Correlations functions of primordial perturbations from symmetries[#1]

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Outline

Motivations

- CMB observations allow the existence of anomalies.
- Vector field perturbations may cause part of the primordial perturbations.
- Vector fields are natural sources for statistical anisotropies, parity violation, among others.
- Statistical descriptors in cosmology relates theory and observation.
- Symmetries gives information about physical systems.

Key idea

1. We exploit the isomorphism between the $4D$ de Sitter space and the $3D$ Conformal space.
2. We will asume that correlation functions of primordial perturbations are invariant under conformal transformations.

The Method

1. Solve the equations of motion of the fields.
2. Find the asymptotic fields in the limit when $-k\tau \to 0$.
3. Find the conformal weight of the fields in $3D$ by using the transformation laws for the fields in $4D$.
4. Propose a general form for the correlation function.
5. Find the form of the correlator using the conditions impose to the fields.
6. Use the Ward identities to constrain the form of the correlation functions.

The Scalar-Vector Coupled System

The action

$$S = -\frac{1}{4}\int d^4x\sqrt{-g}\left[f_1(\phi)F^{\mu\nu}F_{\mu\nu} + f_2(\phi)\tilde{F}^{\mu\nu}F_{\mu\nu}\right] + S_{\mathrm{E-H}} + S_\phi \ .$$

Dynamical equations

$$\nabla_\mu\left(f_1(\phi)F^{\mu\nu} + f_2(\phi)\tilde{F}^{\mu\nu}\right) = 0 \ ,$$

$$\nabla_\mu\tilde{F}^{\mu\nu} = 0 \ .$$

- We use the gauge symmetry to impose the Coulomb gauge.
- We assume that the inflationary dynamic homogenize the scalar field.

Dynamic equations for A_i

$$\left(\frac{\partial^2}{\partial\tau^2} - \nabla^2 + \frac{1}{f_1}\frac{\partial f_1}{\partial\tau}\frac{\partial}{\partial\tau} + \frac{1}{f_1}\frac{\partial f_2}{\partial\tau}\nabla\times\right)\vec{A}(\tau,\vec{x}) = 0 \ .$$

- Dilatation imply that the coupling functions must be proportional and of the same order: $f_1 = \gamma f_2$ and $f_1 \propto (-H\tau)^{-2\alpha}$

Canonical fields

$$A_i(\tau, x_i) \equiv \frac{w_i(\tau, x_i)}{\sqrt{f_1}} \,.$$

The equation to solve

$$\left(\frac{\partial^2}{\partial\tau^2} + k^2 - \frac{\alpha(\alpha+1)}{\tau^2} \pm \frac{2\alpha\gamma k}{\tau}\right) \tilde{w}_{\pm}(\tau, \vec{k}) = 0 \,.$$

The solution

$$\tilde{w}_{\pm}(\tau, \vec{k}) = C_1^{\pm}(k) G_{-\alpha-1}(\pm\xi, -k\tau) + C_2^{\pm}(k) F_{-\alpha-1}(\pm\xi, -k\tau).$$

Taking the asymptotic limit $|8\xi k\tau| \ll 1$ and come back to coordinates:

$$w_{\pm}(\tau,\vec{x}) \approx u_{\pm}(\vec{x})(-\xi H\tau)^{\alpha+1} + v_{\pm}(\vec{x})(-\xi H\tau)^{-\alpha}\ .$$

When $\alpha > -1/2$:

$$\lim_{|\tau|\to 0} w_{\pm}(\tau,\vec{x}) = \tau^{-\alpha} v_{\pm}(\vec{x})\ .$$

$v_{\pm}$ conformal weight

$$\Delta_v = 1-\alpha\ .$$

When $\alpha < -1/2$:

$$\lim_{|\tau|\to 0} w_{\pm}(\tau,\vec{x}) = \tau^{\alpha+1} u_{\pm}(\vec{x})\ .$$

$u_{\pm}$ conformal weight

$$\Delta_u = \alpha+2\ .$$

The Ward Identities

Dilatation Ward identity for a N-point correlation function

$$\left[-3(N-1)+\sum_{a=1}^{N}\Delta_a-\sum_{a=2}^{N}\vec{k}_a\cdot\frac{\partial}{\partial\vec{k}_a}\right]\langle V_i^{(1)}(\vec{k}_1)\cdots V_j^{(N)}(\vec{k}_N)\rangle'=0\,.$$

SCT Ward identity for a N-point correlation function

$$b^m\sum_{a=1}^{N}\left[2(\Delta_a-3)\frac{\partial}{\partial k_a^m}+k_{m(a)}\nabla^2_{k_a}-2\vec{k}_a\cdot\frac{\partial}{\partial\vec{k}_a}\frac{\partial}{\partial k_a^m}\right]\langle V^1_{i_1}(\vec{k}_1)\cdots V^N_{i_N}(\vec{k}_2)\rangle'-$$
$$-2\sum_{a=1}^{N}\sum_{l=1}^{N}\left[\left(b^{j_l}\frac{\partial}{\partial k_a^{i_l}}-b_{i_l}\frac{\partial}{\partial k_{(a)j_l}}\right)\langle V_{i_1}(\vec{k}_1)\cdots V_{j_l}(\vec{k}_l)\cdots V_{i_N}(\vec{k}_{i_N})\rangle'\right]=0\,.$$

The vector two-point correlation functions

General form:

$$\langle \delta A_i(\vec{k})\delta A_j(-\vec{k})\rangle = P_A(k)(\delta_{ij} - b_1(k)\hat{k}_i\hat{k}_j + b_2(k)\epsilon_{ija}k_a) \equiv P_{ij}(\vec{k}) \; .$$

By using the divergenceless condition for A_i and the reality condition:

$$P_{ab}(\vec{k}) \equiv \Pi_{ab}P(k) \equiv [\Delta_{ab} + ib(k)\epsilon_{abc}\hat{k}_c]P(k). \quad \Delta_{ab} \equiv \delta_{ab} - \hat{k}_a\hat{k}_b.$$

Applying the dilatation Ward identity we get:

$$(-3 + 2\Delta_v - \vec{k}\cdot\partial_{\vec{k}})P(k) = 0, \quad \vec{k}\cdot\partial_{\vec{k}}b(k) = 0,$$

“Electric case”:

$$P^v_{ab}(\vec{k}) = A_v k^{2\alpha+1}[\Delta_{ab} + i\beta\epsilon_{abc}\hat{k}_c] \; .$$

“Magnetic case”:

$$P^u_{ab}(\vec{k}) = A_u k^{-(2\alpha+1)}[\Delta_{ab} + i\beta\epsilon_{abc}\hat{k}_c] \; .$$

Vector 3-point correlation functions

We propose the general form:

$$\begin{aligned}\langle\delta\phi(\vec{k}_1)\delta A_i(\vec{k}_2)\delta A_j(\vec{k}_3)\rangle' &= a_1\delta_{ij} + a_2\hat{k}_{2i}\hat{k}_{3j} + a_3\hat{k}_{3i}\hat{k}_{2j} + a_4\hat{k}_{2i}\hat{k}_{2j} + a_5\hat{k}_{3i}\hat{k}_{3j} \\ &+ b_1\epsilon_{ija}\hat{k}_{2a} + b_2\epsilon_{ija}\hat{k}_{3a} + b_3\hat{k}_{2i}\epsilon_{jab}\hat{k}_{2a}\hat{k}_{3b} \\ &+ b_4\hat{k}_{3i}\epsilon_{jab}\hat{k}_{2a}\hat{k}_{3b} + b_5\hat{k}_{2j}\epsilon_{iab}\hat{k}_{2a}\hat{k}_{3b} + b_6\hat{k}_{3j}\epsilon_{iab}\hat{k}_{2a}\hat{k}_{3b}\end{aligned}$$

Following the method and taking the squeeze limit we can write:

$$\langle\delta\phi(\vec{k}_1)\delta A_i(\vec{k}_2)\delta A_j(\vec{k}_3)\rangle' = \frac{1}{k_1^{3-\Delta_{\delta\phi}}k_2^{3-2\Delta_v}}\left[(\alpha_1+\alpha_2)\left(\delta_{ij}-\hat{k}_{2i}\hat{k}_{2j}\right)+2i\alpha_3\epsilon_{ija}\hat{k}_{2a}\right].$$

Tensor perturbations

Equation of motion

$$h''_\lambda - \frac{2}{\tau}h'_\lambda - \nabla^2 h_\lambda = \frac{2}{M_P^2}\Pi_\lambda^{lm} T_{lm}^{EM} \ ,$$

Asymptotic solution

$$h_\lambda(\tau, k) = \gamma_\lambda(k)(-H\tau)^{4+2n}.$$

Conformal weight for the 3D field

$$\Delta_\gamma = 4 + 2n,$$

Solution in terms of the polarization tensor

$$\gamma^{ij}(\vec{k}) = \sum_{\lambda=\pm 2} \Pi_\lambda^{ij}(\vec{k})\gamma_\lambda(\vec{k}) \ .$$

Results

Two point functions

$$\langle\gamma_\lambda^{(0)}(\vec{k})\zeta^{(0)}(-\vec{k})\rangle = 0,$$

$$\langle\gamma_\lambda^{(\delta A)}(\vec{k})\zeta^{(\delta A)}(-\vec{k})\rangle \propto \delta(\vec{k}_{12})k^{-3+\Delta_\gamma+\Delta_\zeta}\left[1-2A\lambda-2B\lambda^2\right]\hat{E}^l\hat{E}^m\Pi_\lambda^{lm},$$

$$\langle\gamma_\lambda^{(0)}(\vec{k})\gamma_{\lambda'}^{(0)}(-\vec{k})\rangle \propto \delta(\vec{k}_{12})k^{-3+2\Delta_\gamma}\left[1-2\lambda B_2-2B_3\lambda^2\right]\delta_{\lambda,\lambda'}\ ,$$

$$\langle\gamma_\lambda^{(\delta A)}(\vec{k})\gamma_{\lambda'}^{(\delta A)}(-\vec{k})\rangle \propto \delta(\vec{k}_{12})k^{-3+2\Delta_\gamma}\hat{E}^k\hat{E}^n\Delta_{kn}\left[1+\lambda U_2+\lambda^2U_3\right]\delta_{\lambda,\lambda'}.$$

3-point functions (squeezed limit)

$$\lim_{k_1\to 0}\langle\gamma_\lambda^{(0)}(\vec{k}_1)\zeta^{(0)}(\vec{k}_2)\zeta^{(0)}(\vec{k}_3)\rangle \propto$$

$$\delta_{\vec{k}_{123}}k_1^{-3+2\Delta_\gamma}k_2^{-3-\Delta_\gamma+2\Delta_\zeta}\left[1-\lambda R_2-\lambda^2R_3\right]2\Pi_\lambda^{ab}(\vec{k}_1)\hat{k}_{2a}\hat{k}_{2b}\ ,$$

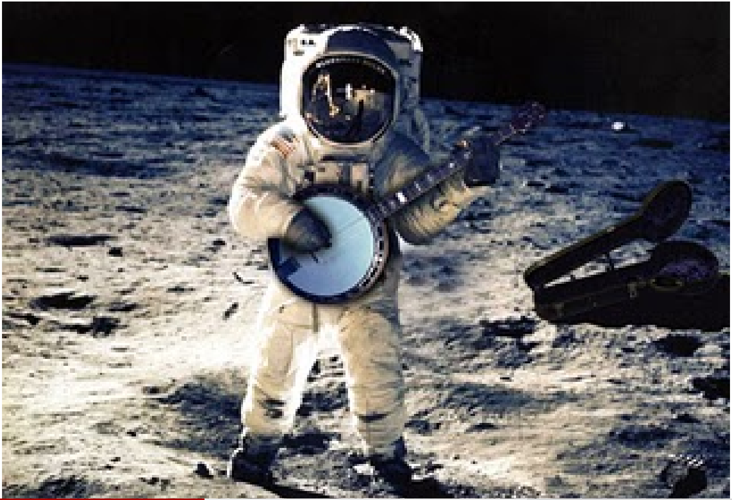
$$\lim_{k_1\to 0}\langle\gamma_\lambda^{(0)}(\vec{k}_1)\zeta^{(\delta A)}(\vec{k}_2)\zeta^{(\delta A)}(\vec{k}_3)\rangle \propto$$

$$\delta_{\vec{k}_{123}}k_2^{-3-\Delta_\gamma+2\Delta_\zeta}k_1^{-3+2\Delta_\gamma}\Pi_\lambda^{ij}(\vec{k}_1)\hat{E}^l\hat{E}^m\Pi_{lj}^{(-)}(\vec{k}_2)\Pi_{mi}^{(+)}(\vec{k}_2)\left[1-\lambda S_2+\lambda^2S_3\right].$$

Summary

- We probed that the coupling $(f_1(\phi)F^2 + f_2(\phi)F\tilde{F})$ is compatible with conformal symmetries in the asymptotic future when the coupling functions are homogeneous.
- Using this method for $f(\phi)(F^2 + \gamma F\tilde{F})$ we found a form for the correlators is in agreement with the literature. With the only down side being that you can't compute the amplitudes with this method.
- The parity violation term in the action generates an explicit dependence of the polarization in the tensor correlators.

THANK YOU!!!



matrices resulting from the conditions over the $\gamma - \zeta$ vector sourced correlation function

$$B^{(1)}_{ijlm} = \Delta_{mj}\Delta_{il} + \Delta_{lj}\Delta_{im} - \Delta_{ij}\Delta_{lm},$$
$$B^{(2)}_{ijlm} = \Delta_{il}\epsilon_{jma}\hat{k}_a + \Delta_{lj}\epsilon_{ima}\hat{k}_a - \Delta_{ij}\epsilon_{lma}\hat{k}_a,$$
$$B^{(3)}_{ijlm} = \epsilon_{ila}\hat{k}_a\epsilon_{jmb}\hat{k}_b + \epsilon_{ima}\hat{k}_a\epsilon_{jlb}\hat{k}_b - \Delta_{ij}\Delta_{lm},$$
$$B^{(4)}_{ijlm} = \Delta_{im}\epsilon_{jla}\hat{k}_a + \Delta_{mj}\epsilon_{ila}\hat{k}_a + \Delta_{ij}\epsilon_{lma}\hat{k}_a.$$

Maxwell dual tensor

$$\tilde{F}_{\mu\nu} = \frac{1}{2\sqrt{-g}}\varepsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$$

Conformal de Sitter metric

$$ds^2 = \frac{1}{(H\tau)^2}\left(-d\tau^2 + d\vec{x}^2\right).$$

Conformal Transformations

$$x_i \to x_i' = a_i + M_{ij}x_j \ ,$$
$$x_i \to x_i' = \lambda x_i \ \ \tau \to \tau' = \lambda\tau \ ,$$
$$x_i \to x_i' = \frac{x_i + b_i\left(-\tau^2 + \vec{x}^2\right)}{1 + 2\vec{b}\cdot\vec{x} + b^2\left(-\tau^2 + \vec{x}^2\right)}$$
$$\tau \to \tau' = \frac{\tau}{1 + 2\vec{b}\cdot\vec{x} + b^2\left(-\tau^2 + \vec{x}^2\right)} \ ,$$