

Consequences of Length
Discretization on Metric
Tensor and Geodesic Equation

Abdel Nasser Tawfik

a.tawfik@cern.ch

Collaborators A. Diab, S. El Shinawy and E. Abou El Dahab

Abstract:

Generalized Uncertainty Principle suggests length discretization. Its possible impacts on Einstein Field Equations shall be studied. We focus on metric tensors, line metric, and geodesic equation.

Questions

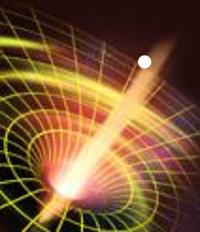
- **Gravity Quantization requires spacetime quantization or fundamental limits on length and momentum.**

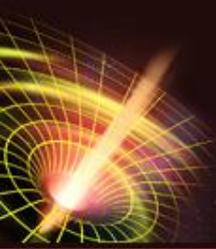
$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- **Various models predict minimum measurable length (space discretization).**
- **Minimum measurable length and/or maximal momentum => generalized uncertainty principle (GUP)**

$$\Delta p \Delta q \geq \frac{1}{2}[1 + \langle \hat{p}^2 \rangle] = \frac{1}{2}[1 + (\Delta p)^2 + \langle \hat{p} \rangle^2]$$

- **$T_{\mu\nu}$ in EFE (rhs) is quantizable!**
- **Would GUP suggest quantization for lhs of EFE?!**
- **How look like the metric tensor, line metric and geodesic equation?**





Results

GUP

$$[\hat{x}_i, \hat{p}_j] \geq \delta_{ij} i \hbar (1 + \beta p^2), \quad \hat{x}_i = \hat{x}_{0i}(1 + \beta p^2),$$
$$\hat{p}_j = \hat{p}_{0j},$$

where $p^2 = g_{ij} p^{0i} p^{0j}$ and g_{ij} is the Minkowski spacetime metric tensor

Metric tensor

$$\tilde{g}_{\mu\nu} = g_{AB} \frac{\partial x^A}{\partial \zeta^\mu} \frac{\partial x^B}{\partial \zeta^\nu} \simeq g_{ab} \left[\frac{\partial x^a}{\partial \zeta^\mu} \frac{\partial x^b}{\partial \zeta^\nu} + \beta \frac{\partial \dot{x}^a}{\partial \zeta^\mu} \frac{\partial \dot{x}^b}{\partial \zeta^\nu} \right] \simeq (1 + \beta \ddot{x}^\lambda \ddot{x}_\lambda) g_{\mu\nu},$$

Line metric

$$d\tilde{s}^2 = g_{\mu\nu} (dx^\mu dx^\nu + \beta^2 d\dot{x}^\mu d\dot{x}^\nu) = (1 + \beta^2 \ddot{x}^\lambda \ddot{x}_\lambda) ds^2$$

Geodesic eq.

$$\ddot{x}^\delta + \beta g_{\mu\nu, \alpha} |\ddot{x}^\delta|^2 = (-\Gamma_{\mu\nu}^\delta + \beta g_{\mu\nu} x^{(4)\delta}) \dot{x}^\mu \dot{x}^\nu,$$

where $x^{(4)\delta} = (\ddot{\ddot{x}})^\delta$, $|\ddot{x}^\delta|^2 = g_{\delta\alpha} \ddot{x}^\delta \ddot{x}^\alpha$ and $g_{\mu\nu, \alpha} = \partial g_{\mu\nu} / \partial x^\alpha$.