

# Testing black-hole near-horizon effects and pseudo-complex general relativity with gravitational Waves



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# Motivation

- Einstein's theory of general relativity is one of the most successful scientific theories of all time.
- Only with very careful measurements can we notice that the Solar System is not described by Newtonian Gravity.
- General relativity predicts its own incompleteness (for black holes and the Big Bang).
- Black hole entropy and the information paradox suggest there may be something going on at (or near) the horizon.
- With binary mergers, observation is now ahead of theory.

# Testing gravity with GWs

- Compact and dynamic

$$\frac{v}{c} \sim \sqrt{\frac{GM}{c^2 r}} \sim \left( \frac{\pi G M f_{GW}}{c^3} \right)^{1/3}$$

- Curvature scale corrections to gravity?

$$R_{abcd} R^{abcd} l_p^4 \sim \frac{3}{4} \frac{\hbar^2 c^2}{G^2 M^4} \ll 1$$

- Horizon scale corrections to gravity?

$$\langle B | \hat{T}_{ab} | B \rangle \Rightarrow \infty$$

# Flux correlations

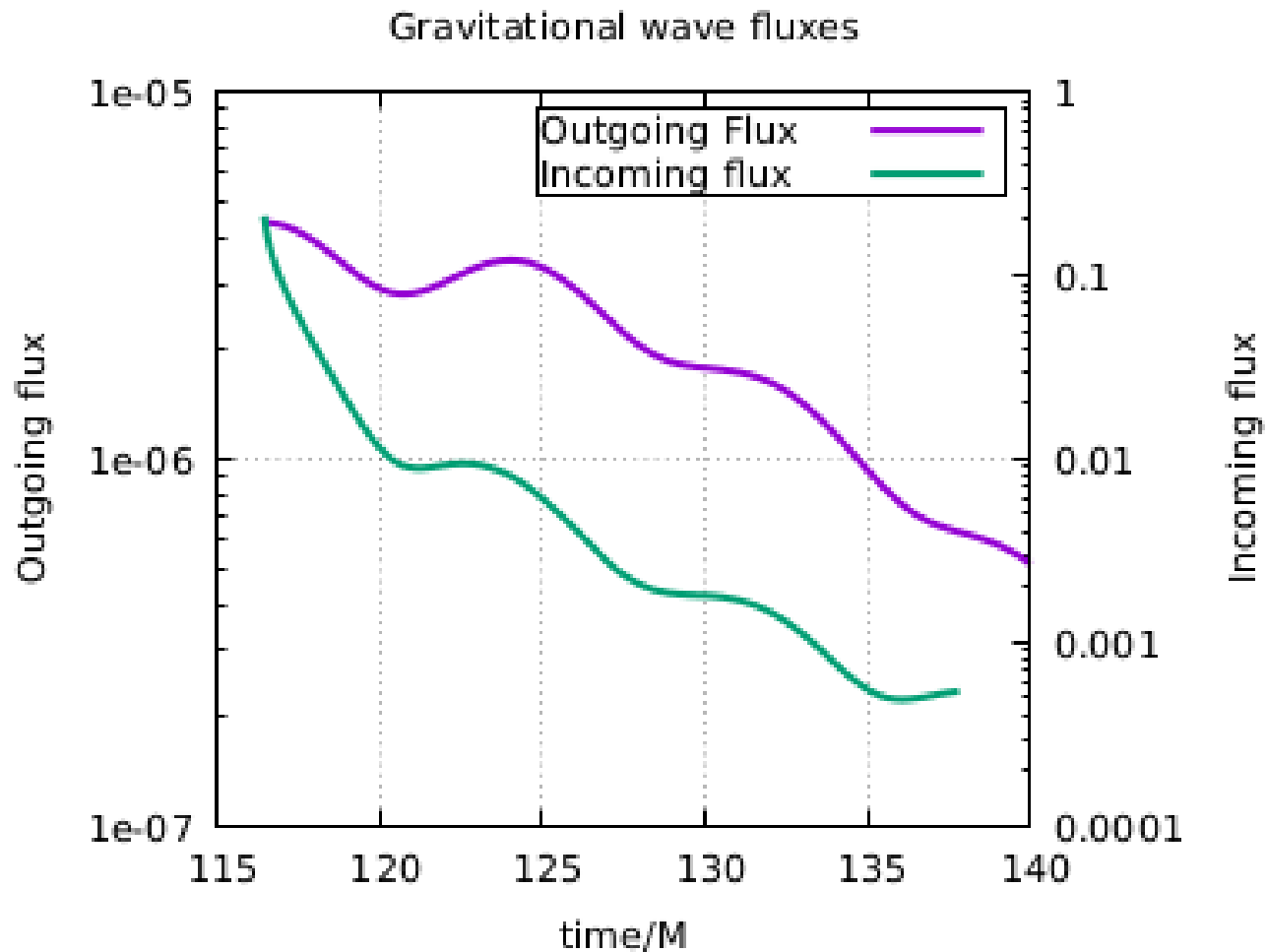


Image: Gupta, Krishnan, Nielsen, Schnetter PRD97 (2018) 084028

# Modified GR solutions and pcGR

$$-g_{tt} = 1 - \frac{2M}{r} + \frac{1}{r} \int \xi dr$$

Full Kerr-like solution derived (Casper et al. 2012 for pcGR)

$$\begin{aligned} g_{tt} &= -\left(1 - \frac{\psi}{\Sigma}\right) , & g_{rr} &= \frac{\Sigma}{\Delta} , & g_{\theta\theta} &= \Sigma , \\ g_{\phi\phi} &= \left( (r^2 + a^2) + \frac{a^2\psi}{\Sigma} \sin^2 \theta \right) \sin^2 \theta , \\ g_{t\phi} &= g_{\phi t} = -a \frac{\psi}{\Sigma} \sin^2 \theta , \end{aligned}$$

Form of  $\xi$  is bound by Solar System tests, but not by near horizon physics....yet

# Solutions without horizons

Take a dimensionless parameter,  $b$ :

$$b = - \left( \frac{r}{M} \right)^n \frac{\int \xi dr}{2M}$$

Provides effective correction to the mass  $M$ :

$$m(r) = M \left( 1 + b \left( \frac{M}{r} \right)^n \right)$$

For sufficiently large  $b$  values there are no horizons and hence no black holes

$$b > \gamma^n \left( 1 - \frac{\chi^2}{2\gamma} - \frac{\gamma}{2} \right) \quad \gamma = \frac{n + \sqrt{n^2 - (n^2 - 1)\chi^2}}{n + 1}$$

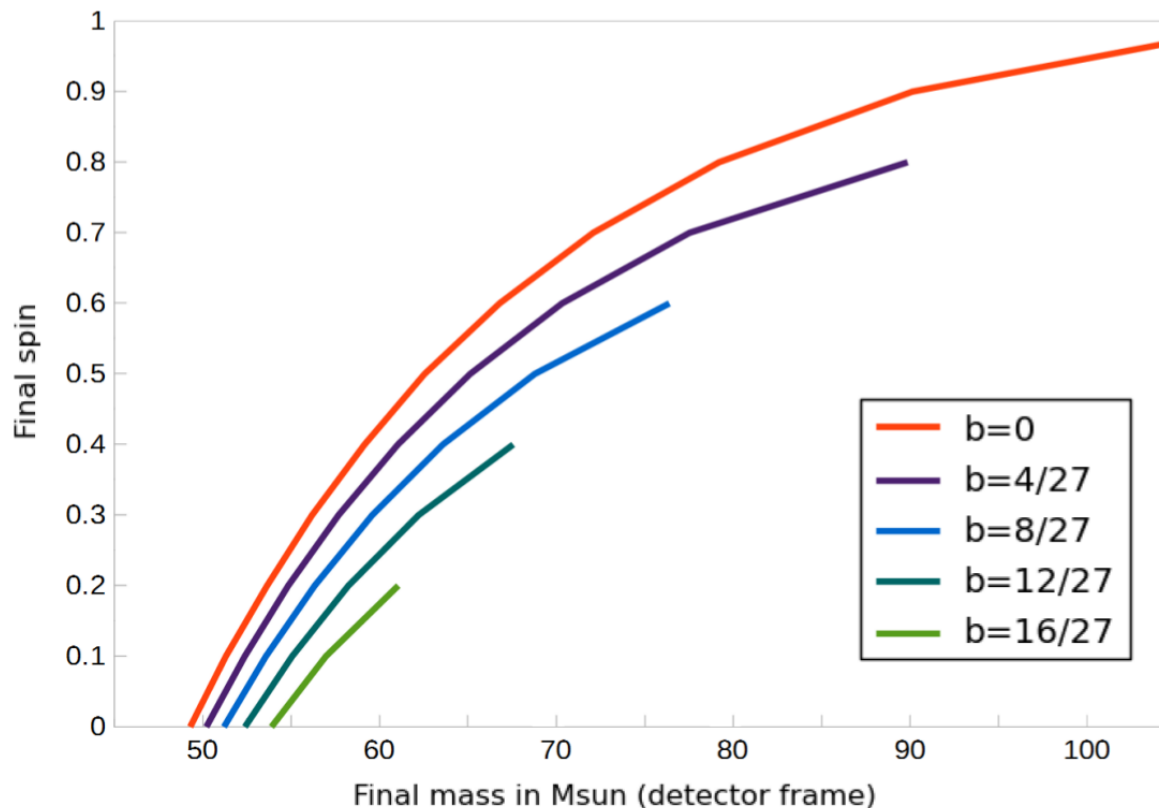
**For  $n=2$  and no spin,  $b_{\text{crit}} = 16/27$**

# Light ring and ringdown

Solve for light ring location:

Geodesic:  $(\omega a + 1)^2 (m - m' r) - \omega^2 r^3 = 0$

Null:  $2m - r + 4\omega ma + \omega^2 (r^3 + ra^2 + 2ma^2) = 0$



# Post-Newtonian terms in inspiral

Expand gravitational wave phase as power series in frequency domain:

$$\Psi(f) = \sum_n p_n \times (\pi M f_{GW})^{n/3}$$

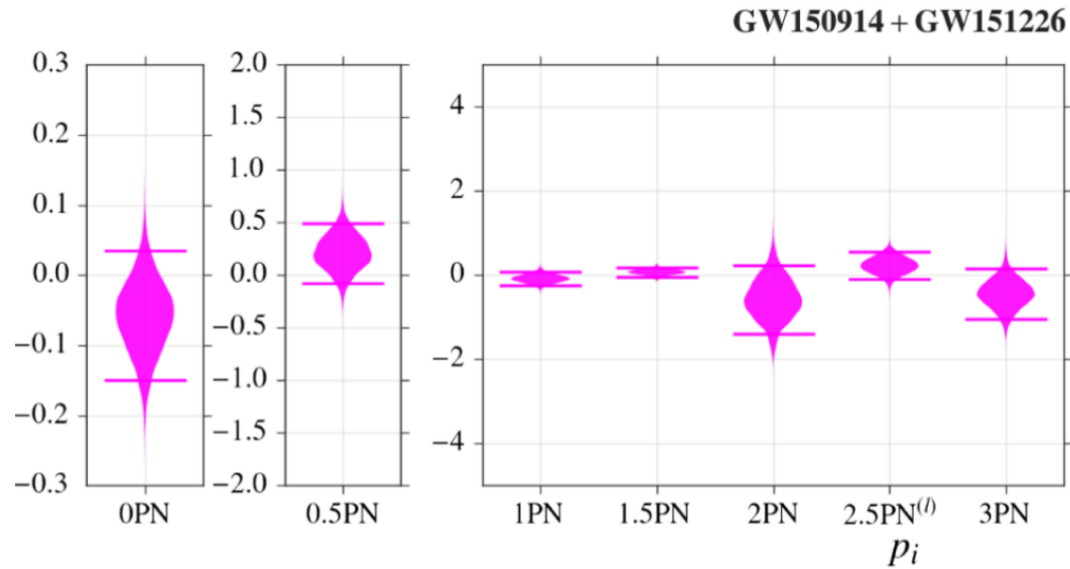
pcGR correction:

$$nPN \text{ term} = \frac{20b(n+2)(n+1)(1+q^n)}{3(n-4)(2n-5)(1+q)^n} (\pi M f_{GW})^{2n/3}$$

**For  $n=2$ ,  $q=1$ , gives about a 25% correction to the value of the GR 2PN term.**



# Bounds on post-Newtonian coefficients



From Fig 7,  
LVC PRX 6 (2016) 041015

n	b_crit	PcGR p_n	GR p_n	Delta p_n	90% confidence range
1	0.5	20/9	6.44	34%	(-20%, 5%)
2	16/27	320/27	46.2	26%	(-130%, 15%)
3	27/32	-225/8	-652	4.3%	(-110%, 10%)

# Summary

- Gravitational wave observations allow tests of near-horizon properties.
- Simple properties can be calculated in many models, including pcGR.
- The LIGO observations already disfavour some regions of pcGR parameter space (while still consistent with vacuum GR).

# Thank you

Further details: Nielsen and Birnholtz, Astron.Nachr. 339 (2018) no.4, 298-305

# Deviations at the horizon

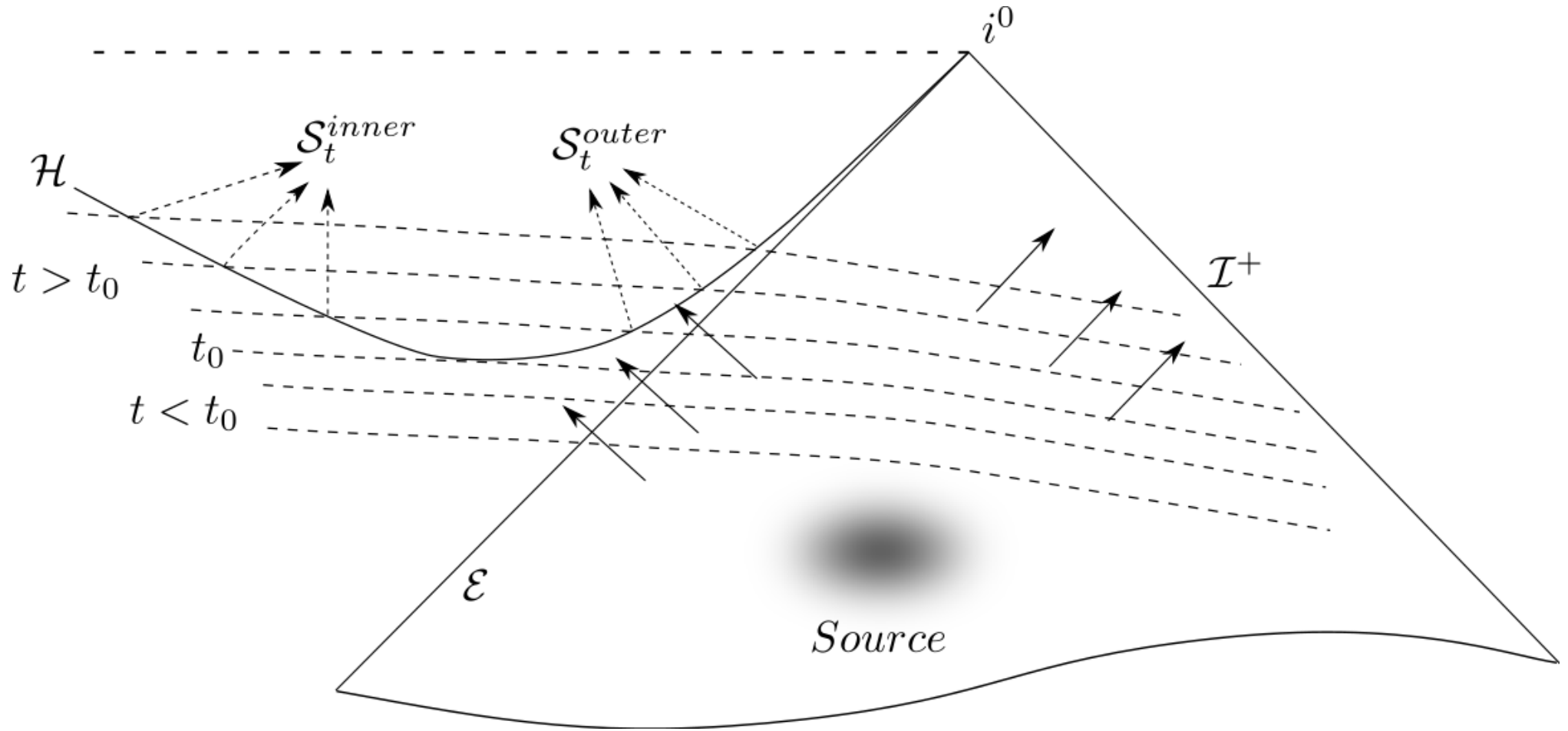
- **Light return time**

$$t_{Schwarzschild} = 2M \ln \left( \frac{r_2 - 2M}{r_1 - 2M} \right) + t_{flat}$$

- **Light scattering time**

$$t_{Scatter} = t_{Hubble} \frac{10^{-22}}{\frac{r_{surface}}{2M} - 1} \frac{M}{M_{Sun}}$$

# Horizon correlations



# EMRIs, ISCO and EHT

Generalisation of  
(circular) Kepler law,  
r-cmpt of geodesic eqn

$$\left(\omega a + 1\right)^2 \left(m - m' r\right) - \omega^2 r^3 = 0$$

Fig 9 of Schönenbach et al 2012

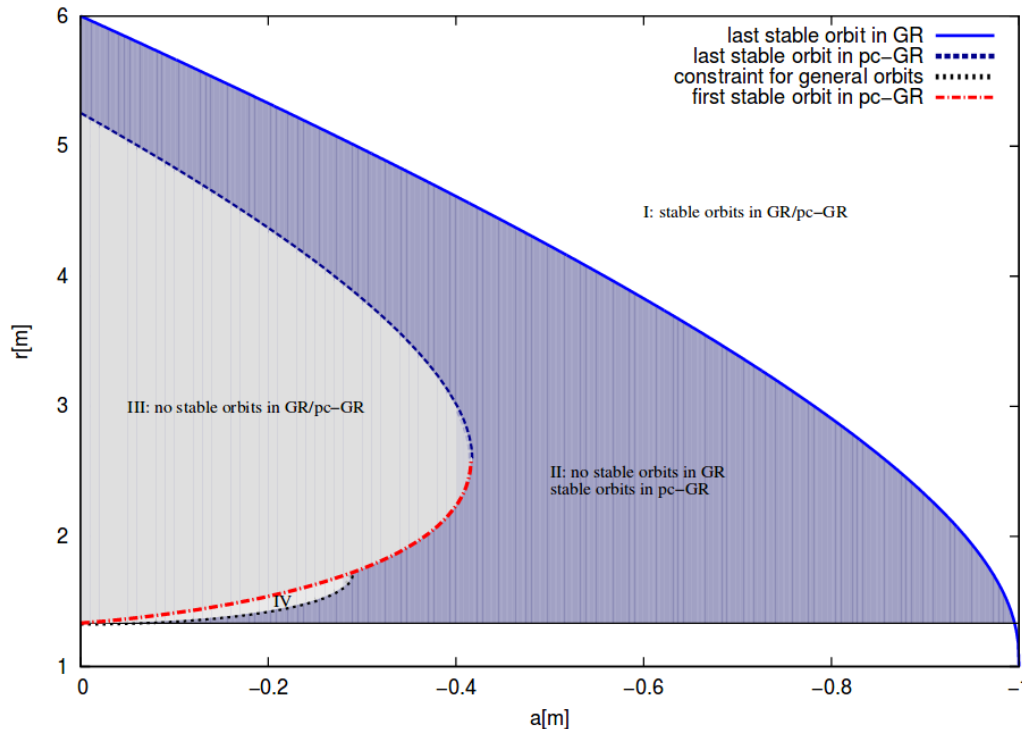
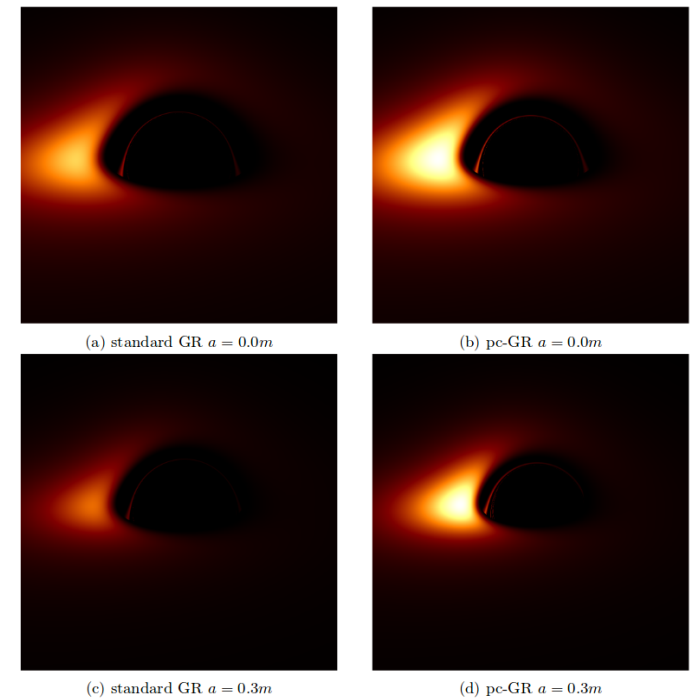


Fig 2 of Schönenbach et al 2014



**Results from EHT due ~ 2018?**