# Exchange potential in KN scattering

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## Abstract

The Fock-Tani formalism is a first principle method to obtain effective interactions from microscopic Hamiltonians. Originally derived for meson-meson or baryon-baryon scattering, we present the corresponding equations for meson-baryon scattering. Then we include the meson-quark acoplament constant, to the interaction potencial between quarks with gluon exchange. In particular, we shall obtain the low energy total cross section for the $K^- + p \to K^- + p$ channel.

## Fock-Tani Formalism

The Fock-Tani formalism uses a unitary operator $U$ to rewrite the particle operators, redefining meson and baryon states as ideals elementary hadron states that satisfy the canonical commutation relations

$$\begin{aligned} |\Omega> \longrightarrow |\Omega) &= U^{-1}|\Omega>, \\ O \longrightarrow O_{\rm FT} &= U^{-1} O U \, . \end{aligned}$$

Once a microscopic interaction Hamiltonian $H$ is defined at the quark level, a new transformed Hamiltonian can be obtained. The transformed Fock-Tani Hamiltonian is a result of the the application of the unitary transformation on the microscopic Hamiltonian

$$H_{\rm FT} = U_B^{-1} U_M^{-1} H \, U_M \, U_B .$$

The transformed Hamiltonian $H_{\rm FT}$ describes all possible processes involving mesons, baryons and quarks. After the applying the Fock-Tani transformation we obtain the following meson-baryon potential with quark and gluon exchange [1]

$$V_{\rm mb}(\alpha\beta;\delta\gamma) = \sum_{i=1}^{4} V_i(\alpha\beta;\delta\gamma)\, m_\alpha^\dagger \, b_\beta^\dagger \, m_\gamma \, b_\delta$$

where

$$\begin{aligned} V_1(\alpha\beta;\delta\gamma) &= -3V_{qq}(\mu\nu;\sigma\rho)\,\Phi_\alpha^{*\mu\nu_2}\,\Psi_\beta^{*\nu\mu_2\mu_3}\,\Phi_\gamma^{\rho\nu_2}\,\Psi_\delta^{\sigma\mu_2\mu_3} \\ V_2(\alpha\beta;\delta\gamma) &= -3V_{q\bar{q}}(\mu\nu;\sigma\rho)\,\Phi_\alpha^{*\mu_1\nu}\,\Psi_\beta^{*\mu\mu_2\mu_3}\,\Phi_\gamma^{\sigma\rho}\,\Psi_\delta^{\mu_1\mu_2\mu_3} \\ V_3(\alpha\beta;\delta\gamma) &= -3V_{qq}(\mu\nu;\sigma\rho)\,\Phi_\alpha^{*\mu\nu_2}\,\Psi_\beta^{*\mu_1\nu\mu_3}\,\Phi_\gamma^{\mu_1\nu_2}\,\Psi_\delta^{\sigma\rho\mu_3} \\ V_4(\alpha\beta;\delta\gamma) &= -6V_{q\bar{q}}(\mu\nu;\sigma\rho)\,\Phi_\alpha^{*\nu_1\nu}\,\Psi_\beta^{*\mu_1\mu\mu_3}\,\Phi_\gamma^{\mu_1\rho}\,\Psi_\delta^{\nu_1\sigma\mu_3} \, . \end{aligned}$$

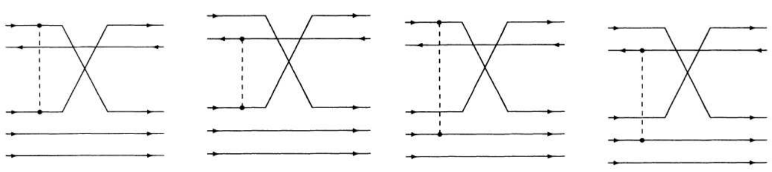
And the direct term without quark exchange [2]

$$V^{dir} \;=\; V_{direct}(\alpha\beta;\delta\gamma)\, m_\alpha^\dagger \, b_\beta^\dagger \, m_\gamma \, b_\delta ,$$

with

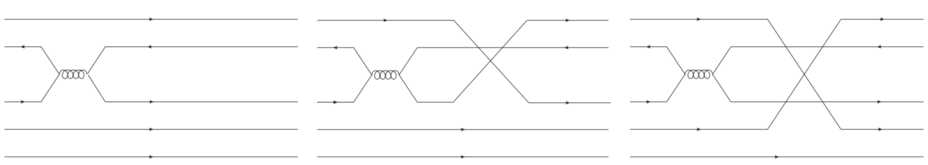
$$V_{direct}(\alpha\beta;\delta\gamma) = 3V_{q\bar{q}}(\mu\nu;\sigma\rho)\Phi_\alpha^{*\mu_1\nu}\Psi_\beta^{*\mu\mu_2\mu_3}\Phi_\gamma^{\mu_1\rho}\Psi_\delta^{\sigma\mu_2\mu_3} \, .$$

The scattering diagrams $V_k$ can be seen in the following figure.



$V_1$ $V_2$ $V_3$ $V_4$

And the annihilation diagrams $V_k$ are



$V_{direct}$ $\overline{V_2}$ $\overline{V_4}$

## Results

The interaction potential between quarks is

$$V_{qq \; or \; q\bar{q}} = \sum_{ij} \frac{g_{mq}^2}{4\pi} \left[ \frac{\alpha_s}{r_{ij}} - \frac{3}{4} b \, r_{ij} - \frac{8\alpha_s}{3\sqrt{\pi}\, m_i m_j} \sigma_1^3 e^{-\sigma_1^2 r_{ij}^2} \vec{S}_i \cdot \vec{S}_j - \frac{\pi \alpha_s}{2} \sigma_2^3 e^{-\sigma_2^2 r_{ij}^2} \left( \frac{1}{m_i^2} - \frac{1}{m_j^2} \right) \right] \vec{T}_i \cdot \vec{T}_j \, ,$$

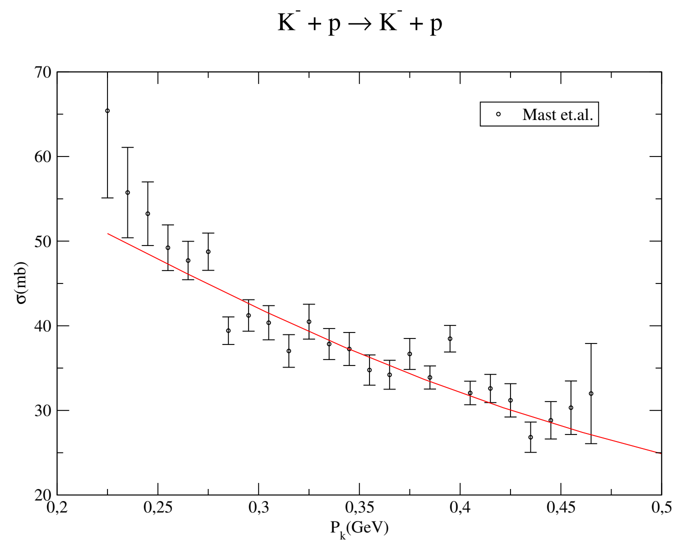
where $g_{mq}^2$ is the meson-quark coupling constant.
Starting with the Hamiltonian and using the first order Born approximation in the matrix-T, we can write $V_{\rm mb}$ as

$$V_{\rm mb}(\alpha\beta;\delta\gamma) = \delta(P_f - P_i)\, h_{fi}$$

and the scattering amplitude $h_{fi}$ is defined by $h_{fi} = \sum_{k=1}^4 V_k$, where $V_k = \omega_k \, I_k^e$.Spatial integrals denoted by $I_k^e$ and $\omega_k$ is the spin-flavor-color part. These amplitudes can be related to the cross section in the laboratory system [3]

$$\frac{d\sigma}{d\Omega} = \frac{16\pi^2 E_{K^-} \, E_p \, E_\Lambda \, E_\eta}{(E_{K^-} + E_p)(E_\Lambda + E_\eta)} \, \frac{|\,\vec{p}_f\,|}{|\,\vec{p}_k\,|} \, |h_{fi}|^2$$

where $\vec{p}_k$ and $\vec{p}_f$ are the initial and final state momenta, $E_k$ are the energy particles and $m_{k^-} = 0.493$ GeV, $m_p = 0.938$.




Total cross sections of the $K^- + p \to K^- + p$ (red line) compared with the data from [3], $\sigma_1 = 0.6$ GeV, $\sigma_2 = 0.01$ GeV, $\alpha_s = 0.45$ GeV, $\alpha_\lambda = 0.4$ GeV, $x = 0.48$, $\beta = 0.3$ GeV and $b = 0.2$ GeV$^2$

## Perspectives

The kaon-nucleon (KN) system has provided an ideal setting for studying short-distance effects of the hadron-hadron force. We intent to do a sistematic study about the non-perturbatives aspects using Schwinger-Dyson equations and higher orders of T-matrix, in low energy interactions of the the kaon-nucleon system $K^+N$, and the charm systems $DN$ and $D^*N$.
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## References

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[2] B. Folador. Interações méson-bárion no formalismo de Fock-Tani aplicado ao sistema Káon- Núcleon. 2017. 80f. Exame de qualificação (Doutorado em Física)-Instituto de Física, Universidade Federal do Rio Grande do Sul, Porto Alegre.

[3] T.S.Mast et al, Phys. Rev. **D14**, 13 (1976).