IWARA 2018

Tidal effect on clock comparison

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Outline

◆Research background

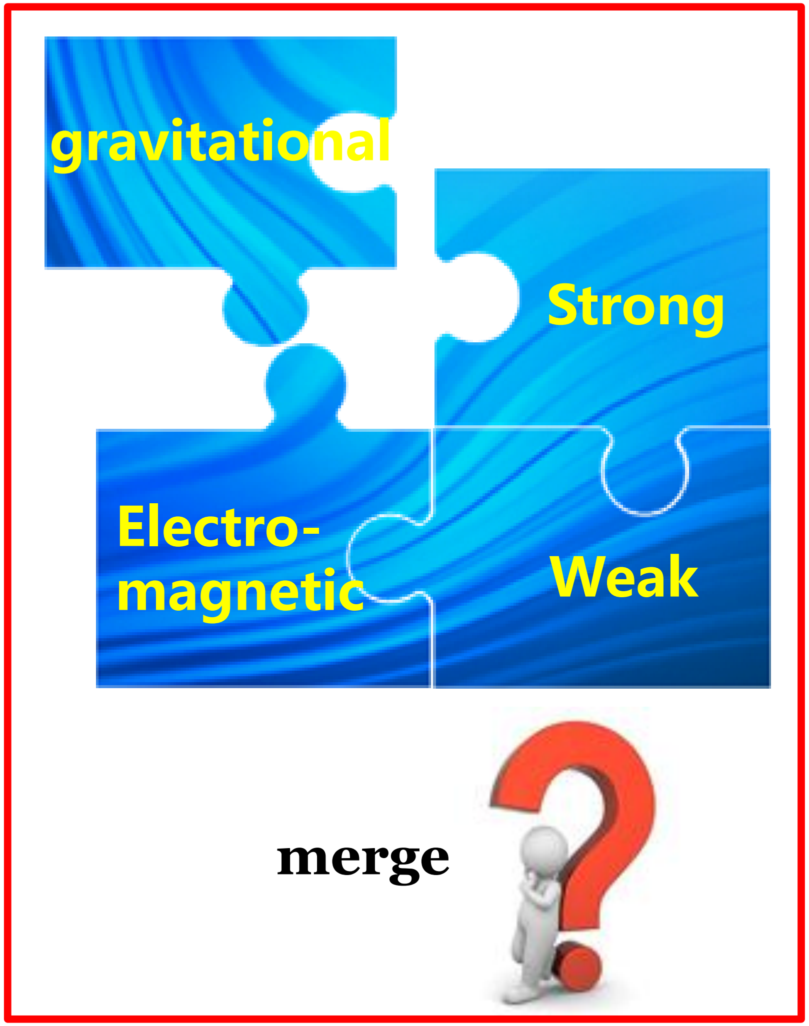
◆Frequency transfer

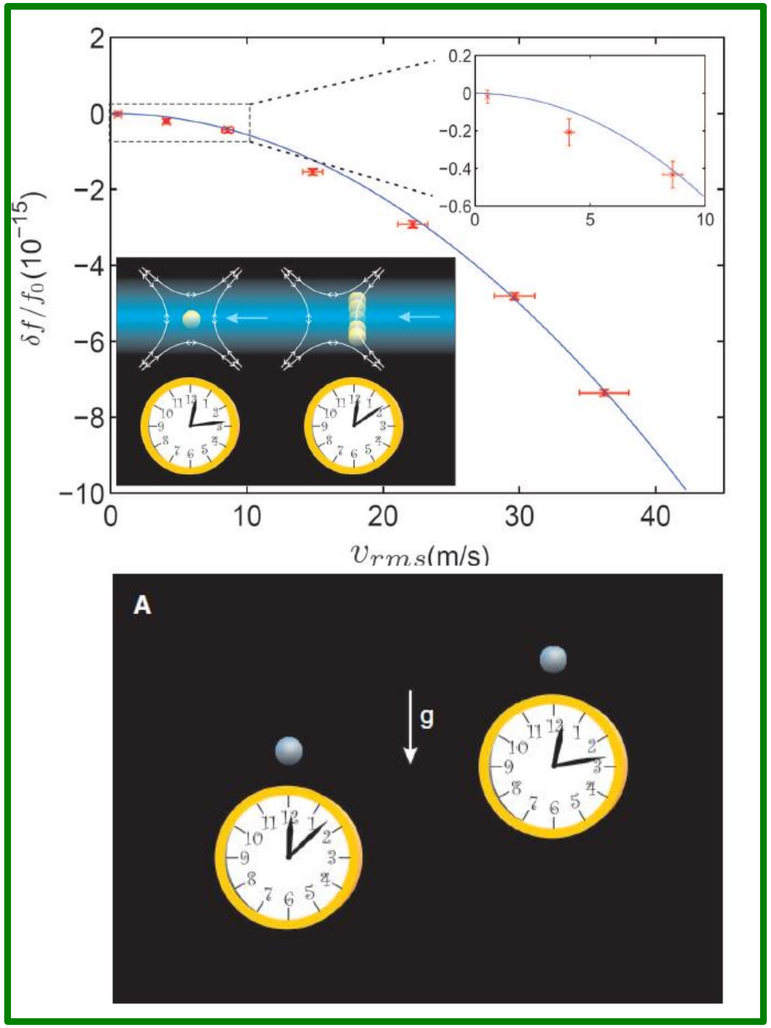
◆Tidal effect on clock comparison

◆conclusion

Background: applications of atomic clocks

➢ Applications of atomic clocks: explore the unified theory

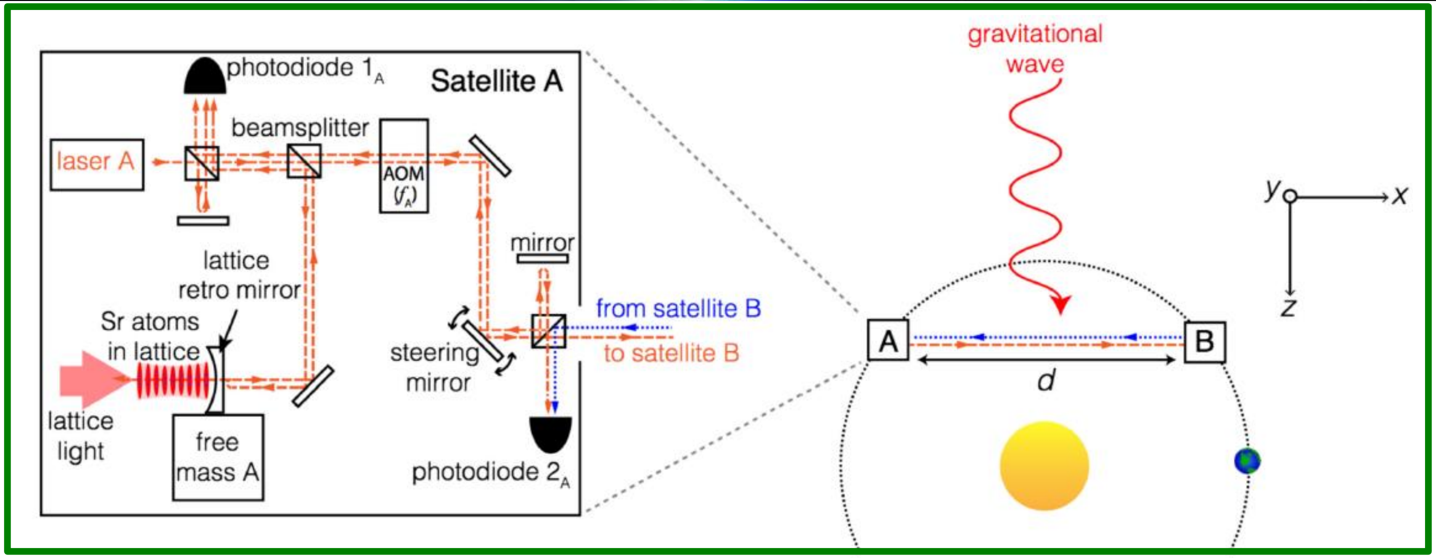




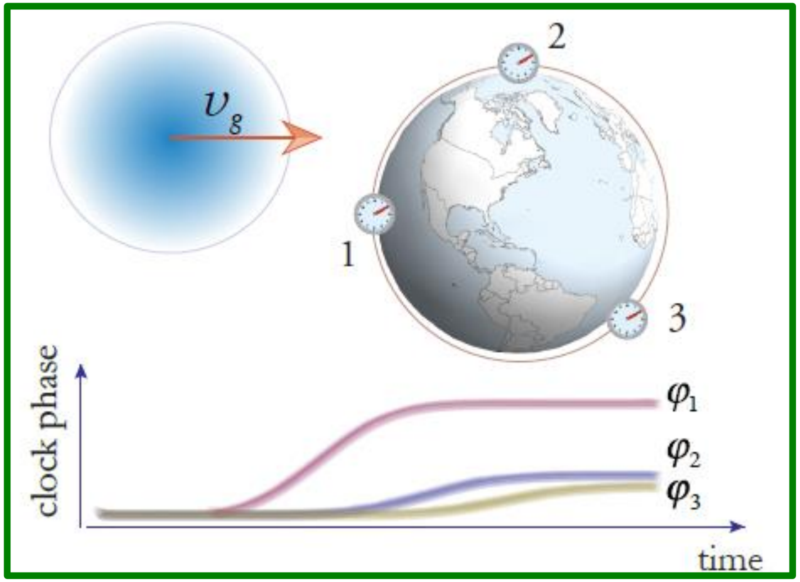
time dilation

C. W. Chou, et al., Science 329 (2010).

Background: applications of atomic clocks

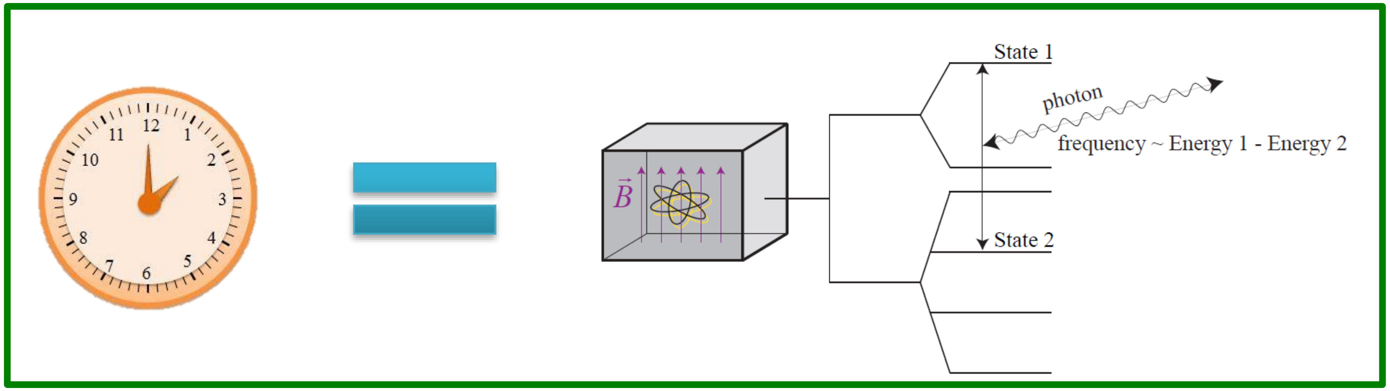


gravitational wave S. Kolkowitz, et al., Phys. Rev. D 94, 124043 (2016).



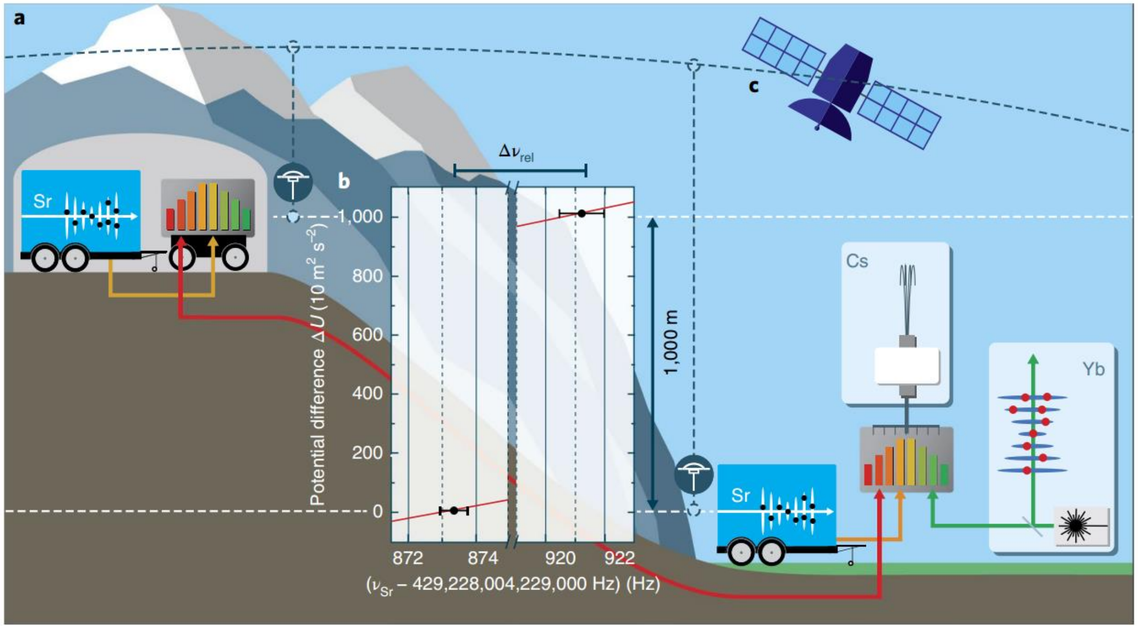
dark matter A. Derevianko, et al., Nat. Phys. 933 (2014).

Background: applications of atomic clocks



Local Lorentz invariance violation Charles D. Lane, Symmetry 9, 245 (2017).

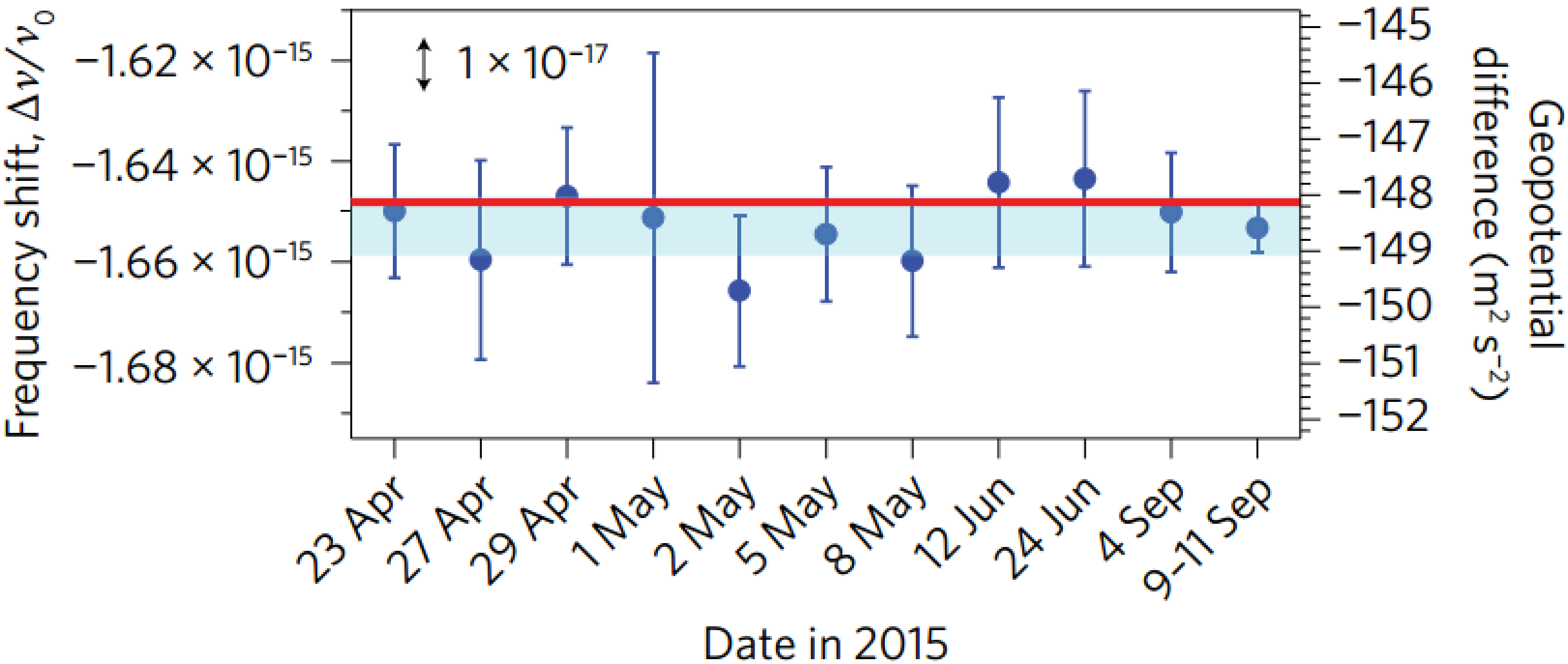
- Applications of atomic clocks: geodesy



Determine the geoid J. Grotti, et al., Nat. Phys., 14, 437-441 (2018).

Background: tidal effect with atomic clocks

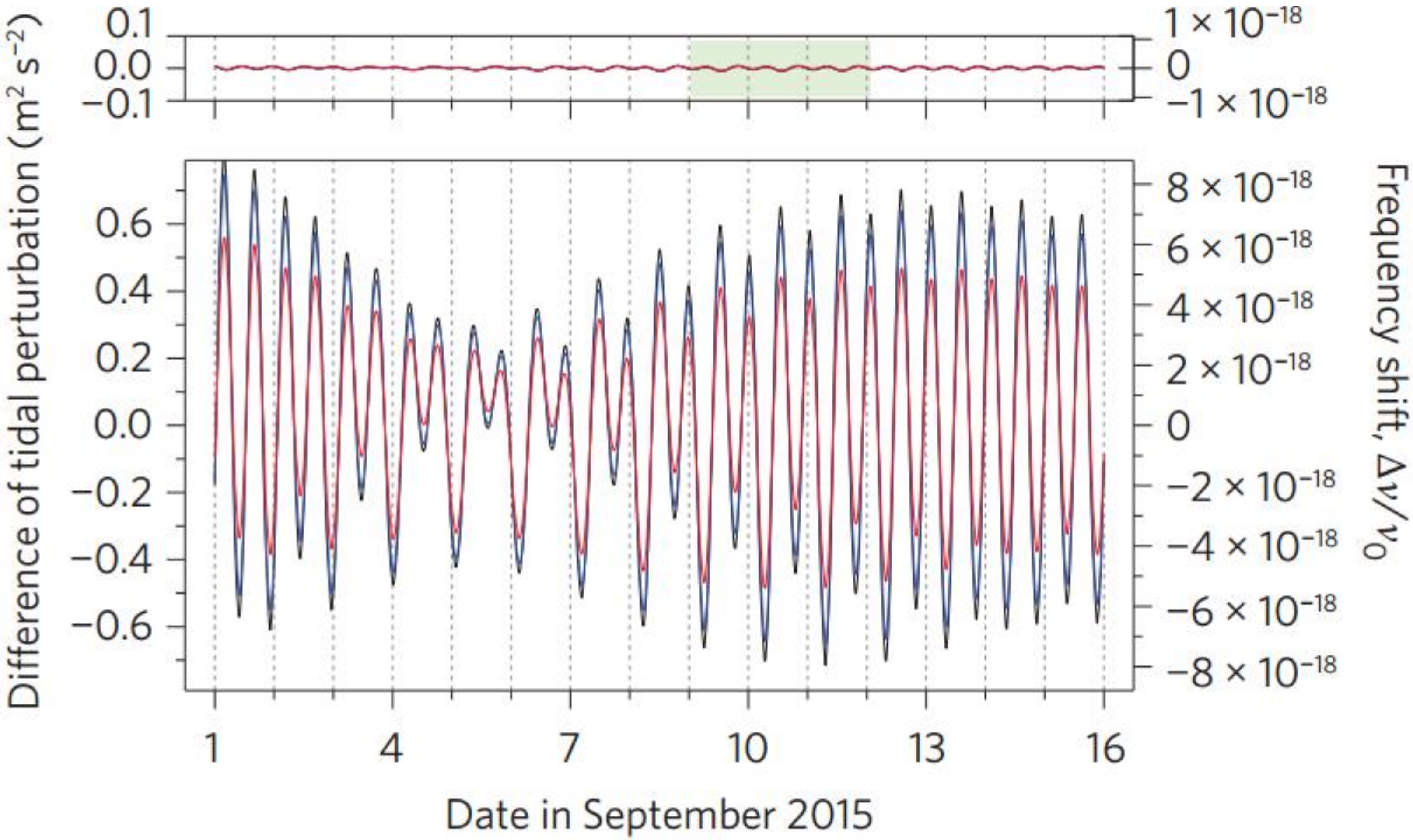
Relativistic geodesy:



relative frequency difference: 1652.9(5.9)×10e-18

T. Takano, et al, Nat. Photonics, 10, 662 (2016).

Background: tidal effect with atomic clocks



15km
2×10e-19

1000km

Tidal effect
- potential W/c^2 measured by clock comparison
- acceleration ∇W measured by gravimeter

T. Takano, et al, Nat. Photonics, 10, 662 (2016).

Frequency transfer

$$\text{clock comparison} \rightarrow \frac{W}{c^2} + ?$$

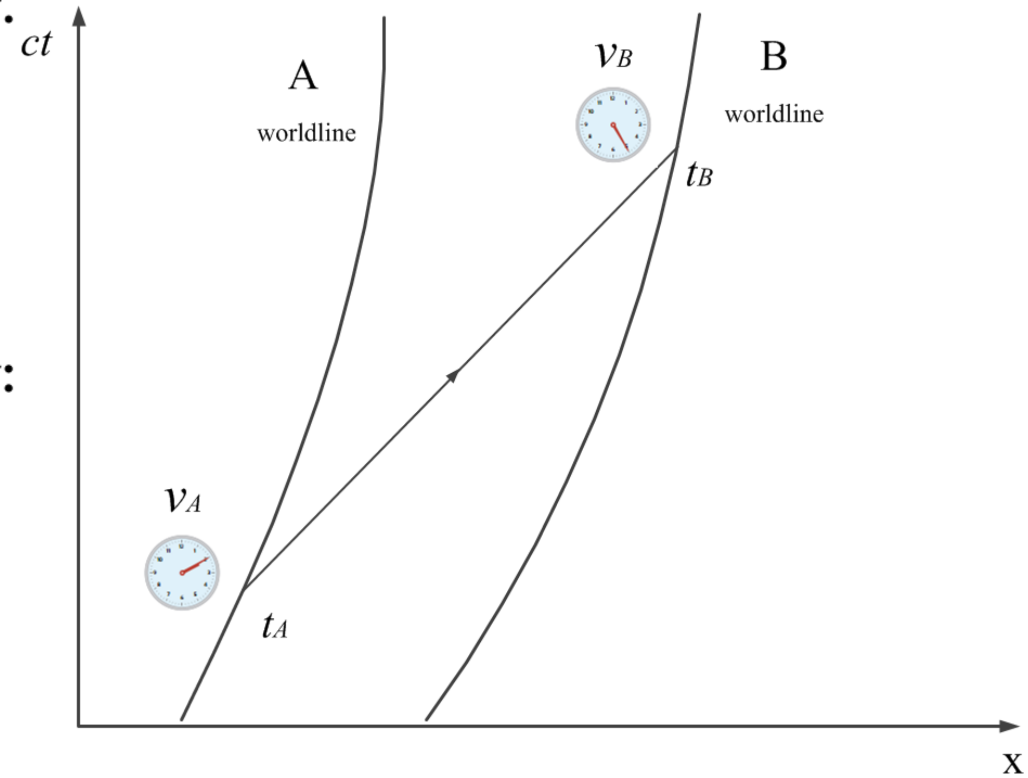
Frequency transfer between clocks

Two clocks A and B are linked by the light signal or optical fiber.

ν_A and ν_B are frequencies measured in clocks A and B, respectively.

Frequency transfer is given by:

$$\frac{\nu_A}{\nu_B} \equiv \frac{k_\mu u_A^\mu}{k_\mu u_B^\mu} = \frac{u_A^0}{u_B^0} \times \frac{q_A}{q_B}$$



Frequency transfer

Part1 $\frac{u_A^0}{u_B^0}$ The second-order Doppler effect, the gravitational red shift due to Earth, tidal potentials of Sun and Moon, tidal response of the Earth…

$$\frac{u_A^0}{u_B^0} = \frac{d\tau_B/dt_B}{d\tau_A/dt_A} = \frac{(g_{00} + 2g_{0i}\beta^i + g_{ij}\beta^i\beta^j)_B^{1/2}}{(g_{00} + 2g_{0i}\beta^i + g_{ij}\beta^i\beta^j)_A^{1/2}}$$

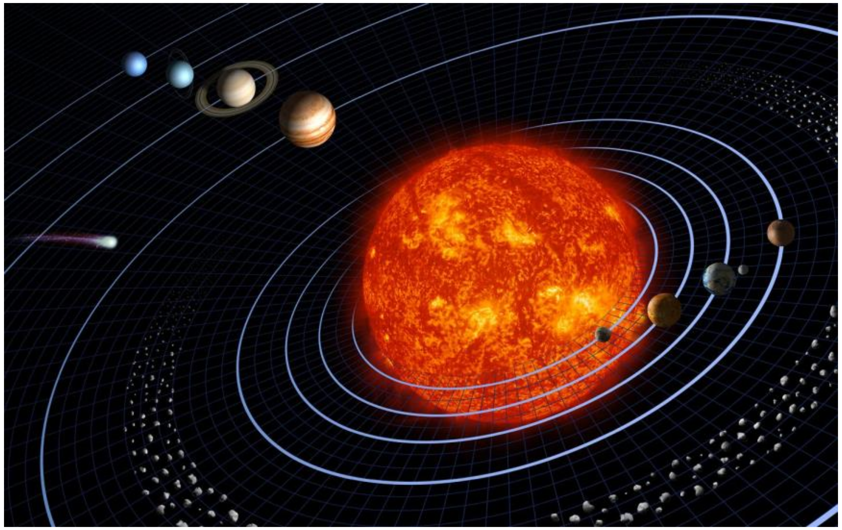
Part2 $\frac{q_A}{q_B}$ The first-order Doppler effect, temperature, medium…

$$\frac{q_A}{q_B} = \frac{dt_B}{dt_A}$$

Tidal effect in clock comparison (BCRS)

BCRS

- The barycentric coordinate reference system is a particular implementation of the barycentric reference system of the solar system.
- It can be sue to describe the motion of bodies in the solar system and li--ght propagation from distant stars.



- The metric components may be :

$$g_{00} = -1 + \frac{2w}{c^2} - \frac{2w^2}{c^4} + O(c^{-5})$$

$$g_{0i} = -\frac{4w^i}{c^3} + O(c^{-5}) \qquad g_{ij} = \delta_{ij}\left(1 + \frac{2w}{c^2}\right) + O(c^{-4})$$

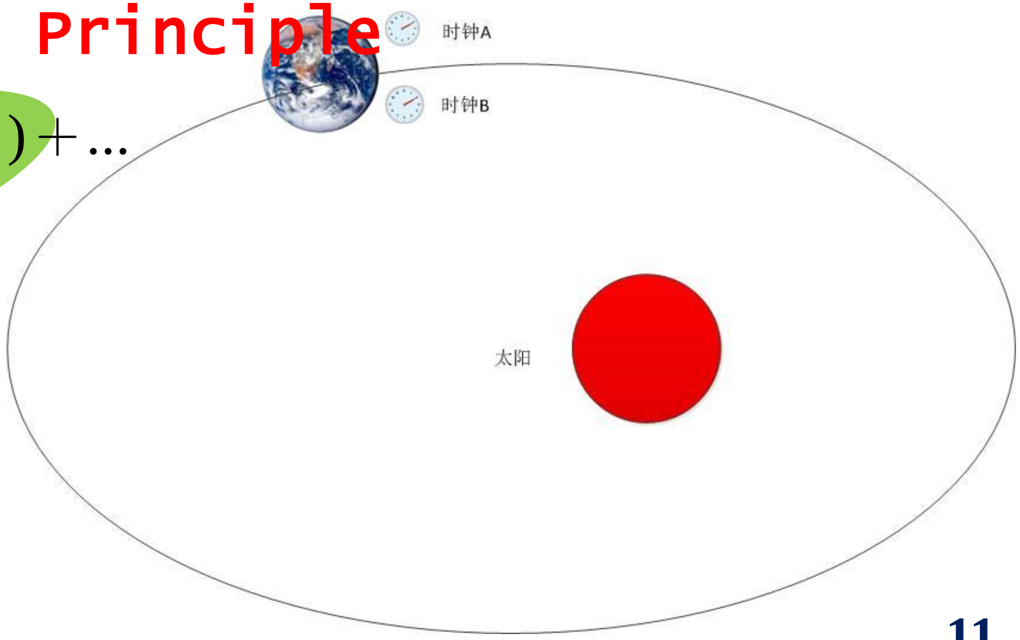
Tidal effect in clock comparison (BCRS)

One-way frequency transfer in BCRS

Part1. the invariance of the Riemannian space-time interval

$$\frac{d\tau_A}{dt_A} = 1 - \frac{1}{c^2}\left(w_{ext}(r_A) + w_E(r_{EA}) + \frac{v_A^2}{2} \right) + O(c^{-4})$$

Einstein's Equivalence Principle



$$w_{ext}(r_A) \rightarrow w_{ext}(r_E) + r_{EA} \cdot \nabla w_{ext}(r_E) + \ldots$$

$$v_A{}^2 \rightarrow v_E{}^2 - r_{EA} \cdot a_E + \ldots$$

Clock comparison tests :

Local Lorentz invariance (LLI)

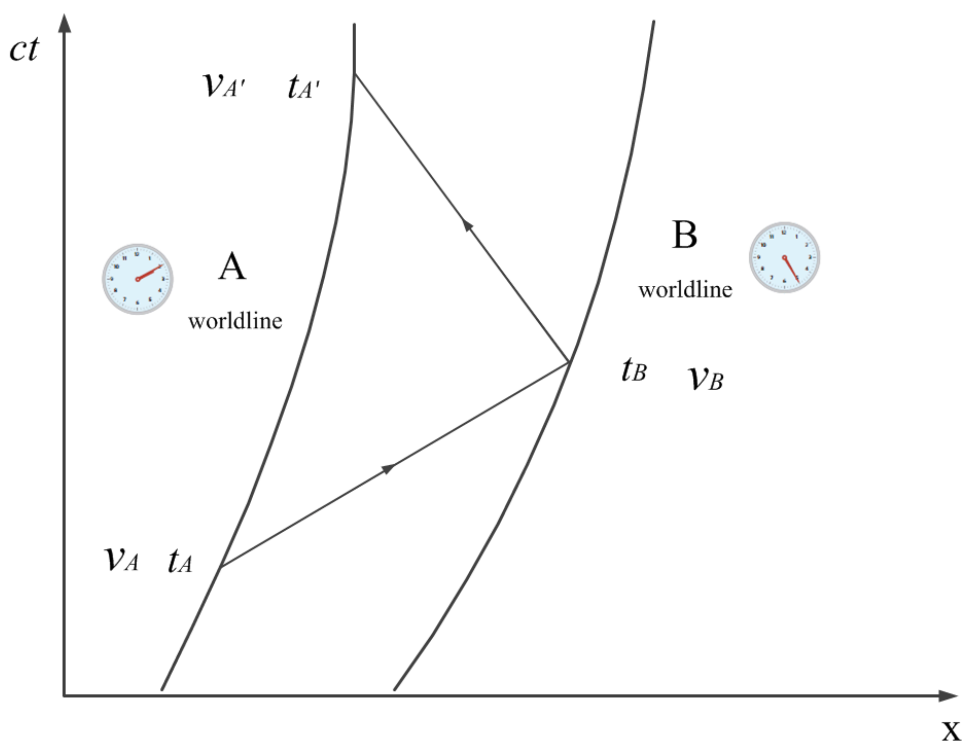
Local Position invariance (LPI)

One-way frequency transfer in BCRS

Part2.

$$\frac{dt_B}{dt_A} = 1 + \frac{d}{dt_A}\left(\frac{r_{AB}}{c} + \frac{r_{AB}\cdot \mathrm{v}_B}{c^2} + \frac{r_{AB}\cdot \mathrm{v}_B \quad \mathrm{n}_{AB}\cdot \mathrm{v}_B}{2c^3} + \frac{\mathrm{v}_B^2 r_{AB}}{c^3} \right) + O \;\; c^{-4}$$

Two-way frequency transfer



Tidal effect in clock comparison (GCRS)

GCRS

- The geocentric coordinate reference system its origin at the mass center of Earth and is physically adequate to describe processes occurring in the vicinity of Earth.
- The gravitational filed of external bodies is given by the form of tidal potential.
- The internal gravitational field coincides with the gravitational field of a corresponding isolated body.
- The metric components may be

$$g_{00} = -1 + \frac{2W}{c^2} - \frac{2W^2}{c^4} + O(c^{-5}) \qquad g_{0i} = -\frac{4W^i}{c^3} + O(c^{-5}) \qquad g_{ij} = \delta_{ij}\left(1 + \frac{2W}{c^2}\right) + O(c^{-4})$$

Tidal effect in clock comparison (GCRS)

One-way frequency transfer in GCRS

Part1.

From the properties of local reference system

$$W(T,X) \approx W_E(T,X) + W_T(T,X)$$

from the invariance of the Riemannian spacetime interval

$$\frac{u_A^0}{u_B^0} = 1 + \frac{1}{c^2}\left(\frac{V_A^2 - V_B^2}{2} + W_E\ T, X_A\ - W_E\ T, X_B\ + W_T\ T, X_A\ - W_T\ T, X_B\ \right) + O\ c^{-4}$$

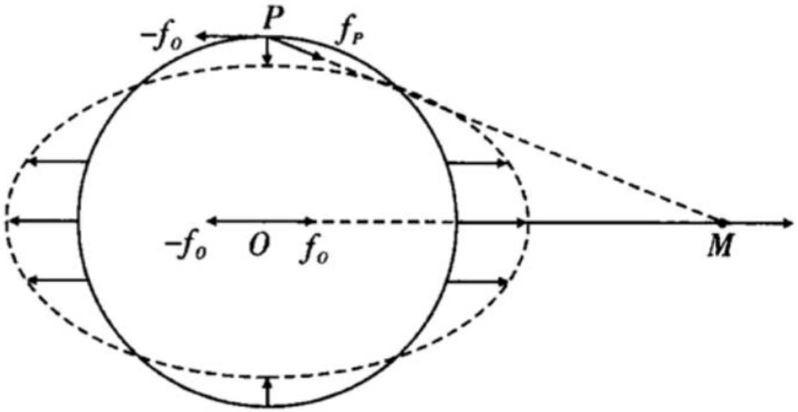
Part2.

$$\frac{dT_B}{dT_A} = 1 + \frac{d}{dT_A}\left(\frac{R_{AB}}{c} + \frac{R_{AB}\cdot \mathrm{V}_B}{c^2} + \frac{R_{AB}\cdot \mathrm{V}_B\ \ \mathrm{N}_{AB}\cdot \mathrm{V}_B}{2c^3} + \frac{\mathrm{V}_B^2 R_{AB}}{c^3}\right) + O\ c^{-4}$$

Tidal effect in clock comparison (GCRS)

Tidal effects --

celestial tidal potential and tidal response of solid Earth (calculated with SNREIO model): Mainly consider tidal potentials due to Moon and Sun.



Solid Earth's tidal response to Moon

.Love number (k_n, h_n, l_n):

redistribution of Earth mass

radial displacement of deformation

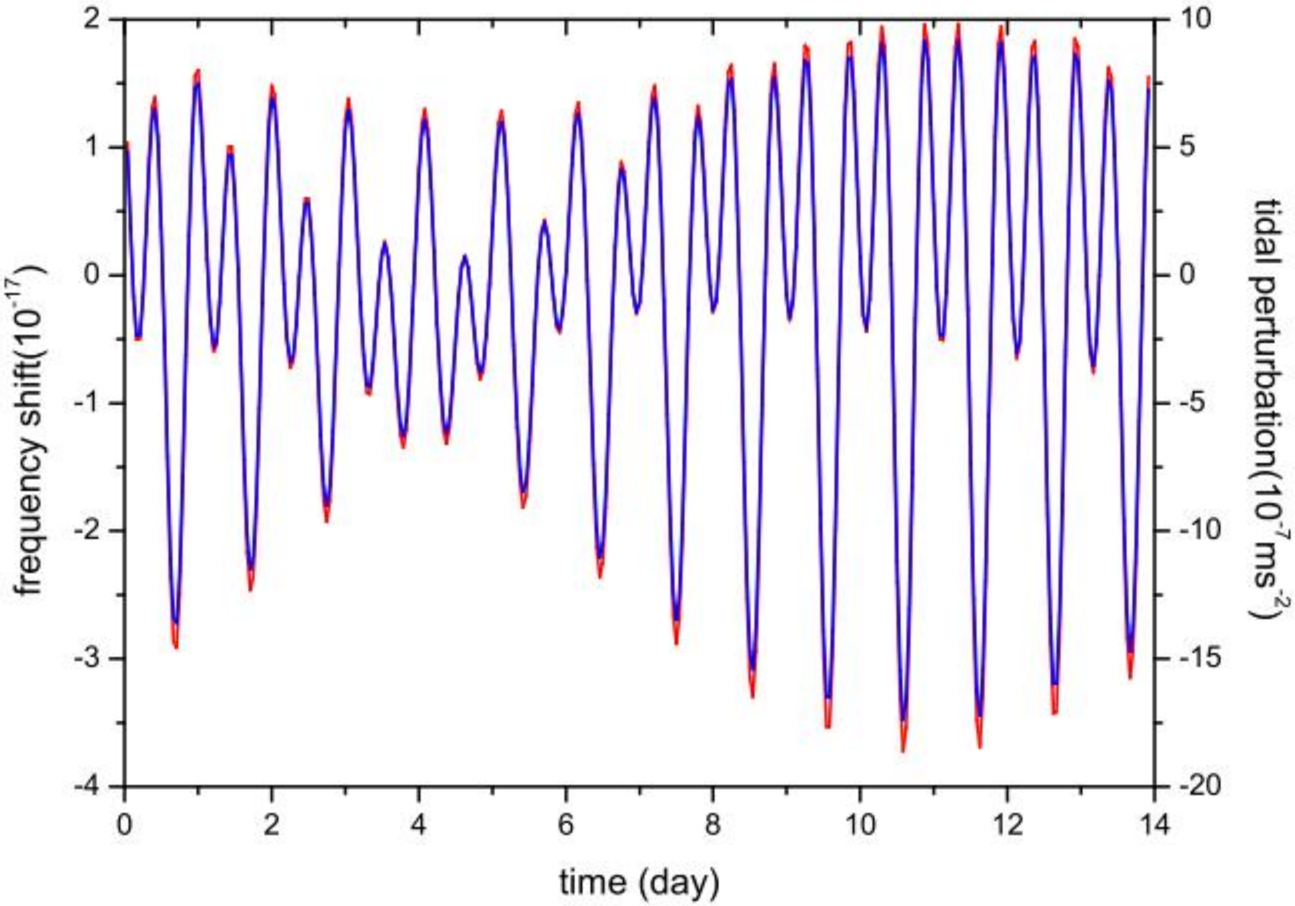
tangential displacement of deformation

$$\left(\frac{\Delta v}{v_0}\right)_T = \frac{1}{c_2}\left\{(1-h_2+k_2)\left[W_T(T,X_A)-W_T(T,X_B)\right]\right\}$$

$$\approx \frac{1}{c^2}\left[\frac{R_E(1-h_2+k_2)}{2+2h_2-3k_2}(\delta g_B-\delta g_A)\right]$$

Tidal effect in clock comparison (GCRS)

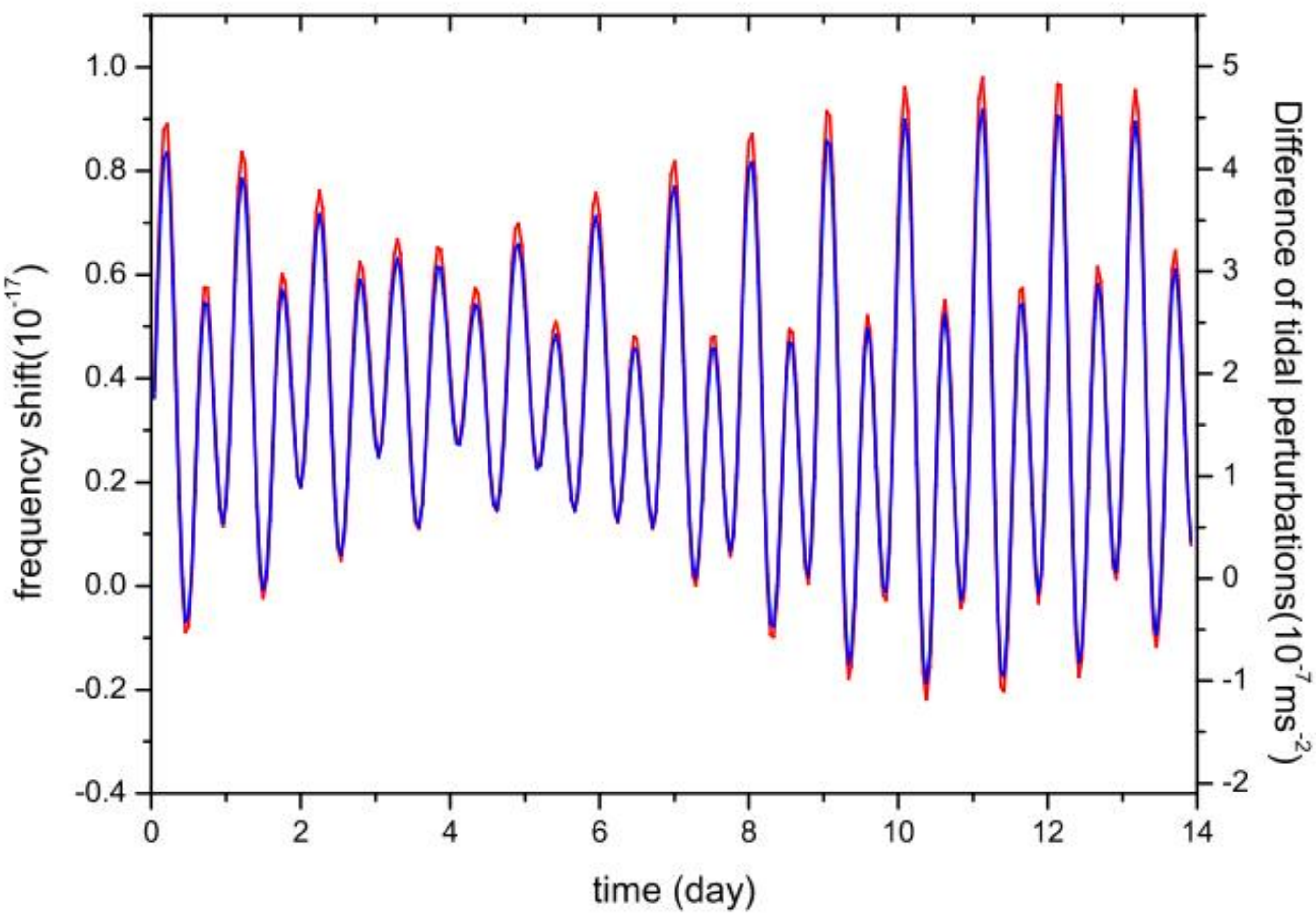
Frequency shift due to Moon's and Sun's tidal potential for one clock at A (E114°, N 30°) :



Tidal potential and acceleration coincide at 1/100 level

Tidal effect in clock comparison (GCRS)

Frequency shift due to Moon's and Sun's tidal potential for two clocks respectively at A (E114, N30) and B (E116, N40), separated by a distance about 1000km:



Tidal effect is observable!

Conclusion

1. Analyze the tidal potential in BCRS, and the result indicates: the tidal potential is very tiny due to EEP, but it can be observed by atomic clock (1000 km-distance clock comparison) with 10^-18 sensitivity.

2. The tidal potential (measured by clocks) and acceleration (measured by gravimeters) coincide at 1/100 level.

Thank you!