Effective Field Theory with Genuine Many-body Forces and Tidal Effect in Neutron Stars

M. Razeira[1], D. Hadjimichef[2], M.V.T. Machado[2], F. Kopp[2], G. Volkmer[2], B. Bodmann[1], G.A. Degrazia[3] and C.A.Z. Vasconcellos[2]

UNIPAMPA[1], UFRGS[2], UFSM[3]

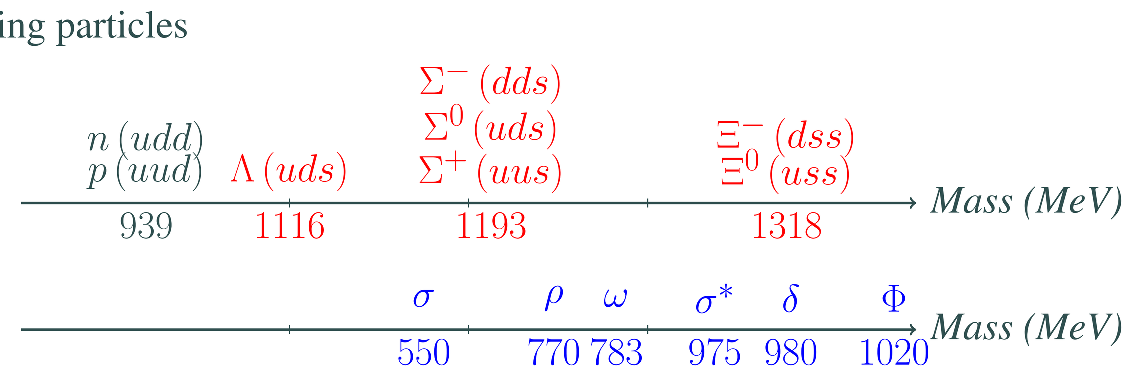
mrazeira@unipampa.edu.br

Abstract

In this contribution we combined our predictions for the tidal parameter with recent gravitational-wave observation of merging system of binary neutron stars of the event GW170817 with quasi- universal relations between the maximum mass of rotating and nonrotating neutron stars. Our results indicate that predictions of the tidal parameter represent an useful constraint of the EoS of neutron star matter.

Effective Field Theory

In our theory the neutron star matter will be composed by the following particles



For our investigations we use the following Lagrangean density[1, 2, 3]

$$\begin{aligned}\mathcal{L} = & \sum_B \bar{\psi}_B \left[i\gamma_\mu\partial^\mu - g_{\omega B}\gamma_\mu\omega^\mu - \frac{1}{2}g_{\varrho B}\gamma_\mu \boldsymbol{\tau}\cdot\boldsymbol{\varrho}^\mu - g_{\phi B}\gamma_\mu\phi^\mu - M_B^* \right]\psi_B \\ & + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - {m_\sigma}^2\sigma^2) + \frac{1}{2}(\partial_\mu\sigma^*\partial^\mu\sigma^* - {m_{\sigma^*}}^2\sigma^{*2}) \\ & - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega{}^2\omega_\mu\omega^\mu - \frac{1}{4}\phi_{\mu\nu}\phi^{\mu\nu} + \frac{1}{2}m_\phi{}^2\phi_\mu\phi^\mu \\ & - \frac{1}{4}\boldsymbol{\varrho}_{\mu\nu}\cdot\boldsymbol{\varrho}^{\mu\nu} + \frac{1}{2}m_\varrho^2\boldsymbol{\varrho}_\mu\cdot\boldsymbol{\varrho}^\mu + \frac{1}{2}(\partial_\mu\boldsymbol{\delta}\cdot\partial^\mu\boldsymbol{\delta} - {m_\delta}^2\boldsymbol{\delta}^2) \\ & + \sum_l \bar{\psi}_l\,(i\gamma_\mu\partial^\mu - m_l)\,\psi_l\end{aligned}$$

where

$$M_B^* = M_B - g_{\sigma B}\, m_B^*\,\sigma - g_{\sigma^* B}\, m_B^*, \sigma^* - \frac{1}{2}g_{\delta B}\, m_B^*\, \boldsymbol{\tau}\cdot\boldsymbol{\delta}$$

is the barion effective mass, with

$$m_B^* = \left(1 + \frac{g_{\sigma B}\sigma + g_{\sigma^* B}\sigma^* + \frac{1}{2}g_{\delta B}\boldsymbol{\tau}\cdot\boldsymbol{\delta}}{\alpha M_B}\right)^{-\alpha}$$

By using the minimum action principle and mean field approximation we obtain the EoS of the neutron star matter ($p = p(\varepsilon)$) in parametric form

$$\begin{aligned}\varepsilon = & \frac{1}{2}m_\sigma^2\sigma_0^2 + \frac{1}{2}m_{\sigma*}^2\sigma_0^{*2} + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_\rho^2\varrho_{03}^2 + \frac{1}{2}m_\delta^2\delta_3^2 \\ & + \frac{1}{\pi^2}\sum_B\int_0^{k_{F,B}} k^2dk\sqrt{k^2 + M_B^{*2}} + \frac{1}{\pi^2}\sum_l\int_0^{k_{F.l}} k^2dk\sqrt{k^2 + m_l^2}\end{aligned}$$

$$\begin{aligned}p = & -\frac{1}{2}m_\sigma^2\sigma_0^2 - \frac{1}{2}m_{\sigma*}^2\sigma_0^{*2} + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_\rho^2\varrho_{03}^2 - \frac{1}{2}m_\delta^2\delta_3^2 \\ & + \frac{1}{3}\frac{1}{\pi^2}\sum_B\int_0^{k_{F,B}} \frac{k^4dk}{\sqrt{k^2 + M_B^{*2}}} + \frac{1}{3}\frac{1}{\pi^2}\sum_l\int_0^{k_{F,l}} \frac{k^4dk}{\sqrt{k^2 + m_l^2}}\end{aligned}$$

Coupling Constants and NS Properties

The meson-nucleon coupling constants are adjusted to reproduce the equilibrium properties of symmetric nuclear matter

$$\varepsilon/\rho - M_N = -16\,MeV \;\;, a_4 = 32.5\,MeV$$

at $\rho_0 = 0.15 fm^{-3}$

Model α	$g_{\sigma N}$	$g_{\omega N}$	$g_{\rho N}$	M_N^*/M_N	K (MeV)	L (MeV)	M_{max} ($M_\odot$)	R (km)	$R_{1.4}$ (km)
0.046	10.515	11.471	8.998	0.665	286	74.16	2.07	12.600	13.566
0.050	10.430	11.294	9.031	0.674	276	74.67	2.04	12.553	13.507
0.055	10.326	11.081	9.069	0.684	266	75.26	2.00	12.427	13.427
0.058	10.265	10.959	9.090	0.690	260	75.58	1.98	12.328	13.392
1.000	7.859	6.680	9.508	0.850	225	83.88	1.47	11.861	12.256

Table 1: Nucleon-meson coupling constants ($g_{\sigma N}, g_{\omega N}, g_{\rho N}$) and nucleon effective mass in units of nucleon mass (M_N^*/M_N) and nuclear matter incompressibility (K) symmetry energy slope (L) and neutron star maximum mass (M_{max}) and its radius (R) and the radius of the canonical neutron star with $1.4\,M_\odot$.

The hidden mesons do not couple with the nucleons, so

$$g_{\sigma^* N} = g_{\Phi\, N} = 0\,.$$

We choose the following value for the delta meson-nucleon coupling constant[4]

$$g_{\delta\, N} = 3.1\,.$$

In order to obtain the coupling constants of the hyperons, we fit the depth of the nuclear potential of hypernuclei in saturated nuclear matter. The corresponding values are given by

$$U_\Lambda^N(\rho_0) = -28MeV, U_\Sigma^N(\rho_0) = +30MeV, U_\Xi^N(\rho_0) = -18MeV$$

See for instance Ref. [5].

In our effective theory the hyperon nuclear potential in nuclear matter is given

$$U_Y^N(\rho_0) = -g_{\sigma Y}^\star\,\sigma_0 + g_{\omega Y}\,\omega_0\,,\;\; Y = \Lambda\,,\Sigma\,,\Xi$$

so we have obtained

Model α	$g_{\sigma\Lambda}$	$g_{\sigma\Sigma}$	$g_{\sigma\Xi}$
0.046	5.745	3.798	2.920
0.050	5.668	3.687	2.881
0.055	5.576	3.554	2.834
0.058	5.523	3.478	2.808
1.000	3.859	0.848	2.053

Table 2: Hyperon-meson coupling constants ($g_{\sigma\Lambda}, g_{\sigma\Sigma}, g_{\sigma\Xi}$).

By using $SU(6)$ symmetry we obtain

$$\begin{aligned}& \frac{1}{2}g_{\delta\,\Sigma} = g_{\delta\,\Xi} = g_{\delta\,N} \\ & \frac{1}{3}g_{\omega\,\Lambda} = \frac{1}{2}g_{\omega\,\Sigma} = \frac{1}{3}g_{\omega\,\Xi} = g_{\omega\,N} \\ & \frac{1}{2}g_{\rho\,\Sigma} = g_{\rho\,\Xi} = g_{\rho\,N} \\ & g_{\rho\,\Lambda} = 0 \\ & 2\,g_{\sigma^*\,\Lambda} = 2\,g_{\sigma^*\,\Sigma} = g_{\sigma^*\,\Xi} = -\frac{2\sqrt{2}}{3}g_{\sigma\,N} \\ & 2\,g_{\Phi\,\Lambda} = 2\,g_{\Phi\,\Sigma} = g_{\Phi\,\Xi} = -\frac{2\sqrt{2}}{3}g_{\omega\,N}\end{aligned}$$

Tidal Deformability of Neutron Stars

The tidal deformability parameter λ of nonrotating neutron star in the leading order perturbation is given by[6]

$$\lambda = -\frac{Q_{i\,j}}{\mathcal{E}_{i\,j}}$$

$$\Lambda = \frac{2\,k_2}{3\,C}\,.$$

In this expression Q_{ij} is the induced quadrupole moment of a star binary, and $\mathcal{E}_{ij}$ is a static external quadrupolar tidal field of the companion star. The tidal deformability parameter depends on the EoS via both the NS radius and a dimensionless quantity k_2, called the second *Love* number. Λ is the dimensionless version of λ, and C is the compactness parameter ($C = M/R$). The electric Love number is given by

$$\begin{aligned}k_2 = & \frac{8}{5}(1-2\,C)^2\,C^5[2\,C\,(y-1) - y + 2]\left\{2\,C\,(4\,(y+1)\,C^4\right. \\ & + \;(6\,y - 4)\,C^3 + (26 - 22\,y)\,C^2 + 3\,(5\,y - 8)\,C - 3\,y + 6) \\ & \left. - \;3\,(1-2\,C)^2\,(2\,C(y-1) - y + 2)\,\log\left(\frac{1}{1-2\,C}\right)\right\}^{-1}.\end{aligned}$$

The value of $y \equiv y(R)$ can be computed by solving the following first order differential equation:

$$r\frac{d\,y(r)}{d\,r} + y(r)^2 + y(r)\,F(r) + r^2\,Q(r) = 0$$

with

$$\begin{aligned}F(r) = & \frac{r - 4\,\pi\,r^3\,[\varepsilon(r) - p(r)]}{r - 2\,M(r)} \\ Q(r) = & \frac{4\,\pi\,r\left(5\,\varepsilon(r) + 9\,p(r) + \frac{\varepsilon(r)+p(r)}{\partial p(r)/\partial\varepsilon(r)} - \frac{6}{4\,\pi\,r^2}\right)}{r - 2\,M(r)} \\ & - \;4\left[\frac{M(r) + 4\,\pi\,r^3\,p(r)}{r^2\,(1 - 2\,M(r)/r)}\right]\end{aligned}$$

To calculate the tidal deformability of a single star, this equation must be integrate simultaneously with the Tolman-Oppenheimer-Volkoff[7] Equations

$$\frac{dM(r)}{dr} = 4\pi r^2\varepsilon(r)\,,$$

$$\frac{dp}{dr} = -\frac{M(r)\varepsilon(r)}{r^2}\left(1 + \frac{p(r)}{\varepsilon(r)}\right)\left(1 + \frac{4\pi r^3 p(r)}{M(r)}\right)\left(1 - \frac{2M(r)}{r}\right)^{-1}$$

for a given EoS and from the boundary conditions $p(0) = p_c$, $M(0) = 0$, and $y(0) = 2$, where p_c, $M(0)$ and $y(0)$ are the central pressure, mass and dimensionless quantity. To obtain the tidal Love number, we solve this set of equations for a given EoS of the star at $r = 0$. The value of $r = R$ where the pressure vanishes defines the surface of the star. Thus, at each central density we can uniquely determine a mass M, a radius R, and a tidal Love number k_2 of the isolated neutron star using the chosen EoS.

The weighted dimensionless tidal deformability of binary neutron stars of mass M_1 and M_2 is given by

$$\begin{aligned}\tilde{\Lambda} = & \frac{8}{3}[(1 + 7\eta - 31\eta^2)(\Lambda_1 + \Lambda_2) + \sqrt{1 - 4\eta} \\ & \times\;(1 + 9\eta - 11\eta^2)(\Lambda_1 - \Lambda_2)]\end{aligned}$$

with tidal correction

$$\begin{aligned}\delta\tilde{\Lambda} = & \frac{1}{2}\left[\sqrt{1-4\eta}\left(1 - \frac{13272}{1319}\eta + \frac{8944}{1319}\eta^2\right)(\Lambda_1 + \Lambda_2)\right. \\ & \left. + \;\left(1 - \frac{15910}{1319}\eta + \frac{32850}{1319}\eta^2 + \frac{3380}{1319}\eta^3\right)(\Lambda_1 - \Lambda_2)\right]\end{aligned}$$

where

$$\eta = \frac{M_1\,M_2}{(M_1 + M_2)^2}$$

is the symmetric mass ratio.

Recently, aLIGO and VIRGO detectors measured a value of $\tilde{\Lambda}$ in the event GW170817 [8] and it is noticed that the values of $\tilde{\Lambda} \leq 800$ in the low-spin case and $\tilde{\Lambda} \leq 700$ in the hight-spin case are within the 90% credible interval.

Finally

$$\begin{aligned}\mathcal{M}_c = & \,(M_1\,M_2)^{3/5}\,(M_1 + M_2)^{-1/5} \\ \mathcal{R}_c = & \,2\,\mathcal{M}_c\,\tilde{\Lambda}^{-1/5}\end{aligned}$$

are the chirp mass and the chirp radius of binary neutron star system.

The precise mass measurements of the neutron stars were reported in Refs.[9, 10] . However, until now no observations has been confirmed regarding the radius of the most massive neutron stars. Recently, aLIGO and VIRGO measured a chirp mass of $1.188^{+0.004}_{-0.002}\,M_\odot$ with very good precision. So we can easily calculate the chirp radius $\mathcal{R}_c$ of the binary system. We find $9.551 \leq \mathcal{R}_c \leq 10.152$ for equal and unequal-mass of the binary system, see table 3.

Results

M_1 ($M_\odot$)	M_2 ($M_\odot$)	R_1 (km)	R_2 (km)	Λ_1	Λ_2	$\tilde{\Lambda}$	$\delta\tilde{\Lambda}$	$\mathcal{M}_c$ ($M_\odot$)	$\mathcal{R}_c$ (km)
1.10	1.10	13.225	13.225	3863.25	3863.25	4185.19	0.000	0.96	10.152
1.37	1.23	13.466	13.361	1174.85	2089.62	1700.49	121.005	1.13	9.999
1.37	1.30	13.466	13.417	1174.85	1560.25	1467.51	52.027	1.16	9.969
1.37	1.37	13.466	13.466	1174.85	1174.85	1272.76	0.000	1.19	9.934
1.43	1.23	13.508	13.361	890.93	2089.62	1485.12	156.946	1.16	9.963
1.43	1.30	13.508	13.417	890.93	1560.25	1280.35	90.124	1.19	9.932
1.43	1.37	13.508	13.466	890.93	1174.85	1109.10	38.986	1.22	9.896
1.43	1.43	13.508	13.508	890.93	890.93	965.18	0.000	1.25	9.857
1.50	1.23	13.543	13.361	679.80	2089.62	1303.38	181.642	1.18	9.925
1.50	1.37	13.543	13.466	679.80	1174.85	971.05	67.847	1.25	9.856
1.50	1.43	13.543	13.508	679.80	890.93	843.78	29.478	1.28	9.815
1.56	1.23	13.571	13.361	521.29	2089.62	1149.18	197.853	1.21	9.886
1.56	1.30	13.571	13.417	521.29	1560.25	988.51	136.703	1.24	9.853
1.56	1.37	13.571	13.466	521.29	1174.85	854.00	88.901	1.27	9.814
1.63	1.23	13.591	13.361	401.33	2089.62	1017.74	207.644	1.23	9.846
1.63	1.30	13.591	13.417	401.33	1560.25	874.41	149.624	1.27	9.812
1.63	1.37	13.591	13.466	401.33	1174.85	754.35	103.902	1.30	9.771
1.70	1.23	13.605	13.361	309.89	2089.62	905.18	212.603	1.26	9.806
1.70	1.30	13.605	13.417	309.89	1560.25	776.77	157.747	1.29	9.770
1.76	1.17	13.610	13.297	239.81	2825.53	946.37	278.735	1.24	9.793
1.76	1.23	13.610	13.361	239.81	2089.62	808.40	213.924	1.28	9.766
1.83	1.10	13.598	13.225	184.20	3863.25	996.45	349.659	1.23	9.766
1.83	1.17	13.598	13.297	184.20	2825.53	847.67	273.815	1.27	9.750
1.90	1.10	13.541	13.225	137.56	3863.25	892.54	339.039	1.25	9.716
1.90	1.17	13.541	13.297	137.56	2825.53	758.40	268.300	1.29	9.699
1.96	1.10	13.389	13.225	96.02	3863.25	796.18	329.187	1.27	9.653
2.03	1.10	12.902	13.225	51.53	3863.25	698.30	322.778	1.29	9.551

Table 3: (Model $\alpha = 0.050$) The binary NS masses (M_1,M_2) and corresponding radii(R_1,R_2), and dimensionless tidal deformabilities (Λ_1,Λ_2). $\tilde{\Lambda}$, $\delta\tilde{\Lambda}$, $\mathcal{M}_c$ and $\mathcal{R}_c$ are the dimensionless tidal deformability, tidal correction, chirp mass and radiuss of the binary NS respectively.

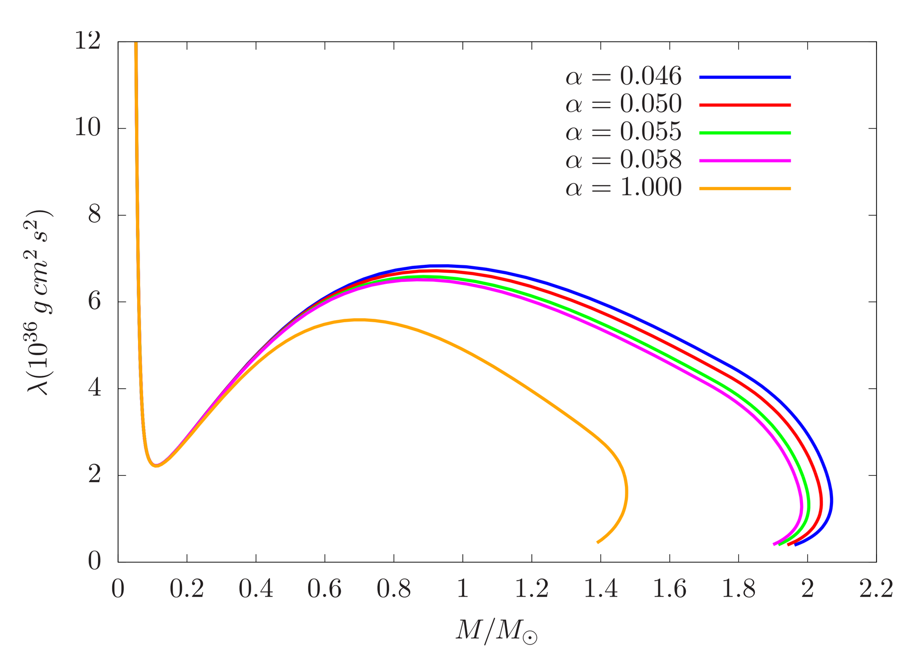


Figure 1: The tidal deformability λ as a function of NS mass with different EoS.

Conclusions

- We have found the maximum neutron star mass to be $2.04 M_\odot$ which is consistent with the current GW170817 observational constraint[8] ($2.01 \pm 0.04 \leq M(M_\odot) \leq 1.16 \pm 0.03$);
- The radius and tidal deformability of a canonical neutron stars of mass $1.4 M_\odot$ are $13.507 km$ and $5.7 \times 10^{36} g\,cm\,s^2$.

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