Strongly interacting matter under intense magnetic fields in NJL models

N. N. Scoccola
Tandar Lab -CNEA– Buenos Aires

PLAN OF THE TALK

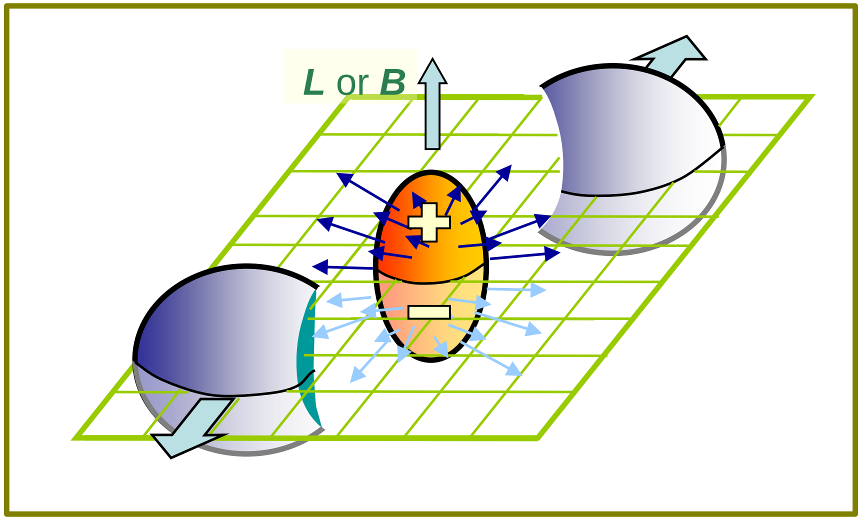
- Introduction
- Magnetic field in NJL-type models
- Magnetized color superconducting matter under compact star conditions
- Outlook

Refs: M. Coppola, P. Allen, A.G. Grunfeld & NNS, Phys.Rev. D 96 (2017) 056013

Introduction

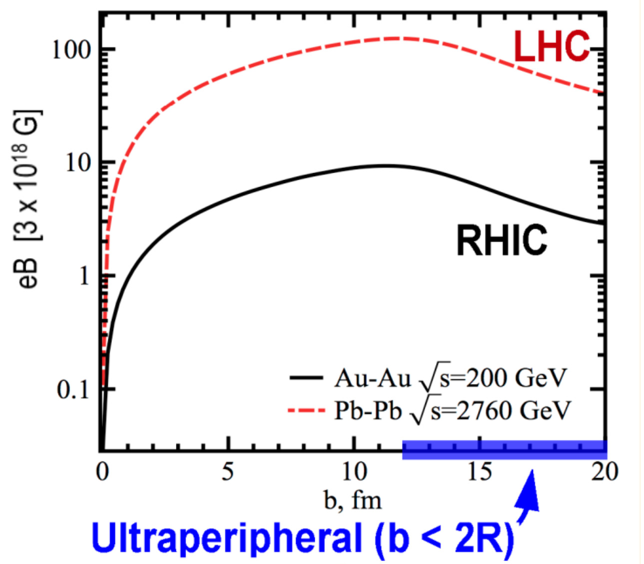
Recently, there has been quite a lot of interest in investigating how the QCD phase diagram is affected by the presence of strong magnetic fields. Motivation: their possible existence in physically relevant situations:

High magnetic fields in non-central relativistic heavy ion collisions



Voloshin, QM2009

Magnetic field at t=0



(A. Bzdak, V. Skokov (12))

Compact Stellar Objects: magnetars are estimated to have B ~10^{14}-10^{15} G at the surface. It could be much higher in the interior (Duncan and Thompson (92/93))

NJL model at finite magnetic field

Recent reviews: Miransky, Shovkovy, Phys. Rept. 576, 1 (2015); Andersen, W. R. Naylor, A. Tranberg, Rev. Mod. Phys. 88, 025001 (2016).

NJL model: simplest model with chiral quark interactions. Local scalar and pseudoscalar four-fermion couplings + UV regularization prescription

NJL (Euclidean) lagrangian

$$\mathcal{L}^E_{NJL} = \bar{\psi}\left(-i\partial\!\!\!/ + m_c\right)\psi(x) - G\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}\ i\gamma_5\vec{\tau}\psi\ \right)^2\right]$$

Nambu, Jona-Lasinio, PR (61)

In the MFA approximation only scalar condensate is non-vanishing

$$S_{MFA} = \frac{M - m_c}{4G} - 4N_c \int \frac{d^3p}{(2\pi)^3}\sqrt{p^2 + M^2} \qquad < \bar{\psi}_f \psi_f >= -\frac{M - m_c}{4G}$$

Integral needs regularization. We use 3D cut-off $|\vec{p}| \leq \Lambda$

Effective quark mass $\boldsymbol{M}$ determined using gap equation $\partial S_{MFA} / \partial M = 0$

The coupling of the quark fields to an external constant and homogenous magnetic field in the z-direction is done using minimal coupling i.e.

$$\vec{\partial} = \vec{\partial} - ie\vec{A} \qquad \vec{A} = (0, Bx, 0)$$ Landau gauge

As well-known, within the Mean Field Approximation that we use in what follows, this leads to the following modifications

$q_{f=u,d}$ Quark charge

$$E_p = \sqrt{p^2 + M^2} \rightarrow E^f_{p_z,k} = \sqrt{p_z^2 + k|q_f B| + M^2}$$

$k = 0,1,2,..$ Landau levels

$$\int \frac{d^3p}{(2\pi)^3} \rightarrow \frac{|q_f B|}{2\pi} \sum_{k=0}^{\infty} \alpha_k \int \frac{dp_z}{2\pi}$$

$\alpha_k = 2 - \delta_{k0}$ Degeneracy

The resulting MFA action is

$$S^{MFA} = \frac{(M - m_c)^2}{4G} - N_c \sum_{f=u,d} \frac{|q_f B|}{2\pi} \sum_{k=0}^{\infty} \alpha_k \int \frac{dp_z}{2\pi} \sqrt{p_z^2 + k|q_f B| + M^2}$$

As before this has to be regularized. Often a function $r(p_z, kB)$ is used. This introduces unphysical oscillations. We use a method (dubbed MFIR) that first separates the second term into a contribution which is explicitly independent of *B* and another one that depends on *B*. Then, only the first is regularized (Menezes et al, PRC79(09))

We get

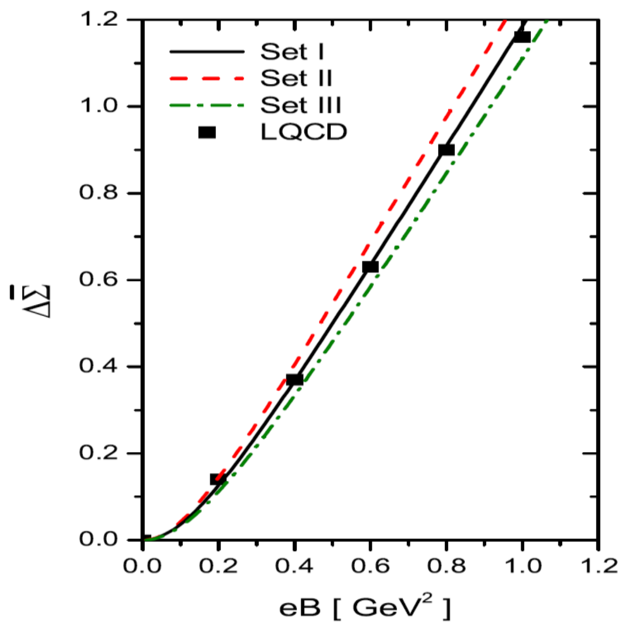
$$S^{MFA} = \frac{(M-m_0)^2}{4G} - \frac{2N_c}{\pi^2}\int_0^{\Lambda} dp\; p^2 \sqrt{p^2+M^2}$$
$$- N_c \sum_{f=u,d} \frac{(q_f B)^2}{2\pi^2}\left\{\xi'(-1,x_f) - \frac{1}{2}(x_f^2 - x_f)\log[x_f] + \frac{x_f^2}{4}\right\}$$

$$x_f = \frac{M^2}{2q_f B}$$

ξ zeta de Riemann

To compare with LQCD results (Bali et al. PRD86(12)) for condensates we introduce

$$\Delta\bar{\Sigma}(B) = \frac{\Delta\Sigma_u(B) + \Delta\Sigma_d(B)}{2} \qquad \text{where } \Sigma_f(B) = -2m_c \frac{<\bar{f}f>_B - <\bar{f}f>_0}{(135\times 86\ \text{MeV}^2)^2}$$



Magnetic catalysis

We consider the following parameterizations which reproduce the empirical vacuum values $m_\pi(B=0) = 138$ MeV and $f_\pi(B=0) = 92.4$ MeV.

Set	M_0 (MeV)	m_0 (MeV)	$G\Lambda^2$	Λ (MeV)
I	350.00	5.66	2.25	613.39
II	320.00	5.42	2.14	639.49
III	380.00	5.79	2.36	596.11

Magnetized color superconducting matter under compact star conditions

Aspects of this problem in the context of NJL-like models have been already discussed in Ferrer, de la Incera '05, Fukushima, Warringa'08, Noronha, Shovkovy'07, Fayazbakhsh, Sadooghi'10, etc. Here, we construct phase diagram including color/charge neutrality and β equilibrium conditions.

We consider the simplest NJL model with additional 2SC interactions

$$\mathcal{L}_{int} = G\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right] + H\left[(i\bar{\psi}^C\ \epsilon_f\ \epsilon_c^3\ \gamma_5\psi)(i\bar{\psi}\ \epsilon_f\ \epsilon_c^3\ \gamma_5\psi^C)\right]$$

Typically, H/G ~ 0.75. $\psi^C = i\gamma_2\gamma_0\bar{\psi}^T$ and ε_f and ε_c^3 are antisymmetric tensors in flavor and color space, respectively.

In the presence of 2SC gap Δ the photon acquires finite mass (Meissner effect). However (Alford et al NPB571(00)) there is a combination of photon and 8th gluon field that remains massless. The corresponding *rotated* charges are

u_r	u_g	u_b	d_r	d_g	d_b
1/2	1/2	1	-1/2	-1/2	0

The rotated electron charge is $\tilde{e} = e\cos\theta$ with θ~1/20 [Gorbar PRD62(00)]

Introducing a chemical potencial for each quark flavor and color the MFA thermodynamical potential a T=0 is

$$\Omega_{\mathrm{MFA}} = \frac{(M-m_c)^2}{4G} + \frac{\Delta^2}{4H} - \sum_{|\tilde{q}|=0,\frac{1}{2},1} P_{|\tilde{q}|} - P_{lep}$$

where

$$\begin{cases} P_{|\tilde{q}|=0} = \int \frac{d^3p}{(2\pi)^3}\left(E_{db}^{+} + \left|E_{db}^{-}\right|\right), \quad P_{|\tilde{q}|=1} = \frac{\tilde{e}B}{8\pi^2}\sum_{k=0}^{\infty}\alpha_k \int_{-\infty}^{\infty} dp_z \left(E_{ub}^{+} + \left|E_{ub}^{-}\right|\right) \\ P_{|\tilde{q}|=1/2} = \frac{\tilde{e}B}{8\pi^2}\sum_{k=0}^{\infty}\alpha_k \int_{-\infty}^{\infty} dp_z \sum_{\lambda,s=\pm}\left|E_{\Delta^s}^{\lambda}\right|, \quad P_{lep} = \sum_{l=e,\mu} P_{|\tilde{q}|=1}\Big|_{\substack{M=m_l \\ \mu_{ub}=\mu_l}} \end{cases}$$

with

$$\begin{cases} E_{db}^{\pm} = \sqrt{p^2+M^2} \pm \mu_{db}, \quad E_{ub}^{\pm} = \sqrt{p_z^2 + 2k\tilde{e}B + M^2} \pm \mu_{ub} \\ E_{\Delta^{\pm}}^{\pm} = \sqrt{\left(\sqrt{p_z^2 + k\tilde{e}B + M^2} \pm \bar{\mu}\right)^2 + \Delta^2} \pm \delta\mu \end{cases}$$

$$\bar{\mu} = \frac{\mu_{dg} + \mu_{ur}}{2}$$

$$\delta\mu = \frac{\mu_{dg} - \mu_{ur}}{2}$$

Under β equilibrium into account (once ν's escape)

$$\mu_{ur} = \mu_{ug} = \mu - \frac{2}{3}\mu_e + \frac{1}{3}\mu_8 \quad ; \quad \mu_{ub} = \mu - \frac{2}{3}\mu_e - \frac{1}{3}\mu_8$$

$$\mu_{dr} = \mu_{dg} = \mu + \frac{1}{3}\mu_e + \frac{1}{3}\mu_8 \quad ; \quad \mu_{db} = \mu + \frac{1}{3}\mu_e - \frac{1}{3}\mu_8$$

Again we regularize using the MFIR scheme. The gap equations are

$$\partial\Omega_{MFA} / \partial M = 0 \quad ; \quad \partial\Omega_{MFA} / \partial\Delta = 0$$

These eqs have to be complemented with those that follow from color and electric charge neutrality

$$\frac{2}{3} n_u - \frac{1}{3} n_d = n_e + n_\mu \quad ; \quad n_r = n_g = n_b$$

where $n_f = \sum_c n_{fc} \quad ; \quad n_c = \sum_f n_{fc}$ with $n_{fc} = \partial\Omega_{MFA} / \partial\mu_{fc}$

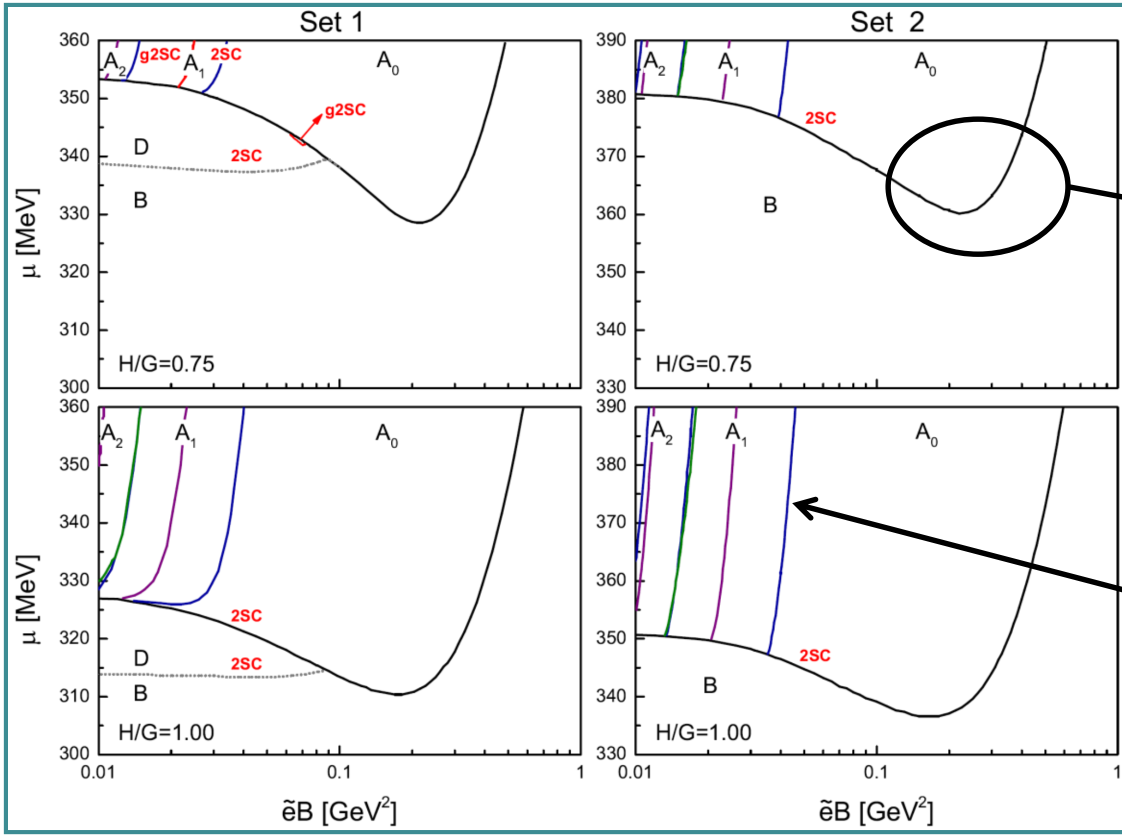
These neutrality conditions are equivalent to demanding

$$\partial\Omega_{MFA} / \partial\mu_e = 0 \quad ; \quad \partial\Omega_{MFA} / \partial\mu_8 = 0$$

We will consider H/G=0.75 and 1, and the parameterization sets

	M_0	m_c	$G\Lambda^2$	Λ	$-<u\bar{u}>^{1/3}$
Set 1	340 MeV	5.59 MeV	2.21	621 MeV	244 MeV
Set 2	400 MeV	5.83 MeV	2.44	588 MeV	241 MeV

Phase diagrams in the B-μ plane

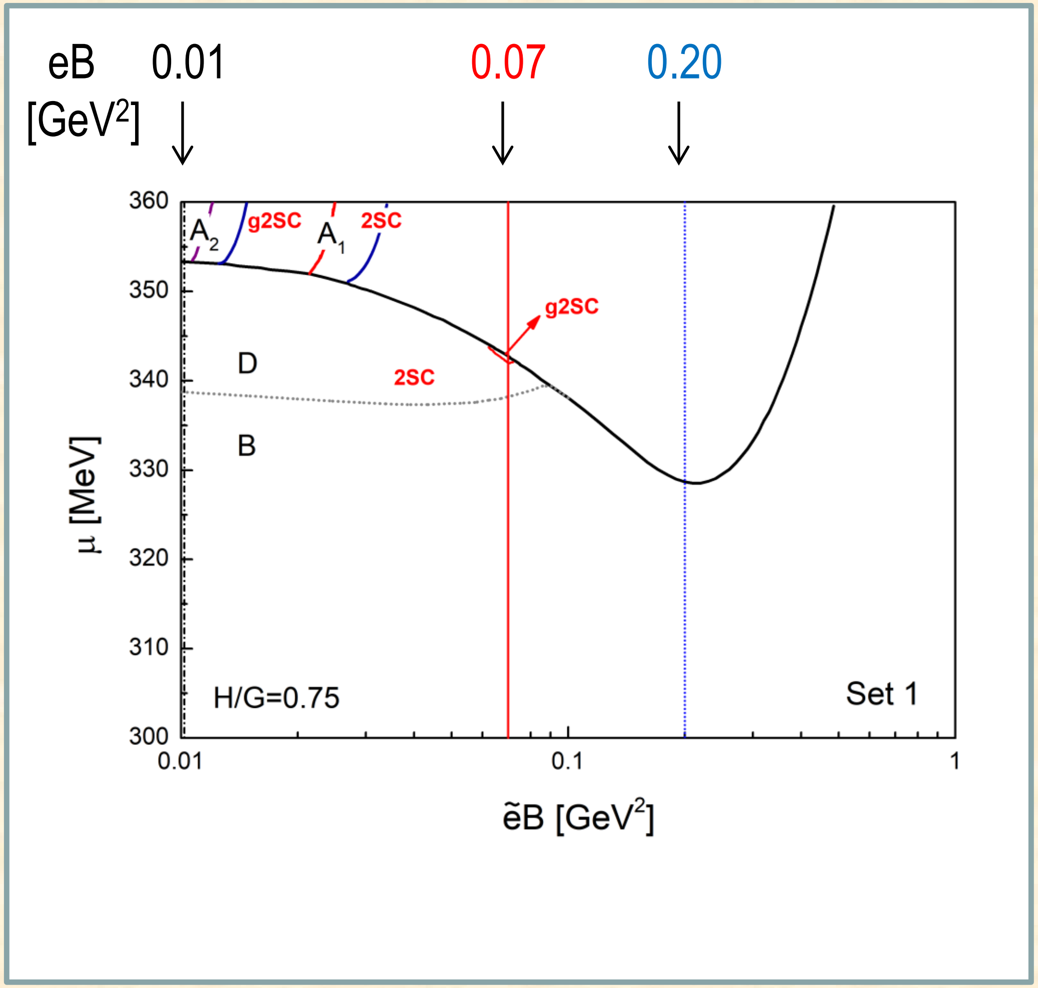


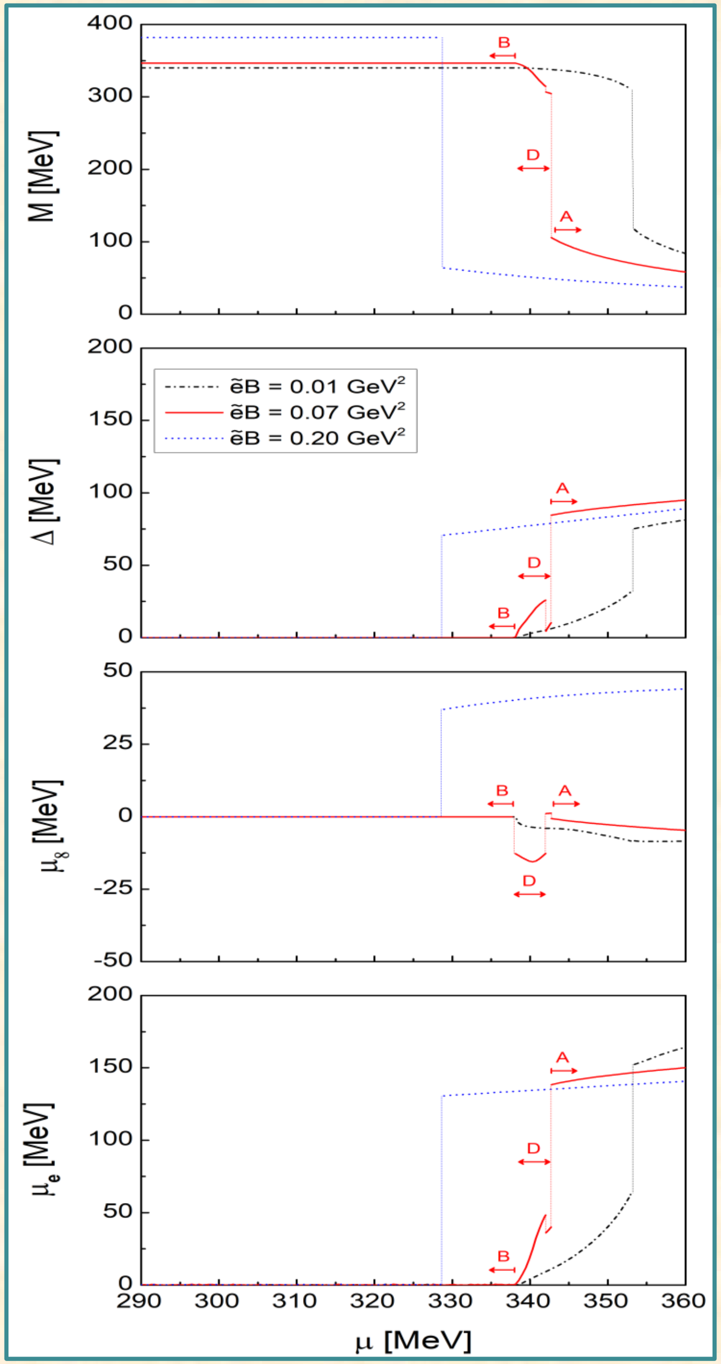
Inverse Magnetic Catalysis (IMC): μ_c decreases for intermediate values of B (Preis, Rebhan,Schmidt '11)

Van Alphen-de Haas (vAdH) transitions (i.e transitions between A phases) are weak 1st order

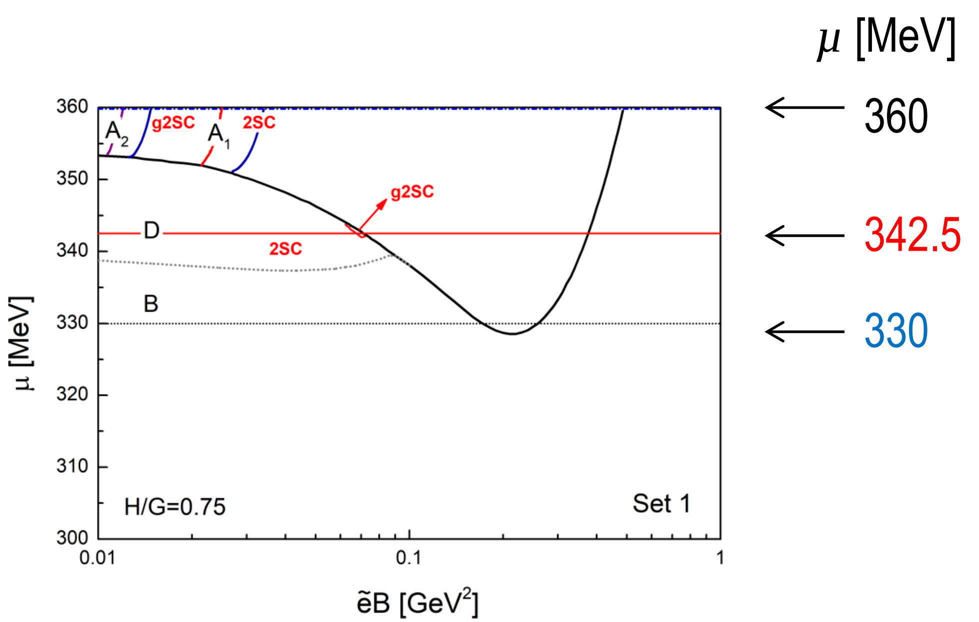
Phase	Characteristics
B (Vacuum phase)	χSB , Mag. Catalysis ; M=M(B,μ=0) ; $\Delta=\mu_3=\mu_8=n=0$
A (CSC phase)	χSym (almost) restored, ; $(\Delta=\mu_3=\mu_8=n) \neq 0$, vA-dH transitions
D (Mixed phase)	$M_B > M_D > M_A$; Inv. Mag. Cat. ; $(\Delta=\mu_3=\mu_8=n)_B > (\Delta=\mu_3=\mu_8=n)_D > (\Delta=\mu_3=\mu_8=n)_B$
2SC	Present in A and D phases ; $\Delta > \delta\mu$, four gapped modes , ndr = nur
g2SC (gapless)	Present only in some cases ; $\Delta < \delta\mu$, two gapped modes , ndr $\neq$ nur

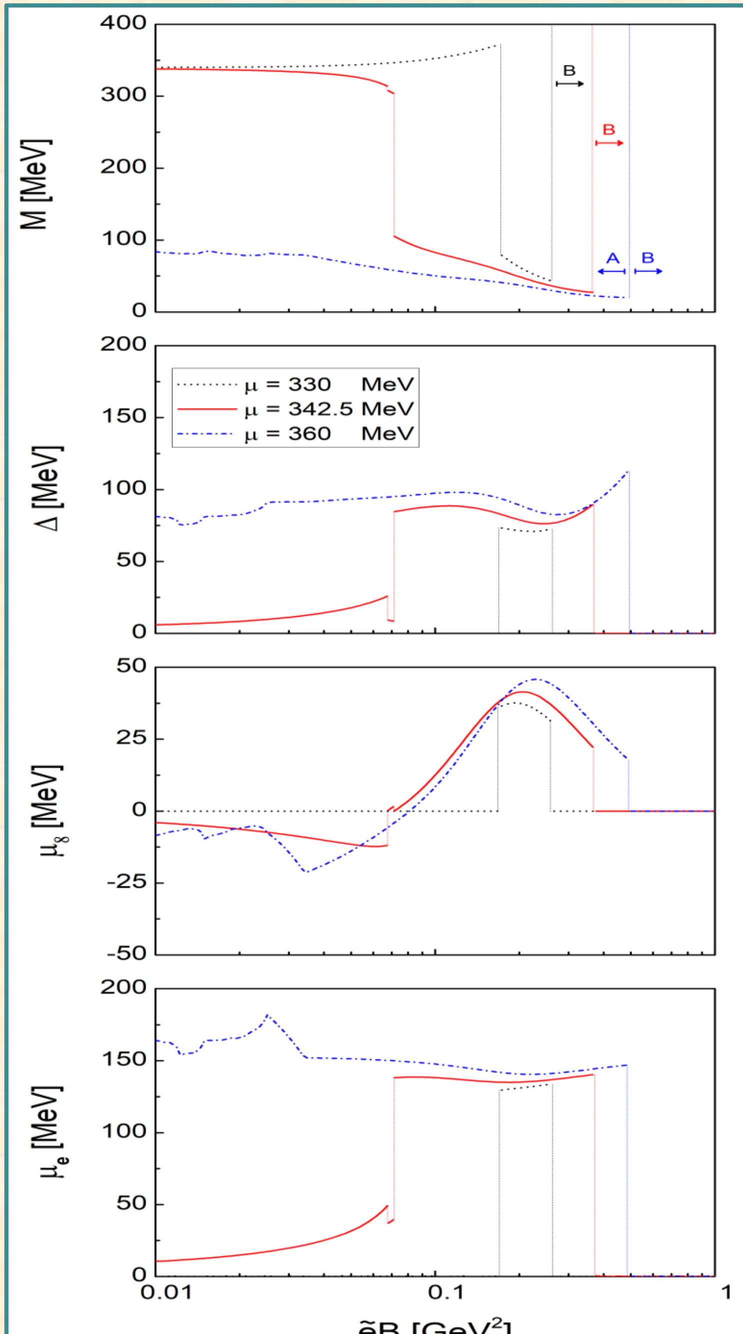
M, Δ, μ_8 and μ_e as functions of μ for fixed B



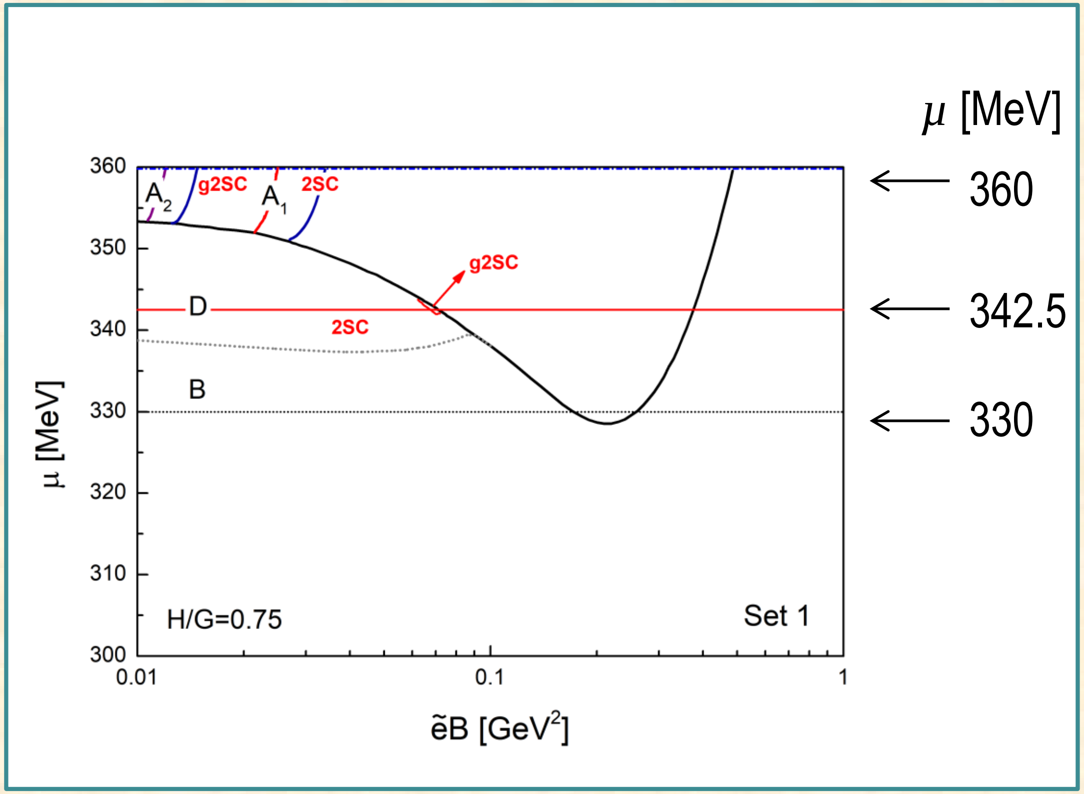


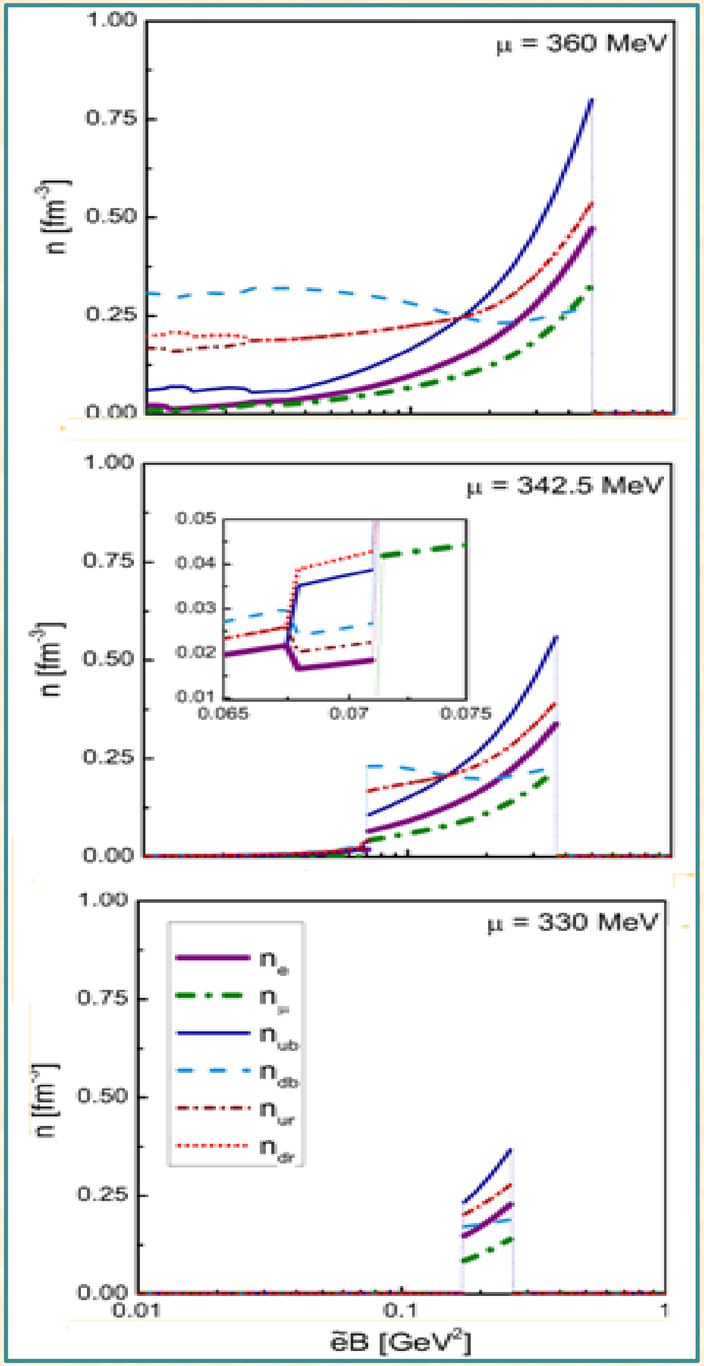
M, Δ, μ_8 and μ_e as functions of B for fixed μ





Quark and lepton densities as functions of B for fixed μ





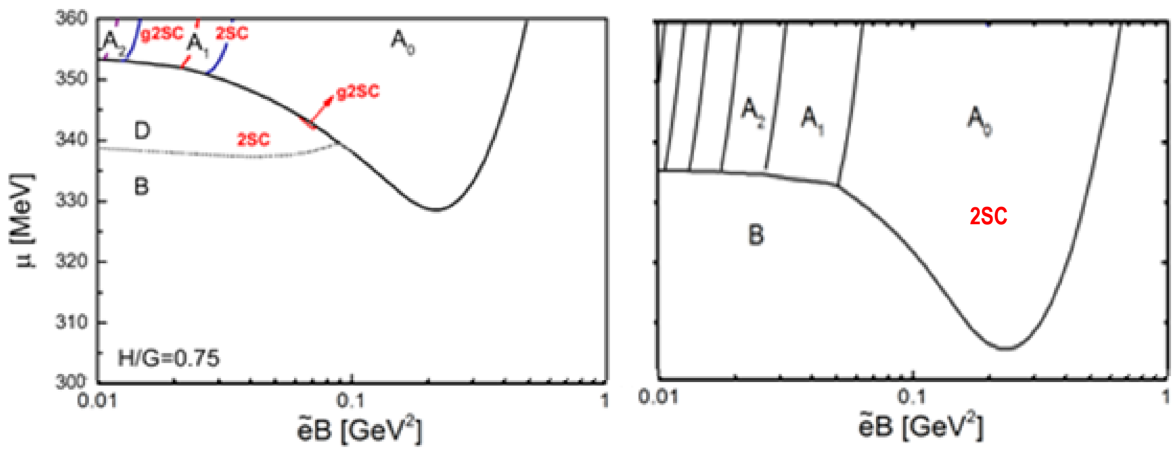
Summary

• Presence of chirally broken B, almost restored A and mixed D phases, the latter two composed of g2SC and/or 2SC regions.

• Based on a B=0 study, for H/G<0.75 the A and D phases are expected to become g2SC and eventually have Δ=0 for H/G<0.65, recovering the C phase.

• Compact star conditions induce the existence of g2SC and 2SC modes, diminishes the maximum LL reached in the vA-dH transitions and reduces the superconducting effect. Critical μ is increased and the magnetic catalysis (MC) effect is attenuated, diminishing the depth of the IMC well and moving the phase diagram upwards.

With CompStar conditions



Without CompStar conditions

• Interesting to see the effect in EoS….
• Inclusion of vector mesons.

Meson masses in the vacuum at finite B

M. Coppola, D. Gómez Dumm & NNS, Phys.Lett. B782 (2018) 155

After bosonization of the usual NJL model and expanding the meson field around the Mean Field (MF) values, the Euclidean action reads

$$S_{\rm bos} = S^{\rm MF}_{\rm bos} + S^{\rm quad}_{\rm bos} + \dots$$

The quadratic contribution is given by

$$S^{\rm quad}_{\rm bos} = \frac{1}{2} \sum_{M=\sigma,\pi^0,\pi^\pm} \int_{x,x'} \delta M(x)^* \left[\frac{1}{2G}\, \delta^{(4)}(x-x') - J_M(x,x') \right] \delta M(x')$$

where $J_{\pi^0}(x,x') = N_c \sum_f {\rm tr}\left[\mathcal{S}^{{\rm MF},f}_{x,x'}\ \gamma_5\ \mathcal{S}^{{\rm MF},f}_{x',x}\ \gamma_5 \right],\quad J_{\pi^\pm}(x,x') = 2N_c\, {\rm tr}\left[\mathcal{S}^{{\rm MF},u}_{x,x'}\ \gamma_5\ \mathcal{S}^{{\rm MF},d}_{x',x}\ \gamma_5 \right]$

The MF quark propagator can be written as $S^{{\rm MF},f}_{x,x'} = e^{i\Phi_f(x,x')} \int_p e^{i\,p\,(x-x')}\, \tilde{S}^f_p$

where

$$\tilde{S}^f_p = \int_0^\infty d\tau\, \exp\left[-\tau \left(M^2 + p_\parallel^2 + p_\perp^2\, \frac{\tanh \tau B_f}{\tau B_f} \right) \right] \left[\left(M - p_\parallel \cdot \gamma_\parallel \right) \left(1 + i s_f\, \gamma_1 \gamma_2 \tanh \tau B_f \right) - \frac{p_\perp \cdot \gamma_\perp}{\cosh^2 \tau B_f} \right]$$

$$B_f = |q_f B|,\ \ s_f = sign(q_f B),\ \ p_\perp = (p_1, p_2),\ \ p_\parallel = (p_3, p_4)$$

and Schwinger phase $\Phi_f(x,x') = \exp\left[i q_f B (x_1 + x'_1)(x_2 - x'_2)/2 \right]$

For charged pions, Schwinger phases do not cancel due to their different quark flavors, and therefore J_{π^+} is not translational invariant. However, it is still diagonal in the Ritus basis. The pion field is expanded as

$$\pi^+(x) = \sum\!\!\!\!\!\!\int_{\bar{q}} \mathbb{F}^+_{\bar{q}}(x)\, \pi^+_{\bar{q}}, \;\; \mathbb{F}^+_{\bar{q}}(x) = N_k\, e^{i(q_2 x_2 + q_3 x_3 + q_4 x_4)}\, D_k(\rho_+)$$

$D_k(x)$ are the cylindrical parabolic functions

$$N_k = (4\pi |q_{\pi^+} B|)^{1/4} / \sqrt{k!}, \;\; \rho_+ = \sqrt{2|q_{\pi^+} B|}\, [x_1 - q_2/\,(q_{\pi^+} B)]\,, \;\; q \equiv (k, q_2, q_3, q_4)$$

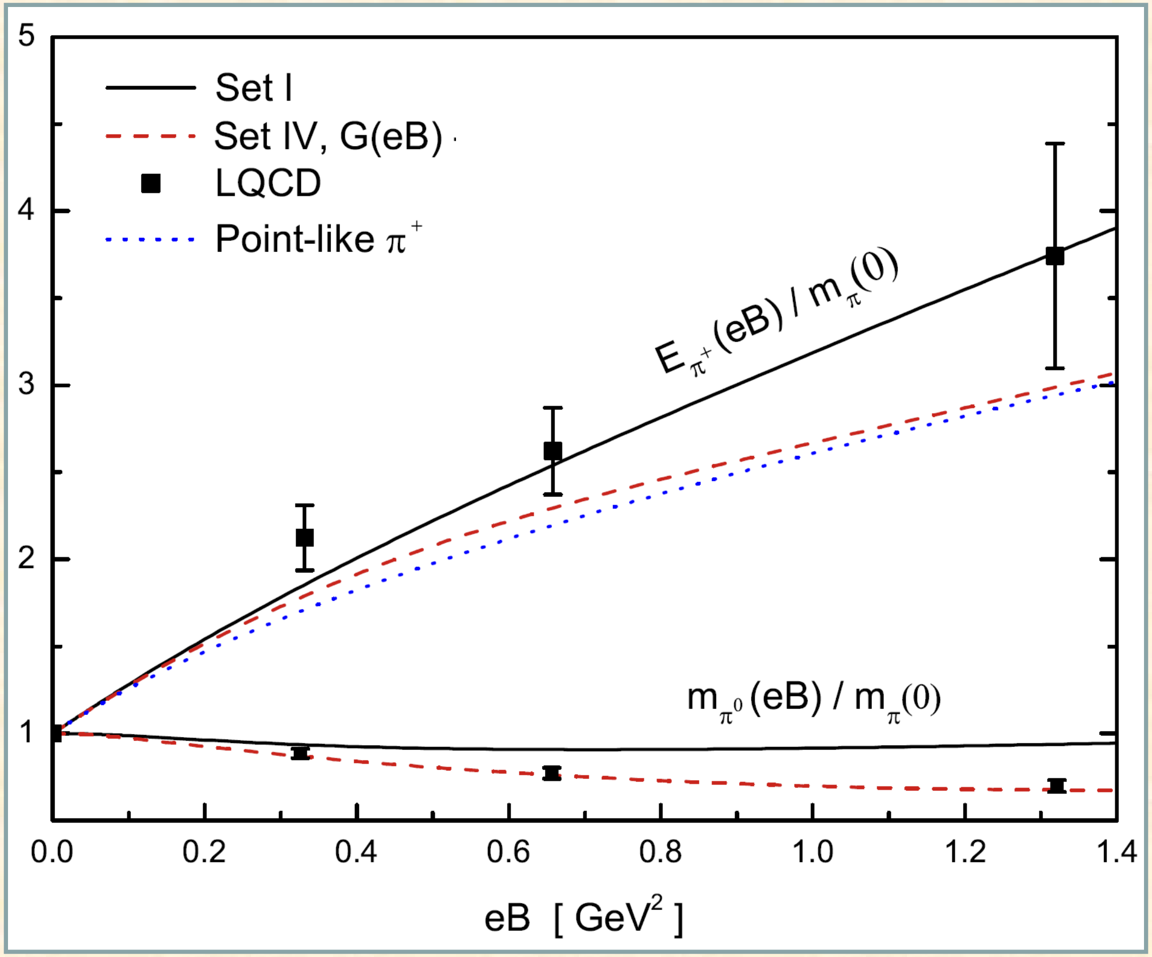
The quadratic action in the Ritus basis reads

$$S^{\rm quad}_{\pi^+} = \frac{1}{2} \sum\!\!\!\!\!\!\int_{\bar{q}} (\delta\pi^+_{\bar{q}})^* \left[\frac{1}{2G} - J_{\pi^+}(k, \Pi^2)\right] \delta\pi^+_{\bar{q}}, \;\; \Pi^2 = (2k+1)\, B_{\pi^+} + q_{\|}^2$$

Again, we regularize the polarization function using the MFIR scheme

$$J^{(\rm reg)}_{\pi^+}(k, \Pi^2) = J^{(\rm reg)}_{\pi, B=0}(\Pi^2) + J^{(\rm mag)}_{\pi^+}(k, \Pi^2) \;\; \Rightarrow \;\; \frac{1}{2G} - J^{(\rm reg)}_{\pi^+}\left(k, -m^2_{\pi^+}(eB)\right) = 0$$

Normalized neutral pion mass and magnetic field-dependent charged pion mass as functions of eB for Set I (solid), Set IV (Avancini et al, PLB 767 (2017)) , the charged point-like case (dotted) and LQCD (squares) (Bali et al, PRD 97 (2018))



$$E_{\pi^+}(eB) = \sqrt{m^2_{\pi^+}(eB) + eB}$$

Minimum energy corresponding to LLL and q_3=0

Note that in NJL calculations m_c has been adjusted to reproduced m_π = 415 MeV used in LQCD calculations

Summary

- Within the NJL model, the charged pion two-point function was diagonalized in the Ritus basis, using the Schwinger form of the quark propagator and regularized in the MFIR scheme.
- When eB is enhanced, the π^0 mass slightly decreases, while E_{π^+} steadily increases, remaining always larger than the one of a point-like pion.
- The results are rather independent of the parametrization.
- For a heavy pion mass, although rescaled parameters show consistency with LQCD (Bali et al, PRD 97 (2018)) for E_{π^+}, the errors are large to be conclusive. For m_{π^0} the results disagree. The agreement is improved if $G(eB)$ is introduced (Avancini et al, PLB 767 (2017)) . Non-local models also provide results in agreement with LQCD for m_{π^0} (Gómez Dumm et al PRD97 (2018))