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Experimental design and progress for testing Lorentz symmetry in gravity

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Outline

- The background of LV in gravity
- ISL experiments in HUST
- Experiments test for LV in short-range gravity
- New experimental design for LV

I. Background of LV in gravity

HARD CCCC

Gravitational phenomena \leftarrow

GR ← Einstein Equivalence Principle (three logical parts):

Weak equivalence principle (WEP)

Local Lorentz invariance (LLI)

Local Position invariance (LPI)

• Lorentz violation in gravity

Lorentz violation ---- Described by the presence of background general tenser fields in spacetime ($s_{\mu\nu}$, $k_{\mu\nu\kappa\lambda}$,...)

The topic :

Recent progress on probing LV in gravity at HUST



General framework: Standard-Model Extension (SME) (developed by Kostelecky and collaborators)

Lagrangian of LV in **gravity**

$$L_{\rm LV} = \frac{\sqrt{g}}{16\pi G} (L_{\rm LV}^{(4)} + L_{\rm LV}^{(5)} + L_{\rm LV}^{(6)} + \cdots) \qquad \text{Bailey, PRD91,022006(2015)}$$

a series involving operators of increasing mass dimension d

 $L_{\rm LV}^{(4)}$ \leftarrow Tested by interaction between Earth and a small test body.

- $L_{\rm LV}^{(5)}$ has no effect on nonrelativistic gravity
- $L_{\rm LV}^{(6)}$ \leftarrow Tested in short-range gravity.

Minimal SME(mSME d=4)



The minimal term with d = 4

The dimensionless coefficient .

- Atom-interferometer
- Lunar laser ranging
- Pulsar-timing observations

$$L_{LV}^{(4)} = (k^{(4)})_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}$$

at tracel

$$-uR + s^{\mu\nu}R_{\mu\nu} + t^{\kappa\lambda\mu\nu}R_{\kappa\lambda\mu\nu}$$

$$\downarrow$$

$$V(r) = -G \frac{m_1m_2}{|\vec{x}_1 - \vec{x}_2|} \left[1 + \frac{1}{2}\hat{x}^j\hat{x}^k\overline{s}_{jk}\right]$$

9 independent components $\overline{s}^{\mu\nu} \sim 10^{-10}$

Laboratory experiments: To measure the acceleration of a free body

Due to the Earth's orbit and rotation

the local acceleration for LV
$$\frac{g_{LV}}{g} \propto \sum_{m} C_m \cos \omega_m t + D_m \sin \omega_m t$$

Six frequencies $\omega_m = (2\omega_{\oplus}, \omega_{\oplus}, 2\omega_{\oplus} + \Omega, 2\omega_{\oplus} - \Omega, \omega_{\oplus} + \Omega, \omega_{\oplus} - \Omega)$
C.G.Shao. etal, PRD97,024019 (2018)

Non-minimal term with d = 6

However, and the second second

LV in short-range gravity. Bailey, PRD91,022006(2015)

Lagrangian includes quadratic couplings of Riemann curvature

$$L_{\rm LV}^{(6)} = \frac{1}{2} \left(k_1^{(6)} \right)_{\alpha\beta\gamma\delta\kappa\lambda} \left\{ D^{\kappa}, D^{\lambda} \right\} R^{\alpha\beta\gamma\delta} + \left(k_2^{(6)} \right)_{\alpha\beta\gamma\delta\kappa\lambda\mu\nu} R^{\alpha\beta\gamma\delta} R^{\kappa\lambda\mu\nu}$$

In nonrelativistic limits

LV is described by the effective coefficients $(\bar{k}_{eff})_{jklm}$

Totally symmetric indices

Modified Possion equation $\nabla^2 U + 4\pi G \rho(\vec{r}) + (\vec{k}_{eff})_{jklm} \partial_j \partial_k \partial_l \partial_m U = 0$

In the case of two point masses $V(r) = -G \frac{m_1 m_2}{r} \left[1 + \frac{\overline{k}(\hat{r})}{r^2} \right]$

Potential between two point masses



$$V_{\rm LV}(\vec{r}) = -Gm_1m_2\frac{k(\hat{r})}{r^3}$$
 $r = |\vec{x}_1 - \vec{x}_2|$

Bailey, PRD91,022006(2015)

anisotropic combination of coefficients $(\overline{k}_{eff})_{iikl}$, function of \hat{r} direction

$$\overline{k}(\hat{r}) = \frac{3}{2} (\overline{k}_{eff})_{iijj} - 9 (\overline{k}_{eff})_{ijkk} \hat{r}^{i} \hat{r}^{j} + \frac{15}{2} (\overline{k}_{eff})_{ijkl} \hat{r}^{i} \hat{r}^{j} \hat{r}^{k} \hat{r}^{l}$$

Compare to usual Yukawa potential $V_{Yuk}(r) = -Gm_1m_2\frac{\alpha e^{-r/\kappa}}{r}$

Distinctive feature of LV : anisotropic cubic potential

depends on sidereal time in lab frame

constrain Yukawa parameter (α, λ) Tests in short-range gravity constrain Lorentz violation $(\overline{k}_{eff})_{jklm}$

Application



$$V(r) = -G \frac{m_{I}m_{2}}{|\vec{x}_{1} - \vec{x}_{2}|} \left(1 + \frac{1}{2}\hat{x}^{\hat{j}}\hat{x}^{\hat{k}}\overline{s}^{-\hat{j}\hat{k}}\right)$$
Gravitational
experiments
in HUST
Measurement of G, g(tidal)
Test of weak equivalence principle (WEP)
Test of Newtonian inverse square law (ISL)

Yukawa-type potential :

$$V(r) = -G \frac{m_1 m_2}{r} \left(1 + \alpha e^{-r/\lambda}\right)$$

Lorentz violation potential :

$$V(r) = -G \frac{m_1 m_2}{r} \left(1 + \frac{\overline{k}(\hat{r})}{r^2}\right)$$

II. ISL experiments in HUST



ISL Experiment, **HUST-2007** PRL 98,201101(2007)

2.1*10⁻¹⁶Nm

Our first result is not very good.

Basic feature: I-shaped pendulum, gold

Source mass platform: facing the pendulum, I-shaped structure

The separation was modulated by driving a motor translation stage

Ranging from 176 to 341 μm



9

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ISL experiment --HUST-2011





ISL Experiment, HUST-2011

PRL 108,081101(2012)

1.59*10⁻¹⁶Nm

At the length scale of several millimeters, we improve the previous bounds by up to a factor of 8.

The material of test and source masses: Tungsten instead of gold

The separation was modulated by driving a motor translation stage

Ranging from 0.4 to 1.0 mm



ISL experiment --HUST-2015





ISL Experiment, HUST-2015

PRL 116, 131101 (2016)

2.0*10⁻¹⁷Nm

This experiment improves the previous bounds by up to a factor of 2 at the length scale $\lambda \approx 160 \ \mu m$.

I-shaped pendulum, tungsten

Source mass platform: facing the pendulum, glass disk structure a rotating eightfold symmetric attractor separation: 0.295 mm



III. Experiments test for LV (d=6)





LV force between two plates

Planar geometry: to suppress the Newtonian background However, it also suppresses the LV signal

 $F_{Newton}^{y}(d)\Big|_{\text{infinite}} = \text{constant} \qquad t_{p} \qquad d$ $F_{LV}^{y}(d)\Big|_{\text{infinite}} = 0 \qquad t$

Force between two finite plates is dominated by the edge effect.

$$\Delta F_{LV}^{y} = F_{LV}^{y}(d_{\min}) - F_{LV}^{y}(d_{\max}) \sim \varepsilon \Delta C(\overline{k}_{eff})_{jkjk} \qquad \text{HUST-2011}$$

$$\Delta C \equiv 2\pi G \rho_{p} \rho A_{p} \left[\ln \frac{(d_{\min} + t_{p})(d_{\min} + t)}{(d_{\min} + t_{p} + t)d_{\min}} - \ln \frac{(d_{\max} + t_{p})(d_{\max} + t)}{(d_{\max} + t_{p} + t)d_{\max}} \right]$$

dimensionless parameter ε : edge effect

Edge effect ε is typically of order ~0.01 or d/\sqrt{A}

Experimental result of LV for HUST-2011

$\tau_{measured}^{z}(t) = \tau_{LV}(T)\cos(2\pi f_s t + \varphi)$ LV torque was modulated by changing d

$$\tau_{LV}(T) = C_0 + \sum_{m=1}^{4} \left[C_m \cos(m\omega_{\oplus}T) + S_m \sin(m\omega_{\oplus}T) \right]$$

	10 ⁻¹⁶ Nm	
C ₀	-0.22±0.95	
C ₁	0.13±0.22	
S ₁	-0.40±0.23	
C ₂	-0.04±0.22	
S ₂	0.20±0.22	
C ₃	-0.30±0.22	
S ₃	-0.25±0.23	
C ₄	-0.06±0.23	
S ₄	0.05±0.23	

		m
	<i>m</i> =1	
	Keff	$10^{-8}m^2$
1	XXXX	-0.2±2.8
2	YYYY	0.4 ± 2.8
3	ZZZZ	-0.9±7.7
4	XXXY	0.4±1.3
5	XXXZ	-0.1±0.5
6	YYYX	0.6±1.3
7	YYYZ	-0.4±0.5
8	ZZZX	-1.3±1.4
9	ZZZY	-0.2±1.3
10	XXYY	-0.1±1.7
11	XXZZ	-0.2±1.0
12	YYZZ	0.2±1.0
13	XXYZ	0.5±0.5
14	YYXZ	-0.2±0.5
15	ZZXY	-0.2±0.5



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Each constraint of $(\overline{k}_{eff})_{JKLM}$ was obtained in turn by setting the other 14 degrees of freedom to be zero.

J.C. Long,et.al. PRD91, 092003 (2015)

Our result: similar to that of IU-2002,2012, a shorter range ISL experiment (80 μ m) ¹⁴

The LV constraint from combined analysis



HUST-2015 separation: 0.295 mm



IU-2002,2012



Combined analysis for HUST-2015, HUST-2011, IU-2012, IU-2002

TABLE II. Independent coefficient values $(2\sigma, \text{ units } 10^{-9} \text{ m}^2)$ obtained by combining HUST and IU data [8–10].

Coefficient	Measurement	
$(\bar{k}_{\rm eff})_{XXXX}$	6.4 ± 32.9	
$(\bar{k}_{\rm eff})_{XXXY}$	0.0 ± 8.1	
$(\bar{k}_{\rm eff})_{XXXZ}$	-2.0 ± 2.6	
$(\bar{k}_{\rm eff})_{XXYY}$	-0.9 ± 10.9	
$(\bar{k}_{\rm eff})_{XXYZ}$	1.1 ± 1.2	
$(\bar{k}_{\rm eff})_{XXZZ}$	-2.6 ± 17.1	
$(\bar{k}_{\rm eff})_{XYYY}$	3.9 ± 8.1	
$(\bar{k}_{\rm eff})_{XYYZ}$	-0.6 ± 1.2	
$(\bar{k}_{\rm eff})_{XYZZ}$	-1.0 ± 1.0	
$(\bar{k}_{\rm eff})_{XZZZ}$	-8.1 ± 10.3	
$(\bar{k}_{\rm eff})_{YYYY}$	7.0 ± 32.9	
$(\bar{k}_{\rm eff})_{YYYZ}$	0.3 ± 2.6	
$(\bar{k}_{\rm eff})_{YYZZ}$	-2.5 ± 17.1	
$(\bar{k}_{\rm eff})_{YZZZ}$	3.6 ± 10.2	

Shao et.al, PRL117,071102(2016)

A spherical decomposition

A convenient formalism for analyzing short-range test of LV

Lagrange density
$$L = L_0 + L_{LV} = L_0 + \frac{1}{4}h_{\mu\nu}(\hat{s}^{\mu\rho\nu\sigma} + \hat{q}^{\mu\rho\nu\sigma} + \hat{k}^{\mu\nu\rho\sigma})h_{\rho\sigma}$$

V. A. Kostelecky. et al, PLB766,137-143(2017)



Cartesian coordinate system (spherical coordinate system

 $\overline{k}(\hat{\mathbf{r}},\mathbf{T}) = \sum Y_{jm}(\theta,\phi) \; k_{jm}^{N(d)lab} \Longrightarrow$ Lorentz violation coefficients - := d) or d ($- \operatorname{Re} k^{N(d)}$

The spherical decomposition provides a clean separation of the observable harmonics in sidereal time.

Transformation matrix(d=6)



Newton spherical coefficients

Effective Cartesian coefficients

		_													_	$(k_{eff})_{XXXX}$
$\begin{bmatrix} k_{2,0} \end{bmatrix}$		36/ √5	0	0	72/√5	0	36/	0	0	0	0	36/	0	36/	0	$\left (\bar{k}_{eff})_{XXXY} \right $
Re <i>k</i> _{2,1}		0	0	$12\sqrt{6/5}$	0	0	0	0	$12\sqrt{6/5}$	0	$12\sqrt{6/5}$	0	0	0	0	$(\bar{k}_{\mu\nu})$
$\operatorname{Im} k_{2,-1}$		0	0	0	0	$-12\sqrt{6/5}$	0	0	0	0	0	0	$-12\sqrt{6/5}$	0	$-12\sqrt{6/5}$	$(\overline{r}_{eff}, XXXZ)$
$\operatorname{Re}k_{2,2}$		$-6\sqrt{6/5}$	0	0	0	0	$6\sqrt{6/5}$	0	0	0	0	$6\sqrt{6/5}$	0	$6\sqrt{6/5}$	0	$(\kappa_{eff})_{XXYY}$
Im <i>k</i> _{2,-2}		0	$12\sqrt{6/5}$	0	0	0	0	$12\sqrt{6/5}$	0	$12\sqrt{6/5}$	0	0	0	0	0	$(k_{eff})_{XXYZ}$
k _{4,0}		-5	0	0	-10	0	$-40\sqrt{10}$	0	0	0	0	-5	0	-40	0	$(k_{eff})_{XXZZ}$
Re <i>k</i> _{4,1}	$\sqrt{\pi}$	0	0	6	0	0	0	0	6√5	0	$-8\sqrt{5}$	0	0	0	0	$(\bar{k}_{eff})_{XYYY}$
$\operatorname{Im} k_{4,-1}$	7	0	0	0	0	$-6\sqrt{5}$	0	0	0	0	0	0	-6\sqrt{5}	0	8√5	$(\bar{k}_{eff})_{XYYZ}$
$\operatorname{Re}k_{4,2}$		-\sqrt{10}	0	0	0	0	-10\sqrt{5}	0	0	0	0	$\sqrt{10}$	0	$-6\sqrt{10}$	0	$(\bar{k}_{eff})_{XYZZ}$
$\operatorname{Im} k_{4,-2}$		0	$2\sqrt{10}$	0	0	0	0	$2\sqrt{10}$	0	-12\sqrt{10}	0	0	0	0	0	$\left (\overline{k}_{aff})_{Y777} \right $
Re <i>k</i> _{4,3}		0	0	$-2\sqrt{35}$	0	0	0	0	6√35	0	0	0	0	0	0	(\overline{k})
$\operatorname{Im} k_{4,-3}$		0	0	0	0	6√35	0	0	0	0	0	0	$-2\sqrt{35}$	0	0	$(\kappa_{eff})_{YYYY}$
Re <i>k</i> _{4,4}		$\sqrt{5/2}$	0	0	$-3\sqrt{70}$	0	0	0	0	0	0	$\sqrt{5/2}$	0	0	0	$\binom{(k_{eff})_{YYYZ}}{-}$
$\operatorname{Im} k_{4,-4}$		0	$-2\sqrt{70}$	0	0	0	0	$2\sqrt{70}$	0	0	0	0	0	0	0	$(k_{eff})_{YYZZ}$
		L													L	$\left[(\bar{k}_{eff})_{YZZZ} \right]$

V. A. Kostelecky. Et al, PLB766,137-143(2017)

Result for spherical coefficients



Cartesian Coefficients

Coefficient	Measurement
$(\bar{k}_{eff})_{XXXX}$ $(\bar{k}_{eff})_{XXXY}$	$6.4 \pm 32.9 \\ 0.0 \pm 8.1$
$ \begin{array}{c} (k_{\text{eff}})_{XXXZ} \\ (\bar{k}_{\text{eff}})_{XXYY} \\ (\bar{k}_{\text{eff}})_{XXYY} \end{array} $	-2.0 ± 2.6 -0.9 ± 10.9 1.1 ± 1.2
$(\bar{k}_{eff})_{XXYZ}$ $(\bar{k}_{eff})_{XXZZ}$ $(\bar{k}_{eff})_{XYYY}$	-2.6 ± 17.1 3.9 ± 8.1
$(\bar{k}_{eff})_{XYYZ}$ $(\bar{k}_{eff})_{XYZZ}$	-0.6 ± 1.2 -1.0 ± 1.0
$(\bar{k}_{eff})_{XZZZ}$ $(\bar{k}_{eff})_{YYYY}$ $(\bar{k}_{eff})_{YYYZ}$	-8.1 ± 10.3 7.0 ± 32.9 0.3 ± 2.6
$(\bar{k}_{eff})_{YYZZ}$ $(\bar{k}_{eff})_{YZZZ}$	$\begin{array}{c} -2.5 \pm 17.1 \\ 3.6 \pm 10.2 \end{array}$

Derived values of Newton spherical coefficients.

Coefficient	Measurement
k ^{N(6)} ₂₀	$(3\pm 23)\times 10^{-8}\ m^2$
Re $k_{21}^{N(6)}$	$(-4\pm 4)\times 10^{-8}\ m^2$
Im $k_{21}^{N(6)}$	$(-2\pm 4)\times 10^{-8}\ m^2$
Re $k_{22}^{N(6)}$	$(0\pm 9)\times 10^{-8}\ m^2$
Im $k_{22}^{N(6)}$	$(1\pm4)\times10^{-8}~m^2$
$k_{40}^{N(6)}$	$(4\pm 25)\times 10^{-8}\ m^2$
Re $k_{41}^{N(6)}$	$(3\pm5)\times10^{-8}~m^2$
Im $k_{41}^{N(6)}$	$(1\pm5)\times10^{-8}~m^2$
Re $k_{42}^{N(6)}$	$(0\pm 12)\times 10^{-8}\ m^2$
Im $k_{42}^{N(6)}$	$(2\pm2)\times10^{-8}~m^2$
Re $k_{43}^{N(6)}$	$(0\pm1)\times10^{-8}~m^2$
Im $k_{43}^{N(6)}$	$(1\pm1)\times10^{-8}~m^2$
Re $k_{44}^{N(6)}$	$(2\pm 9)\times 10^{-8}\ m^2$
Im $k_{44}^{N(6)}$	$(2\pm5)\times10^{-8}~m^2$

C. G. Shao. Et al, PRL117,071102(2016)

V. A. Kostelecky. Et al, PLB766,137-143(2017) 18

IV. New experimental design for LV(d=6)



- Almost all experiments on ISL adopt planar geometry to search for Yukawa-type non-Newton gravity, which also suppressed LV signal
- LV force between two finite flat plates is dominated by edge effect

To increase the signal of LV







14 measurable independently coefficients of LV



Double trace of $(\overline{k}_{eff})_{ijij}$ is a rotational scalar, and can't be measured in short-range gravity



Measured LV torque provides nine components $\tau_{LV} = C_0 + \sum_{m=1}^{4} C_m \cos(m\omega_{\oplus}T) + S_m \sin(m\omega_{\oplus}T)$

Equivalently,
$$\overline{k}(\hat{r}) = c_0 + \sum_{m=1}^{4} c_m \cos(m\omega_{\oplus}T) + s_m \sin(m\omega_{\oplus}T)$$

Nine components in $\overline{k}(\hat{r})$ are functions of the 14 constant coefficients (\overline{k}_{eff}) in Sun-centered frame.

14 measurable coefficients in spherical coordinate system provides a clean separation of the observable harmonics.

$C_i, S_i \leftrightarrow k_{im}$ through functions 14 $\Gamma_i(\theta, \chi)$





Schematic drawing





The feature for test and source masses





shifted up and left half of the width of the strip

Transfer coefficients vary with angle



According to the typical design parameters, we calculate transfer

coefficients as functions of angle

Horizontal stripe-type Vertical stripe-type 20 Г13 20 Γ14 Г14 0 -20 -20 50 20 Г9 Amplitude (10⁻⁹Nm/m²) Г9 Amplituide $(10^{-9}$ Nm/m²) 25 Г10 -20 Г11 -40 Г12 Г12 -50 80 50 $\theta = \pi/7$ $\theta = \pi/2$ $\theta = \pi/6$ $\theta = 3\pi/5$ 40F 25 0 -40 -80 40 20 0 25 40 20 Γ5 -20 -20 Γ6 -40 -40 20 20 $\Gamma 1$ Γ^2 0 -20 -20 $3\pi/2$ 2π $\pi/2$ $\pi/2$ $3\pi/2$ 2π π 0 π $\theta(0-2\pi)$ $\theta(0-2\pi)$

Symmetry 9, 219 (2017)



Compare to the best current constraint[1]

Assuming 3 μm systemic error

Ratio of the total error in the current best constraint to that in our new design

Coefficients	Current constraint $(10^{-8}m^2)$ [21]	Ratio in horizontal stripe-type for $\theta = \pi/7$ and $\pi/2$	Ratio in vertical stripe-type for $\theta = \pi/6$ and $3\pi/5$
k _{2,0}	3 ± 23	4	5
${\rm Re}\dot{k}_{1,1}$	-4 ± 4	16	21
$Im k_{2,1}$	-2 ± 4	16	21
$\operatorname{Re} k_{2,2}$	0 ± 9	67	73
$Im k_{2,2}$	1 ± 4	30	32
$k_{4,0}$	4 ± 25	4	4
$\operatorname{Re} k_{4,1}$	3 ± 5	13	14
$\operatorname{Im} k_{4,1}$	1 ± 5	13	14
$\operatorname{Re} k_{2,2}$	0 ± 12	44	92
$\operatorname{Im} k_{2,2}$	2 ± 2	7	15
$\operatorname{Re} k_{4,3}$	0 ± 1	7	7
$\operatorname{Im} k_{4,3}$	1 ± 1	7	7
$\operatorname{Re} k_{4,4}$	2 ± 9	97	49
$Im k_{4,4}$	2 ± 5	54	27

[1] V. A. Kostelecky. Et al, PLB766,137-143(2017)

Experimental process







The experiment is ongoing

Conclusion and prospect

Conclusion

- For Quadratic couplings of Riemann curvature with d=6, the ISL experiments can be used to constrain the LV
- We suggested experiments with periodic striped geometry, which may improve the current constraints of LV by about one order of magnitude and the experiment is ongoing.

Prospect

The ISL experiments and new design experiments may also be used to test LV effect for d=8, which is now being under study.

