Characterization of Orbits For The MacMillan Problem With Test Particle of Variable Mass

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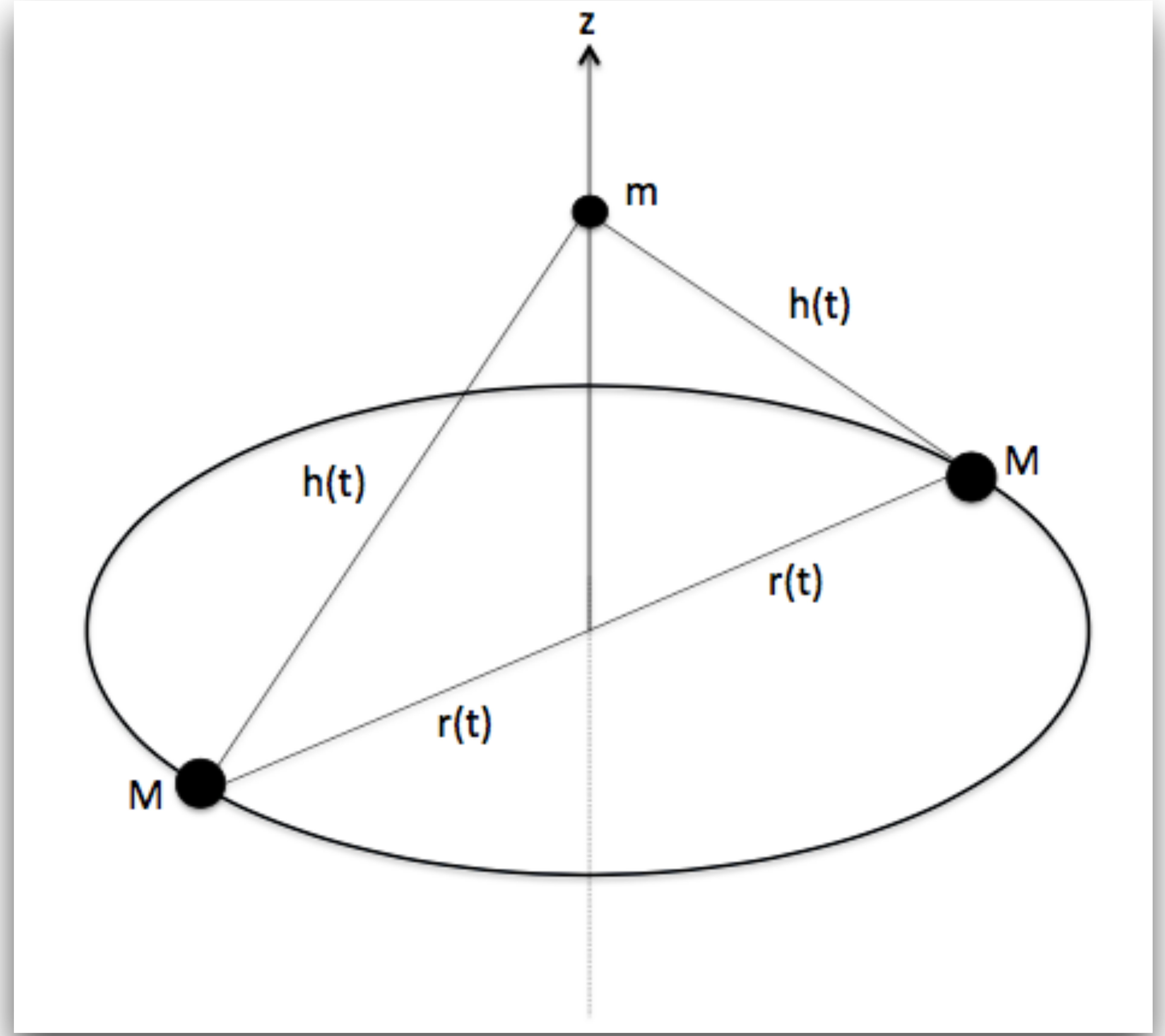
IWARA 2018 - September 10th, 2018

The MacMillan Problem



Motivation

Freistetter & Grützbauch (2018), Sitnikov in Westeros: How Celestial Mechanics can explain once and for all why winter is coming in Game of Thrones

The 'Great Comet' of 1996, Hyakutake. Image credit: NASA

Mahra SULTANÍA - CIRCA 1967. Image credit: Brendan Howard

The Equations of The Problem

- **The Lagrangian**

$$L=\frac{1}{2}m\dot{z}^{2}+\frac{2GMm}{\sqrt{r^{2}+z^{2}(t)}}$$

- **The Jean`s Law**

$$\dot{m}=-\alpha m^{n}$$

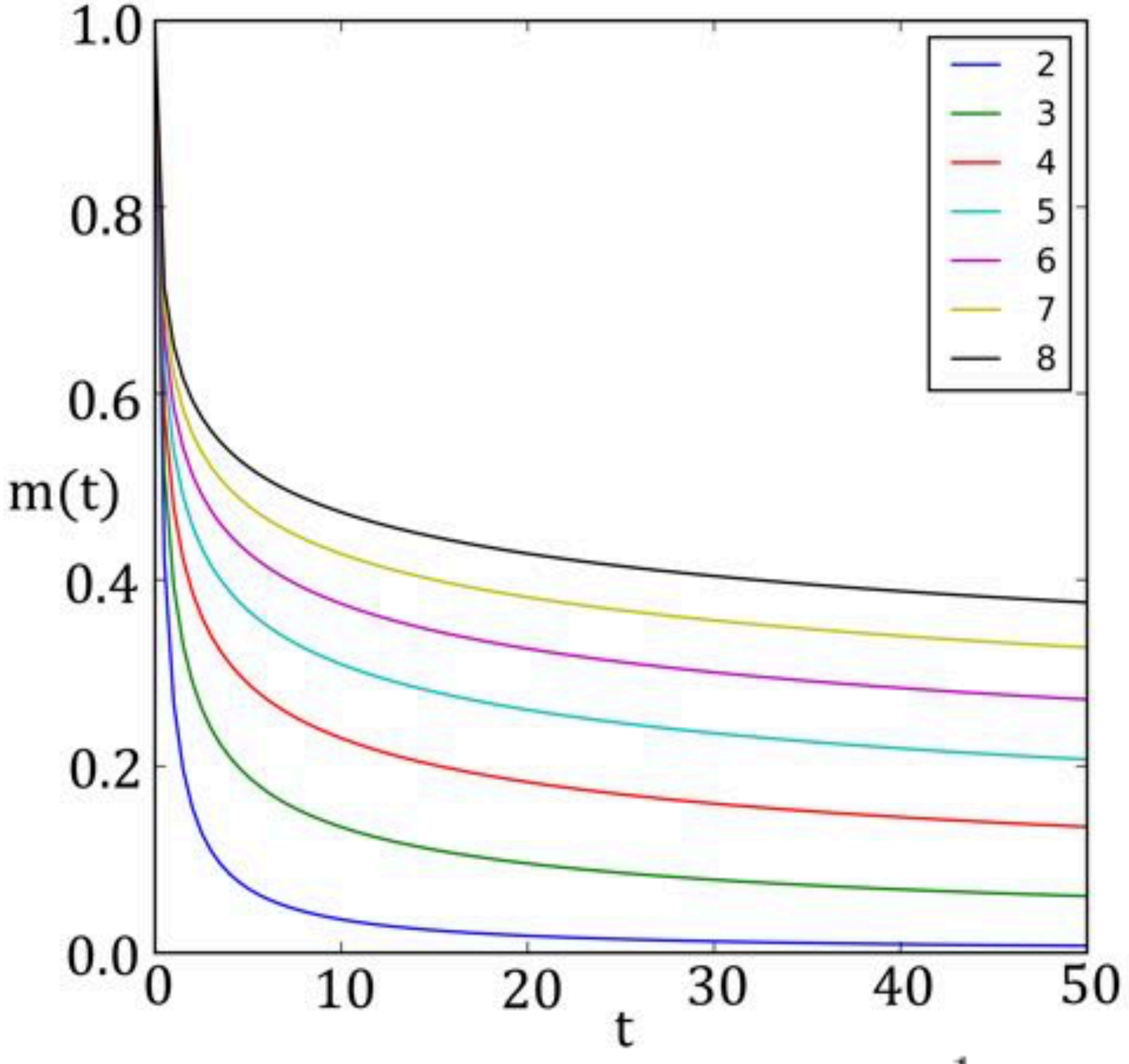
- **The Equation of Motion**

$$\ddot{z}+\frac{\dot{m}}{m}\dot{z}+\frac{2GMz}{(r^{2}+z^{2})^{3/2}}=0$$

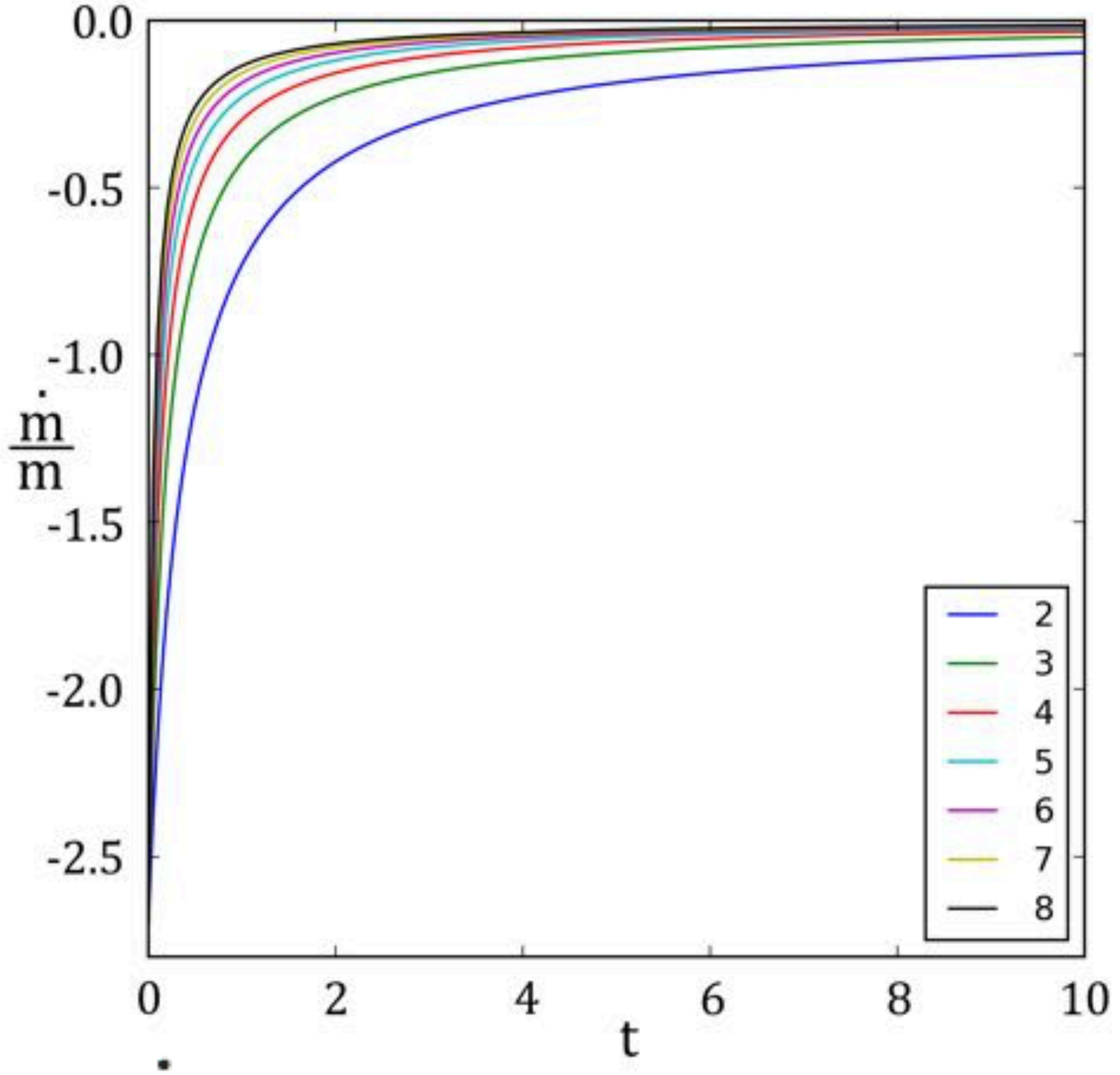
- **The Solution For The Mass**

$$m(t)=\{m_{0}^{n-1}+\alpha(n-1)t\}^{\frac{1}{1-n}}$$

The Results



$$m(t)=\{m_0^{n-1}+\alpha(n-1)t\}^{\frac{1}{1-n}}$$



$$\frac{\dot{m}}{m}=\frac{-\alpha m^n}{\{m_0^{n-1}+\alpha(n-1)t\}^{\frac{1}{1-n}}}$$

Figure shows the behavior of the mass and of the factor $\frac{\dot{m}}{m}$ in time. Each color represents a different value for n.

Solutions For n=2

a)

b)

c)

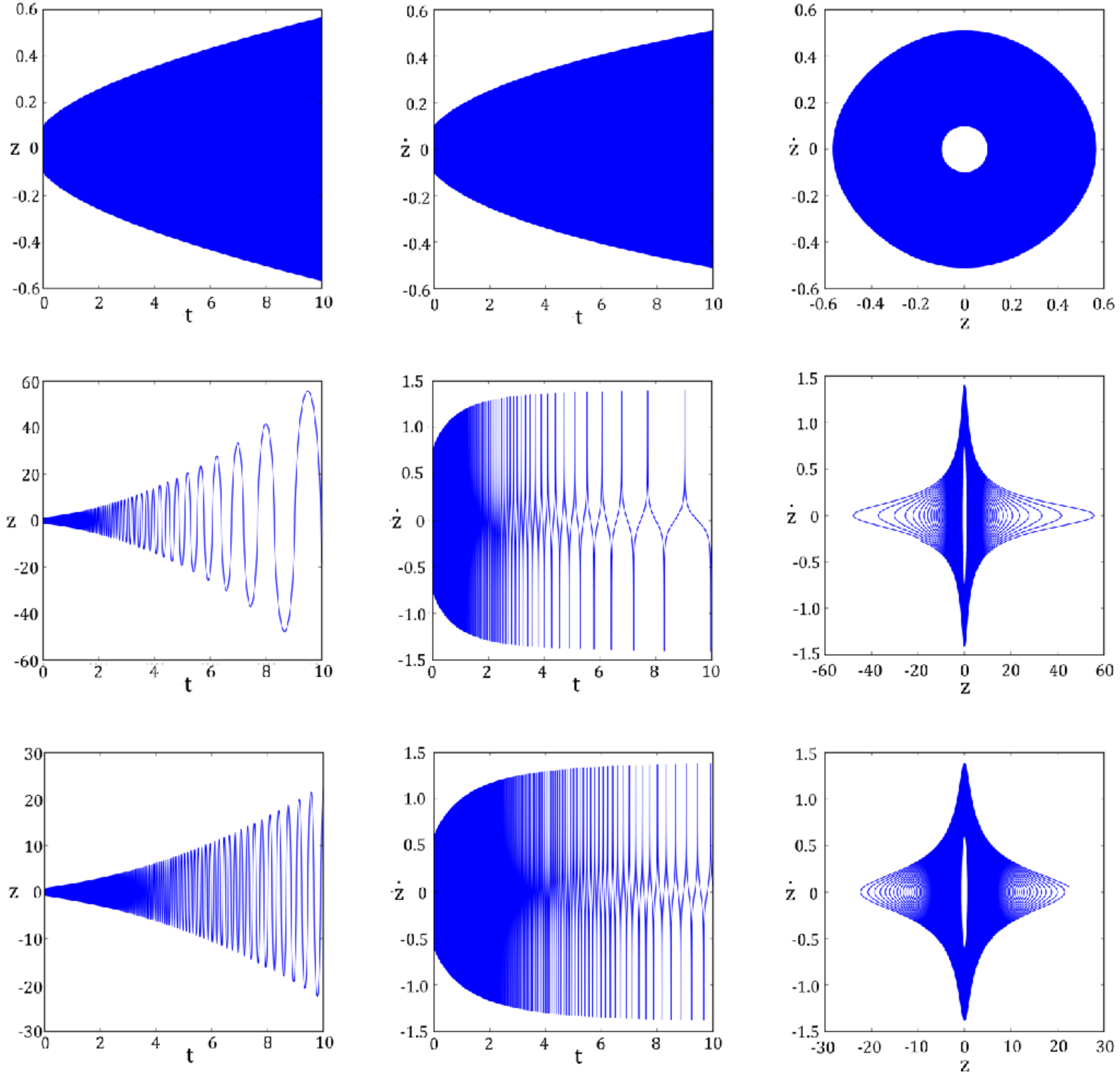
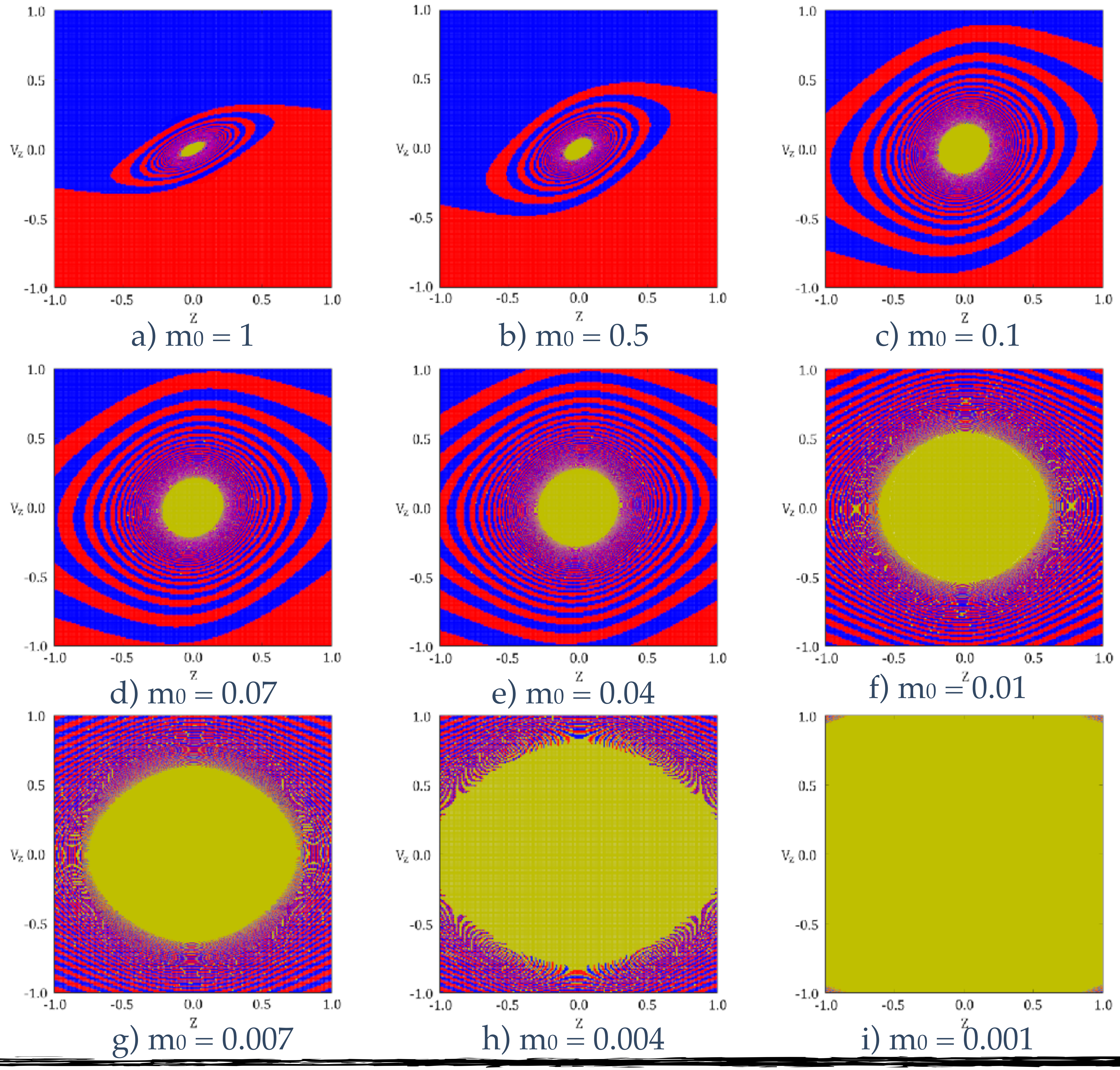


Figure a) shows behavior of z, v_z and phase diagram for initial conditions (0, -0.1)

Figure b) shows behavior of z, v_z and phase diagram for initial conditions (1, 0)

Figure c) shows behavior of z, v_z and phase diagram for initial conditions (0.7, -0.05)



a) $m_0 = 1$ b) $m_0 = 0.5$ c) $m_0 = 0.1$

d) $m_0 = 0.07$ e) $m_0 = 0.04$ f) $m_0 = 0.01$

g) $m_0 = 0.007$ h) $m_0 = 0.004$ i) $m_0 = 0.001$

The figure shows the escape basins for initial mass values in range (0.001 , 1).

Color Convention

Yellow: particle does not escape of the gravitational attraction of the system.

Blue: particle escapes below of the gravitational attraction of the system.

Red: particle escapes above of the gravitational attraction of the system.

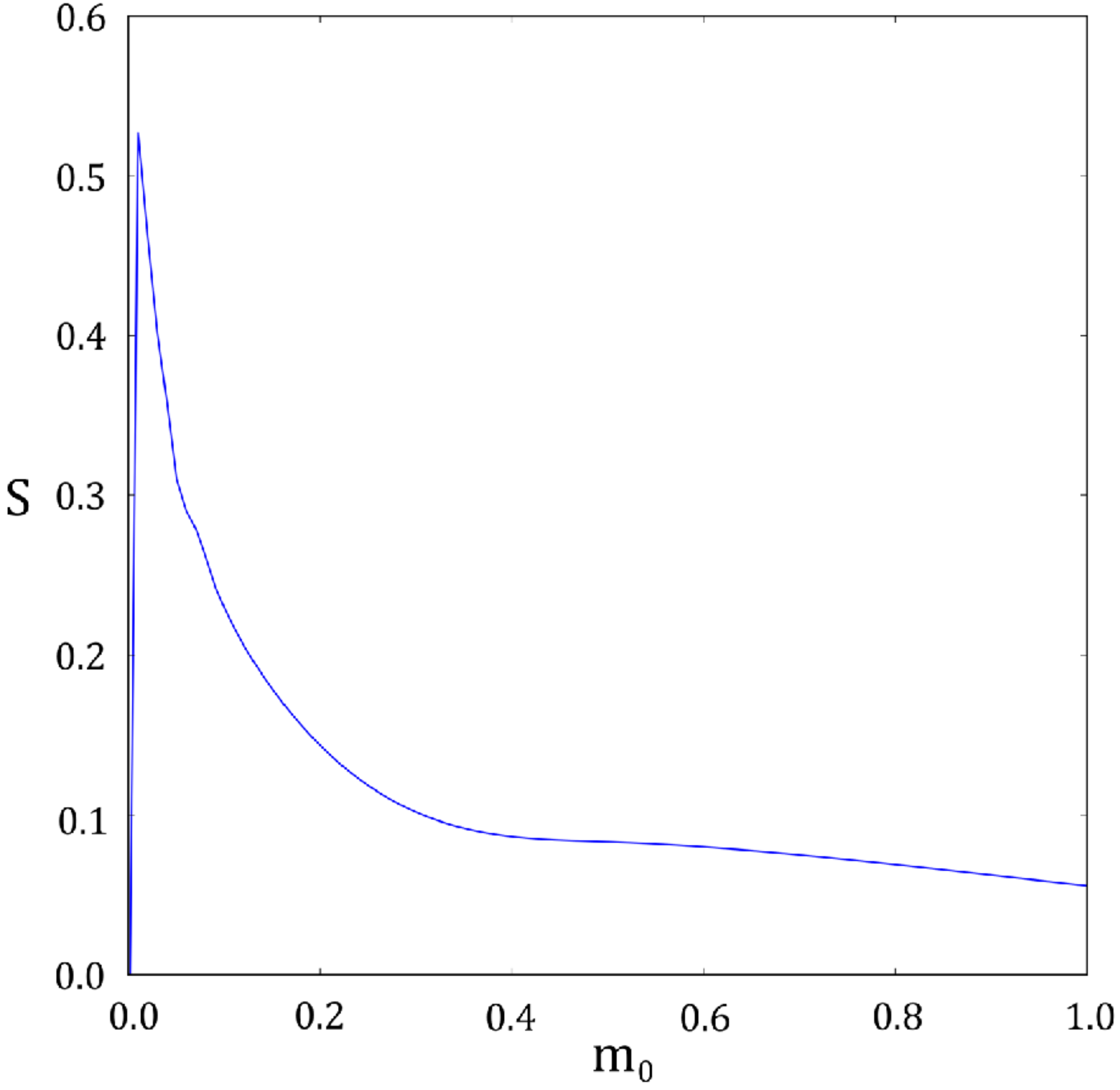
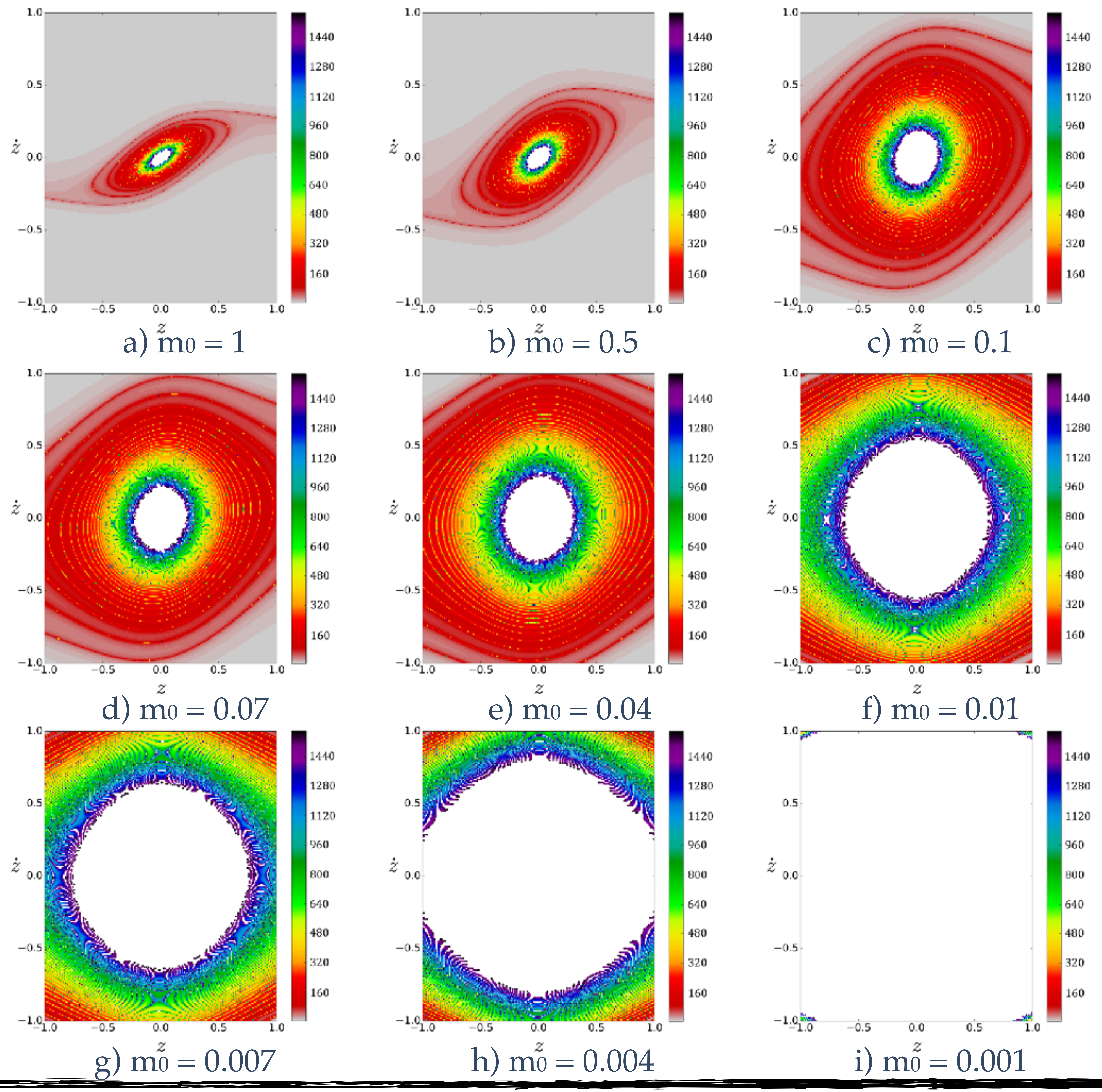


Figure shows the behavior of entropy in function of the initial mass of the test particle.



The figure shows the escape basins for initial mass values in range (0.001 , 1).

Color Convention

The color bar shows the number of orbital periods of the primary masses for which the test particle escapes of the gravitational attraction of the system.

Solutions For n=3

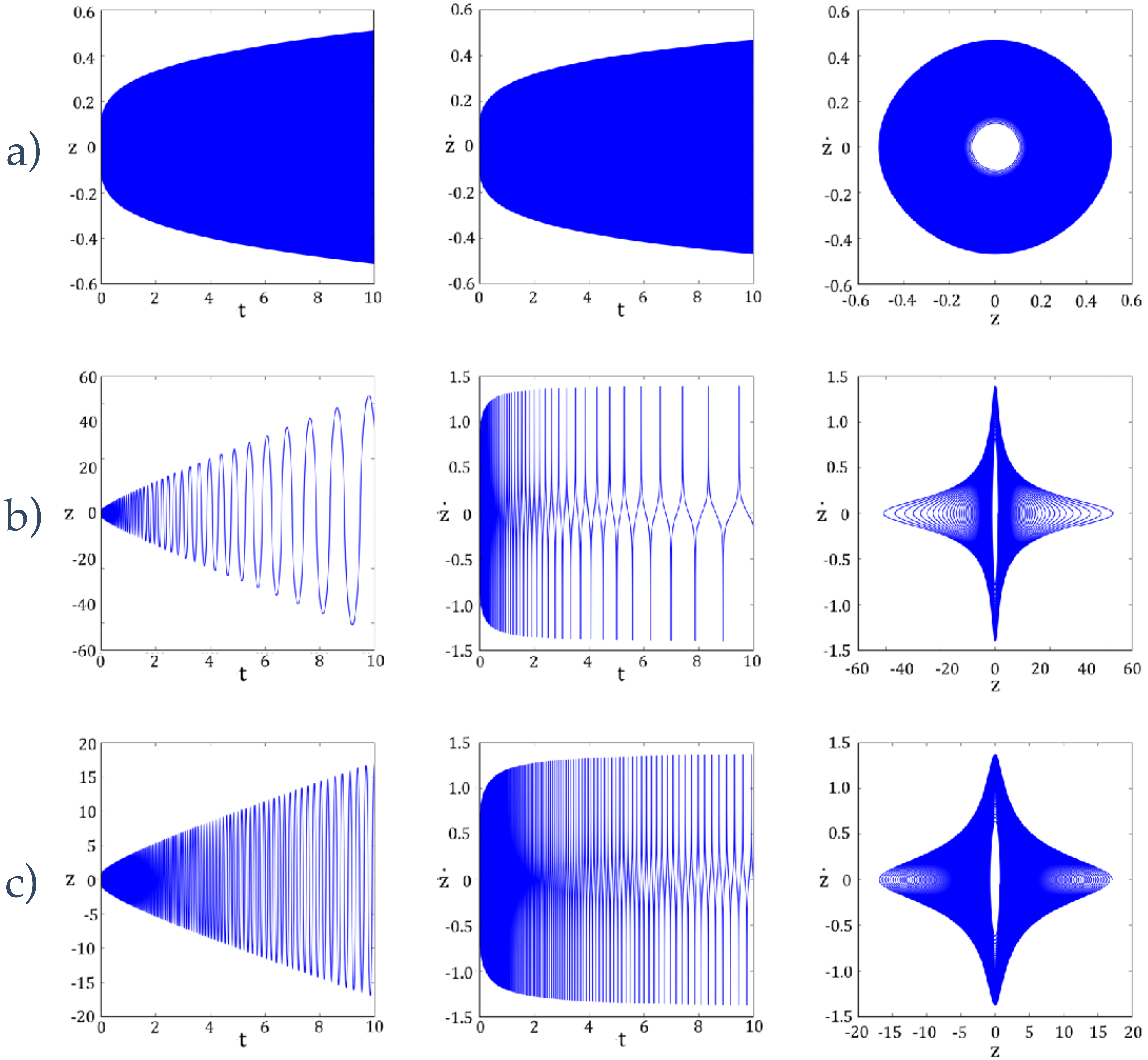


Figure a) shows behavior of z, v_z and phase diagram for initial conditions (0, -0.1)

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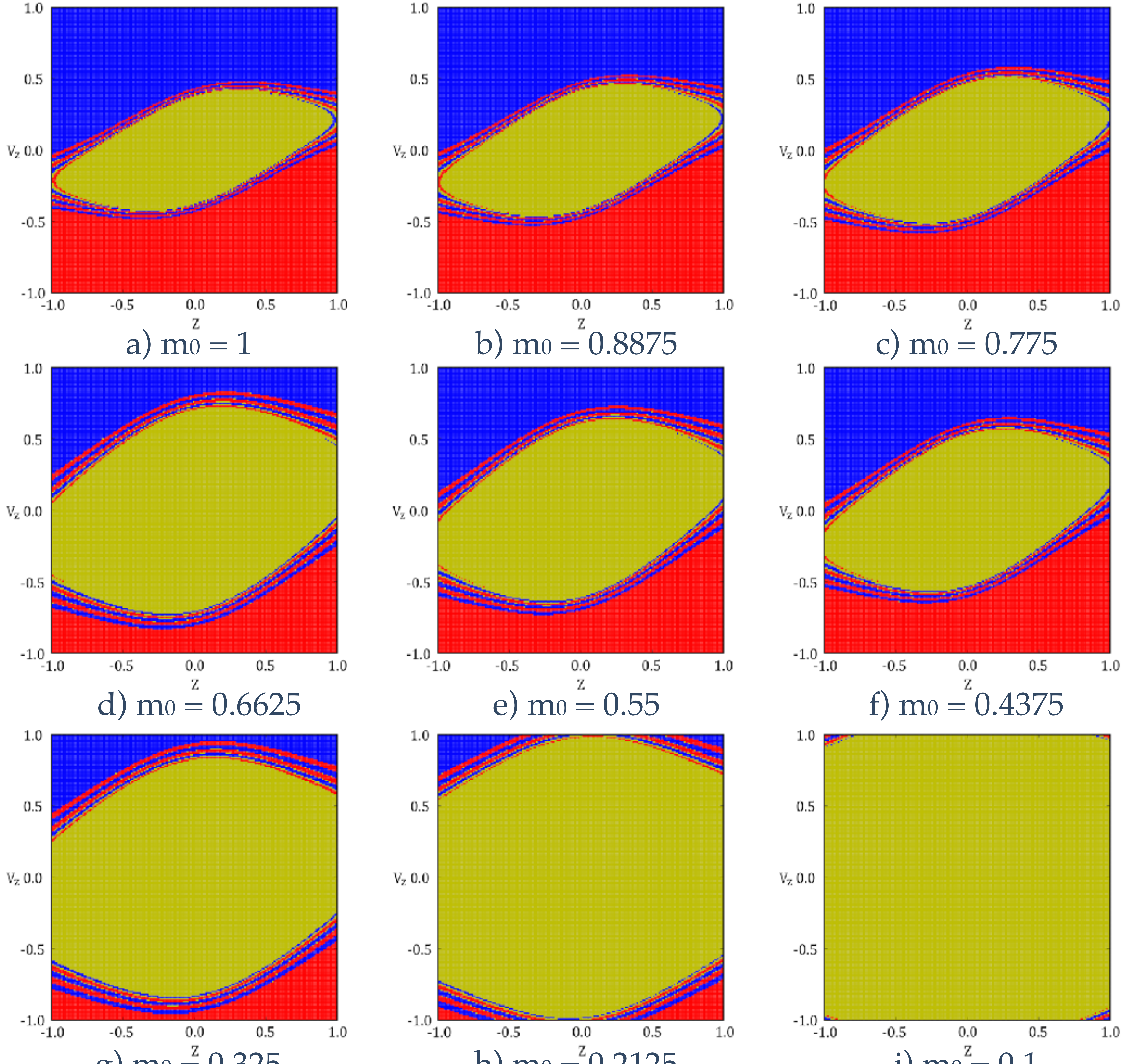

a) $m_0 = 1$ b) $m_0 = 0.8875$ c) $m_0 = 0.775$
d) $m_0 = 0.6625$ e) $m_0 = 0.55$ f) $m_0 = 0.4375$
g) $m_0 = 0.325$ h) $m_0 = 0.2125$ i) $m_0 = 0.1$

The figure shows the escape basins for initial mass values in range (0.1 , 1).

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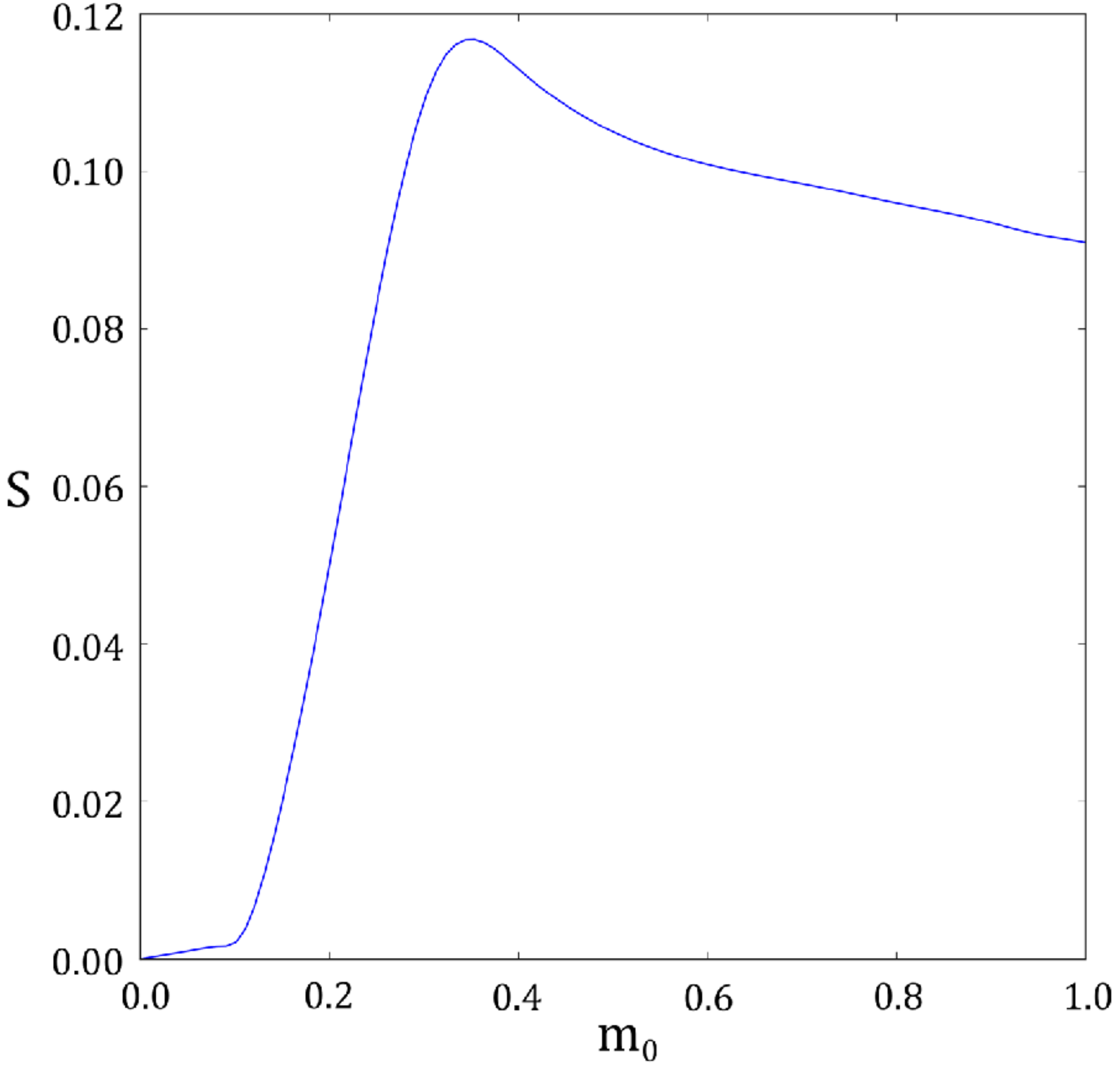

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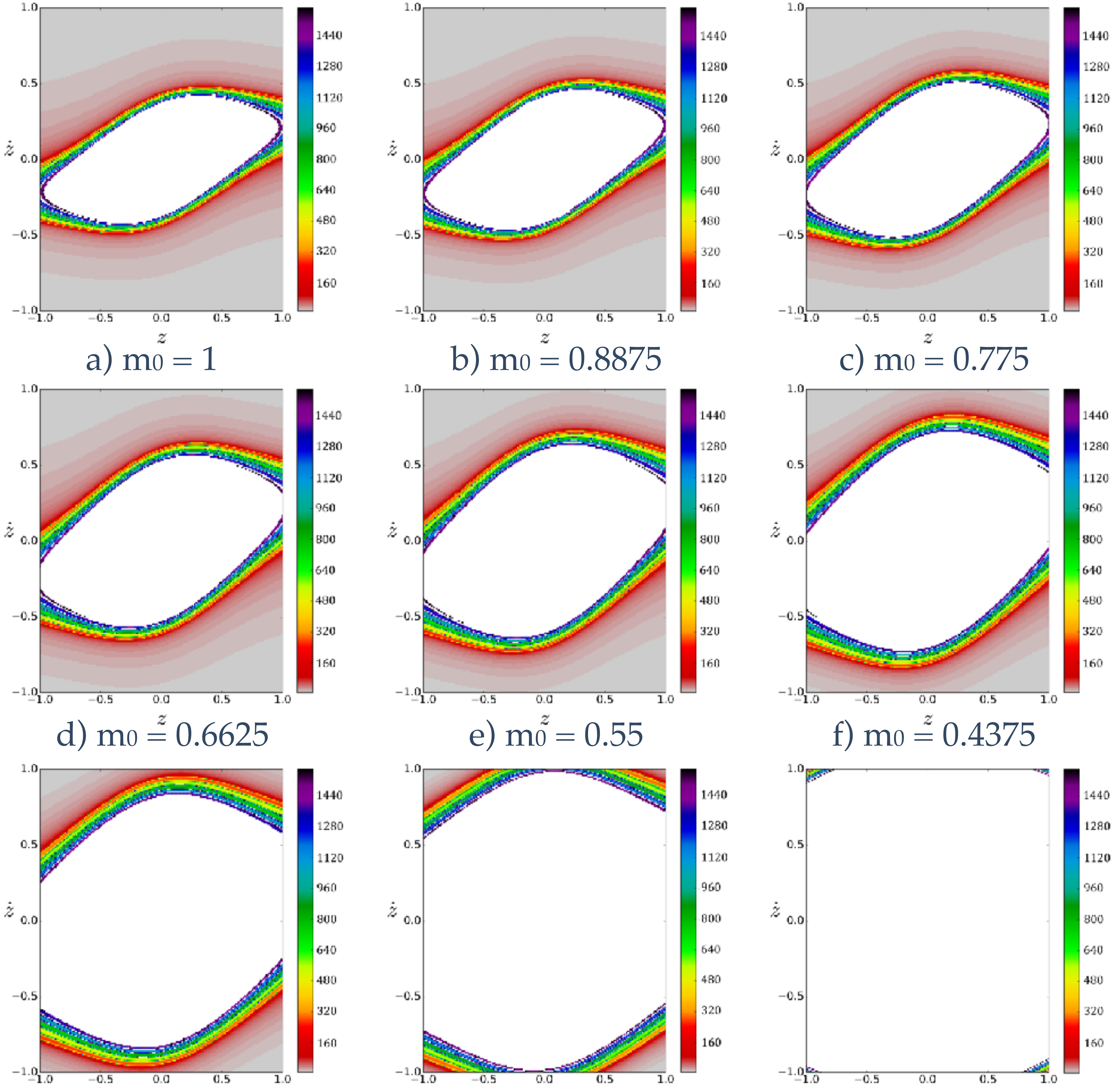

a) $m_0 = 1$

b) $m_0 = 0.8875$

c) $m_0 = 0.775$

d) $m_0 = 0.6625$

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The figure shows the escape basins for initial mass values in range (0.1 , 1).

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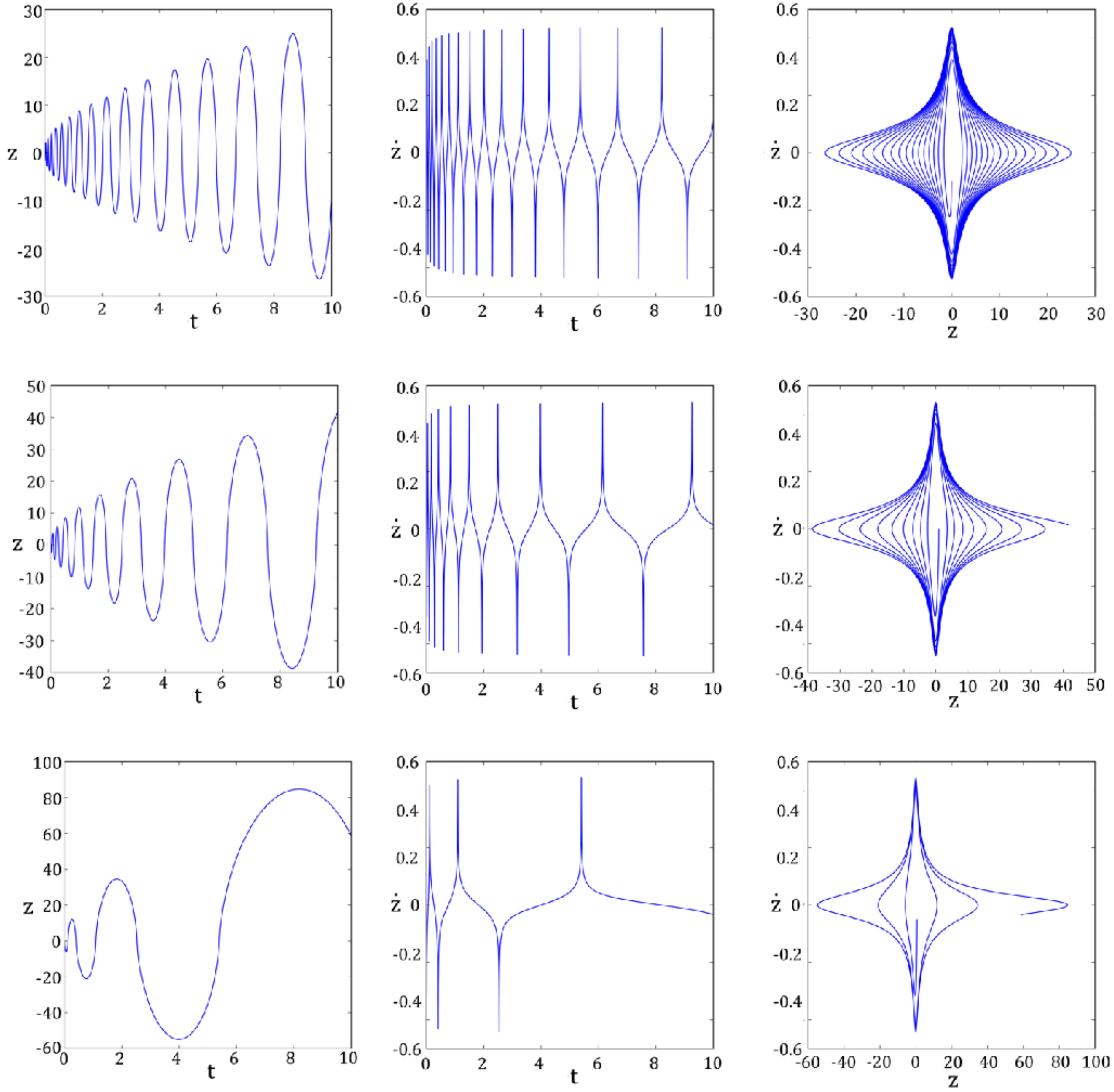
Solutions For n=4



a)

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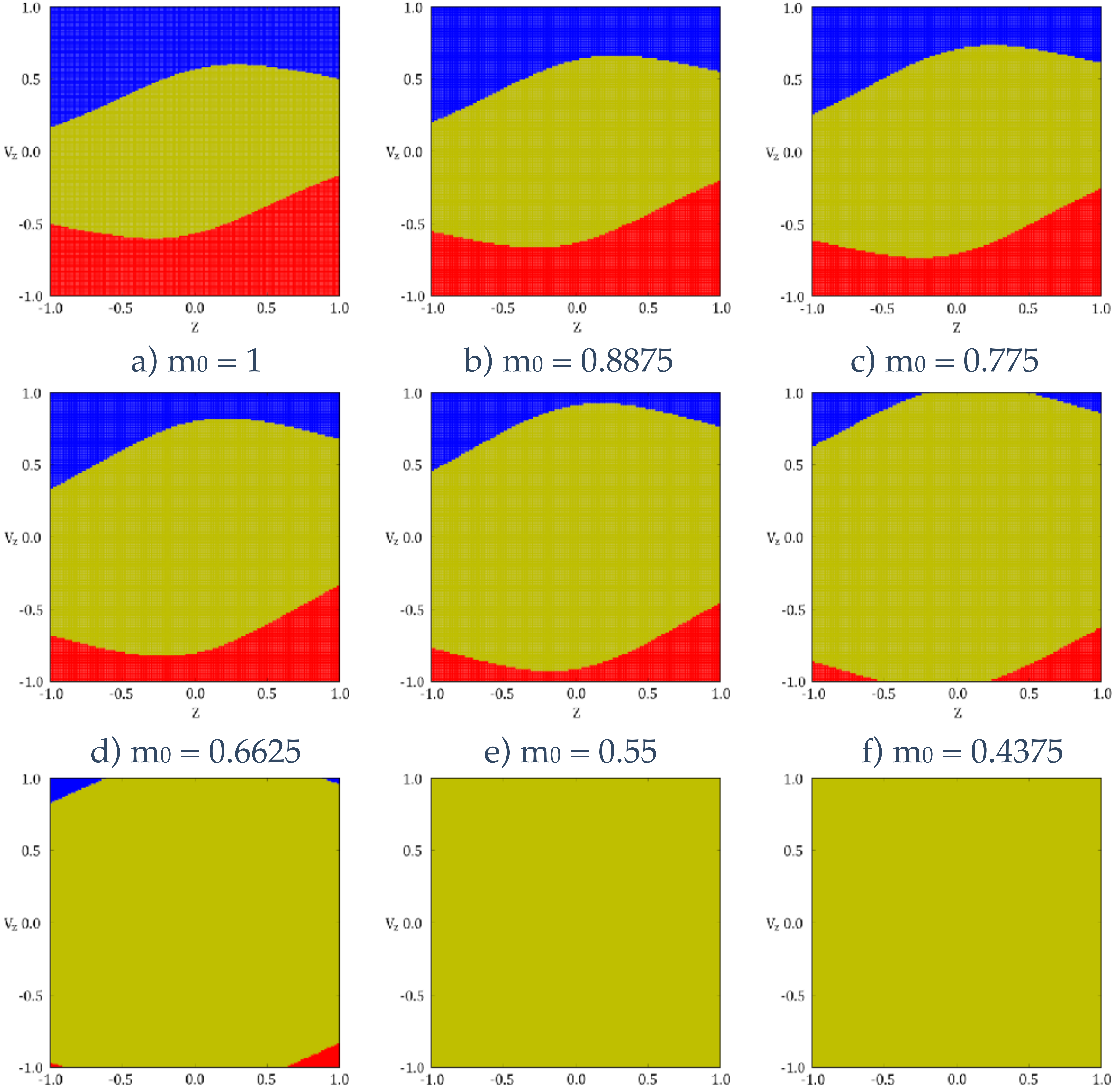

a) $m_0 = 1$ b) $m_0 = 0.8875$ c) $m_0 = 0.775$

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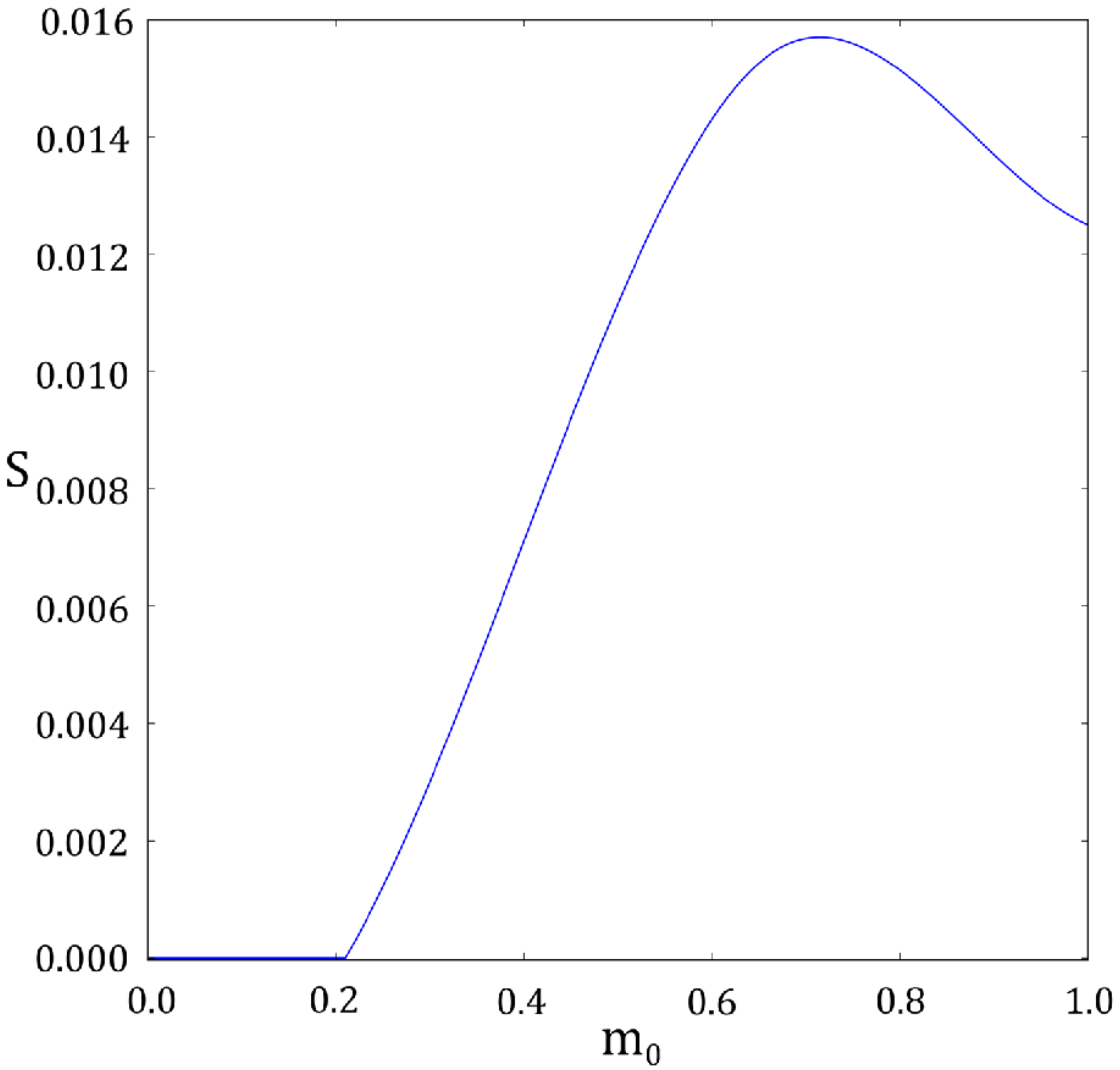

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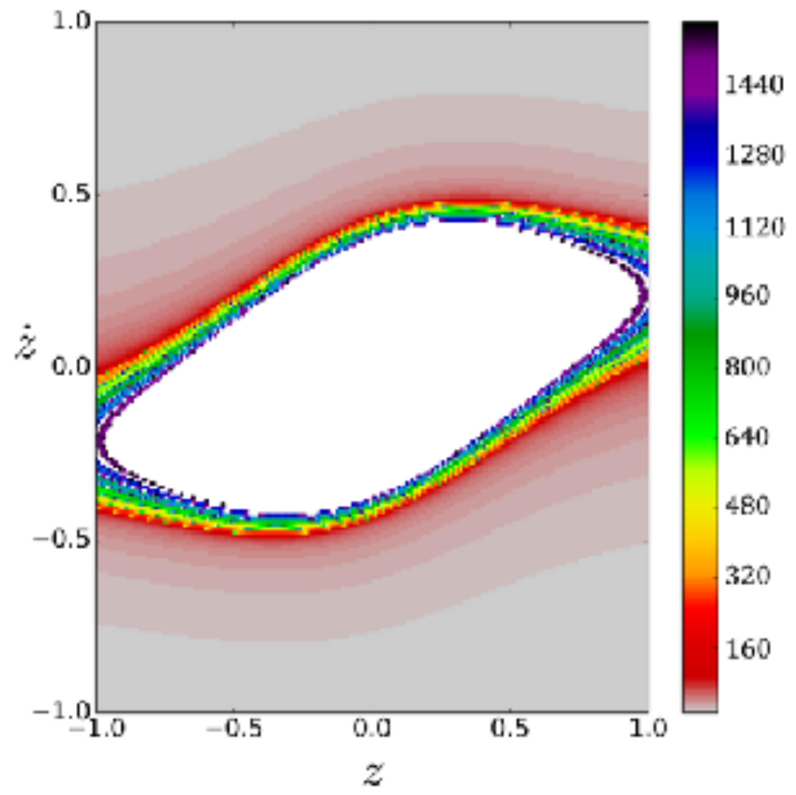
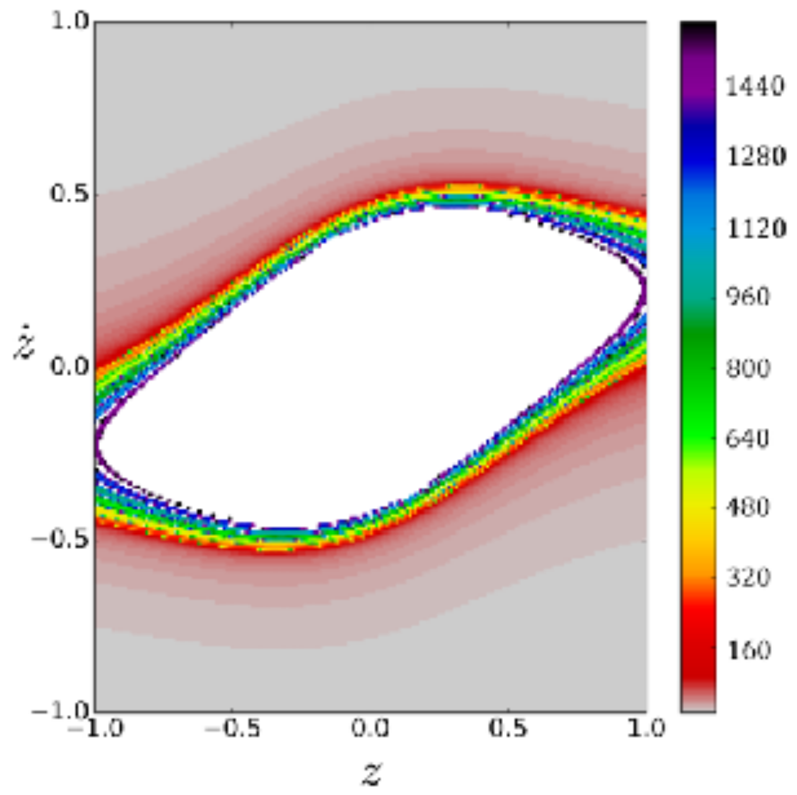
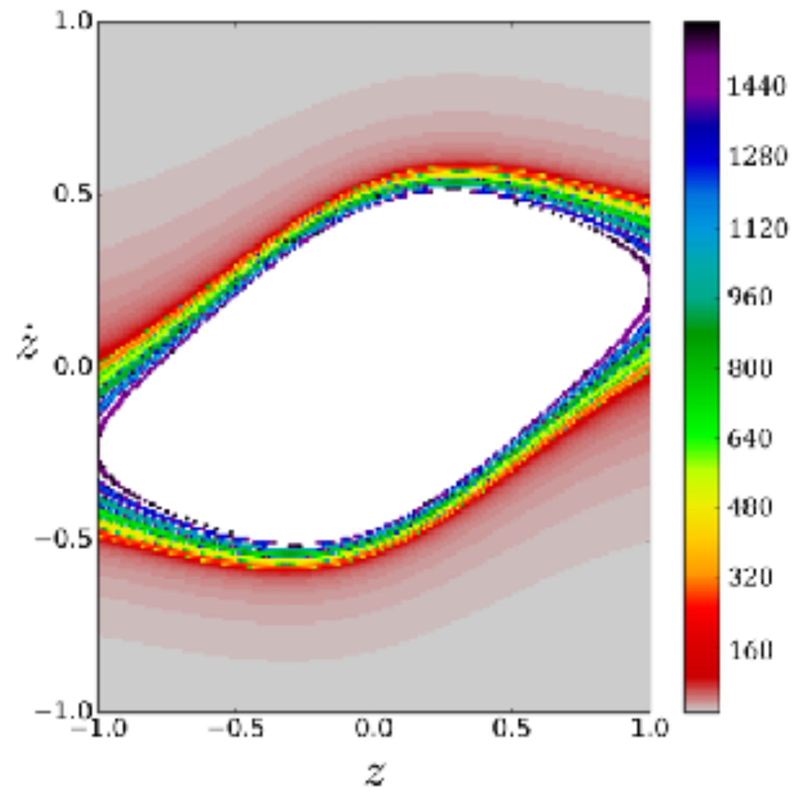


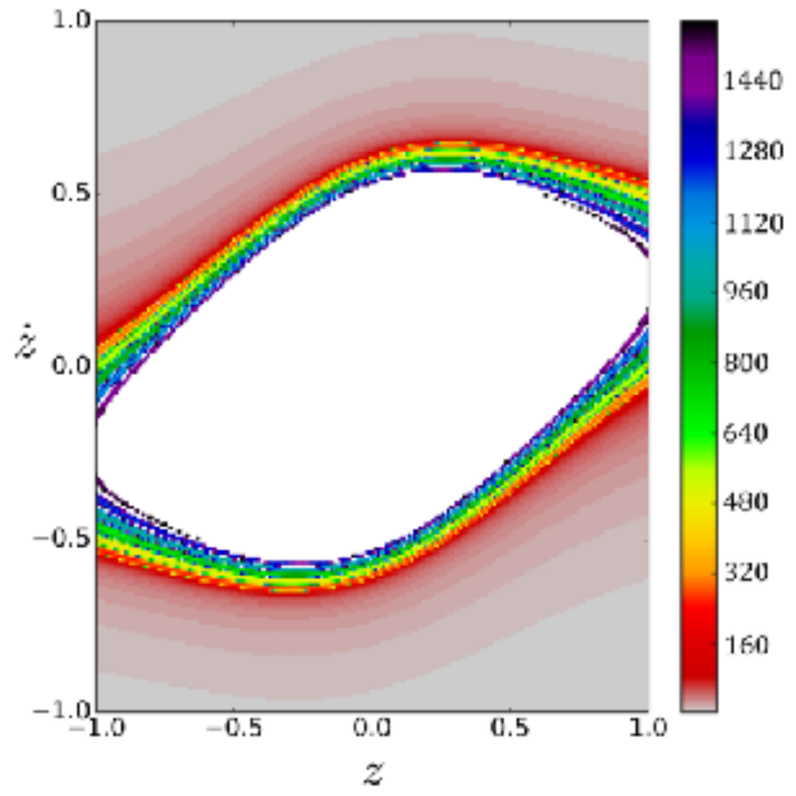
a) $m_0 = 1$



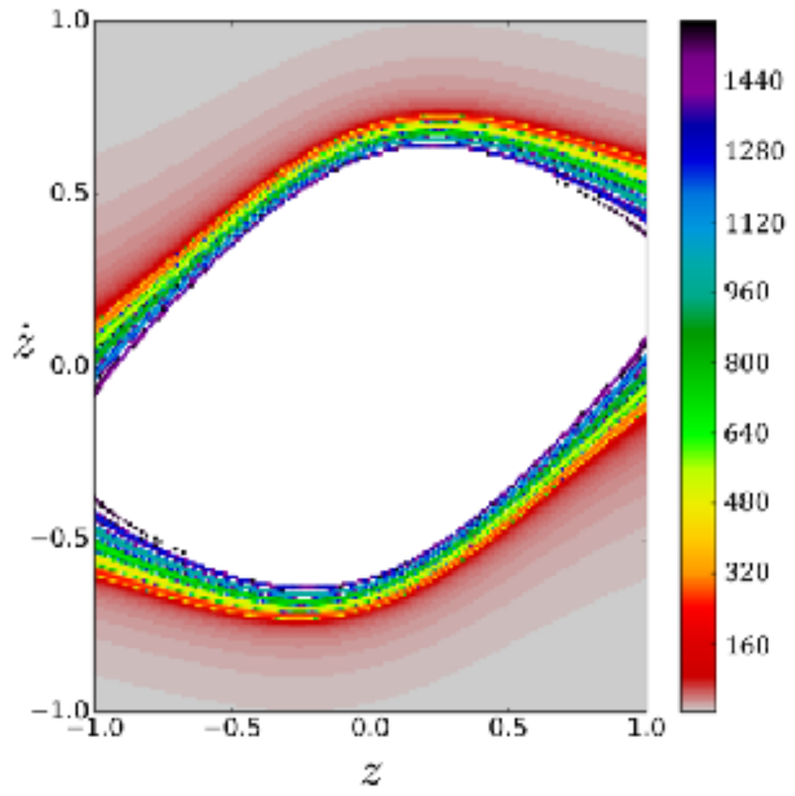
b) $m_0 = 0.8875$



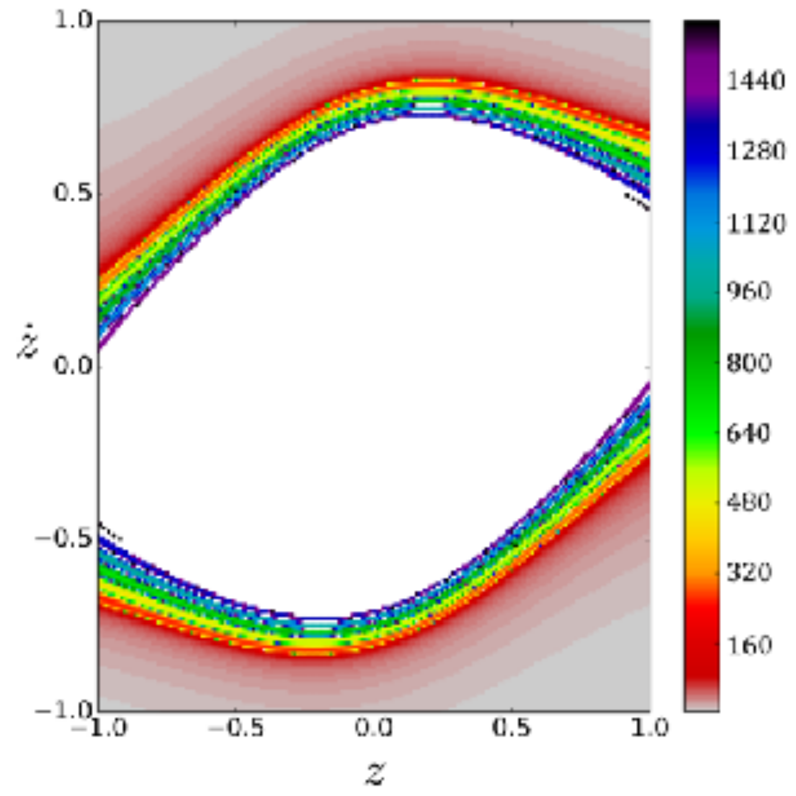
c) $m_0 = 0.775$



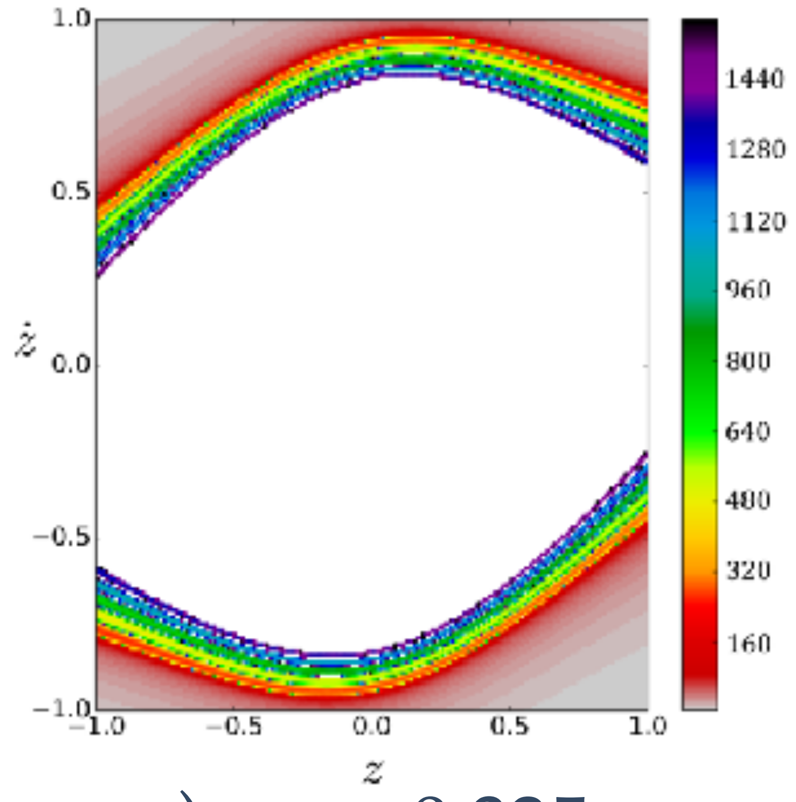
d) $m_0 = 0.6625$



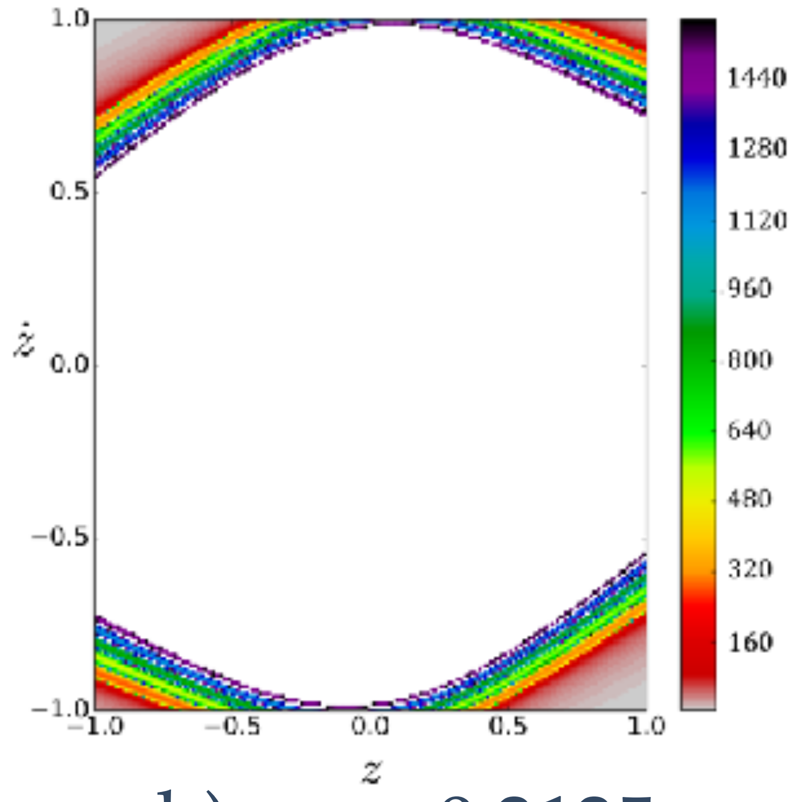
e) $m_0 = 0.55$



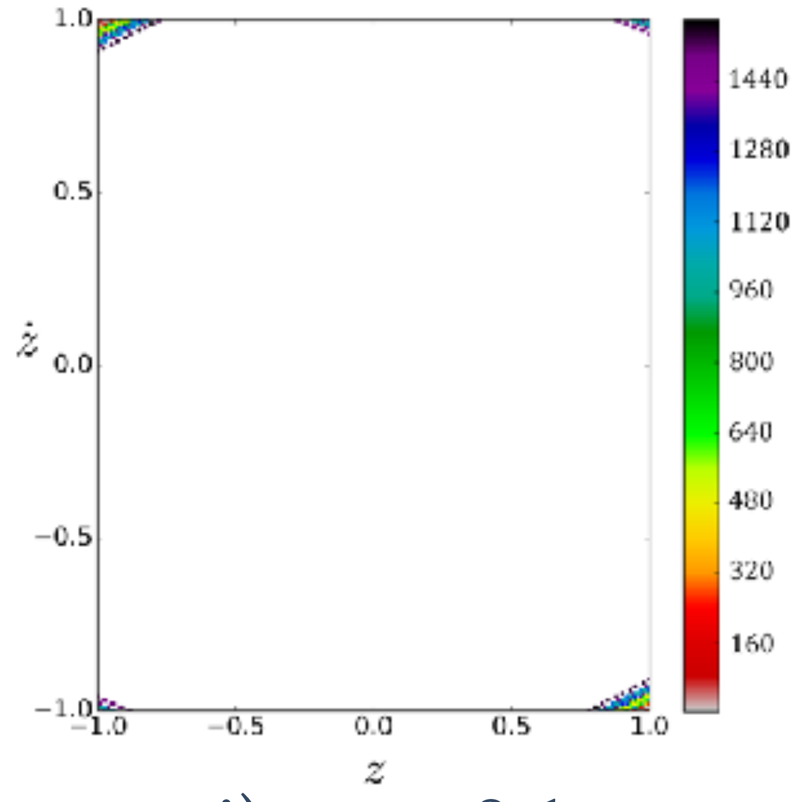
f) $m_0 = 0.4375$



g) $m_0 = 0.325$



h) $m_0 = 0.2125$



i) $m_0 = 0.1$

The figure shows the escape basins for initial mass values in range (0.1 , 1).

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Conclusions

- For the MacMillan problem, when the test particle has a variable mass, the amplitude of the oscillations increases with time due to the behavior of the coefficient that accompanies the velocity in the equation of motion.
- The oscillatory character of the particle is not periodic due to the loss of mass.
- The lower the initial mass value, the greater the number of initial conditions that determine a bounded orbit for the test particle
- Increasing the value of n, the maximum value of entropy becomes smaller, allowing this to conclude that there will come a time when the variation of mass is so small that the entropy of each basin will reach zero for any initial mass value.
- The time it takes to the test particle to escape from the gravitational attraction of the system is inversely proportional to the closeness of the initial conditions to the origin in the phase diagram.

Some References

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Thank you!!!