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# The effects of intrinsic spin of matter in relativistic cosmology and black holes formation

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XV Black Holes Workshop

Lisbon, 19 December 2022

# How to include intrinsic spin?

- The **Einstein-Cartan theory** is an alternative theory of gravity where the connection is not imposed to be the Levi-Civita connection.

  - Given a general metric compatible affine connection:

$$\nabla_\alpha U^\beta = \partial_\alpha U^\beta + C^\beta_{\alpha\gamma} U^\gamma$$

Christoffel Symbols: $$\Gamma^\gamma_{\alpha\beta} = \frac{1}{2}\left(C^\gamma_{\alpha\beta} + C^\gamma_{\beta\alpha}\right)$$

Torsion tensor: $$S_{\alpha\beta}{}^\gamma = \frac{1}{2}\left(C^\gamma_{\alpha\beta} - C^\gamma_{\beta\alpha}\right)$$

- The extra degrees of freedom can be used to consistently include the effects of intrinsic spin of matter in a relativistic gravity theory.

# The Weyl tensor

- In general, the Riemann tensor can be decomposed as a sum of the Ricci and the Weyl tensors.

  - In 4 dimensions:

$$R_{\alpha\beta\gamma\delta} = C_{\alpha\beta\gamma\delta} + R_{\alpha[\gamma}\, g_{\delta]\beta} - R_{\beta[\gamma}\, g_{\delta]\alpha} - \frac{1}{3} R\, g_{\alpha[\gamma}\, g_{\delta]\beta}\,,$$

where $g_{\alpha\beta}$ represents the metric tensor, $C_{\alpha\beta\gamma\delta}$ the Weyl tensor, $R_{\alpha\beta} \equiv R_{\alpha\mu\beta}{}^{\mu}$ the Ricci tensor and $R$ the Ricci scalar.

# The Weyl tensor

- In an orientable, four-dimensional space-time, the Weyl tensor itself can be decomposed as

$$C_{\alpha\beta\gamma\delta} = -\varepsilon_{\alpha\beta\mu}\varepsilon_{\gamma\delta\nu}E^{\nu\mu} - 2u_\alpha E_{\beta[\gamma}u_{\delta]} + 2u_\beta E_{\alpha[\gamma}u_{\delta]} - 2\varepsilon_{\alpha\beta\mu}H^{\mu}{}_{[\gamma}u_{\delta]} - 2\varepsilon_{\mu\gamma\delta}\bar{H}^{\mu}{}_{[\alpha}u_{\beta]}\,,$$

where

$$E_{\alpha\beta} = C_{\alpha\mu\beta\nu}\,u^\mu u^\nu$$

$$H_{\alpha\beta} = \frac{1}{2}\varepsilon_\alpha{}^{\mu\nu}C_{\mu\nu\beta\delta}u^\delta$$

$$\bar{H}_{\alpha\beta} = \frac{1}{2}\varepsilon_\alpha{}^{\mu\nu}C_{\beta\delta\mu\nu}u^\delta$$

# Field equations

- Given the Einstein-Hilbert action

$$A = \frac{1}{8\pi}\int (R - \Lambda)\sqrt{-g}d^4x + \int \mathcal{L}_{\mathrm{matter}}\sqrt{-g}d^4x$$

we find the field equations for the Einstein-Cartan theory

$$R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R + \Lambda g_{\alpha\beta} = 8\pi T_{\alpha\beta}\,,$$
$$S^{\alpha\beta\gamma} + 2g^{\gamma[\alpha}S^{\beta]}{}_{\mu}{}^{\mu} = -8\pi\Delta^{\alpha\beta\gamma}\,,$$

where $T_{\alpha\beta}$ represents the canonical stress-energy tensor, $\Delta^{\alpha\beta\gamma}$ the intrinsic hypermomentum and $\Lambda$ the cosmological constant.

# The cosmological model

➡ We were interested in studying solutions of the theory where:

- the space-time manifold is foliated by space-like, orientable 3-hypersurfaces;
- the 3-hypersurfaces are spatially homogeneous and isotropic;
- the space-time is permeated by a perfect fluid whose constituents have non-vanishing, randomly oriented intrinsic spin.

↳ The torsion tensor is given by $S_{\alpha\beta}{}^{\gamma} = \varepsilon_{\alpha\beta\mu} S^{\mu} u^{\gamma}$

# Field equations

- As in GR, the metric tensor is described by the Robertson Walker model.

- Contrary to GR, due to torsion (sourced by intrinsic spin) the Weyl tensor does not vanish, in general.

  - Intrinsic spin contributes to the tidal interaction.

  - Intrinsic spin may source gravitational waves.

  - Intrinsic spin restricts the allowed spatial geometry and topology.

# Influence of intrinsic spin in the geometry

**Theorem.** Let ${}^3R$ represent the Ricci scalar of each spacial 3-hypersurface. In the considered setup, ${}^3R \leq 0$. Moreover, each 3-hypersurface is flat, that is ${}^3R = 0$, if and only if there are no spin-induced gravitational waves.

# Influence of intrinsic spin in the topology

**Theorem.** In the considered setup, for a non-vanishing torsion tensor, the spacial 3-hypersurfaces cannot be closed.

# Gravitational waves

**Proposition.** In the considered setup, if the intrinsic spin density is a differentiable function of the energy density and the pressure of the fluid, then the traceless part and the trace of the magnetic part of the Weyl tensor $\bar{H}$ verify the following equations:

$$\tilde{\Box}\bar{H}_{\langle\alpha\beta\rangle} + {}^{3}R\,\bar{H}_{\langle\alpha\beta\rangle} + \frac{4}{3}\bar{H}_{\langle\alpha\beta\rangle}\left(\dot{\theta} - \frac{4}{3}\theta^{2}\right) = 0\,,$$

$$\frac{D}{d\tau}\bar{H}_{\alpha}{}^{\alpha} + \frac{4}{3}\bar{H}_{\alpha}{}^{\alpha}\theta = 0\,,$$

where $\tilde{\Box}\bar{H}_{\alpha\beta}$, defined in terms of the Levi-Civita connection, represents the wave operator, $\tau$ is the proper time measured by an observer comoving with the fluid and $\theta$ is the expansion scalar.

# Gravitational waves

- According to the previous Theorem
  - If the spacial hypersurfaces are Ricci flat, the only consistent solution is the trivial one.
  - If the spacial hypersurfaces are hyperbolic, the solutions are non-trivial.

# Gravitational waves

- The equation for the trace $\bar{H}_{\alpha}{}^{\alpha}$ can be readily integrated, finding

$$\bar{H}_{\alpha}{}^{\alpha} = \frac{C}{\ell^4}$$

where $\ell$ represents the scale factor.

- To integrate the wave equation, we assume that the space and proper time dependencies of $\bar{H}_{\langle\alpha\beta\rangle}$ are separable and employ a harmonic decomposition over the eigenfunctions of the covariant Laplace-Beltrami operator.

$$\bar{H}_{\langle\alpha\beta\rangle} = \sum_k \mathrm{h}_k^{(0)} Q_{\alpha\beta}^{(0),k} + \mathrm{h}_k^{(1)} Q_{\alpha\beta}^{(1),k} + \mathrm{h}_k^{(2)} Q_{\alpha\beta}^{(2),k} \,,$$

  - This type of decomposition is known as scalar-vector-tensor decomposition due to some properties of the harmonics.

# Gravitational waves

- From the field equations, we find that $\bar{H}_{\langle\alpha\beta\rangle}$ is characterized only by the *scalar* harmonics, such that

$$\bar{H}_{\langle\alpha\beta\rangle} = \sum_k \mathrm{h}_k^{(0)} Q_{\alpha\beta}^{(0),k} \, .$$

- Using the ansatz $\mathrm{h}_k^{(0)} = \frac{f_k}{\ell^4}$, where $f_k$ are arbitrary smooth functions, replacing the harmonic decomposition above in the wave equation yields

$$\frac{d^2 f_k}{dt^2} - 9\ell\mathrm{H}\frac{df_k}{dt} + k^2 f_k = 0\,,$$

where $t$ , defined as $dt = \ell^{-1} d\tau$ represents the conformal time and $\mathrm{H} \equiv \frac{\dot{\ell}}{\ell}$ is the Hubble parameter.

# Gravitational waves

$$\frac{d^2 f_k}{dt^2} - 9\ell\mathrm{H}\frac{df_k}{dt} + k^2 f_k = 0\,,$$

- To continue the analysis in detail, a matter model must be imposed.
  - However, assuming $f_k$ and its derivatives up to second order are bounded, we can consider two regimes:

$$\frac{k^2}{\ell\mathrm{H}} \gg 1 \quad \text{or} \quad \frac{k^2}{\ell\mathrm{H}} \ll 1$$

# Gravitational waves

$$\frac{d^2 f_k}{dt^2} - 9\ell \mathrm{H} \frac{df_k}{dt} + k^2 f_k = 0\,,$$

For $\frac{k^2}{\ell \mathrm{H}} \gg 1$ the solutions for the harmonic coefficients are:

$$\mathrm{h}_k^{(0)} \approx \frac{c_1 \cos(kt) + c_2 \sin(kt)}{\ell^4}\,,$$

where the integration constants $c_1$ and $c_2$ might change with the harmonic index $k$.

# Gravitational waves

$$\frac{d^2 f_k}{dt^2} - 9\ell \mathrm{H} \frac{df_k}{dt} + k^2 f_k = 0\,,$$

As for the other regime, first notice that the quantity in the 2º term is proportional to the comoving Hubble radius:

$$R_{\mathrm{H}} = (\ell \mathrm{H})^{-1}\,,$$

such that $\dot{R}_{\mathrm{H}} = -\ddot{\ell} R_{\mathrm{H}}^2\,.$

Hence, the regime $\frac{k^2}{\ell \mathrm{H}} \ll 1$ represents the late-time behavior of the lower order modes of the spin induced gravitational waves in an accelerating expanding Universe.

In this case, the solutions for the harmonic coefficients read

$$\mathrm{h}_k^{(0)} \approx \frac{\text{const.}}{\ell^4}$$

# Influence of intrinsic spin in singularities

- A. Trautman, “Spin and torsion may avert gravitational singularities.”, Nature Phys. Sci. **242**, 7 (1973).

- J. Stewart and P. Hájíček, “Can spin avert singularities?”, Nature Phys. Sci. **244**, 96 (1973).

→ These articles asserted, for the first time, that the presence of intrinsic spin sourced torsion could prevent the formation of singularities.

# Influence of intrinsic spin in singularities

- In M. Tsamparlis, "Methods for deriving solutions in generalized theories of gravitation: The Einstein-Cartan theory", Phys. Rev. D **24**, 1451 (1981), it was shown that under certain conditions the symmetries of the metric tensor are also symmetries of the torsion.

  - Under those condition, of course, it was asserted that there are no consistent cosmological solutions of the Einstein-Cartan theory with intrinsic spin sourced torsion.

  - Consequently, the study of intrinsic spin sourced torsion was mostly disregarded in the literature.

# Influence of intrinsic spin in singularities

- However, the key condition in that article is imposed *ad hoc*: it does not follow from the field equations.

  - As it was shown, we can in fact have intrinsic spin sourced torsion in a space-time permeated by an isotropic and homogenous fluid.

  - These result re-opens the possibility to consider the effects of intrinsic spin of matter in the formation of singularities due to gravitational collapse of massive compact objects.

# Conclusion

- The Einstein-Cartan theory allows solutions of the Robertson-Walker type for an intrinsic spin sourced torsion.
- The allowed solutions can not be foliated by closed spacial hypersurfaces, limiting the allowed topology.
- In the case where the spacial hypersurfaces are hyperbolic, there are spin induced gravitational waves.
- In the case where the spacial hypersurfaces are Ricci flat, the solution differs from the GR ones in the tidal forces.