Classical features of polynomial higher-derivative gravities

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Based in collaboration with

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CBPF

Conselho Nacional de Desenvolvimento Científico e Tecnológico

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(Regular solutions in polynomial higher-derivative gravities)

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Contents:

a. The model: polynomial higher-derivative gravity

b. Classical singularities in the linear regime

    Cancellation of the Newtonian singularity

    Regularity of curvature invariants

    Collapse of null shells

c. Other (phenomenological) developments

Towards quantum gravity

- General Relativity (GR): Einstein, 1915.
- Physical singularities for high curvatures!
- Way out: GR is not valid at all scales.
- The "true" gravity theory should coincide with GR in the large distances & low energy realm.
- Natural extension via higher-derivatives (curvature-squared terms), required already in the semiclassical approach.

$$\mathcal{L} = \sqrt{-g}\left[\frac{2R}{\kappa^2} + \frac{\alpha}{2}R^2 + \frac{\beta}{2}R_{\mu\nu}^2\right]$$

- Fourth-order gravity is renormalizable (but contains a massive spin-2 ghost) [Stelle, *PRD* 77]

$$G_{\text{spin}-2}(k) \sim \left(\frac{1}{k^2} - \frac{1}{k^2+m^2}\right)$$

Polynomial higher-derivative gravity

- Super-renormalizable HDG [Asorey, López & Shapiro, *Int. Jour. Mod. Phys. A*, 1997]

$$
\begin{aligned}
S &= \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R-2\Lambda) + \int d^4x\sqrt{-g}\,\Big\{c_1 R^2_{\mu\nu\alpha\beta} + c_2 R^2_{\mu\nu} + c_3 R^2 \\
&+ d_1 R_{\mu\nu\alpha\beta}\Box R^{\mu\nu\alpha\beta} + d_2 R_{\mu\nu}\Box R^{\mu\nu} + d_3 R\Box R + d_4 R^3 + d_5 R R^{\mu\nu} R_{\mu\nu} + \dots \\
&+ f_1 R_{\mu\nu\alpha\beta}\Box^k R^{\mu\nu\alpha\beta} + f_2 R_{\mu\nu}\Box^k R^{\mu\nu} + f_3 R\Box^k R + \dots + f_{\dots} R^{k+2}_{\dots}\Big\},
\end{aligned}
$$

- An attempt to move the ghost further to the UV region
- Particle content: graviton and k+1 massive particles for each spin
- Actually, ghosts and healthy modes alternate:

$$0 < m_1^2 < m_2^2 < m_3^2 < \cdots < m_{k+1}^2$$

- Remarkable quantum properties:

  - ❖ The theory is super-renormalizable…
    - Propagators behave like $\sim(\text{momentum})^{-(2k+4)}$ in the UV limit.
    - Power counting yields, to the most divergent diagrams:

$$D = 4 + 2k - 2kp$$

k = 1: divergences for 1, 2 and 3 loops;
k = 2: divergences for 1 and 2 loops;
k ≥ 3: divergences for 1 loop.

Counterterms have at most 4 derivatives.

  - ❖ …and Lee-Wick unitary
    - If k is odd ≥ 1 it is possible to have all massive poles complex, and the S-matrix becomes unitary in the Lee-Wick sense. [Modesto & Shapiro, *PLB* 2016]

- Other possibility for dealing with (avoiding) ghosts: non-local theories

  [Krasnikov, 1987; Kuz'min, 1989; Tomboulis, 1997;

  Modesto, *PRD* 2012; Biswas, Gerwick, Koivisto & Mazumdar, *PRL* 2012]

$$S=-\frac{1}{2\kappa}\int d^4x\sqrt{-g}\,\left\{R+G_{\mu\nu}\,\frac{a(\Box)-1}{\Box}\,R^{\mu\nu}\right\},\qquad a(\Box)=e^{-\Box/m^2}$$

  - (Super-renormalizable), non-singular Newtonian limit, ghost-free at the tree-level.
  - Quantum corrections can lead to an infinite number of ghosts, all of them complex. [Shapiro, *PLB* 2015]

➢ Higher-derivatives and the role they play should be investigated.

Classical and quantum singularities I: Newtonian potential

❖ Is renormalizability related to the cancellation of the Newtonian singularity? [Accioly, Helayël-Neto et al., *IJMPD* 2013; Modesto, Paula-Netto & Shapiro, *JHEP* 2015; Shapiro, *PLB* 2015]

- Stelle, 1977: four derivatives ($R + R^2 + R_{\mu\nu}^2$), finite potential

$$V \;= MG\left(-\frac{1}{r}+\frac{4}{3}\frac{e^{-m_2 r}}{r}-\frac{1}{3}\frac{e^{-m_0 r}}{r}\right)$$

- Modesto, Paula-Netto & Shapiro, 2015: $R + RF_1(\Box)R + R_{\mu\nu}F_2(\Box)R^{\mu\nu}$

[with real poles; renormalizable models, polynomials $F_{1,2}$ of the same order]

$$V = MG\left(-\frac{1}{r}+\frac{4}{3}\frac{C_2(r)}{r}-\frac{1}{3}\frac{C_0(r)}{r}\right) \quad \text{with} \quad C_{0,2}(r) = 1 + \mathcal{O}(r)$$

Classical and quantum singularities I: Newtonian potential

❖ Is renormalizability related to the cancellation of the Newtonian singularity?

- BLG, *PLB* 2017: general model of the type $R + RF_1(\Box)R + R_{\mu\nu}F_2(\Box)R^{\mu\nu}$

$$\phi_{\rm div}(r) = -\frac{GM}{r}\left[1 - \frac{4}{3}\sum_{i=1}^{n_2}\prod_{j\neq i}\frac{m^2_{(2)j}}{m^2_{(2)j} - m^2_{(2)i}} + \frac{1}{3}\sum_{i=1}^{n_0}\prod_{j\neq i}\frac{m^2_{(0)j}}{m^2_{(0)j} - m^2_{(0)i}}\right]$$

$$\sum_i \prod_{j\neq i}\frac{a_j}{a_j - a_i} = \frac{\prod_{j\neq i}(a_i - a_j)}{\prod_{j\neq i}(a_i - a_j)} = 1 \qquad \Longrightarrow \qquad \phi_{\rm div}(r) \equiv 0$$

Classical and quantum singularities I: Newtonian potential

❖ Is renormalizability related to the cancellation of the Newtonian singularity?

- The Newtonian singularity can be cancelled even in non-renormalizable models.
- To cancel the singularity it suffices to have at least one massive mode in each sector.
- Hence, the simplest form (avoiding tachyons) is Stelle's 4th order (renormalizable) gravity.

[BLG, *Phys.Lett.B* **766** (2017) arXiv:1609.05432]

Classical and quantum singularities II: Curvature

- Stelle's 4th order gravity has a finite Newtonian potential but still has curvature singularities [Stelle, 1978; Lü, Perkins, Pope & Stelle, 2015]

- Isotropic static spherically symmetric metric:

$$ds^2 = -(1+2\varphi)dt^2 + (1-2\psi)(dx^2+dy^2+dz^2)$$

$$\begin{aligned}\varphi(r) &= \varphi_0 + \varphi_1 r + \varphi_2 r^2 + \varphi_3 r^3 + O(r^4)\\ \psi(r) &= \psi_0 + \psi_1 r + \psi_2 r^2 + \psi_3 r^3 + O(r^4)\end{aligned}$$

Kretschmann scalar:

$$R^2_{\mu\nu\alpha\beta} = \frac{8(\varphi_1^2 + 3\psi_1^2)}{r^2} + \frac{32(\varphi_1\varphi_2 + 4\psi_1\psi_2)}{r} + 48\left(\varphi_2^2 + 4\psi_2^2 + \varphi_1\varphi_3 + 5\psi_1\psi_3\right) + O(r)$$

$$S_{grav} = \frac{1}{4\kappa}\int d^4x\sqrt{-g}\left\{-2R + R_{\mu\nu}\,F_1(\Box)\,R^{\mu\nu} + R\,F_2(\Box)\,R\right\}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

❖ Linearised equations of motion + delta source

$$a(\Box)\,(\Box h_{\mu\nu} - \partial_\rho\partial_\mu h^\rho_\nu - \partial_\rho\partial_\nu h^\rho_\mu) + c(\Box)\,(\eta_{\mu\nu}\partial_\rho\partial_\omega h^{\rho\omega} - \eta_{\mu\nu}\Box h + \partial_\mu\partial_\nu h)$$
$$+ [a(\Box) - c(\Box)]\,\frac{1}{\Box}\,\partial_\mu\partial_\nu\partial_\rho\partial_\omega h^{\rho\omega} = -2\kappa\,T_{\mu\nu},$$

$$a(\Box) = 1 - \frac{1}{2}F_1(\Box)\,\Box \qquad c(\Box) = 1 + 2F_2(\Box)\,\Box + \frac{1}{2}F_1(\Box)\,\Box\,.$$

- At least 4 derivatives: *finite* Newtonian potentials.
- At least 6 derivatives: *regular* Newtonian limit.

[BLG & Paula-Netto, arXiv: 1806.05664]

$$\varphi(r) = \varphi_0 + \varphi_1 r + \varphi_2 r^2 + \varphi_3 r^3 + O(r^4)$$
$$\psi(r) = \psi_0 + \psi_1 r + \psi_2 r^2 + \psi_3 r^3 + O(r^4)$$

$$\varphi = \frac{1}{3}(2\chi + \omega)\,, \qquad \psi = \frac{1}{3}(\chi - \omega)$$

$$\chi(r) = -\frac{2Gm}{r} + \frac{2Gm}{\sqrt{\pi}} \sum_{i=1}^{N} \sum_{j=1}^{\alpha_i} A_{i,j} \left(\frac{r}{2m_i}\right)^{j-\frac{3}{2}} K_{j-\frac{3}{2}}(m_i r)\,.$$

$$\chi_1 = Gm(S_1 - S_2)\,, \qquad S_1 = \sum_{i=0}^{N} A_{i,1} m_i^2\,, \quad S_2 = \sum_{i=0}^{N} A_{i,2}$$

- Divergences are softened when going from 2 to 4 and to 6+ derivatives:
  - Non-renormalizable → renormalizable → super-renormalizable
  - Singular potential and curvature → finite potential → finite potential and regular curvature

Spherical collapse of small masses

[Frolov, Zelnikov & Paula-Netto, *JHEP* 2015; Frolov, *PRL* 2015]

- Gyraton: apply a boost to Newtonian limit solution and take the Penrose limit (keeping relativistic mass constant).
  - Dominant contribution comes from the spin-2 sector.
- Collapsing *thin* null shell: spherical superposition of gyratons.
- Collapsing *thick* null shell: spherical superposition of thin shells.

Density at the origin: $\rho(t) = \begin{cases} 0\,, & \text{if } |t| > \tau/2\,, \\ M/\tau\,, & \text{if } -\tau/2 < t < \tau/2\,, \end{cases}$

Field depends on

$$F_\Omega(z) = -E_1\left(\frac{z}{4\Omega^2}\right) + 2\sum_{i=0}^{N}\sum_{j=1}^{\alpha_i} A_{i,j}\left(\frac{\sqrt{z}}{2m_i}\right)^{j-1} K_{j-1}(m_i\sqrt{z})\,.$$

For GR: $F_\Omega^{GR}(z) \approx \ln\left(\frac{z}{\Omega^2}\right)$

Spherical collapse of small masses

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$$F_\Omega(z) = -E_1\left(\frac{z}{4\Omega^2}\right) + 2\sum_{i=0}^{N}\sum_{j=1}^{\alpha_i} A_{i,j}\left(\frac{\sqrt{z}}{2m_i}\right)^{j-1} K_{j-1}(m_i\sqrt{z})\,.$$

For polynomial HDG:

$$F(z) = -\frac{z}{4}\left[(\ln z + 2\gamma - 2\ln 2 - 1)(S_1 - S_2) - S_1 + S\right] + c' + O(z^2)$$

- Kretschmann scalar $R^2_{\mu\nu\alpha\beta}$:

6th- and higher-order models:

$$R^2_{\mu\nu\alpha\beta} = \frac{32G^2M^2\left(S - S_2\right)^2}{3\tau^2} + O(r^2) \ ,$$

4th-order gravity:

$$R^2_{\mu\nu\alpha\beta} = \frac{32G^2M^2m_1^4}{27\tau^2}\left[5 + 9c^2 + 36c\ln r + 36(\ln r)^2\right] + O(r^2)$$

- It is also possible to show that for a general polinomial higher-derivative gravity there is a mass gap for the formation of mini black holes. [Frolov, *PRL* 2015]

[BLG & Paula-Netto, arXiv: 1806.05664]

Other developments

- Phenomenological aspects of higher-derivatives
  - Bending of light

  Accioly, BLG & Shapiro, *PRD* 2017; BLG & Shapiro, *PLB* 2018;

  - Torsion balance experiments of the inverse-square force law: complex poles introduce oscillations in the potential

  BLG, *PLB* 2017; Perivolaropoulos, *PRD* 2017; Boos, 2018.

  - Gravitational seesaw-like mechanism, to avoid the Planck suppression

  Accioly, BLG & Shapiro, *EPJC* 2017.

Summary & Perspectives

- Theories with more than four derivatives in both spin-2 and spin-0 sectors have not only a finite Newtonian potential, but also a regular non-relativistic limit.
- Generalization of previous consideration [Frolov, *PRL* 2015] on the collapse of null shells, so as to account for the possibility of complex and degenerate poles. Regular solution for theories with more than four derivatives; mass gap.
- This contrasts to Stelle's fourth-order gravity [Stelle, *GRG* 1978; Lü, Perkins, Pope & Stelle, *PRL* 2015, *PRD* 2015].
- Theories with 6+ derivatives seem to share the same regularity properties of the non-local ghost-free theories. [Frolov, *PRL* 2015; Buoninfante, Koshelev, Lambiase, Marto & Mazumdar, *JCAP* 2018; Boos, Frolov & Zelnikov, *PRD* 2018; Frolov & Zelnikov, *PRD* 2016; Edholm, Koshelev & Mazumdar, *PRD* 2016]
- Motivation for the investigation of the full theory (non-linear regime), *e.g.*,
    - Numerical search of spherically symmetric solutions in theories with 6, 8 and 10 derivatives only found regular solutions [Holdom, *PRD* 2002].
- Applications to astrophysics and astroparticle physics.