Oblique Magnetic Fields

and the

Role of Frame Dragging

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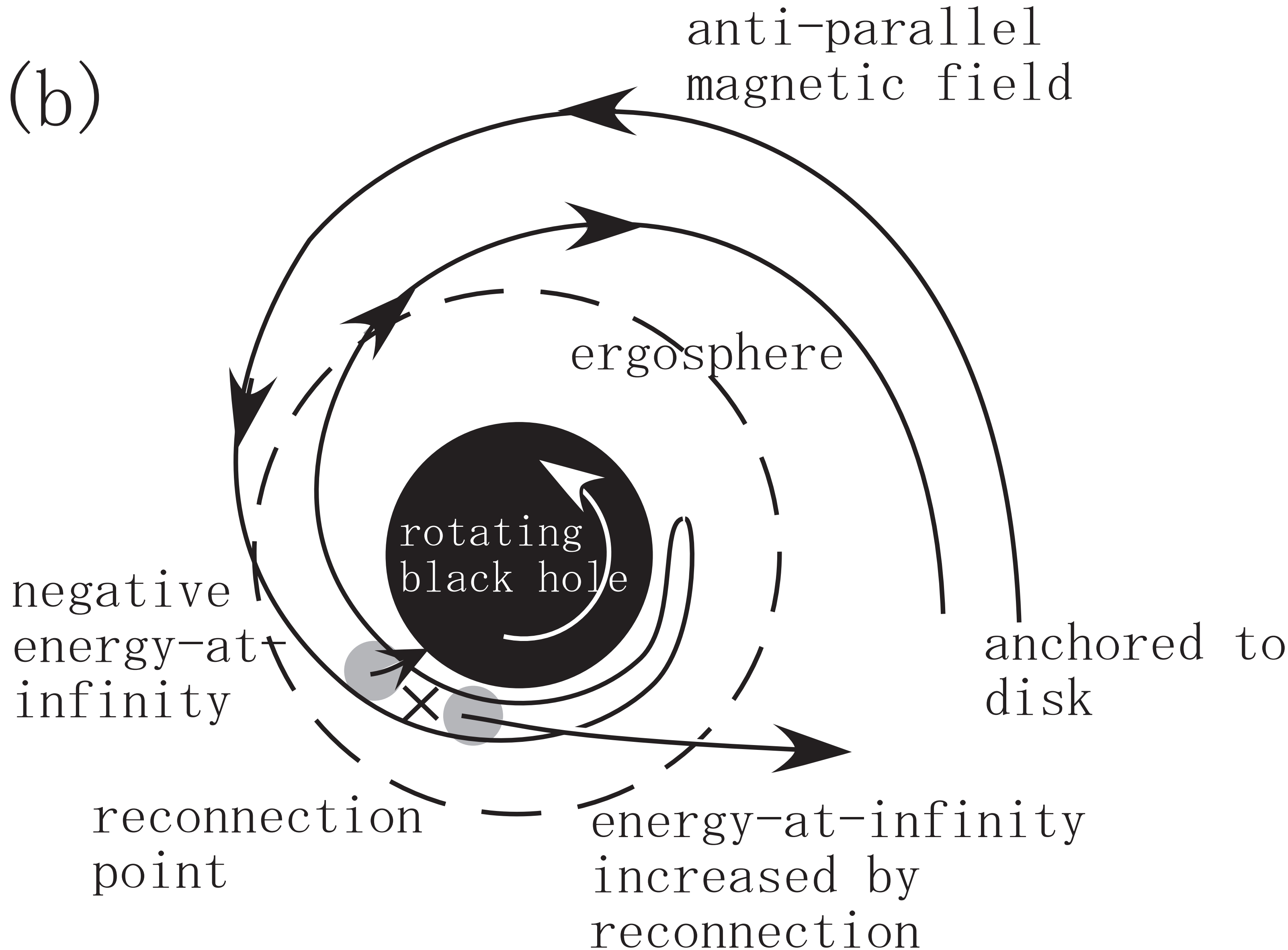
Motivation: magnetic reconnection in ergosphere?



Koide & Arai (ApJ, 2008); Lyutikov (PRD, 2011); Morozova et al. (2014)

Wald's axisymmetric field

$$F = \tfrac{1}{2} B_0 \left(\mathrm{d}\tilde{\xi} + \tfrac{2J}{M} \, \mathrm{d}\xi \right)$$

Magnetic flux surfaces (magnetic field lines lie in these surfaces):

$$4\pi\Phi_{\mathcal{M}} = \int_{\mathcal{S}} \boldsymbol{F} \;=\; \text{const.}$$

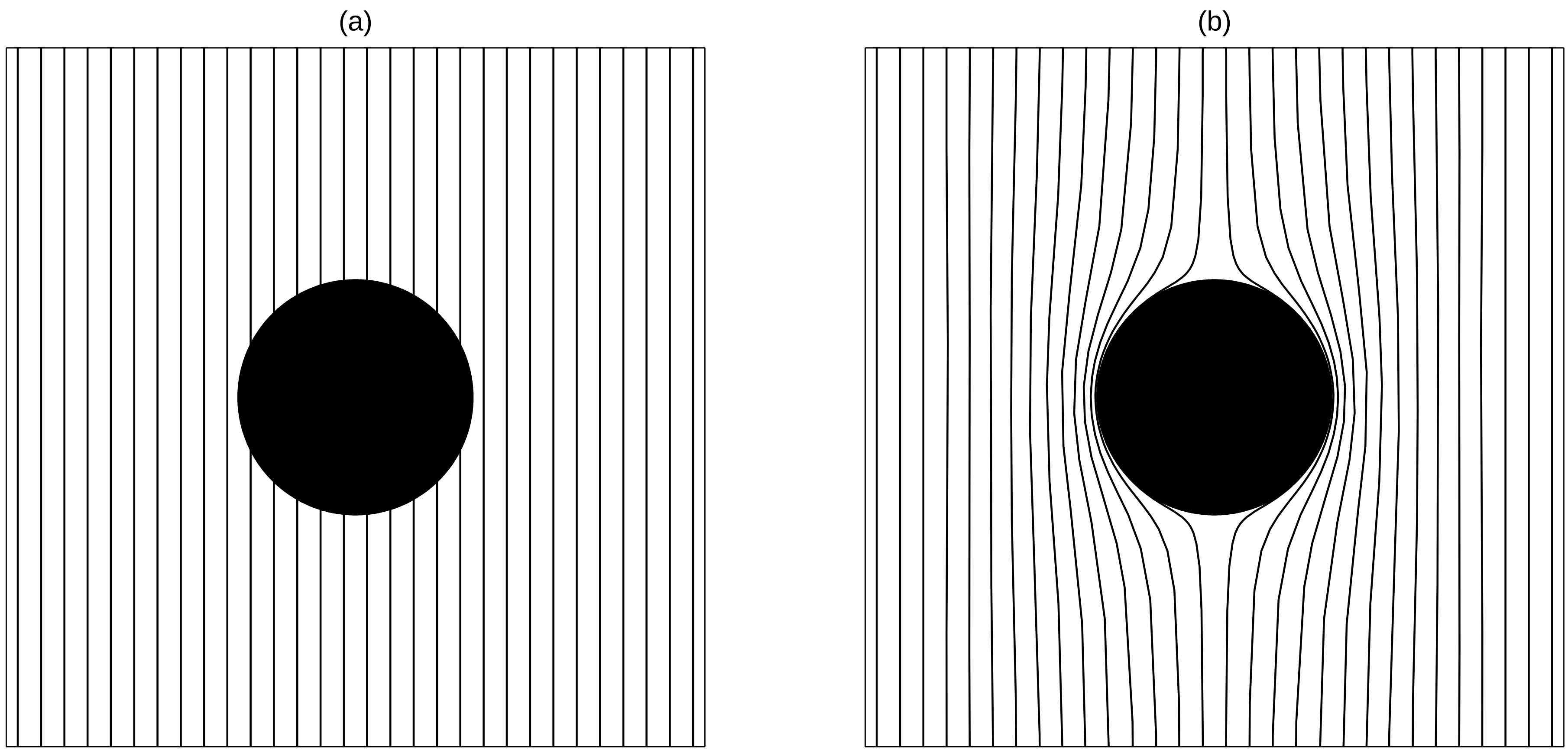
Magnetic/electric Lorentz force:

$$m\dot{\boldsymbol{u}} = q_{\mathrm{m}} {}^{\star}\boldsymbol{F}.\boldsymbol{u}, \qquad m\dot{\boldsymbol{u}} = q_{\mathrm{e}}\boldsymbol{F}.\boldsymbol{u}.$$

Magnetic field lines (aligned case):

$$\frac{\mathrm{d}r}{\mathrm{d}\theta} = \frac{B_r}{B_\theta},$$

Rotating black hole in vacuum, aligned magnetic field



An axisymmetric case: (a) $a = 0$; a non-rotating (Schwarzschild) black hole;

(b) $a = M$ a maximally rotating Kerr black hole – Meissner effect.

Magnetic/electric lines of force

Lorentz force acting on electric/magnetic monopole at rest

$$\frac{\mathrm{d}u^{\mu}}{\mathrm{d}\tau} \propto {}^{\star}F^{\mu}_{\nu}\,u^{\nu}, \qquad \frac{\mathrm{d}u^{\mu}}{\mathrm{d}\tau} \propto F^{\mu}_{\nu}\,u^{\nu}.$$

Magnetic lines:

$$\frac{\mathrm{d}r}{\mathrm{d}\theta} = -\frac{F_{\theta\phi}}{F_{r\phi}}, \qquad \frac{\mathrm{d}r}{\mathrm{d}\phi} = \frac{F_{\theta\phi}}{F_{r\theta}}.$$

Magnetic flux (axially symmetric case):

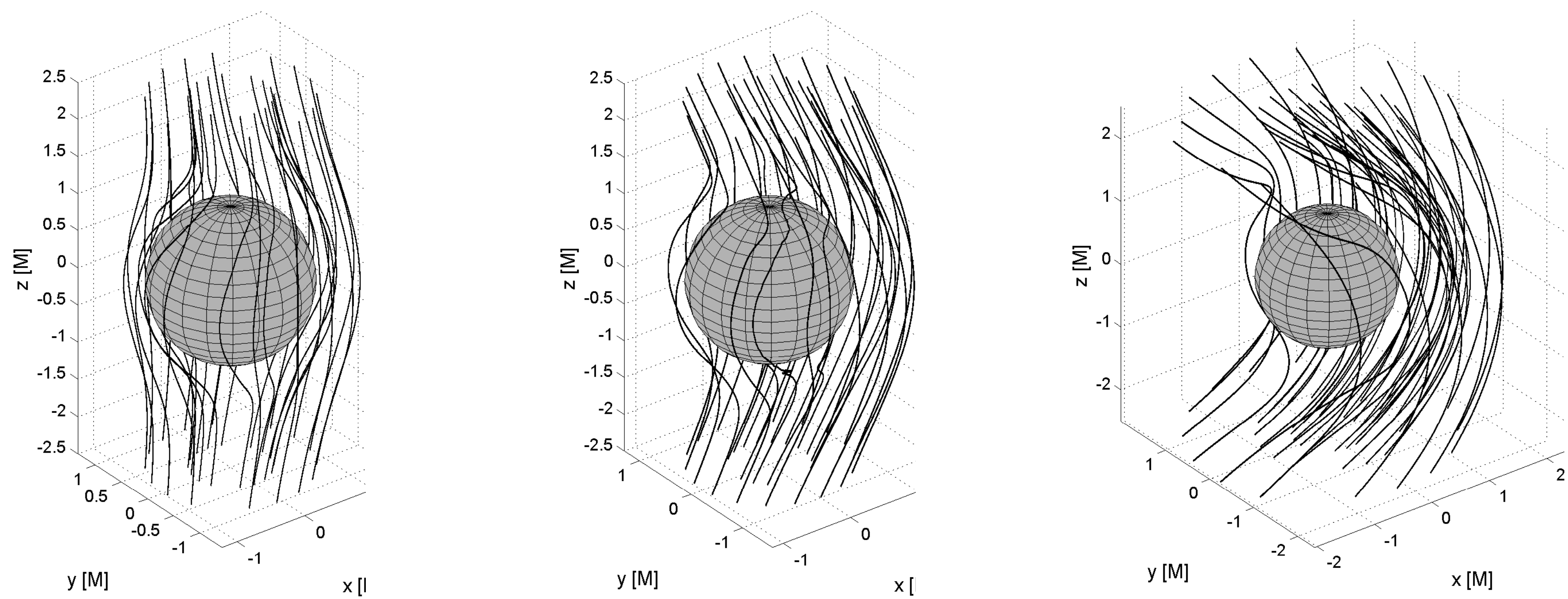
$$\Phi_{\mathrm{m}} = \pi B_0 \left[r^2 - 2Mr + a^2 + \frac{2Mr}{r^2 + a^2\,\cos^2\theta}\left(r^2 - a^2\right)\right]\sin^2\theta$$

Expulsion of magnetic flux out of fast rotating black hole:

$\Phi_{\mathrm{m}} = 0$ on hemisphere $r = r_+$, $a = M$ (“Meissner effect”).

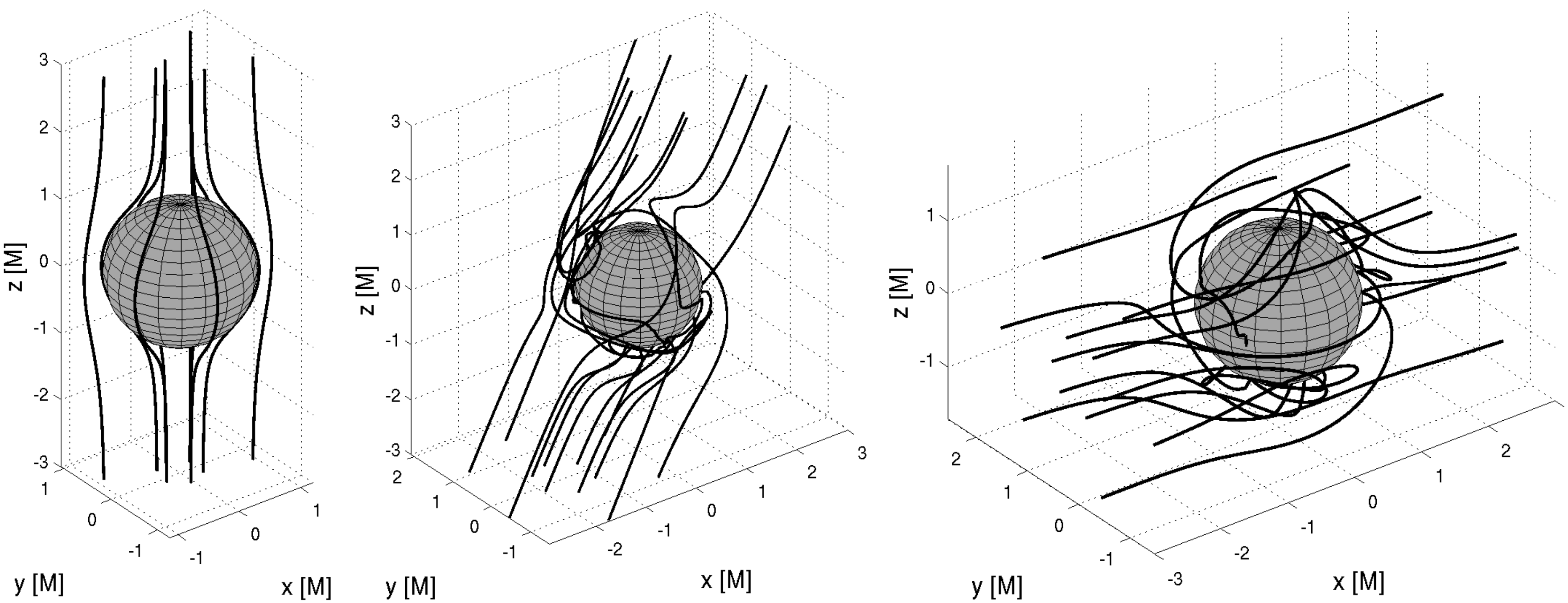
Rotating black hole, translation boost

$v = 0.1$ $\qquad$ $v = 0.3$ $\qquad$ $v = 0.7$



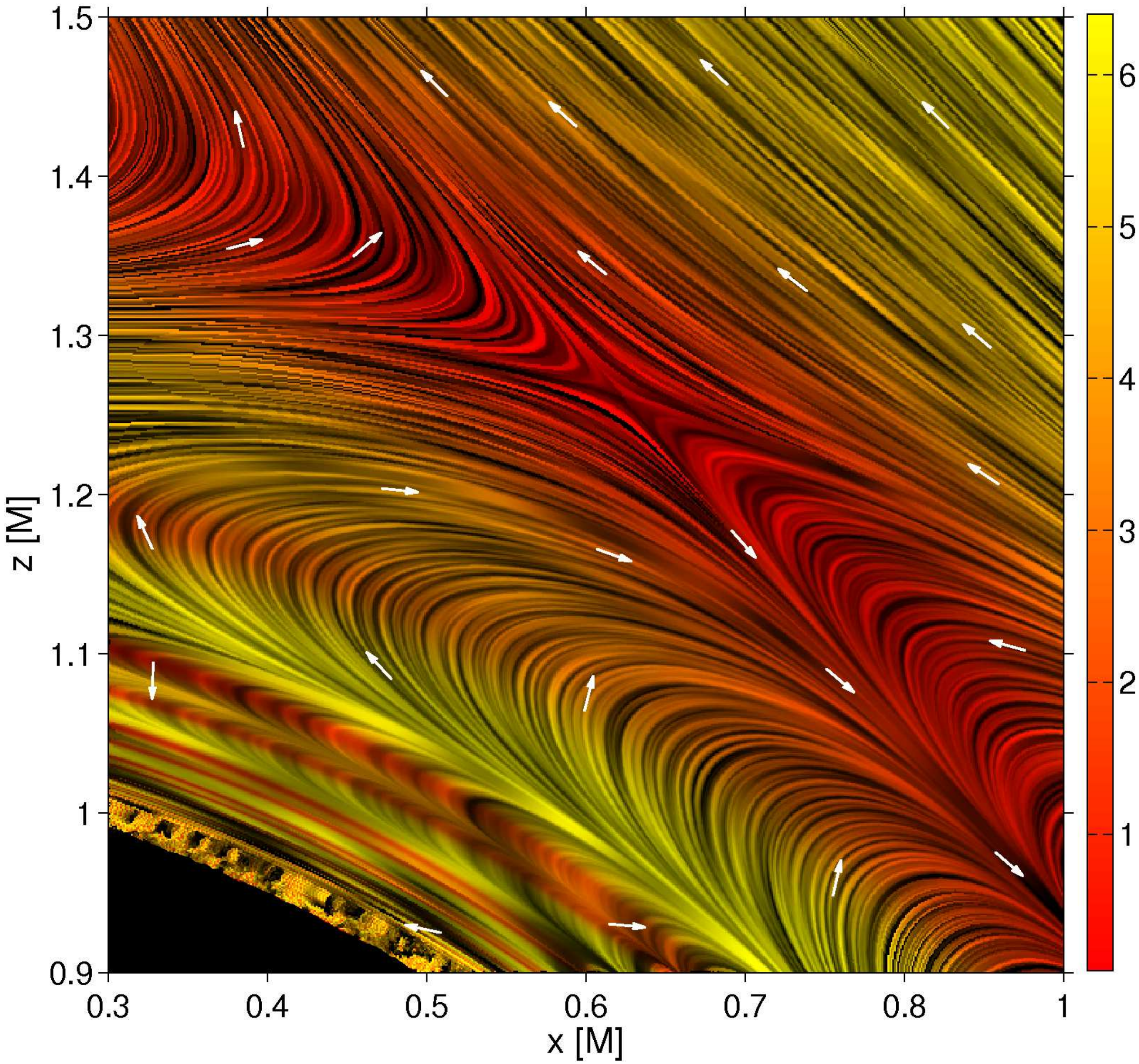
Effect of translatory motion (linear boost).

Rotating black hole in vacuum, oblique magnetic field



Effect of misalignement.

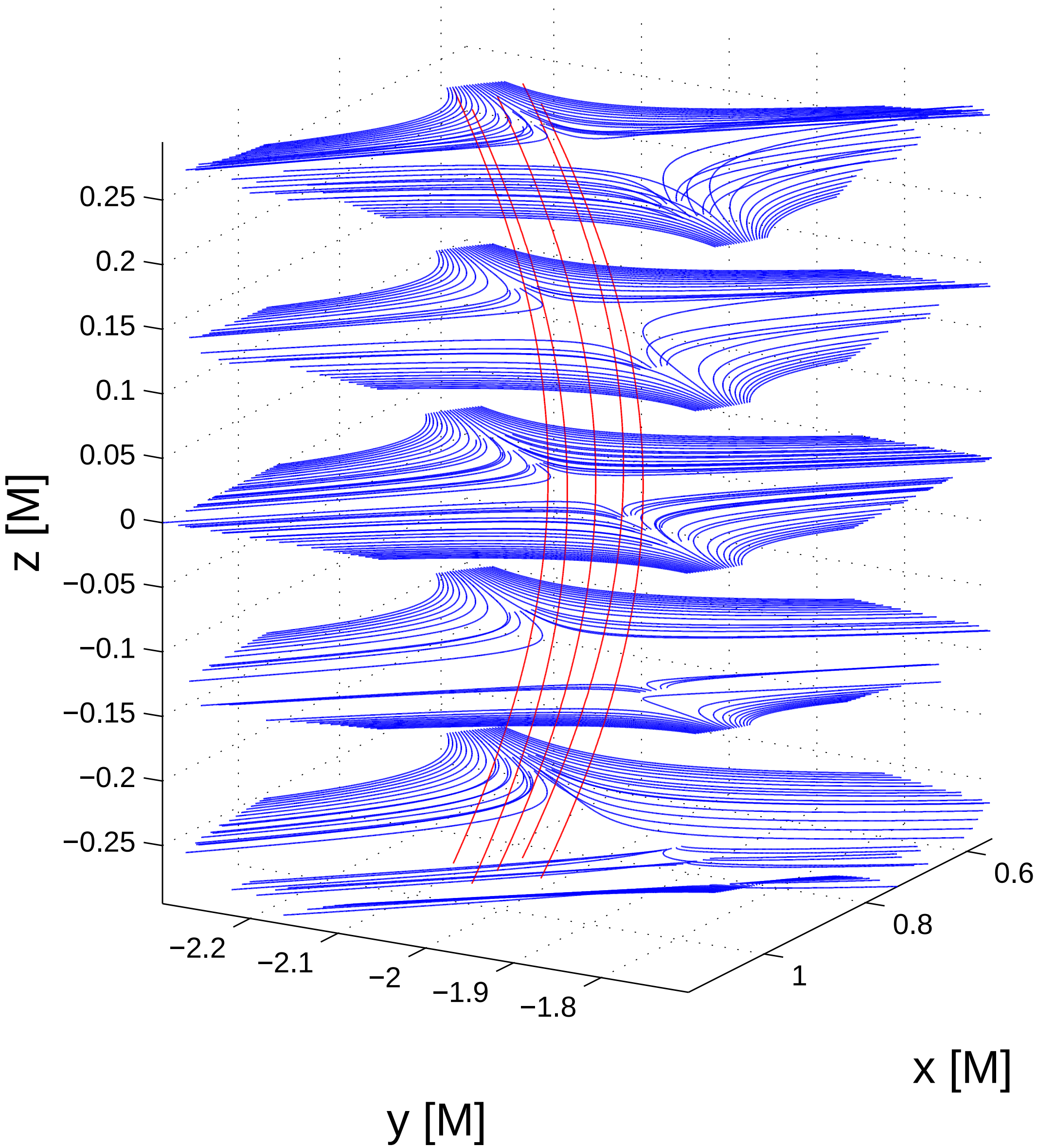
Magnetic null points

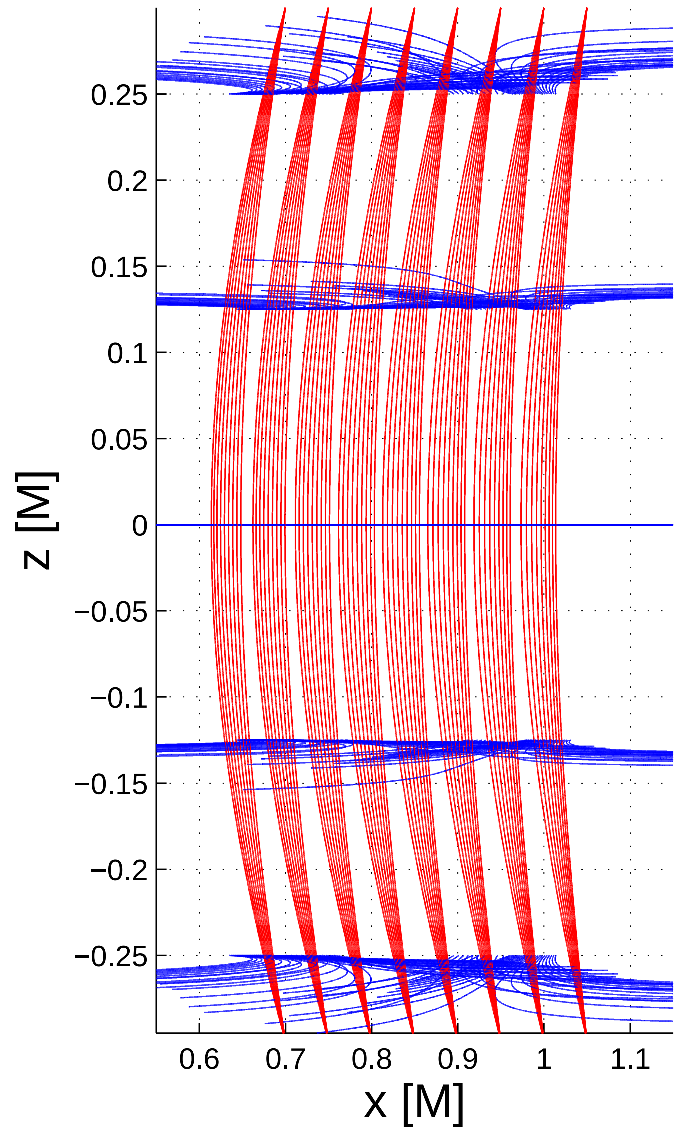


Field lines in equatorial plane

Karas et al. (CQG, 2009, 2012)

Magnetic (blue) and electric (red) field lines





Magnetic dipole in Rindler approximation

BIG BLACK HOLE, LITTLE NEUTRON STAR: MAGNETIC . . . PHYSICAL REVIEW D **88,** 064059 (2013)

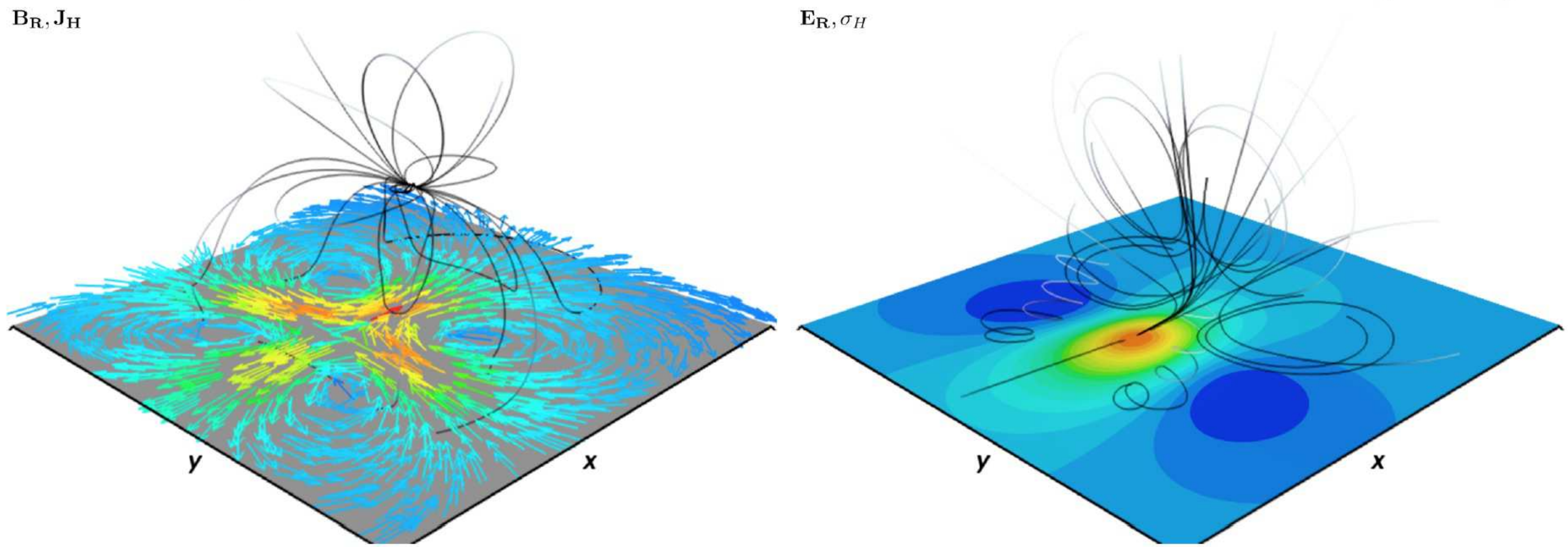


FIG. 14 (color online). A 3D visualization of the magnetic fields lines and corresponding horizon currents $\mathbf{J}_{\mathcal{H}}$ (left) and electric field lines with the corresponding horizon charges $\sigma_{\mathcal{H}}$ (right) for the boosted Rindler dipole. The case shown is for $v_{S,x} = 0.2$.

D'Orazio & Levin (Phys. Rev. D, 2013)

Magnetic dipole in Rindler approximation

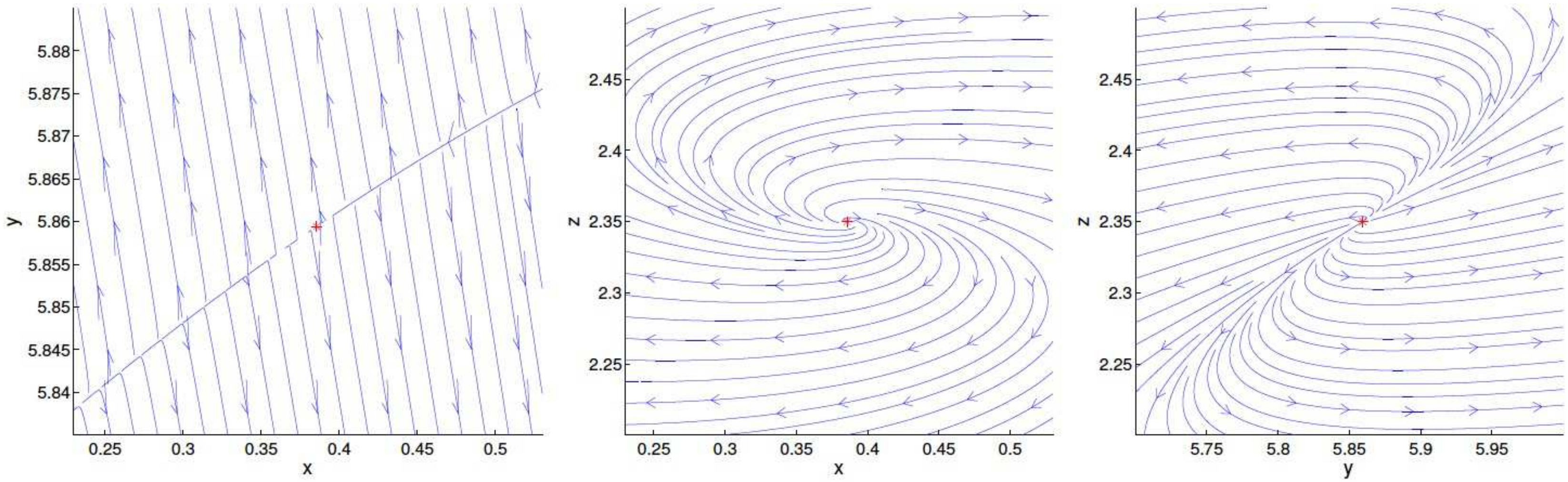


Figure 5. Two-dimensional sections of the magnetic field lines in the vicinity of the null point (red mark) located at $x_0 = 0.39$, $y_0 = 5.86$ and $z_0 = 2.35$. Same values of parameters as in Fig. 4 are used.

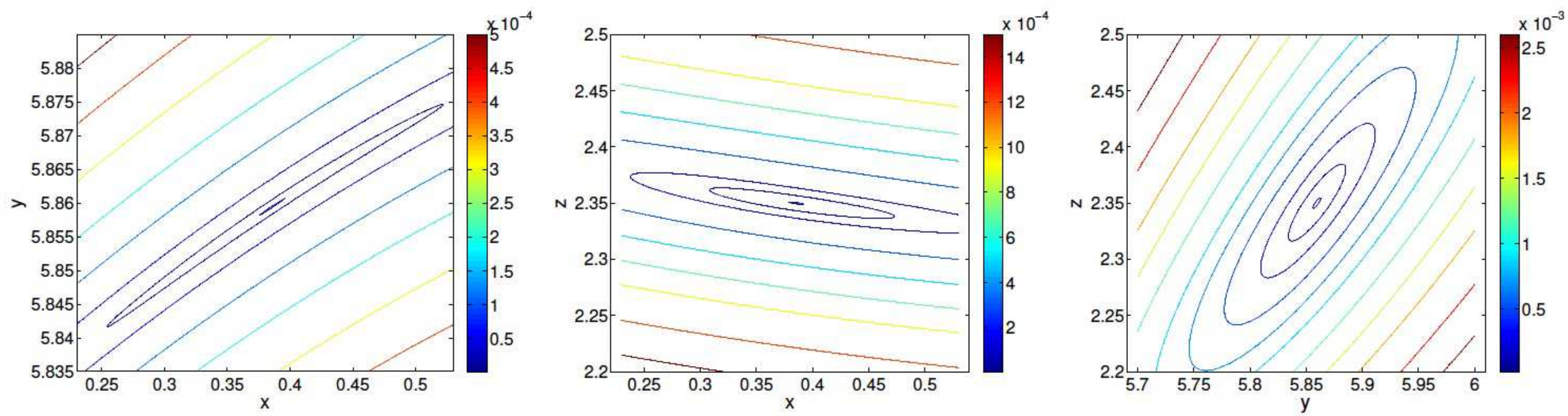


Figure 6. Iso-contours of the magnetic field strength B in the vicinity of the null point located at $x_0 = 0.39$, $y_0 = 5.86$ and $z_0 = 2.35$. Same values of parameters as in Fig. 4 and same section planes as in Fig. 5 are used.

Kopáček, Tahamtan & Karas (2018)

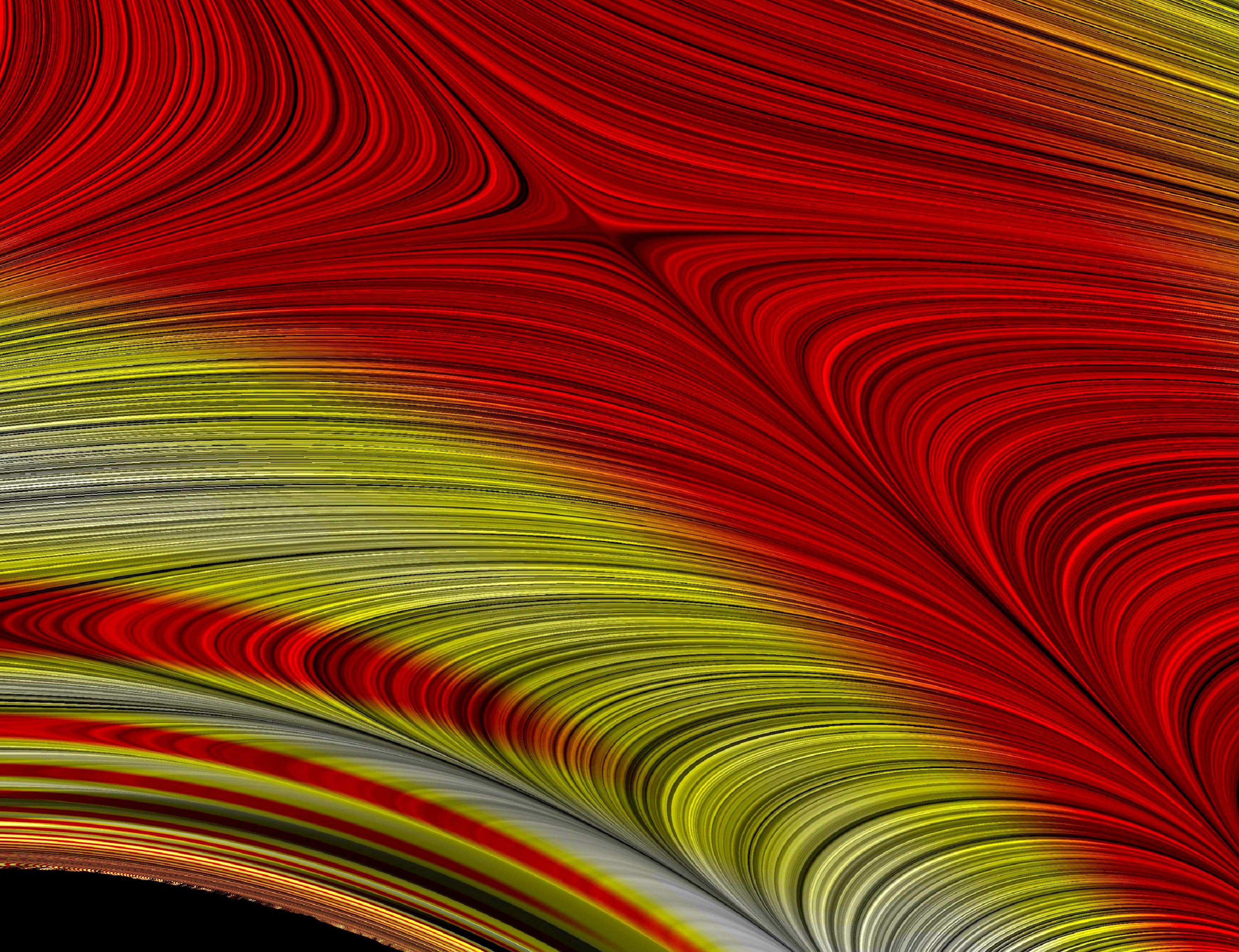
Conclusions

Because of the combined effect of frame dragging and boost, a rotating BH forms magnetic null points.

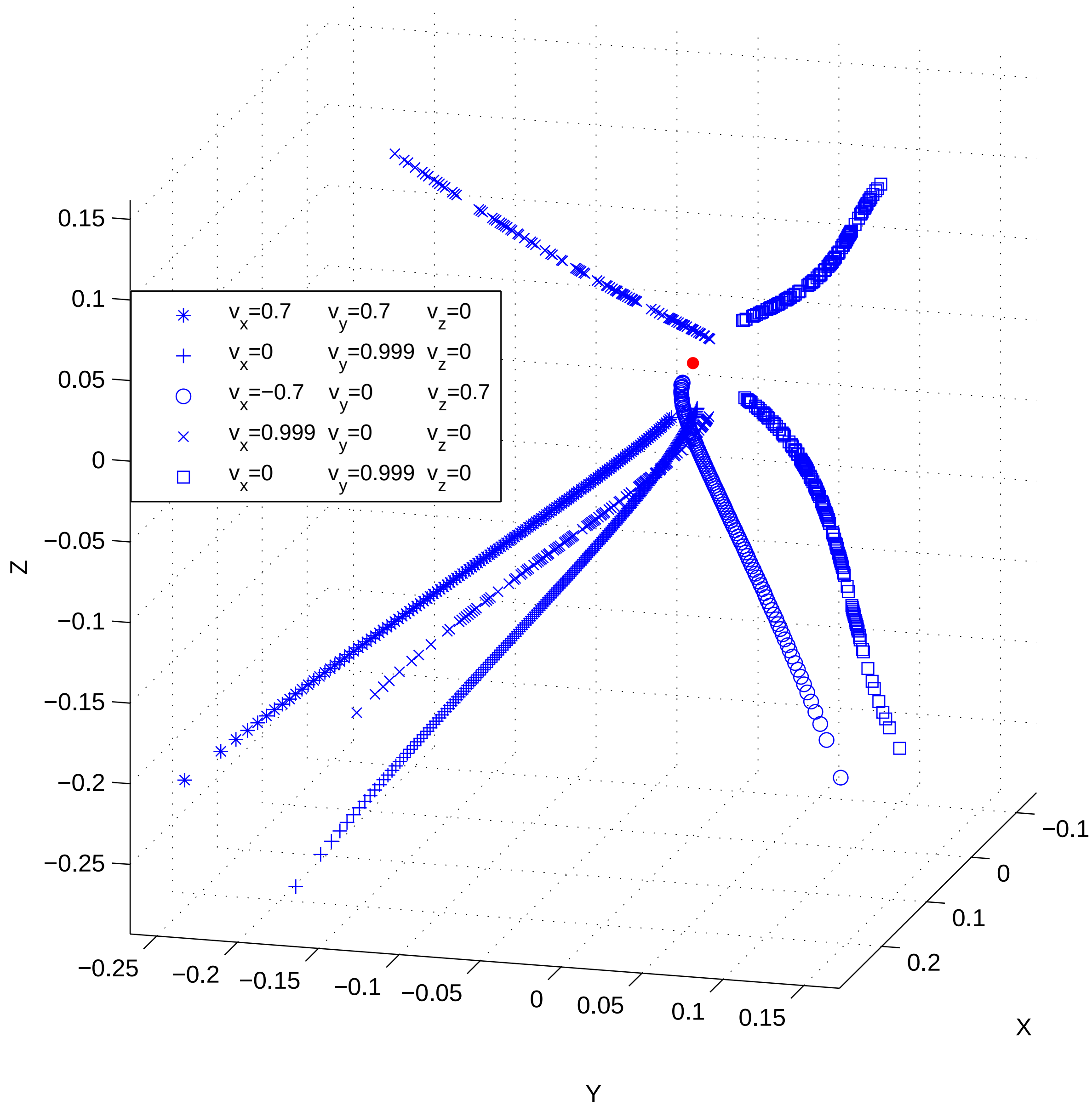
Charged particles can be efficiently accelerated by electric field passing through magnetic nulls.

Karas, Kopáček, & Kunneriath (2013), Int. J. Astron. Astrophys., 3, 18

Kopáček, Tahamtan, & Karas (2018), Phys. Rev. D, submitted



Supplementary slides



Killing vectors

in a vacuum spacetime generate a test-field solution of Maxwell equations:

$$\xi_{\mu;\nu} + \xi_{\nu;\mu} = 0$$

We *define*

$$F_{\mu\nu} = 2\xi_{\mu;\nu}.$$

Then, using the Killing equation and the definition of Riemann tensor,

$$\boxed{F^{\mu\nu}{}_{;\nu} = 0.}$$

Field invariants:

$$\boldsymbol{E.B} = \frac{1}{4}\,{}^{\star}F_{\mu\nu}F^{\mu\nu}, \qquad B^2 - E^2 = \frac{1}{2}F_{\mu\nu}F^{\mu\nu}.$$

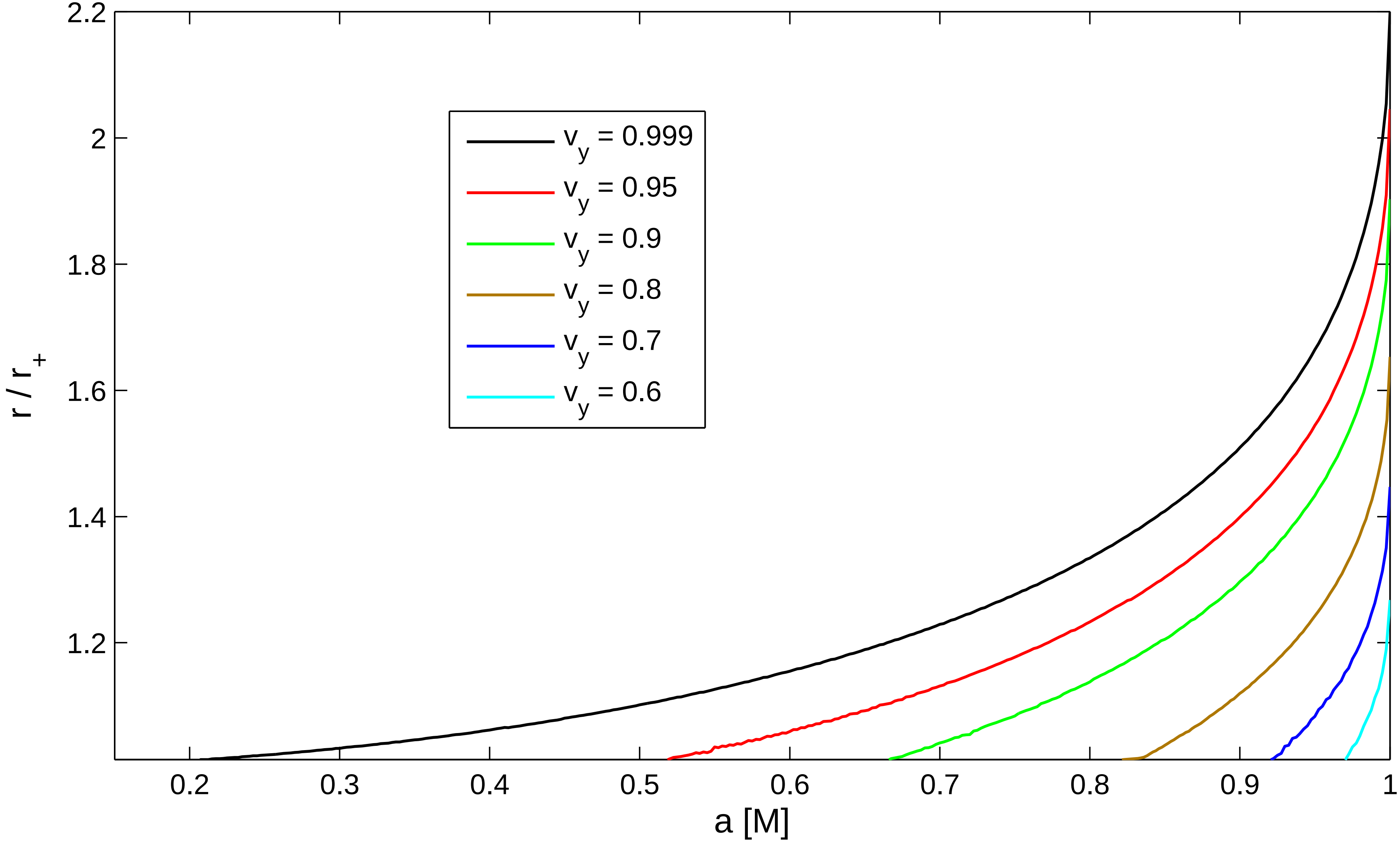
Two examples of non-diverging elmg. test field

1. A spherically symmetric electric field. A unique solution that is well-behaving both at $r = r_+$ and at $r \to \infty$. This term describes a weakly charged Reissner-Nordström black hole.

2. An asymptotically uniform magnetic field:

$$
\begin{aligned}
F_{\mu\nu} \quad &\to \quad B_{\parallel} \boldsymbol{e_z} + B_{\perp} \boldsymbol{e_x}, \\
\text{i.e.} \quad & F_{r\theta} \to -B_{\perp}\, r \sin\phi, \\
& F_{r\phi} \to B_{\parallel}\, r\, \sin^2\theta - B_{\perp}\, r\, \sin\theta\, \cos\theta\, \cos\phi, \\
& F_{\theta\phi} \to B_{\parallel}\, r^2\, \sin\theta \cos\theta + B_{\perp}\, r^2\, \sin^2\theta\, \cos\phi.
\end{aligned}
$$

Magnetic null points – II



Magnetic null points – III