Non-linear perturbation of black branes at large D

Umpei Miyamoto
Akita Prefectural University

JHEP06(2017)033 [arXiv:1705.00486]

IWARA2018@Peru 2018.9.9

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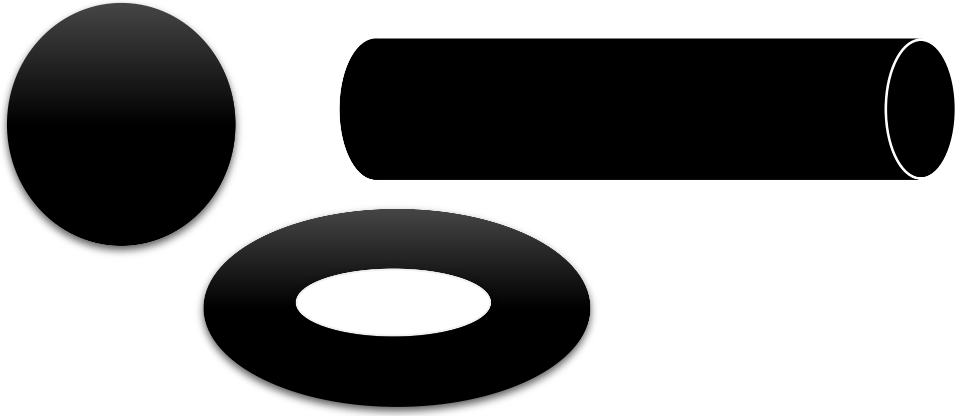
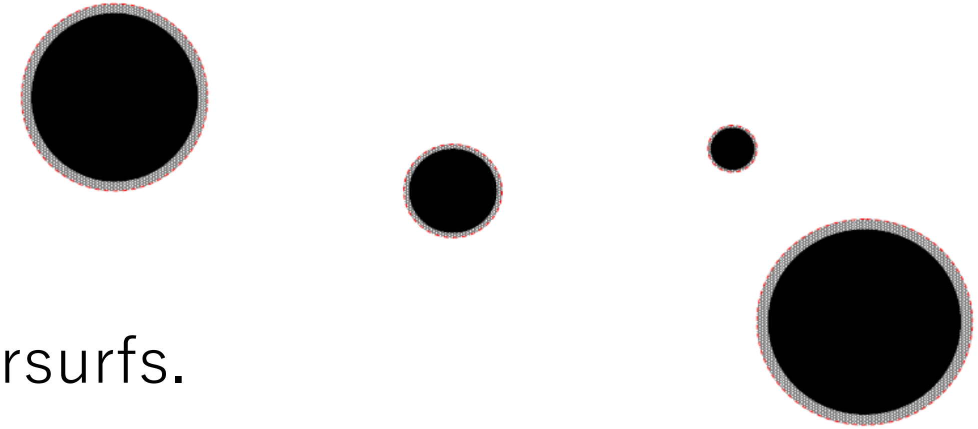
Background



- Higher-dim. BHs
  - Instability & various phase structures
    (e.g. Gregory-Laflamme inst.)
  - Gauge/Gravity correspondence
    (e.g. AdS/CFT, AdS/CMP, Fluid/Gravity)

Holes cut out in Minkowski space

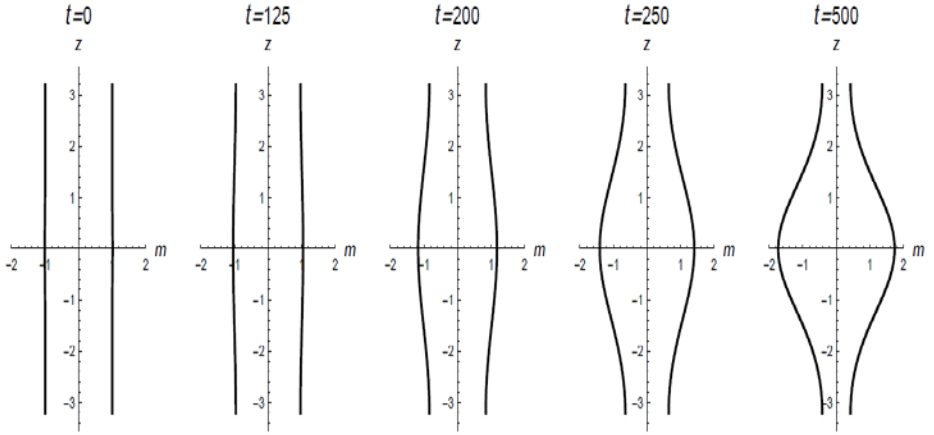


- Large-D approach
  (R. Emparan, R. Suzuki, K. Tanabe, and more)
  - 1/D expansion of GR
  - Horizons are Constant-Mean-Curvature hypersurfs.

http://www2.yukawa.kyoto-u.ac.jp/ws/2013/string13/Emparan.pdf

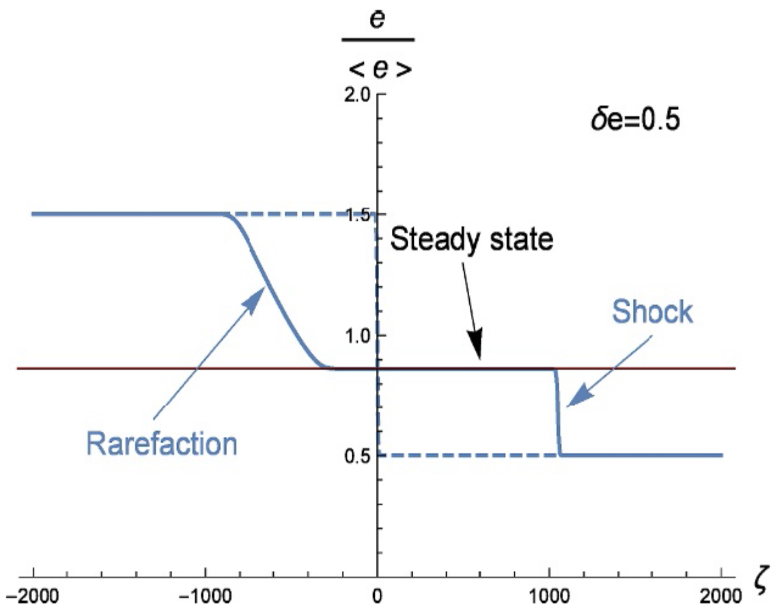
Preceding works：Black branes at large D

(Emparan-Suzuki-Tanabe PRL2015)



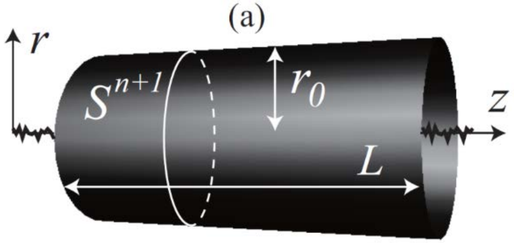
- Leading-order EOMs of BBs in 1/D (Asympt. flat & AdS)
  - 1+1 diffusion eqs. for m(t,z) and p(t,z)
  - GL-unstable Black String converges to a non-uniform BS（Sorkin 2004）

- Related work (Herzog-Spillane-Yarom 2016)
  - Holographic dual of Riemann problem ➔Non-Equilibrium Steady States (NESSs)



- Question: Can we say anything analytically using simplified EOMs?

My work : Non-linear pert. of large-D black branes (UM JHEP2017)



$$(\partial_t - \partial_z^2)m + \partial_z p = 0,$$

$$(\partial_t - \partial_z^2)p - \partial_z m = -\partial_z(\frac{p^2}{m}),$$

- Diffusion eqs. for m(t,z) & p(t,z)

$$m(t,z;\epsilon) = 1 + \sum_{\ell=1}^{\infty} m_\ell(t,z)\epsilon^\ell,$$

- Expand them around a uniform brane

$$p(t,z;\epsilon) = \sum_{\ell=1}^{\infty} p_\ell(t,z)\epsilon^\ell,$$

- Laplace & Fourier transf. w.r.t. t and z, respectively
  - Solve algebraic eqs.
  - Inverse transf.
  - General solutions

$$\dot{m}_\ell - m_\ell'' + p_\ell' = 0,$$

$$\dot{p}_\ell - p_\ell'' - m_\ell' = \psi_\ell,$$

$$\psi_2 = -2p_1p_1',$$

$$\psi_3 = 2m_1p_1p_1' + m_1'p_1^2 - 2p_1p_2' - 2p_1'p_2.$$

General form of solutions at O(ε ^l)

$$\begin{pmatrix} m_\ell(t,z) \\ p_\ell(t,z) \end{pmatrix} = \sum_{\sigma=+,-} \boldsymbol{B}_\sigma \begin{pmatrix} \mathcal{F}^{-1}[e^{s_\sigma(k)t}\bar{m}_\ell(0,k)] \\ \mathcal{F}^{-1}[e^{s_\sigma(k)t}\bar{p}_\ell(0,k) + e^{s_\sigma(k)t} * \bar{\psi}_\ell(t,k)] \end{pmatrix}$$

$$\boldsymbol{B}_\sigma := \frac{1}{2}\begin{pmatrix} 1 & -\sigma i \\ \sigma i & 1 \end{pmatrix},$$

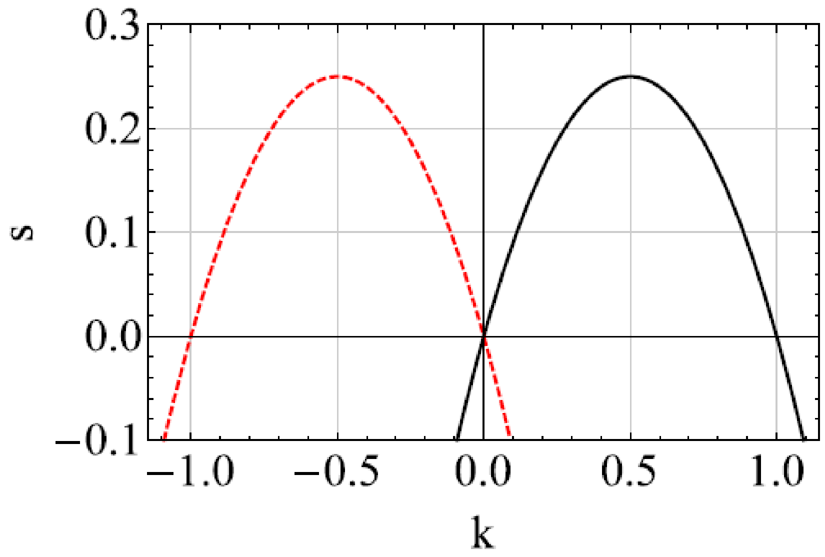
$$s_\sigma(k) := k(\sigma 1 - k).$$

I have obtained the soln. at every order as the linear combinations of inverse Fourier tr. of initial spectra m_l(0,k) & p_l(0,k).

Applicat'n to GL inst.

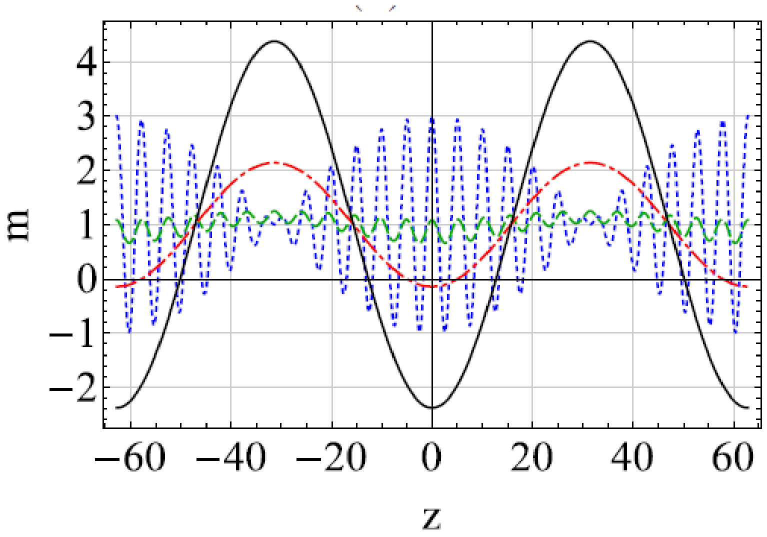
$$m_1(0,z) = \sum_{n=1}^{N} a_n \cos(k_n z + \varphi_n)$$

- Superposit'n of sinusoidal waves as initial condition
  ➔Obtain solns. up to 2nd order



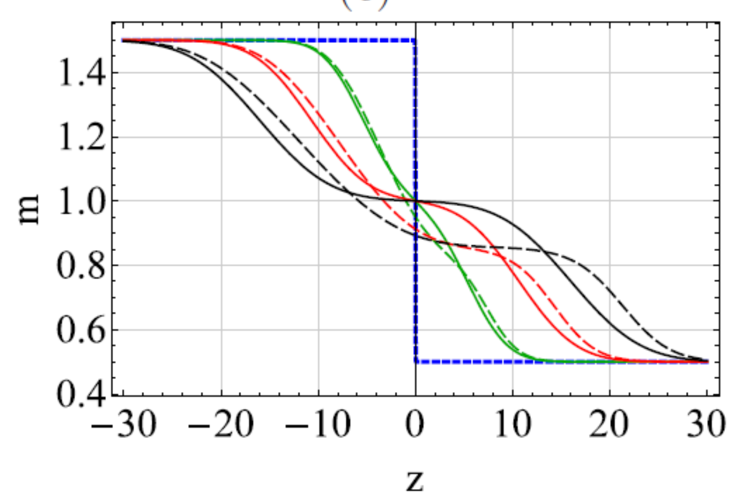
- A couple of GL-stable modes constitute a "beat"

- The beat grows if its size is greater than GL critical length



Applicat'n to Riemann problem

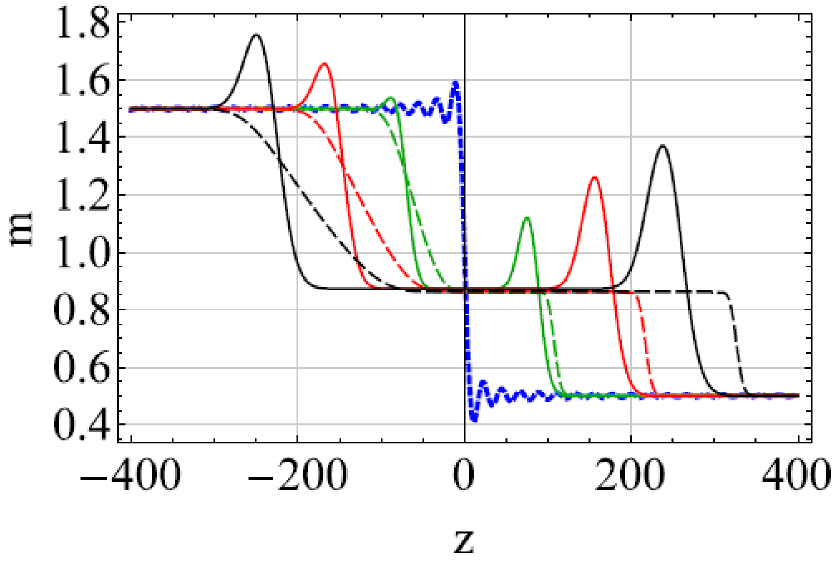
- Step-funct'n as m1(t=0,z)
  - 1st order soln. in terms of Gauss error fun.
  - Semi-analytic descript'n of NESS



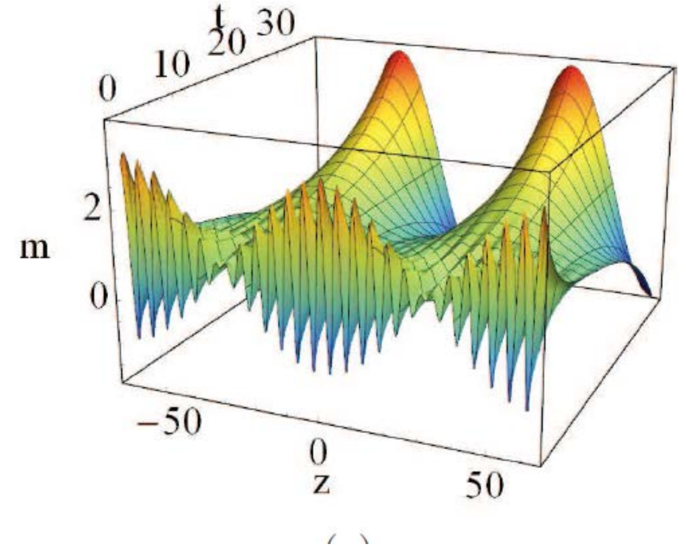
$$m_1(t,z) = \frac{\alpha}{2}\left[\mathrm{erf}\left(\frac{t+z}{2\sqrt{t}}\right) - \mathrm{erf}\left(\frac{t-z}{2\sqrt{t}}\right)\right],$$

$$p_1(t,z) = -\frac{\alpha}{2}\left[\mathrm{erf}\left(\frac{t-z}{2\sqrt{t}}\right) + \mathrm{erf}\left(\frac{t+z}{2\sqrt{t}}\right)\right].$$

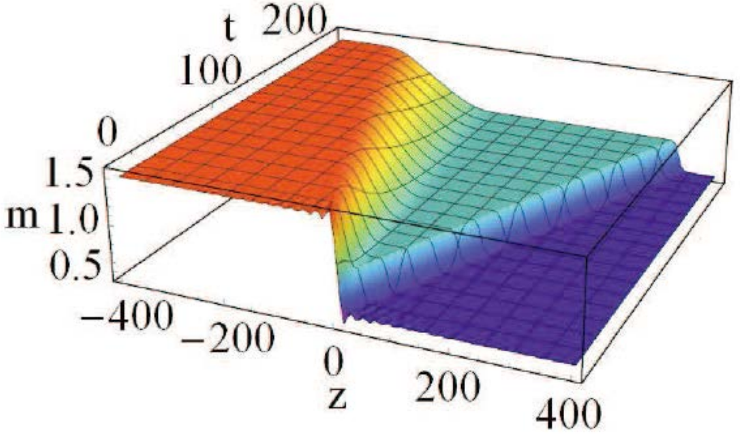
- Superposit'n of sinusoidal waves as m1(t=0,z)
  - Obtain solutions up to 2nd order
  - Reproduce details of NESS



Conclusion



- Perturb BBs at large-D around the uniform soln. and obtain the general form of soln. at every order
- Write down the explicit form of solns. for several initial conditions.
  - Property of GL inst. in non-linear regime
  - Analytic description of NESS



- Future prospects
  - 1/D corrections & BBs with charges
  - Application to turbulence (Rozali et al. 2017)