A (axial)vector meson extended quark-meson model to describe quark matter in the core of neutron star

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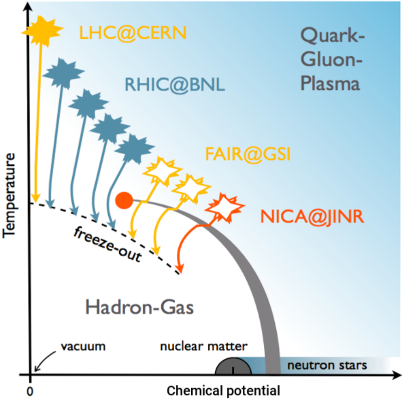
Based on: P. Kovács, Zs. Szép, Gy. Wolf, PRD93 (2016) 114014, P. Kovács, J. Takátsy, J. Schaffner-Bielich, Gy. Wolf, Phys. Rev. D105 (2022) 103014

Overview

Dense strongly interacting matter



What is the phase diagram and EOS for dense strongly interacting matter?

At $\mu = 0$: lattice and experiments (STAR/PHENIX and ALICE).

For $\mu >> 0$ no precise theory and no heavy ion experiment.

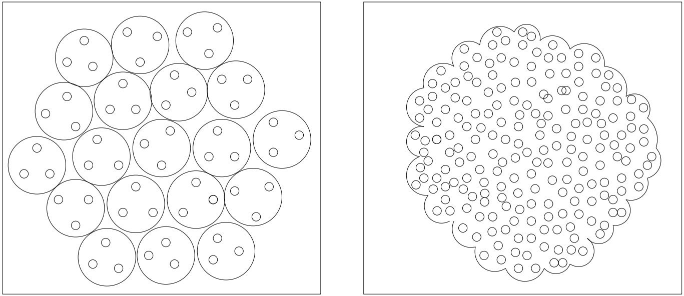
Dense matter at T=0

Is there a phase transition at T=0? If yes, at which density? What are the phases?

$V_{proton} = 2.48 fm^3$ (with R_{em}), densest packing with spheres: 74% $\rightarrow \rho_{max} = 0.3 fm^{-3} \approx 1.8\rho_0$ by maximal packing

Reid hard core potential: $R_{hc} \approx 0.5 R_{em} \rightarrow \rho_{max} \approx 15\rho_0$ at hard core overlap

heavy ion collisions: no sharp transition until 2-3 ρ_0

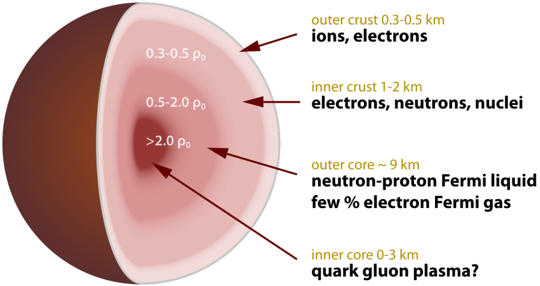


Neutron Stars a challenge and a possibility

Neutron stars are contain the densest matter in the Universe (Laboratory for strong interaction)

What is the structure of neutron stars (what are the constituents), hybrid stars? Superfluids?

Strange matter is unlikely, three-body repulsion for Λ, Σ (Weise)



Tolman-Oppenheimer-Volkoff (TOV) equation

Solving the Einstein's equation for spherically symmetric case and homogeneous matter $\longrightarrow$ TOV eqs.:

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{[p(r)+\varepsilon(r)]\,[M(r)+4\pi r^3 p(r)]}{r[r-2M(r)]} \quad (1)$$

with

$$\frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2 \varepsilon(r)$$

These are integrated numerically for a specific $p(\varepsilon)$

- For a fixed ε_c central energy density Eq. (1) is integrated until $p = 0$
- Varying ε_c a series of compact stars is obtained (with given M and R)
- Once the maximal mass is reached, the stable series of compact stars ends

EOS

hadronic matter - soft: SFHo

(Steiner, A. W., Hempel, M., Fischer, T. Astrophys. J. 774 (2013) 17)

hadronic matter - stiff: DD2

(Typel, S., Ropke, G., Klahn, T., Blaschke, D., Wolter, H. Phys. Rev. C81, (2010) 015803 and Hempel, M., Schaffner-Bielich, J. Nucl. Phys. A837 (2010) 210)

Quark matter: Quark-meson model

Hadron-quark crossover with polynomial interpolation ($\rho = \rho_B$):

$$\begin{aligned} \varepsilon(\rho_B) &= \varepsilon_{hadronic}(\rho_B) \qquad \rho_B < \rho_{BL}, \\ \varepsilon(\rho_B) &= \sum_{k=0}^{5} C_k \rho_B^k \qquad \rho_{BL} \leq \rho_B \leq \rho_{BU} \\ \varepsilon(\rho_B) &= \varepsilon_{qm}(\rho_B) \qquad \rho_{BU} < \rho_B. \end{aligned}$$

C_k is determined by the requirement that the energy density, ε and its first two derivatives with respect to ρ_B, pressure and sound velocity is continuous at the boundaries.

Quark-meson model

- Build an effective model having the right global symmetry pattern. (Extended Quark-Meson model)
- Compare the thermodynamics of the model with lattice at $\mu = 0$
- Extrapolate to high μ.
- particle content (pseudo)scalar and (axial)vector nonets, quarks
- problems of mixing in scalar and axial vector sectors

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, D.H. Rischke, Phys. Rev. D87 (2013) 014011

Determination of the parameters of the Lagrangian

16 unknown parameters ($m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F, g_V, g_A$) $\longrightarrow$ Determined by the min. of χ^2:

$$\chi^2(x_1,\ldots,x_N) = \sum_{i=1}^{M}\left[\frac{Q_i(x_1,\ldots,x_N) - Q_i^{\exp}}{\delta Q_i}\right]^2,$$

where $(x_1,\ldots,x_N) = (m_0, \lambda_1, \lambda_2, \ldots)$, $Q_i(x_1,\ldots,x_N)$ calculated from the model, while $Q_i^{\exp}$ taken from the PDG

multiparametric minimalization $\longrightarrow$ MINUIT

- PCAC $\to$ 2 physical quantities: f_π, f_K
- Tree-level masses $\to$ 15 physical quantities:
  $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- Decay widths $\to$ 12 physical quantities:
  $\Gamma_{\rho\to\pi\pi}, \Gamma_{\Phi\to KK}, \Gamma_{K^\star\to K\pi}, \Gamma_{a_1\to\pi\gamma}, \Gamma_{a_1\to\rho\pi}, \Gamma_{f_1\to KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\to K\pi},$
  $\Gamma_{f_0^L\to\pi\pi}, \Gamma_{f_0^L\to KK}, \Gamma_{f_0^H\to\pi\pi}, \Gamma_{f_0^H\to KK}$
- $T_c = 155$ MeV from lattice

T/μ_B dependence of the condensates

Ω: grand canonical potential

$$\frac{\partial \Omega}{\partial \Phi} = \frac{\partial \Omega}{\partial \bar{\Phi}}\Big|_{\varphi_N=\phi_N,\varphi_S=\phi_S} = 0$$

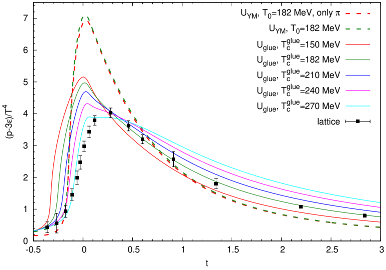
$$\frac{\partial \Omega}{\partial \phi_N} = \frac{\partial \Omega}{\partial \phi_S}\Big|_{\Phi,\bar{\Phi}} = 0, \qquad \text{(after the SSB)}$$

$$\frac{\partial \Omega}{\partial v_0} = 0 \qquad \text{(only contribute at} \quad \mu > 0)$$

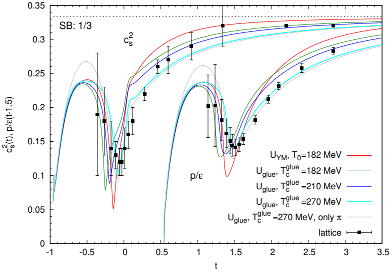
five order parameters:
$(\phi_N, \phi_S, \Phi, \bar{\Phi}, v_0) \rightarrow$ five T/μ-dependent equations

Observables

interaction measure

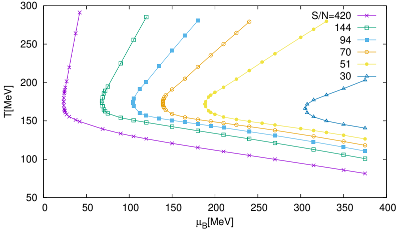


speed of sound



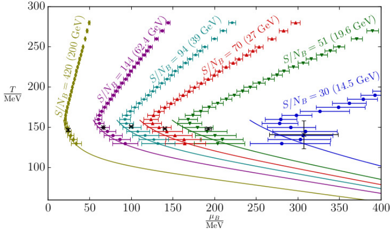
Isentropic trajectories in the $T - \mu_B$ plane

our model, where $\mu_B^{CEP} > 850\text{MeV}$



lattice (analytic continuation)

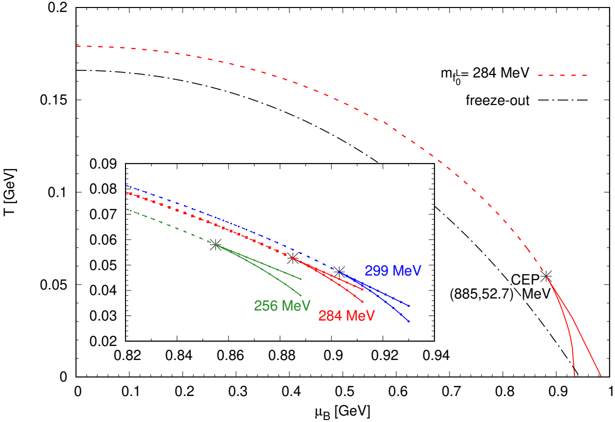
Günther *et al.*, arXiv:1607.02493



same qualitative behavior of the isentropic trajectories for $\mu_B \leq 400$ MeV

$\Longrightarrow$ indication that in the lattice result there is no CEP in this region of μ_B

$T - \mu_B$ phase diagram



- we use U^{glue} with $T_c^{glue} = 210$ MeV
- freeze-out curve from Cleymans et al., J.Phys.G32, S165
- Curvature at $\mu_B = 0$ $\kappa = 0.0193$, close to the lattice value $\kappa = 0.020(4)$

(Cea *et al.*,PRD93, 014507)

Bayesian inference

Unsetted parameters:

$m_\sigma, g_v, \quad \overline{\rho_B} \equiv 0.5(\rho_{BL} + \rho_{BU}), \quad \Gamma \equiv 0.5(\rho_{BU} - \rho_{BL})$

$290 \text{ MeV} \leq m_\sigma \leq 700 \text{ MeV}$

$0 \leq g_v \leq 10$

$2\rho_0 \leq \overline{\rho_B} \leq 5\rho_0$

$\rho_0 \leq \Gamma \leq 4\rho_0$ with the constraint: $\rho_{BL} = \overline{\rho_B} - \Gamma > \rho_0$

Bayes theorem:

θ is a parameter set, $p(\theta)$ is the prior probability for θ, $p(data|\theta)$ is the probability that for given θ, the data is measured. Then

$$p(\theta|data) = \frac{p(data|\theta)p(\theta)}{p(data)}$$

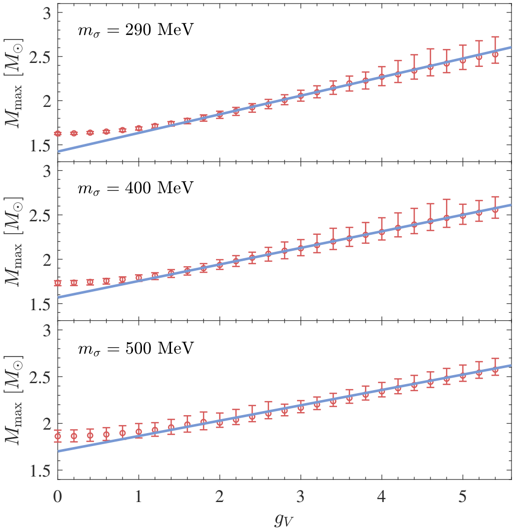
$p(data)$ is a normalization constant. We assume $p(\theta)$ is uniform in the allowed hypersurface. For independent observations:

$$p(data|\theta) = p(M_{max}|\theta)p(NICER|\theta)p(\overline{\Lambda}|\theta)$$

Data

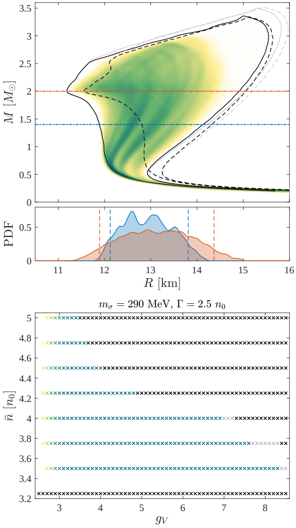
- M_{max}: PSR J0348+0432 with a mass $2.01 \pm 0.04 M_{\odot}$, PSR J1614-2230 with a mass $1.908 \pm 0.016 M_{\odot}$
- EOS should converge to perturbative QCD keeping $v_s < 1$
- NICER: (M,R) values for PSR J0030+0451, PSR J0740+6620 (Miller)
- GW170817: tidal deformability, $70 < \Lambda(1.4 M_{\odot}) < 720$ Abbot (2019)
- GW170817: no prompt black hole formation, timescales: $2.01 \geq M_{TOV}/M_{\odot} \geq 2.16$ (Rezzola)

g_v dependence of the M(R) curve

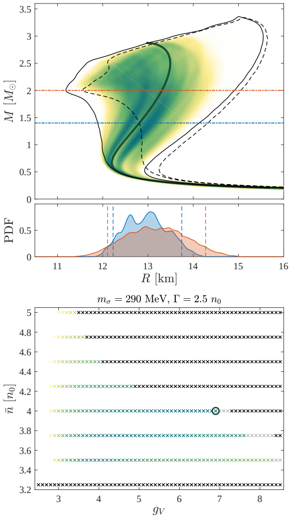


The errorbars are obtained by varying $\overline{\rho}$ and Γ.

Bayesian analysis

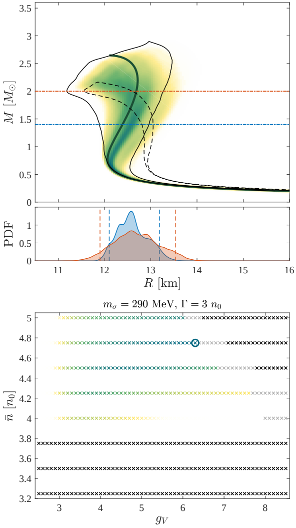


prior (including M_{max})

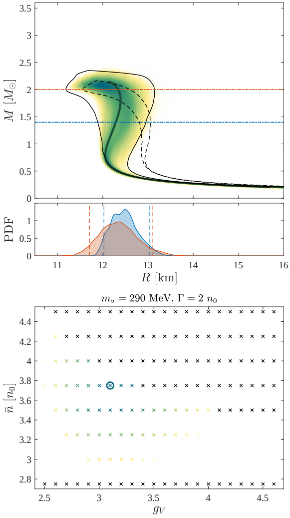


prior + NICER

Bayesian analysis



prior + NICER+ Λ



prior + NICER+ Λ + M_{TOV}

Summary and Conclusions

- Our model can reproduce the lattice calculations at $\mu = 0$
- With our model we can fulfill the present astronomical constraints
- hadronic and quark phase ought to be handled with the same model to drop ad-hoc parameters
- understand neutron star cooling
- damping of r-mode oscillations

Meson fields - pseudoscalar and scalar meson nonets

$$\Phi_{PS} = \sum_{i=0}^{8} \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix} (\sim \bar{q}_i \gamma_5 q_j)$$

$$\Phi_{S} = \sum_{i=0}^{8} \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{pmatrix} (\sim \bar{q}_i q_j)$$

Particle content:

Pseudoscalars: $\pi(138), K(495), \eta(548), \eta'(958)$

Scalars: $a_0(980 \text{ or } 1450), K_0^\star(800 \text{ or } 1430)$,

(σ_N, σ_S) : 2 of $f_0(500, 980, 1370, 1500, 1710)$

Included fields - vector meson nonets

$$V^\mu = \sum_{i=0}^{8} \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star+} \\ \rho^- & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star-} & \bar{K}^{\star 0} & \omega_S \end{pmatrix}^\mu$$

$$A^\mu = \sum_{i=0}^{8} b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

Particle content:

Vector mesons: $\rho(770), K^\star(894), \omega_N = \omega(782), \omega_S = \phi(1020)$

Axial vectors: $a_1(1230), K_1(1270), f_{1N}(1280), f_{1S}(1426)$

Lagrangian (2/1)

$$\mathcal{L}_{\text{Tot}} = \text{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2$$
$$- \frac{1}{4}\text{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \text{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + \text{Tr}[H(\Phi + \Phi^\dagger)]$$
$$+ c_1(\det \Phi + \det \Phi^\dagger) + i\frac{g_2}{2}(\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\})$$
$$+ \frac{h_1}{2}\text{Tr}(\Phi^\dagger \Phi)\text{Tr}(L_\mu^2 + R_\mu^2) + h_2 \text{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2h_3 \text{Tr}(L_\mu \Phi R^\mu \Phi^\dagger).$$
$$+ \bar{\Psi} i \not{\partial} \Psi - g_F \bar{\Psi}(\Phi_S + i\gamma_5 \Phi_{PS})\Psi + g_V \bar{\Psi}\gamma^\mu \left(V_\mu + \frac{g_A}{g_V}\gamma_5 A_\mu\right)\Psi$$
$$+\text{Polyakov loops}$$

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, D.H. Rischke, Phys. Rev. D87 (2013) 014011

Lagrangian (2/2)

where

$$D^\mu\Phi = \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu) - ieA_e^\mu[T_3,\Phi]$$

$$\Phi = \sum_{i=0}^{8}(\sigma_i + i\pi_i)T_i, \quad H = \sum_{i=0}^{8} h_i T_i \qquad T_i : U(3) \text{ generators}$$

$$R^\mu = \sum_{i=0}^{8}(\rho_i^\mu - b_i^\mu)T_i, \quad L^\mu = \sum_{i=0}^{8}(\rho_i^\mu + b_i^\mu)T_i$$

$$L^{\mu\nu} = \partial^\mu L^\nu - ieA_e^\mu[T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu[T_3, L^\mu]\}$$

$$R^{\mu\nu} = \partial^\mu R^\nu - ieA_e^\mu[T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu[T_3, R^\mu]\}$$

$$\bar\Psi = (\bar u, \bar d, \bar s)$$

non strange – strange base:

$$\varphi_N = \sqrt{2/3}\varphi_0 + \sqrt{1/3}\varphi_8,$$

$$\varphi_S = \sqrt{1/3}\varphi_0 - \sqrt{2/3}\varphi_8, \qquad \varphi \in (\sigma_i, \pi_i, \rho_i^\mu, b_i^\mu, h_i)$$

broken symmetry: non-zero condensates $\langle\sigma_N\rangle, \langle\sigma_S\rangle \longleftrightarrow \phi_N, \phi_S$

Spontaneous symmetry breaking

Interaction is approximately chiral symmetric, spectra not, so SSB:

$$\sigma_{N/S} \to \sigma_{N/S} + \phi_{N/S} \qquad \phi_{N/S} \equiv < \sigma_{N/S} >$$

For tree level masses we have to select all terms quadratic in the new fields. Some of the terms include mixings arising from terms like $\mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)]$:

$$\begin{aligned}
\pi_N - a^\mu_{1N} &: \quad -g_1\phi_N a^\mu_{1N}\partial_\mu\pi_N, \\
\pi - a^\mu_1 &: \quad -g_1\phi_N(a^{\mu+}_1\partial_\mu\pi^- + a^{\mu 0}_1\partial_\mu\pi^0) + \mathrm{h.c.}, \\
\pi_S - a^\mu_{1S} &: \quad -\sqrt{2}g_1\phi_S a^\mu_{1S}\partial_\mu\pi_S, \\
K_S - K^\star_\mu &: \quad \frac{ig_1}{2}(\sqrt{2}\phi_S - \phi_N)(\bar{K}^{\star 0}_\mu\partial^\mu K^0_S + K^{\star -}_\mu\partial^\mu K^+_S) + \mathrm{h.c.}, \\
K - K^\mu_1 &: \quad -\frac{g_1}{2}(\phi_N + \sqrt{2}\phi_S)(K^{\mu 0}_1\partial_\mu\bar{K}^0 + K^{\mu+}_1\partial_\mu K^-) + \mathrm{h.c.}.
\end{aligned}$$

Diagonalization→ Wave function renormalization

Inclusion of the vector meson- quark interaction

$$\mathcal{L}_{\mathrm{Vq}} = -g_V\sqrt{6}\bar{\Psi}\gamma_\mu V_0^\mu \Psi$$
$$V_0^\mu = \frac{1}{\sqrt{6}}\mathrm{diag}(v_0 + \frac{v_8}{\sqrt{2}}, v_0 + \frac{v_8}{\sqrt{2}}, v_0 - \sqrt{2}v_8) \qquad (2)$$

vector fields: like Walecka model, nonzero expectation values are built up at nonzero chemical potential. For simplicity

$$< v_0^\mu >= v_0\delta^{0\mu}, \quad < v_8^\mu >= 0$$

Modification of the grand canonical potential:

$$\Omega(T=0,\mu_q,g_v) = \Omega(T=0,\tilde{\mu}_q,gv=0) - \frac{1}{2}m_v^2 v_0^2,$$

with $\quad \tilde{\mu}_Q = \mu_q - g_v v_0$

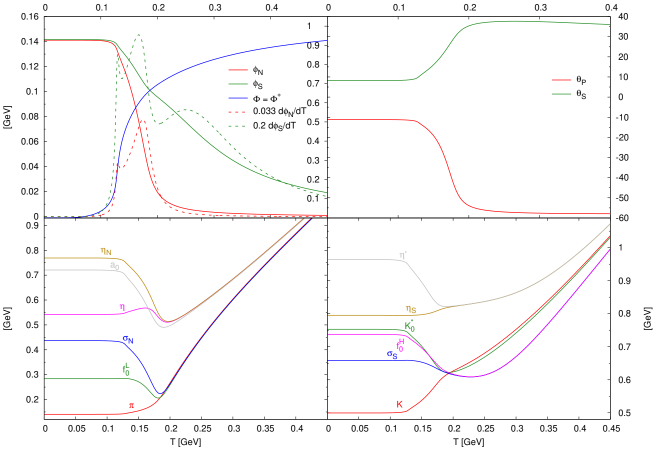
Features of our approach

- D.O.F's: scalar, pseudoscalar, vector, axial vector nonets,
- Polyakov loop variables, $\Phi, \bar{\Phi}$ with U^{glue}
- u,d,s constituent quarks, $(m_u = m_d)$
- mesonic fluctuations included in the grand canonical potential:

$$\Omega(T, \mu_q) = -\frac{1}{\beta V} ln(Z)$$

- Fermion vacuum and thermal fluctuations
- Five order parameters $(\phi_N, \phi_S, \Phi, \bar{\Phi}, v_0) \rightarrow$ five T/μ-dependent equations

With low mass scalars, $m_{f_0^L} = 300$ MeV



chiral symmetry is restored at high T as the chiral partners (π, f_0^L), (η, a_0) and $(K, K_0^\star)$, (η', f_0^H) become degenerate

$U(1)_A$ symmetry is not restored, as the axial partners (π, a_0) and (η, f_0^L) do not become degenerate

Thermodynamical Observables

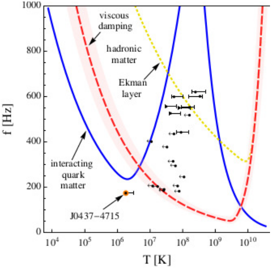
We include mesonic thermal contribution to p for (π, K, f_0^l)

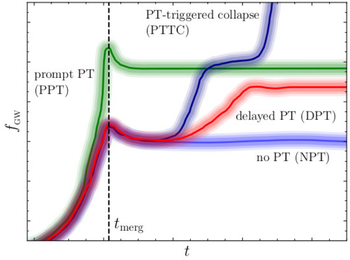
$$\Delta p(T) = -nT \int \frac{d^3q}{(2\pi)^3} ln(1 - e^{-\beta E(q)}), \quad E(q) = \sqrt{q^2 + m^2}$$

- pressure: $p(T, \mu_q) = \Omega_H(T = 0, \mu_q) - \Omega_H(T, \mu_q)$
- entropy density: $s = \frac{\partial p}{\partial T}$
- quark number density: $\rho_q = \frac{\partial p}{\partial \mu_q}$
- energy density: $\epsilon = -p + Ts + \mu_q \rho_q$
- scaled interaction measure: $\frac{\Delta}{T^4} = \frac{\epsilon - 3p}{T^4}$
- speed of sound at $\mu_q = 0$: $c_s^2 = \frac{\partial p}{\partial \epsilon}$

How to observe phases inside neutron stars

- Static observables: M, R Λ
  EOS is needed, difficult to differentiate between sructures, but there are (and will be even better) constraints, R depends substantially on the crust (complicated)
- r-mode oscillation: gravitational wave probe the volume, viscosities
- cooling neutrino emission, triangle inequality
- merger postmerger GW emission





Polyakov loops in Polyakov gauge

Polyakov loop variables: $\Phi(\vec{x}) = \frac{\mathrm{Tr}_c L(\vec{x})}{N_c}$ and $\bar{\Phi}(\vec{x}) = \frac{\mathrm{Tr}_c \bar{L}(\vec{x})}{N_c}$ with

$$L(x) = \mathcal{P} \exp\left[i \int_0^\beta d\tau A_4(\vec{x}, \tau) \right]$$

$\longrightarrow$ signals center symmetry ($\mathbb{Z}_3$) breaking at the deconfinement

low T: confined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle = 0$
high T: deconfined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle \neq 0$

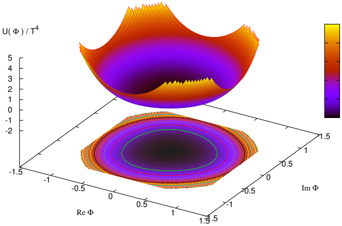
Polyakov gauge: the temporal component of the gauge field is time independent and can be gauge rotated to a diagonal form in the color space

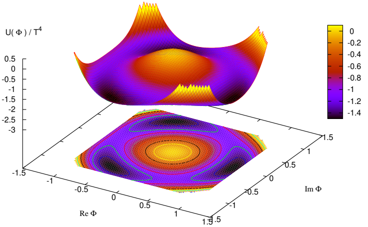
Effects of the gauge fields:

- In this gauge the effect of the gauge field on the quarks acts like an imaginary chemical potential
  $\longrightarrow$ modified quark distribution function.
- Polyakov potential: $\mathcal{U}\left(\Phi, \bar{\Phi}\right)$ models the free energy of a pure gauge theory, parameters are fitted to the pure gauge lattice data

Polyakov loop potential

"Color confinement"	"Color deconfinement"
$\langle\Phi\rangle = 0 \longrightarrow$ no breaking of $\mathbb{Z}_3$	$\langle\Phi\rangle \neq 0 \longrightarrow$ spontaneous breaking of $\mathbb{Z}_3$
one minimum	minima at $0, 2\pi/3, -2\pi/3$
	one of them spontaneously selected





from H. Hansen et al., PRD75, 065004 (2007)

Effects of Polyakov loops on FD statistics

Inclusion of the Polyakov loop modifies the Fermi-Dirac distribution function

$$f(E_p-\mu_q) \longrightarrow f_\Phi^+(E_p) = \frac{\left(\bar{\Phi}+2\Phi e^{-\beta\left(E_p-\mu_q\right)}\right) e^{-\beta\left(E_p-\mu_q\right)} + e^{-3\beta\left(E_p-\mu_q\right)}}{1+3\left(\bar{\Phi}+\Phi e^{-\beta\left(E_p-\mu_q\right)}\right) e^{-\beta\left(E_p-\mu_q\right)} + e^{-3\beta\left(E_p-\mu_q\right)}}$$

$$f(E_p+\mu_q) \longrightarrow f_\Phi^-(E_p) = \frac{\left(\Phi+2\bar{\Phi} e^{-\beta\left(E_p+\mu_q\right)}\right) e^{-\beta\left(E_p+\mu_q\right)} + e^{-3\beta\left(E_p+\mu_q\right)}}{1+3\left(\Phi+\bar{\Phi} e^{-\beta\left(E_p+\mu_q\right)}\right) e^{-\beta\left(E_p+\mu_q\right)} + e^{-3\beta\left(E_p+\mu_q\right)}}$$

$\Phi, \bar{\Phi} \to 0 \Longrightarrow f_\Phi^\pm(E_p) \to f(3(E_p \pm \mu_q))$ $\quad \Phi, \bar{\Phi} \to 1 \Longrightarrow f_\Phi^\pm(E_p) \to f(E_p \pm \mu_q)$

three-particle state appears: mimics confinement of quarks within baryons

at $T = 0$ there is no difference between models with and without Polyakov loop

Thank you for your attention!