Pions near condensation under compact star conditions

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Phys. Rev. D 95, 096013 (2017)

Outline

- Motivation
- Pion decay width and metastability
- Pion-lepton chemical equilibrium
- Neutrino emission
- Conclusions and outlook

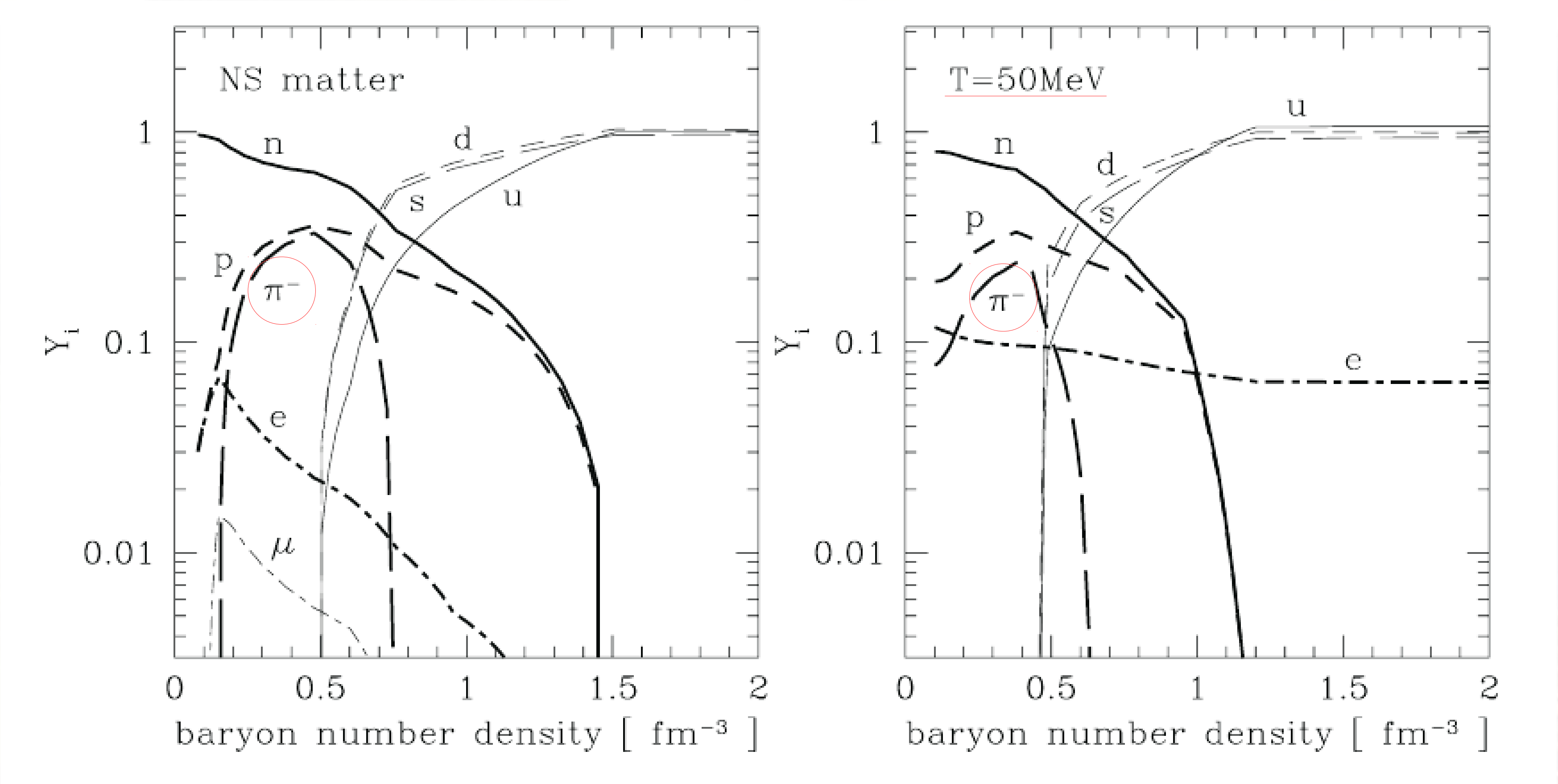
Motivation

The role of pions in compact stars

Pion condensation → direct influence in NS cooling process

- pion-nucleon s-wave repulsive interaction increases pion mass.
- But p-wave is attractive
- Hyperon formation reduces electron chemical potential
- Kaon condensation is more favorable to condense

Pions in normal phase?



K. Nakazato, K.Sumiyoshi, S. Yamada, Phys. Rev. D **77**, 103006 (2008)

Pions in normal phase (without condensate)

- Number temperature depends on temperature

$$\rho_i \sim (m_\pi T)^{3/2} e^{-(m_\pi - \mu_\pi)/T}$$

- Short lifetime

$$\tau \approx 2.6 \times 10^{-8}\ \mathrm{s}$$

- production mechanism (low)

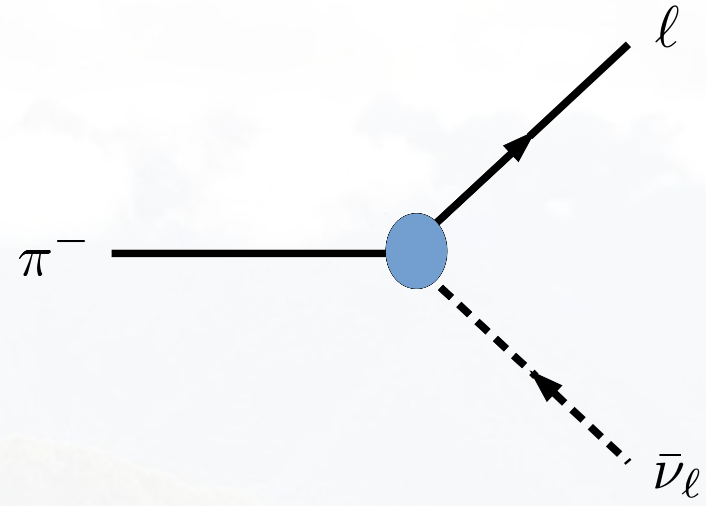
$$N + n \to N + p + \pi^-$$

Decay rate reduction?

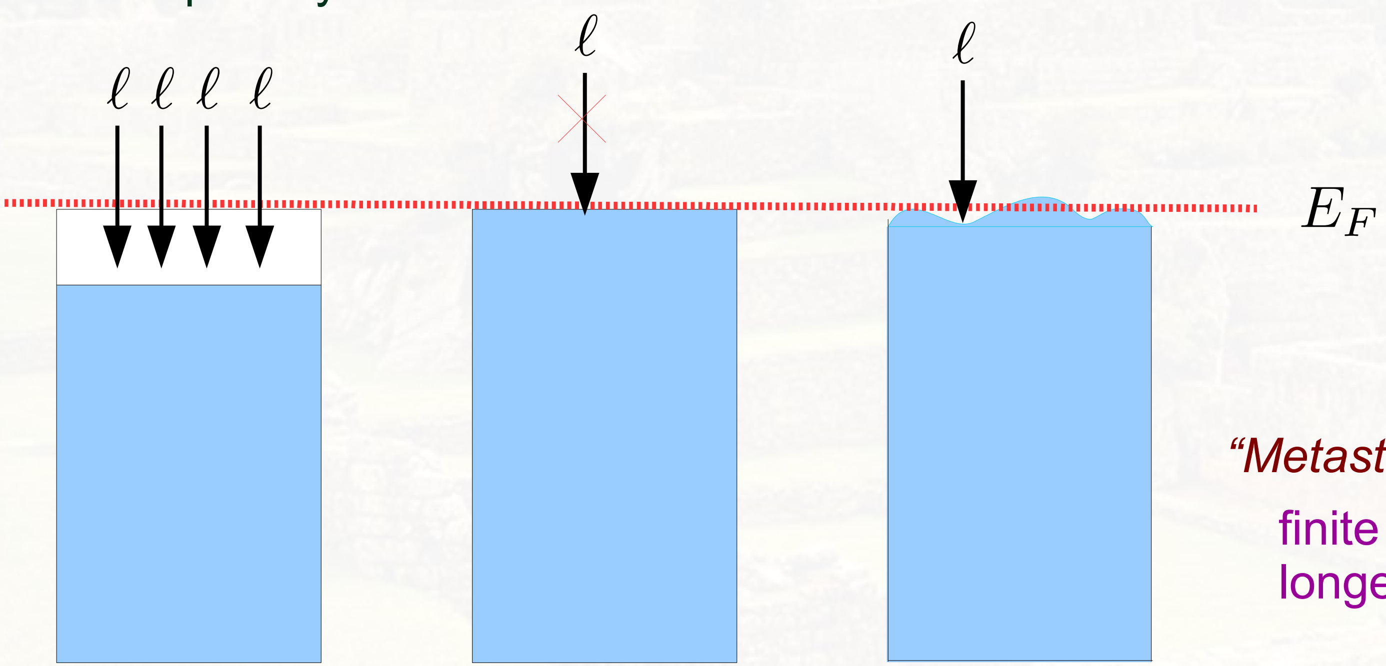
Leptonic weak decay of negative pions

$$\pi^- \to \mu + \bar{\nu}_\mu \qquad \sim 99.99\%$$

$$\pi^- \to e + \bar{\nu}_e \qquad < 0.001\%$$



Dense lepton system



$\mu_\ell < m_\ell$	$\mu_\ell > m_\ell$	
Non- degenerated	**degenerated**	**Thermal fluctuations**

"Metastable" (in nuclear physics):

finite lifetime but considerably longer than usual

Pion decay width

$$\Gamma = 1/\tau$$

Model

Free particles

$$\mathcal{L}_\pi = (D\pi^+)\cdot(D\pi^-) - m_\pi^2(\rho_B)\,\pi^+\pi^-$$

$$\mathcal{L}_\ell = \bar{\psi}_\ell[i\partial\!\!\!/ + \mu_\ell\gamma_0 - m_\ell]\psi_\ell$$

$$D\pi^\pm = (\partial_0 \pm i\mu_\pi, \boldsymbol{\nabla})\pi^\pm$$

$$\mathcal{L}_{\nu_\ell} = \bar{\psi}_{\nu_\ell}[i\partial\!\!\!/ + \mu_{\nu_\ell}\gamma_0]\psi_{\nu_\ell}$$

The interacting model

$$\mathcal{L}_{\pi\ell} = f_\pi(\rho_B)\,G_F[\bar{\psi}_{\nu_\ell} D\!\!\!\!/\,\pi^+(1-\gamma_5)\psi_\ell + \bar{\psi}_\ell D\!\!\!\!/\,\pi^-(1-\gamma_5)\psi_{\nu_\ell}]$$

Chemical equlibrium

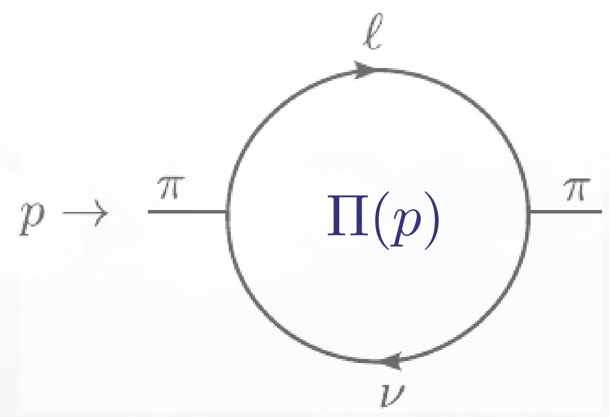
$$\mu_\mu - \mu_{\nu_\mu} = \mu_e - \mu_{\nu_e}$$

leptonic

$$\mu_\pi = \mu_\ell - \mu_{\nu_\ell}$$

beta

Retarded propagator and decay rate



$$D^{\rm ret}_{\pi^-}(p) = \left. \frac{i}{(p_0+\mu_\pi)^2 - E_\pi^2 - \Pi(p)} \right|_{p_0 \to p_0 + i\epsilon}$$

$$\approx \frac{i}{(p_0+\mu_\pi)^2 - E_\pi^2 + i E_\pi \Gamma_{\pi^-}}$$

$$\Gamma_{\pi^-} = -\frac{1}{E_\pi} \mathrm{Im}\, \Pi(E_\pi - \mu_\pi + i\epsilon,\ \boldsymbol{p})$$

Non-relativistic
Breit-Wigner distribution

⇨ At *T=0* is the decay rate

meaning of the imaginary part of the thermal self-energy
→ Particles slightly out of thermal equilibrium

Bosons

$$f_B(E) = n_B(E) + c(E)e^{-\Gamma(E)t}$$

$$\Gamma = \Gamma_d - \Gamma_i \qquad \Gamma_d = (1 + n_B)\Gamma \qquad \Gamma_i = n_B\Gamma$$

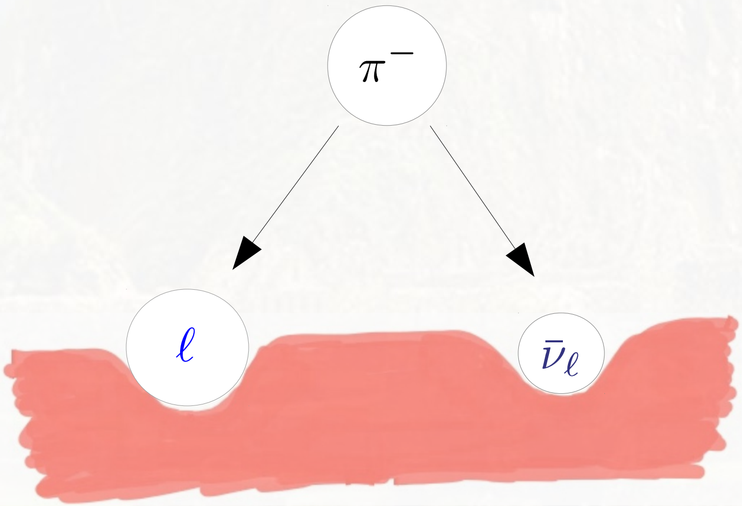
Fermions

$$f_F(E) = n_F(E) + c(E)e^{-\Gamma(E)t}$$

$$\Gamma = \Gamma_d + \Gamma_i \qquad \Gamma_d = (1 - n_F)\Gamma \qquad \Gamma_i = n_F\Gamma$$

H.A. Weldon, Phys.Rev. D **28**, 2007 (1982)

$$\Gamma_{\pi^-} \sim \int \Big[\underbrace{(1-n_\ell)(1-n_{\bar{\nu}_\ell})}_{\Gamma_d} - \underbrace{n_\ell n_{\bar{\nu}_\ell}}_{\Gamma_i}\Big] = \int \big[1 - n_\ell - n_{\bar{\nu}_\ell}\big]$$



direct

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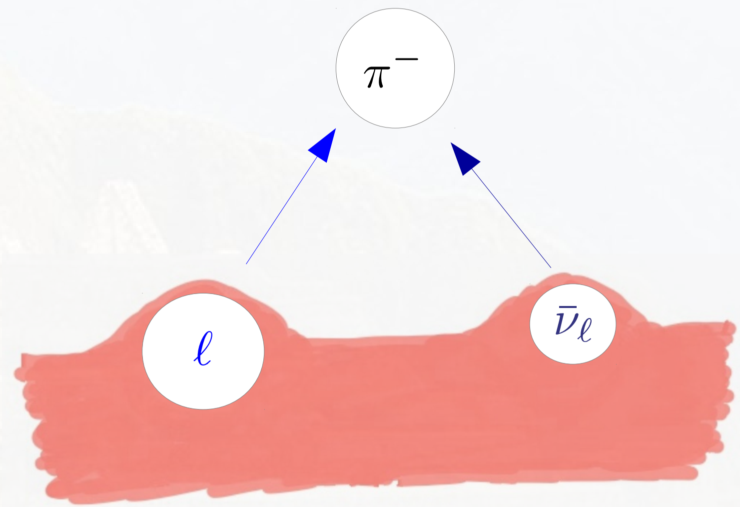


inverse

$$\Gamma_d = (1 + n_{\pi^-})\Gamma_{\pi^-} \qquad \Gamma_i = n_{\pi^-}\Gamma_{\pi^-}$$

$$n_\ell = n_F(E_\ell - \mu_\ell), \qquad n_{\bar{\nu}_\ell} = n_F(E_{\nu_\ell} + \mu_{\nu_\ell}), \qquad n_{\pi^-} = n_B(E_\pi - \mu_\pi)$$

Considering chemical equilibrium $\mu_\pi = \mu_\ell - \mu_\nu$

$$\Gamma_{\pi^-} = \bar{\Gamma}_{\pi\ell}\frac{m_\pi}{E_\pi}\left[1 + \frac{T}{2a_\ell|\boldsymbol{p}|}\ln\left(\frac{1+e^{-(E_\ell^+ - \mu_\ell)/T}}{1+e^{-(E_\ell^- - \mu_\ell)/T}}\right) + \frac{T}{2a_\ell|\boldsymbol{p}|}\ln\left(\frac{1+e^{-(E_{\nu_\ell}^+ + \mu_{\nu_\ell})/T}}{1+e^{-(E_{\nu_\ell}^- + \mu_{\nu_\ell})/T}}\right)\right]$$

$$\mathrm{E}_\ell^\pm = (1-a_\ell)E_\pi \pm a_\ell|\boldsymbol{p}| \qquad a_\ell = \frac{m_\pi^2 - m_\ell^2}{2m_\pi^2}$$

$$\mathrm{E}_{\nu_\ell}^\pm = a_\ell(E_\pi \pm |\boldsymbol{p}|) \qquad E_\pi = \sqrt{\boldsymbol{p}^2 + m_\pi^2}$$

$\bar{\Gamma}_{\pi\ell}$ Is the decay width at $T = \mu = 0$

In rest frame

$$\Gamma_{\pi^-} = \bar{\Gamma}_{\pi\ell} \left[1 - n_F\left(\frac{m_\pi^2 + m_\ell^2}{2m_\pi} - \mu_\ell \right) - n_F\left(\frac{m_\pi^2 - m_\ell^2}{2m_\pi} + \mu_{\nu_\ell} \right) \right]$$

At zero temperature

$$\Gamma_{\pi^-} = \bar{\Gamma}_{\pi\ell}\, \theta\left(\frac{m_\pi^2 + m_\ell^2}{2m_\pi} - \mu_\ell \right)$$

Vanishes for $$\mu_\ell > \frac{m_\pi^2 + m_\ell^2}{2m_\pi} \approx 109.74\ \mathrm{MeV}$$

(considering $m_\pi = 139.5$ MeV
and $m_\ell = m_\mu = 105.66$ MeV)

Metastability condition at chemical equilibrium

$$\Gamma_{\pi^-} = \Gamma_0 + \delta\Gamma(T) \qquad\qquad \delta\Gamma \ll \bar{\Gamma}_{\pi\ell}$$

$$\Gamma_0 = \bar{\Gamma}_{\pi\ell}\frac{m_\pi}{E_\pi}\left[1 + \frac{\mu_\ell - E_\ell^+}{2a_\ell\,|\boldsymbol{p}|}\theta(\mu_\ell - E_\ell^+) - \frac{\mu_\ell - E_\ell^-}{2a_\ell\,|\boldsymbol{p}|}\theta(\mu_\ell - E_\ell^-)\right]$$

$\Gamma_0 = 0 \Rightarrow$

$$|\boldsymbol{p}| < \frac{m_\pi^2 + m_\ell^2}{2m_\ell^2}q_F - \frac{m_\pi^2 - m_\ell^2}{2m_\ell^2}\mu_\ell \equiv p_c$$

and

$$\mu_\pi > \frac{m_\pi^2 + m_\ell^2}{2m_\pi} \equiv \mu_c$$

$$q_F \equiv \sqrt{\mu_\ell^2 - m_\ell^2}$$

Other interesting phenomena:

if $$\mu_{\nu_\ell} > \frac{m_\pi^2 - m_\ell^2}{2m_\pi} \approx 29.9\ \mathrm{MeV} \qquad (\text{for } m_\pi = 139.5\ \mathrm{MeV})$$

→ π^+ turns metastable

Chemical equilibrium

Chemical equilibrium

$$\pi^- \leftrightarrow \ell + \bar{\nu}_\ell$$

$$\Gamma_{\pi^-} \sim \Gamma_\ell$$

Pion decay

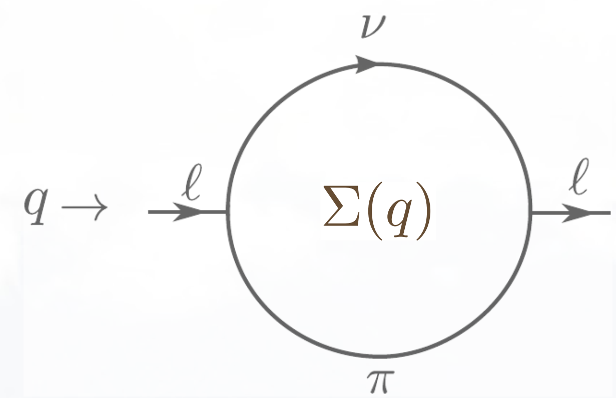
recombination of
Leptons and anti-neutrinos

The slightly out of equilibrium system of particles equilibrates at the same time

$$\mu_\pi = \mu_\ell - \mu_\nu$$

I. Kuznetsova and J. Rafelski, Phys. Rev. C **82**, 035203 (2010)

Lepton decay rate: recombination



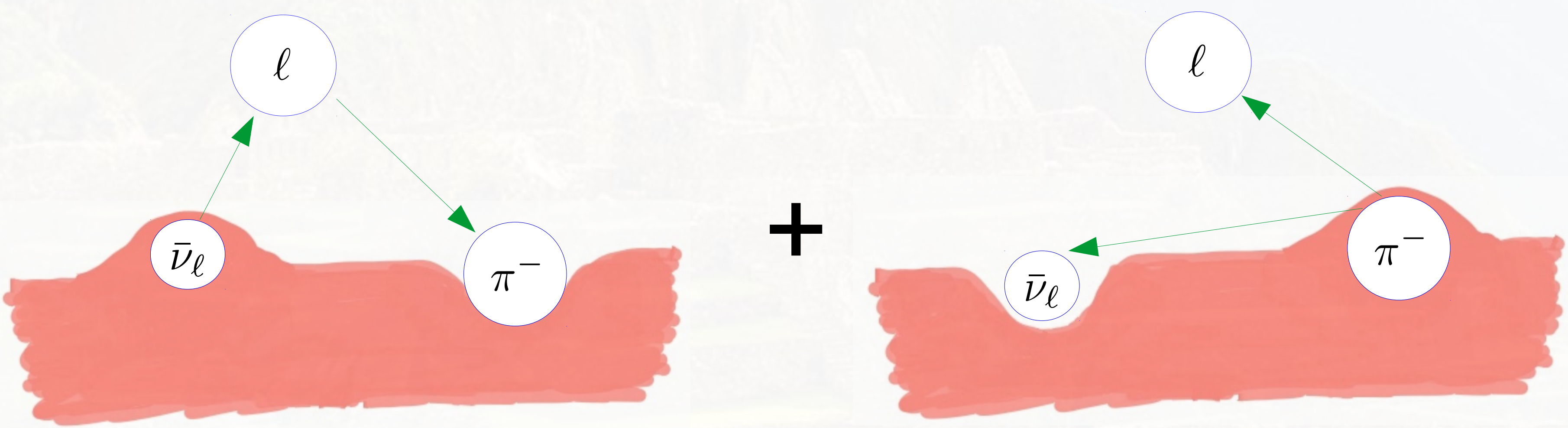
$$S_\ell^{\text{ret}}(q) = \left.\frac{i}{\not{q} + \mu_\ell\gamma_0 - m_\ell - \Sigma(q)}\right|_{q_0 \to q_0 + i\epsilon}$$

$$= (\not{q} + \mu_\ell\gamma_0 + m_\ell - \Sigma(q)) \left[\frac{\mathcal{P}_+}{(q_0+\mu_\ell)^2 - E_\ell^2 - \Pi_+} + \frac{\mathcal{P}_-}{(q_0+\mu_\ell)^2 - E_\ell^2 - \Pi_-}\right]\Bigg|_{q_0 \to q_0 + i\epsilon}$$

$$\Gamma_\pm = -\frac{1}{E_\ell}\text{Im}\,\Pi_\pm(E_\ell - \mu_\ell + i\epsilon,\ \boldsymbol{q})$$

$$\Gamma_\ell = \Gamma_+ + \Gamma_-$$

$$\Gamma_\ell = \Gamma_d + \Gamma_i \quad \sim \int \Big[\underbrace{n_{\bar{\nu}_\ell}(1 + n_{\pi^-})}_{\Gamma_d} + \underbrace{(1 - n_{\bar{\nu}_\ell})n_{\pi^-}}_{\Gamma_i}\Big] \; = \int \left[n_{\bar{\nu}_\ell} + n_{\pi^-}\right]$$



direct

inverse

$$\Gamma_d = (1 - n_\ell)\Gamma_\ell$$

$$\Gamma_i = n_\ell \, \Gamma_\ell$$

lepton decay rate in chemical equilibrium

$$\Gamma_\ell = \bar{\Gamma}_{\pi\ell}\left(\frac{m_\pi}{2m_\ell}\right)^3 \frac{m_\ell}{E_\ell}\frac{T}{2b_\ell|\boldsymbol{q}|}\left[\ln\left(\frac{1-e^{-(E_\pi^+-\mu_\pi)/T}}{1-e^{-(E_\pi^--\mu_\pi)/T}}\right) - \ln\left(\frac{1+e^{-(\tilde{E}_{\nu_\ell}^+ +\mu_{\nu_\ell})/T}}{1+e^{-(\tilde{E}_{\nu_\ell}^- +\mu_{\nu_\ell})/T}}\right)\right]$$

$$\mathrm{E}_\pi^\pm = (1+b_\ell)E_\ell \pm b_\ell|\boldsymbol{q}|$$

$$b_\ell = \frac{m_\pi^2 - m_\ell^2}{2m_\ell^2}$$

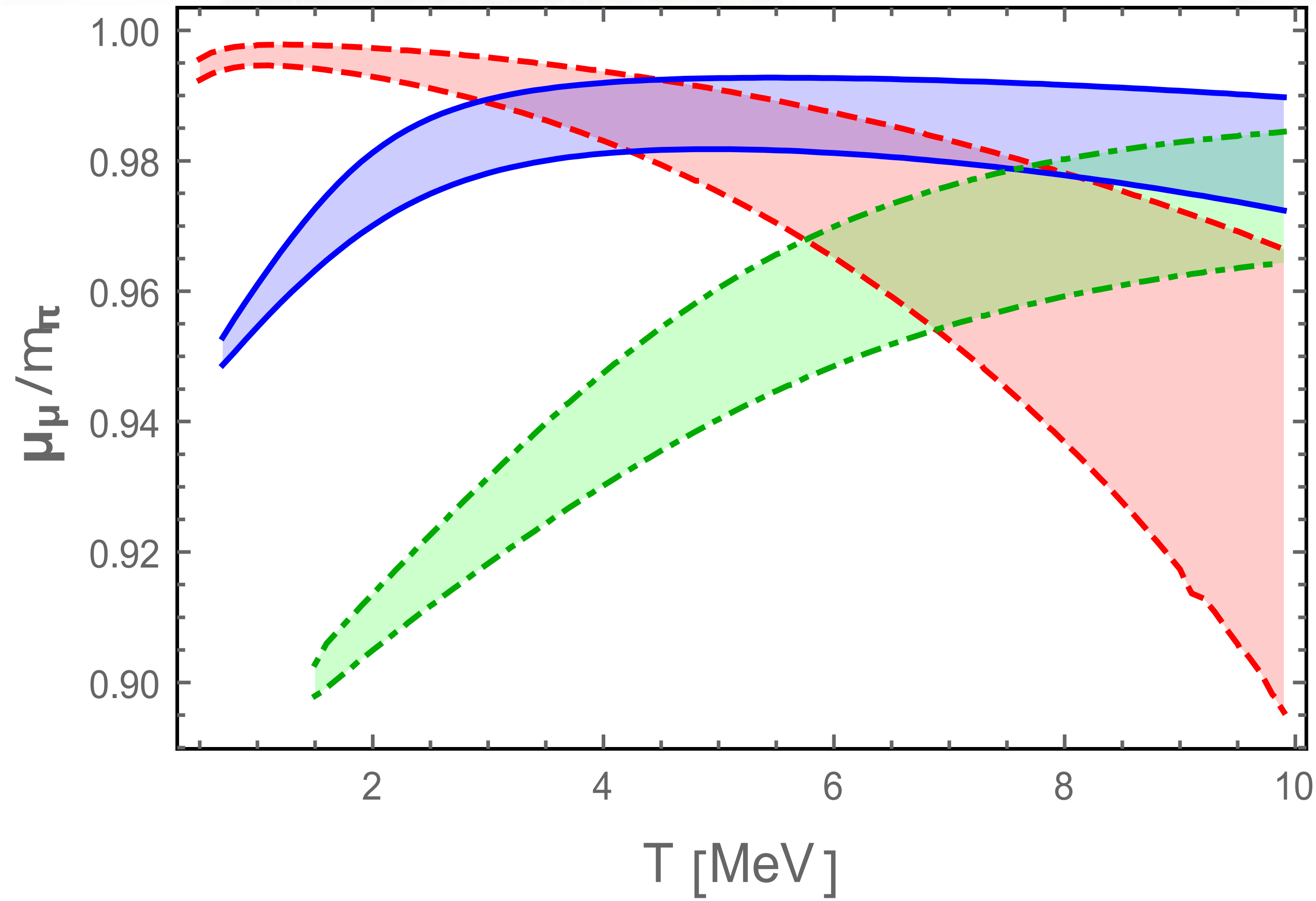
$$\tilde{E}_{\nu_\ell}^\pm = b_\ell E_\pi \pm b_\ell|\boldsymbol{q}|$$

$$E_\ell = \sqrt{\boldsymbol{q}^2 + m_\ell^2}$$

On the Fermi surface:

$$\Gamma_\ell = \bar{\Gamma}_{\pi\ell}\left(\frac{m_\pi}{2m_\ell}\right)^3 \frac{m_\ell}{\mu_\ell}\frac{T}{2b_\ell\, q_F}\ln\left(\frac{\sinh[(b_\ell\mu_\ell+\mu_{\nu_\ell})/T]+\sinh[b_\ell q_F/T]}{\sinh[(b_\ell\mu_\ell+\mu_{\nu_\ell})/T]-\sinh[b_\ell q_F/T]}\right)$$

$$\Gamma_{\pi^-}(|\boldsymbol{p}| = 0) \quad \sim \quad \Gamma_\ell(|\boldsymbol{q}| = q_F)$$



$m_\pi = 115$ MeV

$m_\pi = 140$ MeV

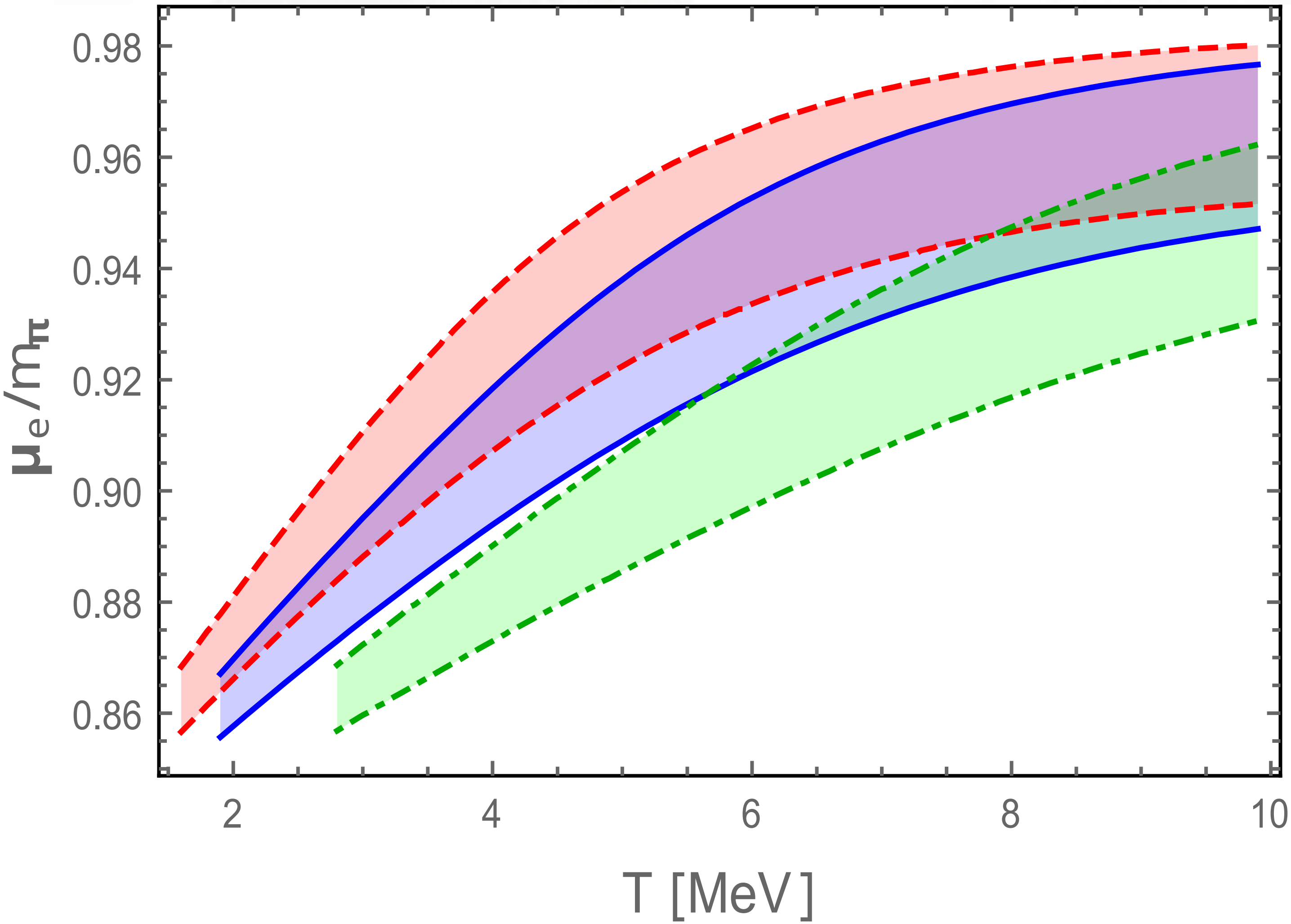
$m_\pi = 200$ MeV

$\mu_c = 0.91 m_\pi$

$\mu_c = 0.78 m_\pi$

$\mu_c = 0.64 m_\pi$

$$\Gamma_{\pi^-}(|\boldsymbol{p}| = 0) \quad \sim \quad \Gamma_\ell(|\boldsymbol{q}| = q_F)$$



$m_\pi = 115$ MeV

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$m_\pi = 200$ MeV

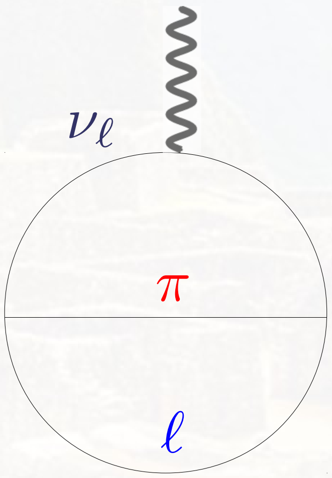
$\mu_c \approx 0.5 m_\pi$

Neutrino emission

Neutrino emissivity

$$\epsilon_\pi = \int \bar{d}p\, \bar{d}q\, \bar{d}k \sum_{\text{spin}} |\mathcal{M}|^2\, k_0\, n_B(p_0)\, [1 - n_F(q_0)] (2\pi)^4 \delta^4(p - q - k)$$

$$\bar{d}p = \frac{d^4p}{(2\pi)^3} \theta(p_0 + \mu) \delta^4((p_0 + \mu)^2 - E^2)$$



$$\langle \ell\, \bar{\nu} | \int d^4x\, \mathcal{L}_{\pi\ell}\, |\pi^-\rangle = i\mathcal{M}(2\pi)^4 \delta^4(p - q - k).$$

for $p_c > T$

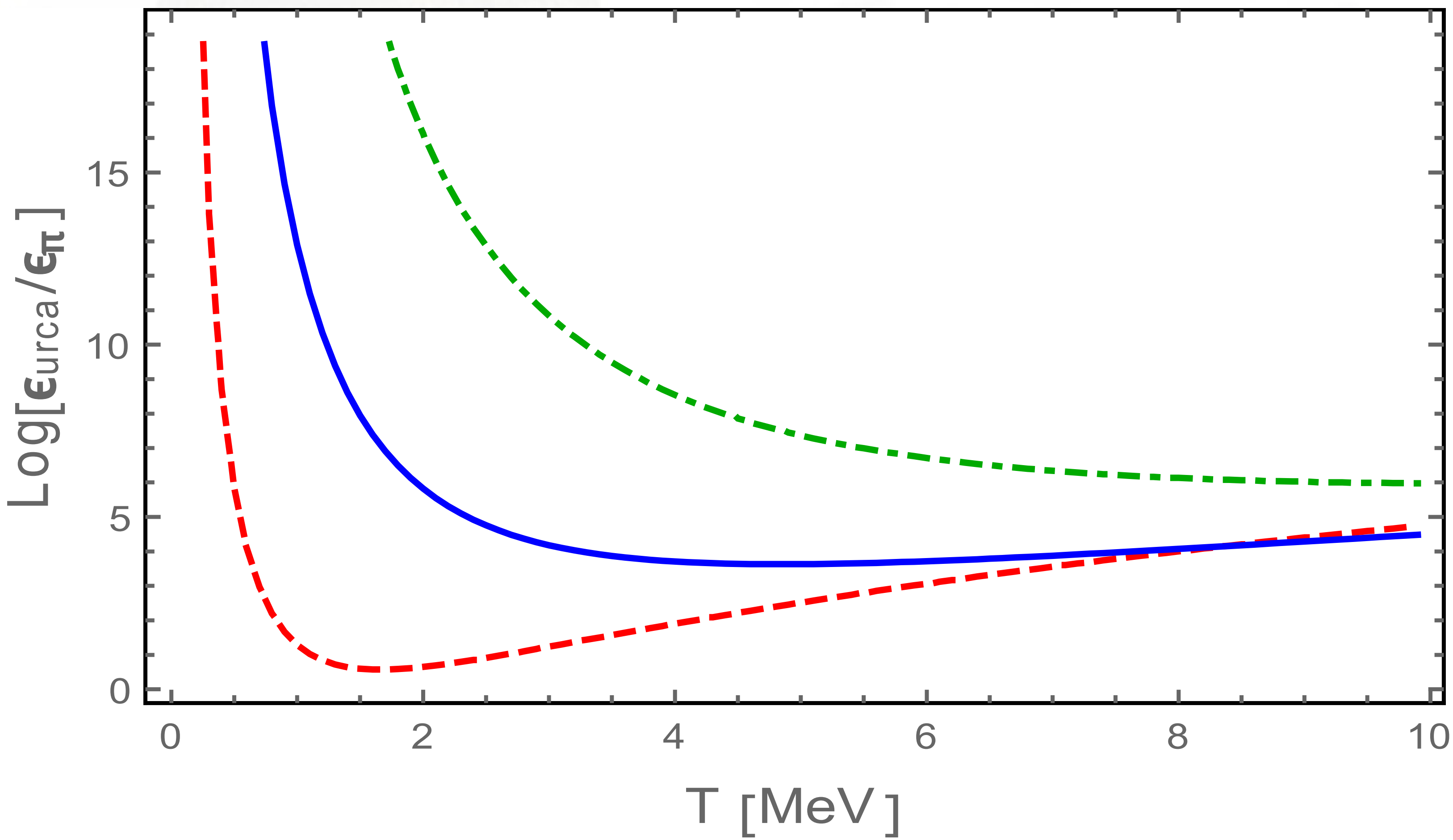
$$\epsilon_\pi \approx \bar{\Gamma}_{\pi\ell}\, m_\pi^4\, g(\mu_\ell) \left(\frac{T}{2\pi m_\pi}\right)^2 e^{-(E_c-\mu_\pi)/T}$$

with $E_c = \sqrt{p_c^2 + m_\pi^2}$

Simple pion gas $\epsilon_\pi \approx \bar{\Gamma}_{\pi\ell}\, m_\pi^4 \left(\frac{T}{2\pi m_\pi}\right)^{3/2} e^{-(m_\pi-\mu_\pi)/T}$

URCA $\epsilon_{\rm URCA} \sim T^6$

$\bar{\nu}_\mu$ emission



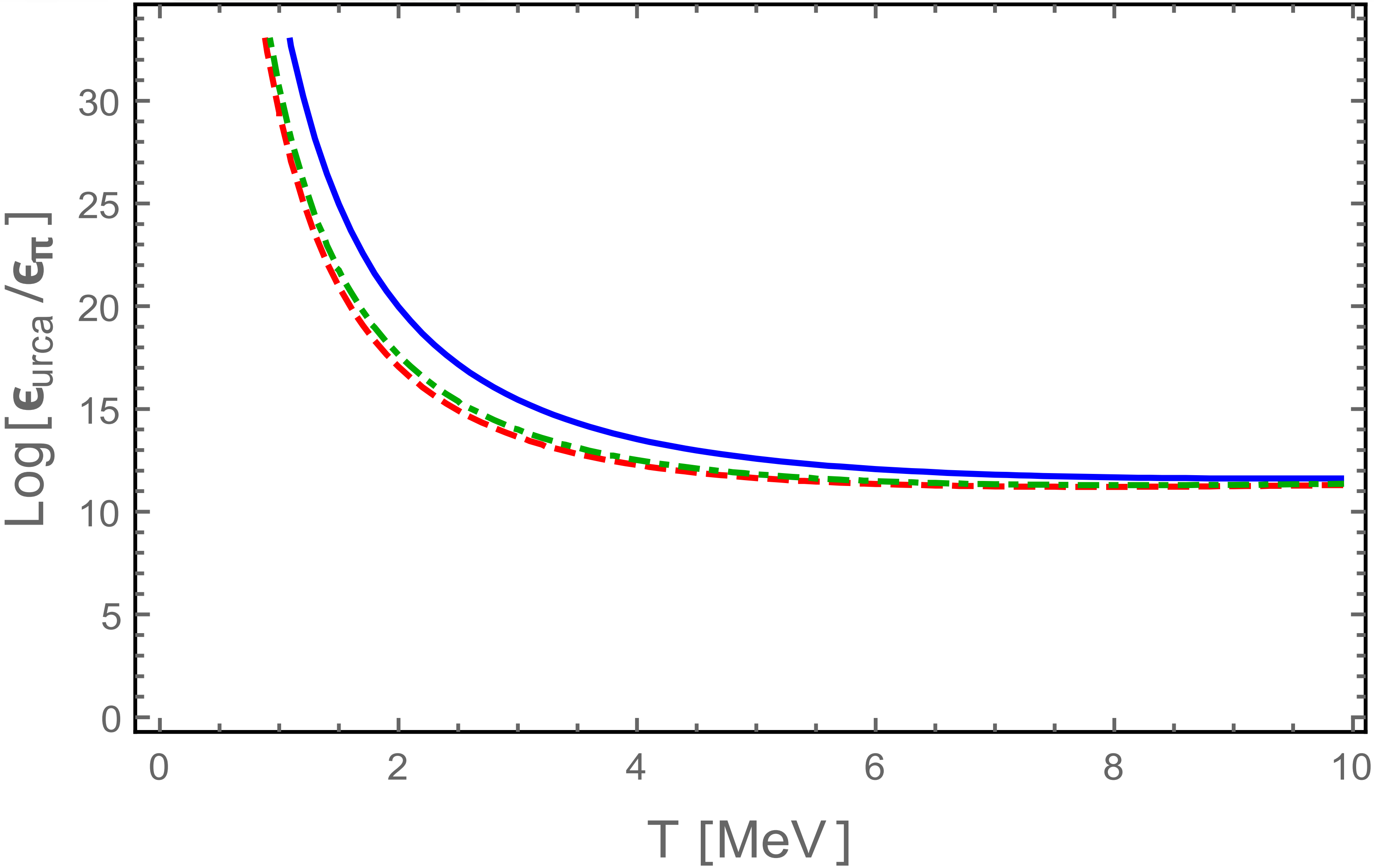
$m_\pi = 115$ MeV

$m_\pi = 140$ MeV

$m_\pi = 200$ MeV

$\mu_\ell = 0.95 m_\pi$ $\quad f_\pi^* = 0.8 f\pi$ $\quad m_N^* \approx 0.8 m_N$

$\bar{\nu}_e$ emission



$m_\pi = 115$ MeV

$m_\pi = 140$ MeV

$m_\pi = 200$ MeV $\quad \mu_\ell = 0.95 m_\pi \quad f_\pi^* \approx 0.8 f\pi \quad m_N^* \approx 0.8 m_N$

Conclusions

- Metastable pion is a possible relevant state in proto neutron stars
- Acceptable window in parameter space for pion-lepton chemical equilibrium
- Source of neutrino emission in the NS initial stages.

Outlook

- abundance considering metastable pions
- Metastable Kaons
- Trapped neutrinos

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THANKS!