Born-Infeld Magnetars: larger than classical toroidal magnetic fields and implications for gravitational-wave astronomy

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Nonlinear theories in the vicinities of stars

- Currently, the activity of Soft Gamma Repeaters (SGR) and Anomalous X-ray Pulsars (AXP) is mainly understood with the presence of very large surface magnetic fields ($10^{14} - 10^{15}$ G) [Kaspi and Beloborodov (2017)].
- Thus, nonlinear electrodynamics might influence some magnetar observables.
- Born-Infeld (BI) theory is a low energy limit of string theory and laboratory constraints of its scale field fail for values around $10^{15} - 10^{23}$ G [Carley and Kiessling (2006), Ellis et al. (2017)].
- So, opportunity arises to probe BI theory with magnetars! Could nonlinearities of electrodynamics also leave a trace in the gravitational waves coming from these stars?

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Nonlinear electrodynamics within ideal MHD

- In the context of ideal magnetohydrodynamics,

$$\vec{E} = -\frac{\vec{v}}{c} \times \vec{B} = -\frac{\vec{\omega} \times \vec{r}}{c} \times \vec{B}. \tag{1}$$

- For crossed fields

$$\nabla \cdot (L_F \vec{E}) = 4\pi\rho, \tag{2}$$

$$\nabla \cdot \vec{B} = 0, \tag{3}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{4}$$

$$\nabla \times (L_F \vec{B}) = \frac{1}{c}\frac{\partial L_F \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{j}, \tag{5}$$

with $L_F \doteq \partial L/\partial F$, $F \doteq F_{\mu\nu}F^{\mu\nu}$, L: nonlinear Lagrangian of electrodynamics, ρ: charge density and $\vec{j}$: current vector

Nonlinear electrodynamics within ideal MHD-II

- If one takes as reference Maxwell's electromagnetism ($L = F$),

$$L_F \vec{E} = \vec{E}_{Ma} + \nabla \times \vec{C}, \tag{6}$$

$$L_F \vec{B} = \vec{B}_{Ma} + \nabla f, \tag{7}$$

  where $\vec{C}$ is an arbitrary vector, likewise to f.

- Consistency with MHD demands that

$$\nabla \times \vec{C} = -\frac{\vec{\omega} \times \vec{r}}{c} \times \nabla f. \tag{8}$$

- When the hypothesis of very conductive media is taken into account ($|\vec{E}| \ll |\vec{B}|$) (ρ: energy density),

$$\rho \approx \frac{L}{16\pi}. \tag{9}$$

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Magnetars dominated by toroidal fields

- Low-poloidal magnetic fields in magnetars also support this view [Rea et al. (2010), (2012), (2014)].
- Within the modified twisted magnetic field, ratios of the toroidal energy to the total magnetic energy in stars could be up to $90\,\%$ [Ciolfi and Rezzolla (2013)].
- Assuming $\vec{B} \equiv \vec{B}_t = B_\phi(r,\theta)\hat{\phi}$, where $\hat{\phi}$ is the azimuthal unit vector ($\nabla \cdot \vec{B}_t = 0$), and defining the z-axis such that $\vec{\omega} = \omega\hat{z}$ ($\vec{v} = v\hat{\phi}$),

$$\vec{E} = \vec{0} \to \nabla \times \vec{C} = \vec{0} \to \nabla f = \vec{0}, \tag{10}$$

  and then

$$L_F B_\phi = B_\phi^{Ma}. \tag{11}$$

- It suffices solving an algebraic equation to find B_ϕ in terms of B_ϕ^{Ma}.

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Born-Infeld Lagrangian and toroidal fields

- Born-Infeld Lagrangian for crossed fields is [Born and Infeld (1934)]

$$L_{B.I} \doteq 4b^2\left(\sqrt{1+\frac{F}{2b^2}}-1\right), \tag{12}$$

where b: scale field of the theory and $F = 2(\vec{B}^2 - \vec{E}^2)$.

- For toroidal fields,

$$B^{BI}_{\phi} = \frac{bB^{Ma}_{\phi}}{\sqrt{b^2-(B^{Ma}_{\phi})^2}}. \tag{13}$$

- Thus, the nonlinear field B_ϕ is always larger than B^{Ma}_{ϕ}!
- Likewise to their energy density ratio,

$$\frac{\rho_{BI}}{\rho_{Ma}} = 2\left(\frac{b}{B^{Ma}_{\phi}}\right)^2\left\{\left[1-\left(\frac{B^{Ma}_{\phi}}{b}\right)^2\right]^{-\frac{1}{2}}-1\right\} > 1. \tag{14}$$

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Larger magnetic ellipticities

- For $\epsilon = c_1 \langle B_\phi^2 \rangle$ [Ostriker and Gunn (1969)], c_1 a constant,

$$\frac{\epsilon_{BI}}{\epsilon_{Ma}} = \left(\frac{\dot{E}_{GW}^{BI}}{\dot{E}_{GW}^{Ma}} \right)^{\frac{1}{2}} \approx \left(\frac{\Delta E_{GW}^{BI}}{\Delta E_{GW}^{Ma}} \right)^{\frac{1}{2}} \approx \left[1 - \left(\frac{B_\phi^{Ma}}{b} \right)^2 \right]^{-1}, \qquad (15)$$

- The upper-limit to the ellipticity, $|\epsilon_{ul}|$, could be constrained if a minimum value for b, b_{min}, was given, since

$$|\epsilon_{ul}| = \left[1 - \left(\frac{B_\phi^{Ma}}{b_{min}} \right)^2 \right]^{-1} |\epsilon_{Ma}|. \qquad (16)$$

Energetics and upper limits

- An estimate to c_1 and B_ϕ^{Ma} would be (E_{fl}: flare energy)

$$|c_1| \approx \frac{R^4}{GM^2}, \quad (B_\phi^{Ma})^2 = \frac{6E_{fl}}{R^3}. \qquad (17)$$

- From H constraints [Carley and Kiessling (2006)], $b_{min} \approx 4 \times 10^{15}$ G, while from fiducial stellar parameters and flares with $E_{fl} = 10^{47}$ erg (SGR 1806–20), $|\epsilon_{Ma}| = 1{,}20 \times 10^{-6}$. Thus, $|\epsilon_{ul}| \approx 1{,}24 \times 10^{-6}$ and $1 < \Delta E_{GW}^{BI}/\Delta E_{GW}^{Ma} \lesssim 1{,}08$.
- $E_{fl} = 10^{47} - 10^{48}$ erg, say $E_{fl} = 5 \times 10^{47}$ erg, $|\epsilon_{ul}| \approx 7{,}4 \times 10^{-6}$ [$B_\phi^{Ma} = 1{,}7 \times 10^{15}$ G] and $1 < \Delta E_{GW}^{BI}/\Delta E_{GW}^{Ma} \lesssim 1{,}53$.
- If just $B_\phi^{Ma} < 10^{15}$ G are possible in magnetars' surfaces, one has that $1 < \Delta E_{GW}^{BI}/\Delta E_{GW}^{Ma} \lesssim 1{,}14$ and $|\epsilon_{ul}| \approx 2 \times 10^{-6}$.

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Conclusions and perspectives

- In nonlinear electrodynamics and ideal MHD, unexpectedly enough, fields could be larger than in Maxwell's theory.
- Nonlinear electrodynamics might increase GW production.
- The upper limit to the magnitude of the ellipticity should be within the range $10^{-6} - 10^{-5}$ for current observations of magnetars.
- BI's GW energy for giant flare events of around 10^{47} erg could be at most $10\,\% - 20\,\%$ larger than their classical counterparts.
- For more energetic events, the maximum BI GW increase percentage could be much higher than $50\,\%$. This might be relevant for GRBs associated with magnetars.
- When GW detectors are able to better constrain magnetar ellipticities, minimum values for the Born-Infeld's scale field could be inferred astrophysically.
- It's still pending stability analysis for magnetic fields within nonlinear electrodynamics.