace, and to exit Julia you can type *exit()*.



More details about the methods and results can be found in
arXiv:2212.07293