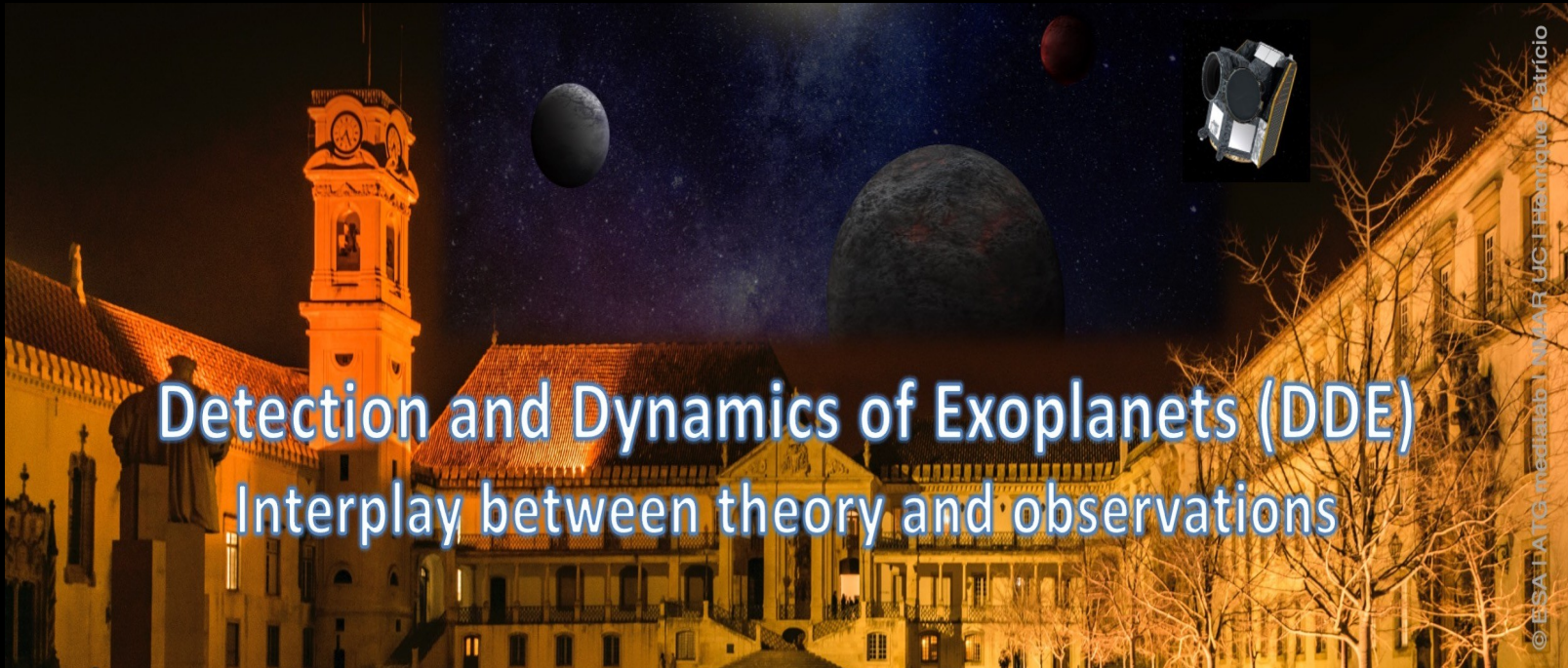




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Stability of Circumbinary Planets



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Coimbra, 09/07/2025

PLANETS IN BINARY SYSTEMS (INDEPENDENT DATABASE & INFORMATION PAGE)

(THIS PAGE IS FOR PLANETS ON **S-TYPE** ORBITS THAT ORBIT ONE OF THE STARS. FOR **CIRCUMBINARY PLANETS**, [CLICK HERE](#))

.... CURRENTLY **739** SYSTEMS IN THE DATABASE

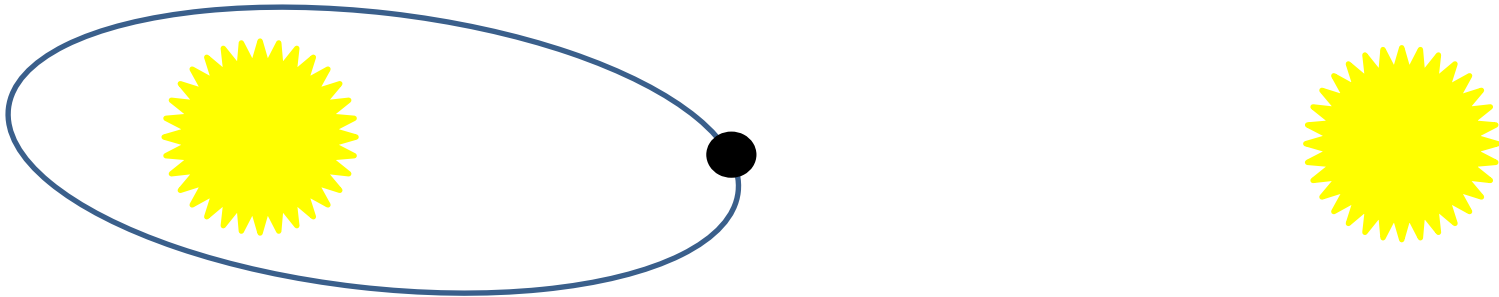
CIRCUMBINARY PLANETS

(THIS PAGE IS FOR PLANETS ON **P-TYPE** ORBITS THAT ORBIT BOTH STARS. FOR PLANETS ON **S-TYPE** ORBITS, [CLICK HERE](#))

.... CURRENTLY **31** SYSTEMS IN THE DATABASE

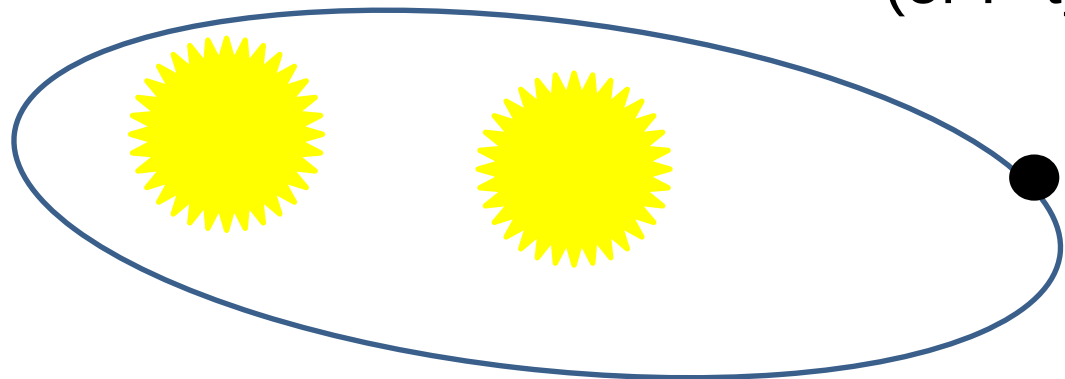
Circumstellar configuration

(or S-type)

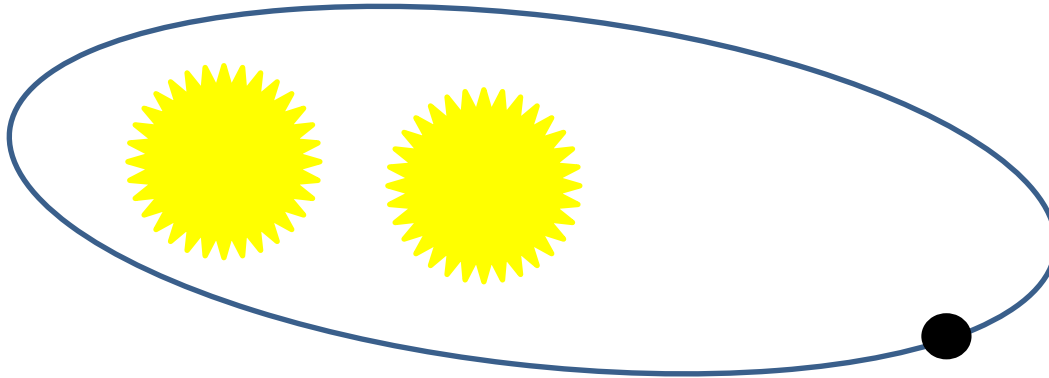


Circumbinary configuration

(or P-type)



Circumbinary planets



We want to find out where a planet is allowed to be around the binary without its orbit being destabilized



Important for many processes:

planet formation
planet detection
habitability etc.

Past research on planetary stability in binaries:

Harrington (1977)

Dvorak (1986)

Rabl & Dvorak (1988)

Dvorak (1989)

Holman & Wiegert (1999)

Quarles et al. (2018, 2020)

Hong & van Putten (2021)

Adelbert et al. (2023)



Stability criteria proposed

Motivations for this work:

- i) The above circumbinary stability criteria only cover parts of the parameter space.
- ii) The various definitions of stability used in past works may result in misclassification of circumbinary planetary orbits as stable while they are actually unstable or vice versa.

Aims of this work:

- i) To extend and homogenize the results of previous studies on the dynamical stability of circumbinary planetary orbits
- ii) To remedy the limitations and inconsistencies that arise from combining stability estimates from different works by carrying out a self-consistent set of numerical simulations over long timescales.

Numerical Simulations of Circumbinary systems:

Masses:

$$M_b = \frac{m_2}{m_1 + m_2} \text{ and } M_p = \frac{m_p}{m_1 + m_2}$$

and

$$M_b \in \{0.5, 0.3, 0.1, 0.05, 0.02, 0.01\}$$

$$M_p \in \{10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}, 10^{-7}\}$$

Eccentricities:

$$e_b, e_p \in \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$$

Mutual Inclination:

$$I_m \in \{0^\circ, 18^\circ, 36^\circ, 54^\circ, 72^\circ, 90^\circ, 108^\circ, 126^\circ, 144^\circ, 162^\circ, 180^\circ\}.$$

Planetary slowly varying angles:

$$\varpi_p, \omega_p, \Omega_p \in \{0^\circ, 90^\circ, 180^\circ\}$$

Planetary true anomaly:

$$f_p \in \{0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ, 225^\circ, 270^\circ, 315^\circ\}$$

Binary true anomaly:

$$f_b \in \{0^\circ, 180^\circ\}$$

Integration time=1000000 planetary orbital periods

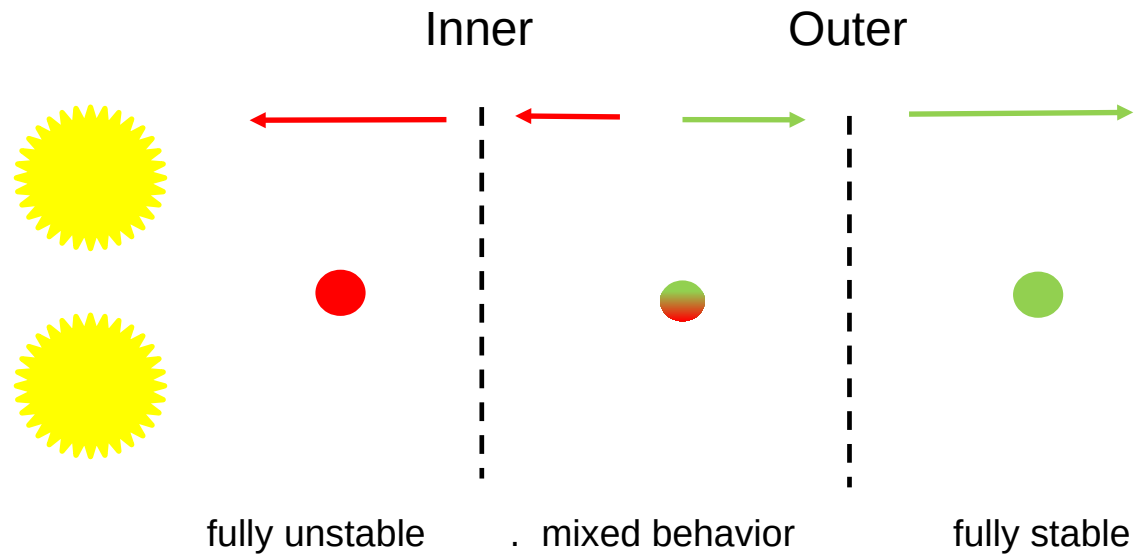
Semi-major axis resolution=0.1

Criteria for instability

For any initial position of the planet:

- i) either of the binary or planetary orbital eccentricity becomes ≥ 1
- ii) orbit crossing
- iii) $a_b \leq 0.001$ or $a_b \geq 100$
- iv) $a_p \geq 1000$

Looking for two critical borders:



Results

Effect of each parameter on the stability borders:

- binary mass ratio: moderate effect
- planetary mass ratio: insignificant effect
- binary eccentricity: moderate effect
- planetary eccentricity: strongest effect
- mutual inclination: moderate effect
- planetary pericenter: small effect
- node: insignificant effect

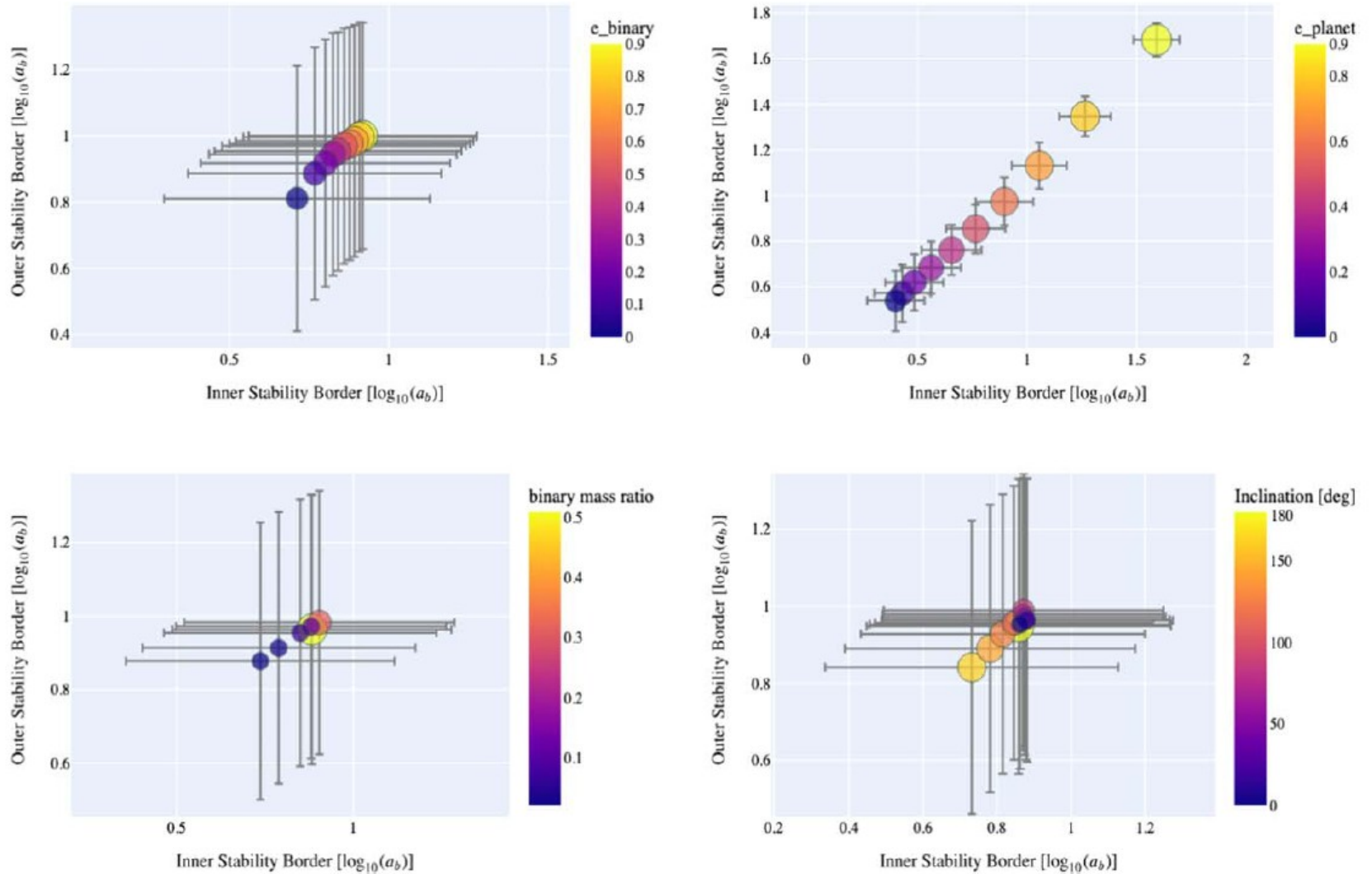


Figure 2. Mean and standard deviation of outer vs. inner stability borders in units of \log_{10} of the binary semimajor axis. The color scale refers to the binary orbit eccentricity (top left), the planet's orbital eccentricity (top right), the binary mass ratio (bottom left), and the mutual inclination (bottom right). Stability limits depend strongly on the planetary orbital eccentricity, which accounts for most of the variance in the system. Stability borders also show roughly the same sensitivity to the orbital eccentricity of the binary star, the binary mass ratio, and the inclination of the system.

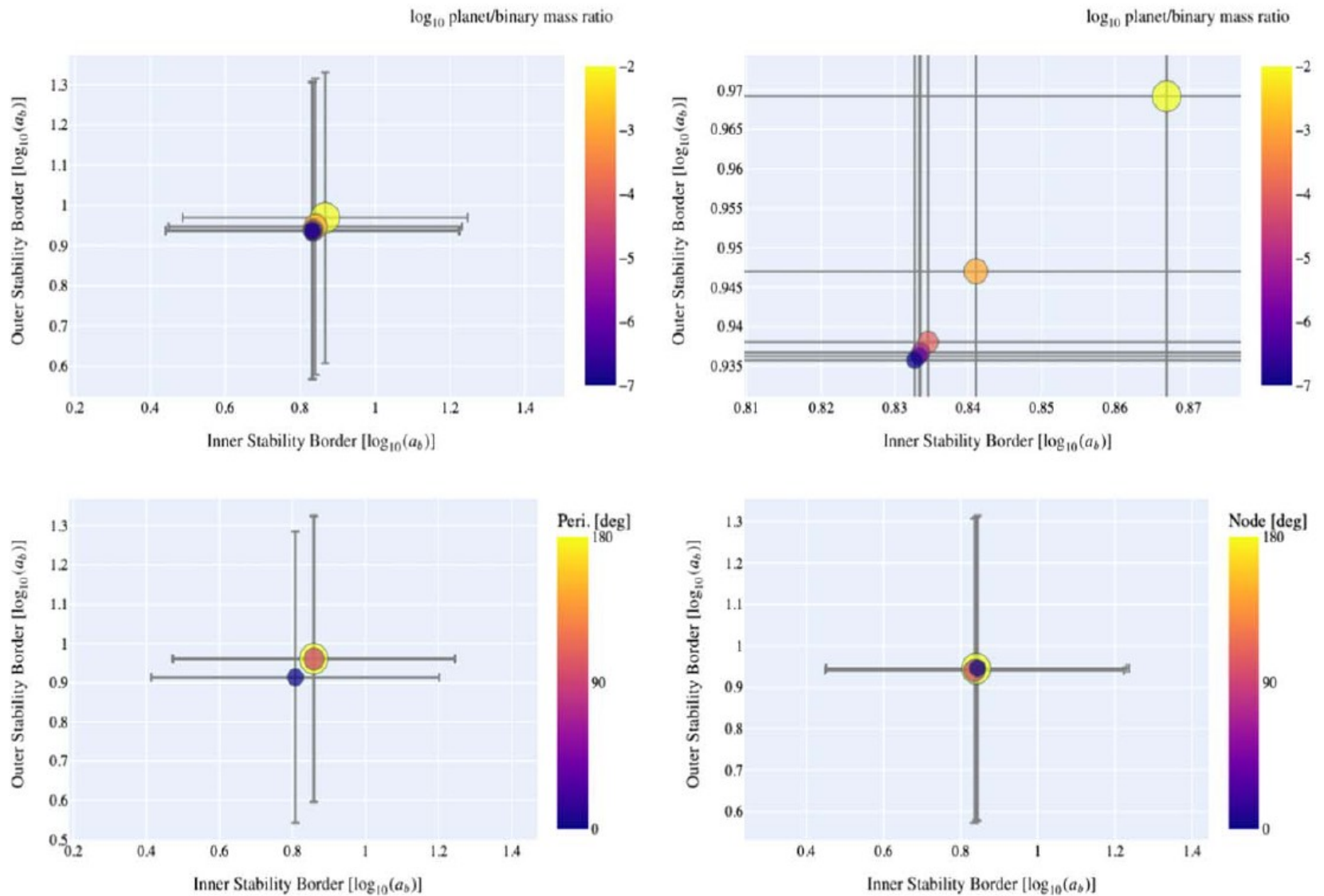


Figure 3. Same as Figure 2, but for the planet to binary mass ratio $M_p = m_p/(m_1 + m_2)$ (top left), a zoomed-in plot of the same (top right), the pericenter (bottom left), and the longitude of the ascending node (bottom right). In the parameter regime we have chosen for this study, the planet’s mass does not substantially affect the stability limits. Aligned pericenters lead to lower instability in a system. The relative position of the nodes does not significantly impact the location of stability limits.

Fitting formulae

- For every set of values $(M_b, M_p, e_b, e_p, I_m)$, we had 9 critical distance values for different combinations of (Ω_p, ω_p) . We retained the largest value for the outer critical border and the smallest value for the inner critical border (better markers for our borders – 2 variables down).
- Planetary mass dropped.
- Binary mass ratio and critical distances were rescaled using \log_{10} . Mutual inclination in radians.
- Third order polynomial fit selected. χ^2 testing was used to control the quality of the fits.
- Two fits constructed: one for $e_p \leq 0.8$ and one for all e_p .

B coefficient vector, C uncertainties vector, X parameter vector

$$a_i^{\text{cr}} = a_b \times 10^{(B_i \pm C_i) \cdot X_i} \quad \text{and} \quad a_o^{\text{cr}} = a_b \times 10^{(B_o \pm C_o) \cdot X_o}$$

For $e_p \leq 0.8$

Inner border

$$B_i = (0.20729, -0.32875, 0.10339, 0.58433, 0.36623, \\ -0.25569, -0.06425, -0.38387, 1.01951, 0.26910, \\ 0.38912, -0.19863, -0.25361, -0.30333, \\ 0.09080, -0.05955),$$

$$C_i = (0.003763, 0.01015, 0.00224, 0.00922, 0.00978, \\ 0.00982, 0.00069, 0.00947, 0.01176, 0.00687, \\ 0.00759, 0.00420, 0.00735, 0.00913, \\ 0.00129, 0.00280),$$

$$X_i = (1, M_{lb}, I_m, e_b, e_p, M_{lb}^2, I_m^2, e_b^2, e_p^2, M_{lb}e_b, \\ M_{lb}e_p, I_me_b, M_{lb}e_b^2, M_{lb}e_p^2, I_me_b, M_{lb}^3).$$

Outer border

$$B_o = (0.23612, -0.29377, 0.22710, 1.06753, 0.62109, \\ -0.21512, -0.06648, -1.52936, -0.4748, -0.31329 \\ -0.00869, 0.11846, -0.03932, -0.00933, \\ 0.87506, 1.25895),$$

$$C_o = (0.00317, 0.00927, 0.00313, 0.00905, 0.00975, \\ 0.00910, 0.00202, 0.02330, 0.02893, 0.00389, \\ 0.00116, 0.00119, 0.00260, 0.00041, \\ 0.01699, 0.02373),$$

$$X_o = (1, M_{lb}, I_m, e_b, e_p, M_{lb}^2, I_m^2, e_b^2, e_p^2, I_me_b, I_me_p, I_m^2e_b, \\ M_{lb}^3, I_m^3, e_b^3, e_p^3).$$

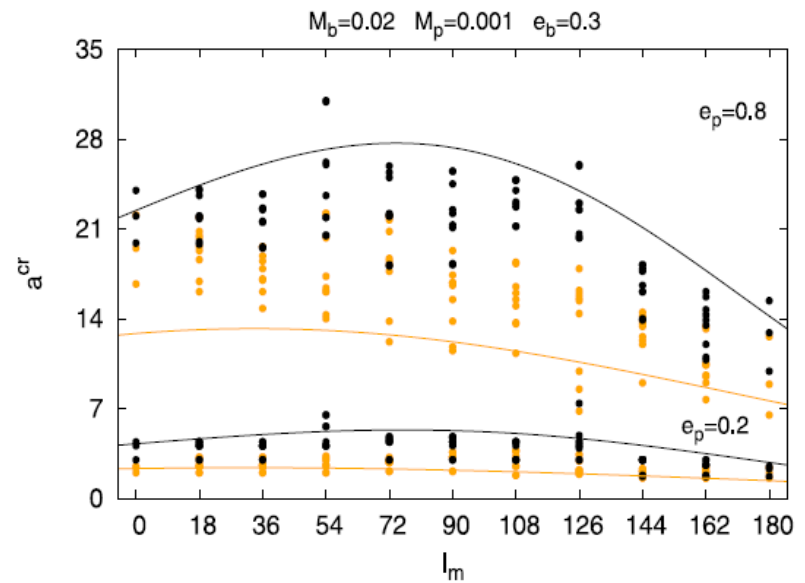
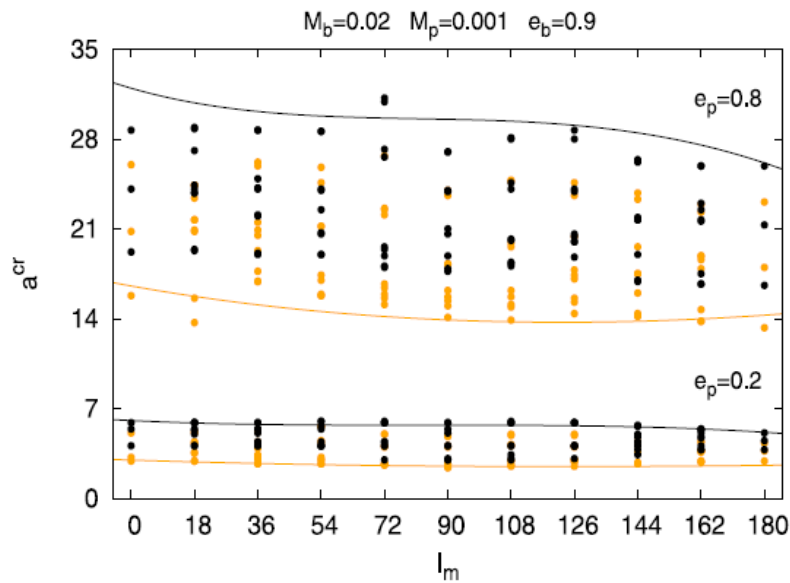
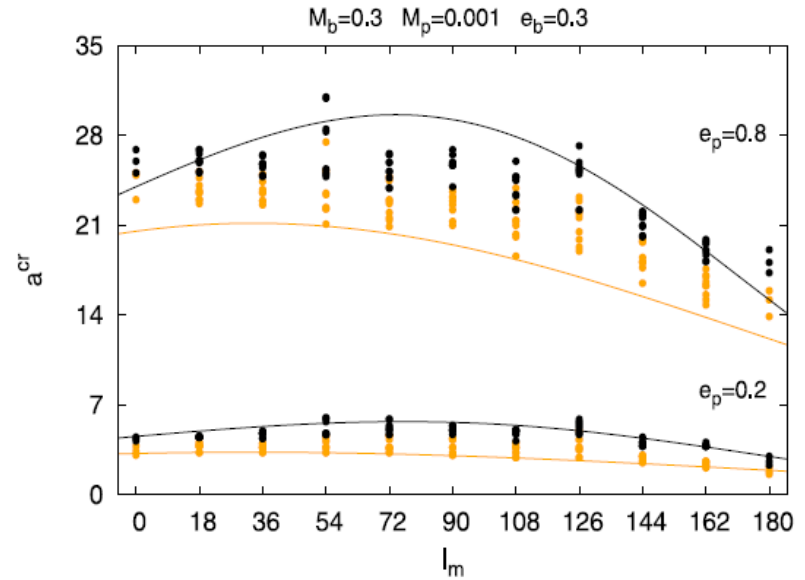
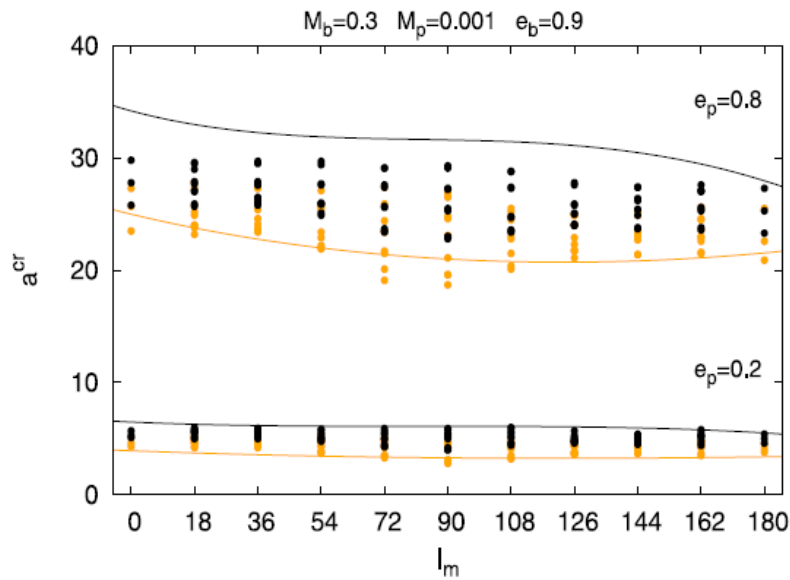


Figure 9. Critical semimajor axis against mutual inclination for a variety of systems. The orange color refers to the inner boundary, while the black color denotes the outer stability border. The continuous lines are our empirical fits as given in Section 3.2. The points are the output from the numerical simulations for the specific systems. Note that the majority of the points lie between the two curves, as they ideally should.

Fit performance against random simulations

In order to test the quality of our fitting formulae, we carried out a number of additional, randomly generated, simulations. We drew parameter values for our random systems from a uniform distribution within the ranges used for the creation of the simulation dataset. The planetary semi-major axis was sampled using rejection sampling upon distributions created from our simulation dataset.

50000 systems were created in total.

Application to known circumbinary systems

Parameter Values of Circumbinary Systems Used for the Validation of the Empirical Stability Fits

System	m_1 (M_\odot)	m_2 (M_\odot)	m_p (M_J)	I_m (deg)	a_b (au)	a_p (au)	e_b	e_p
Kepler-16	0.6897	0.20255	0.333	0.4	0.22431	0.7048	0.15944	0.00685
Kepler-34	1.0479	1.0208	0.22	1.81	0.22882	1.0896	0.52087	0.182
Kepler-35	0.8876	0.8094	0.127	1.28	0.17617	0.60345	0.1421	0.042
Kepler-38	0.949	0.249	0.384	0.182	0.1469	0.4646	0.1032	0.032
Kepler-47 b	0.957	0.342	0.006513	0.166	0.08145	0.2877	0.0288	0.021
Kepler-47 c	0.957	0.342	0.05984	1.165	0.08145	0.6992	0.0288	0.024
Kepler-47 d	0.957	0.342	0.00997	1.38	0.08145	0.9638	0.0288	0.044
Kepler-64	1.528	0.378	0.531	2.814	0.1744	0.634	0.2117	0.0539
Kepler-413	0.82	0.5423	0.21	4.073	0.10148	0.3553	0.0365	0.1181
Kepler-453	0.944	0.1951	0.05	2.258	0.18539	0.7903	0.0524	0.0359
Kepler-1647	1.21	0.975	1.52	2.9855	0.1276	2.7205	0.1593	0.0581
Kepler-1661	0.841	0.262	0.053	0.93	0.187	0.633	0.112	0.057
TIC 172900988	1.2388	1.2023	2.74	1.45	0.191928	0.89809	0.44793	0.088
TOI-1338 b	1.127	0.3313	0.0685	0	0.1321	0.4607	0.155522	0.088
TOI-1338 c	1.127	0.3313	0.205	0-180	0.1321	0.794	0.155522	0.16

Comparison with other results

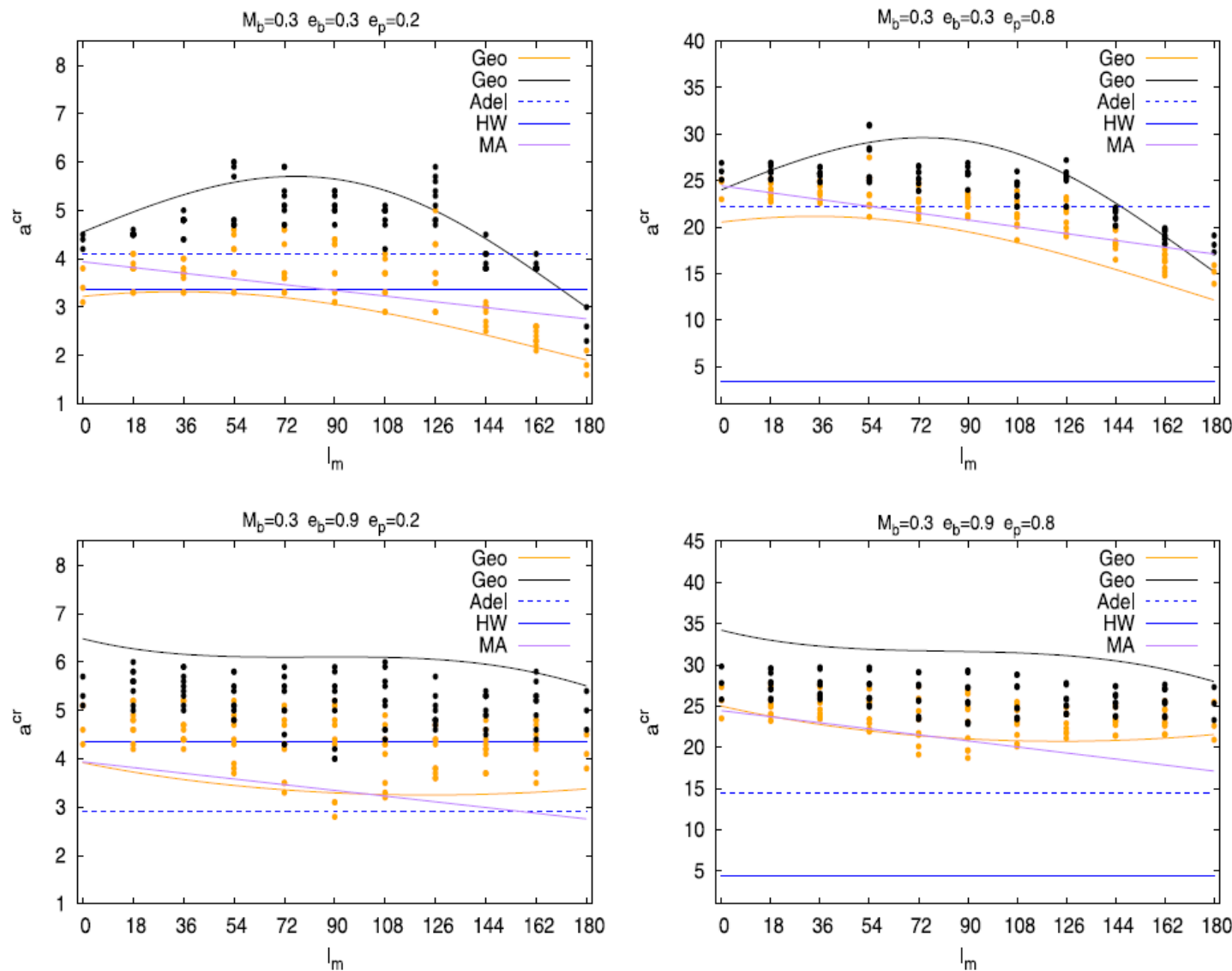
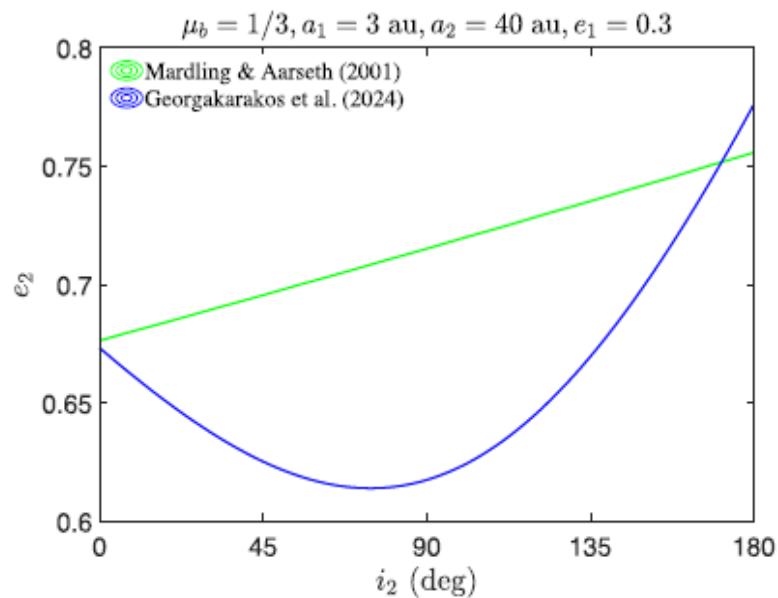
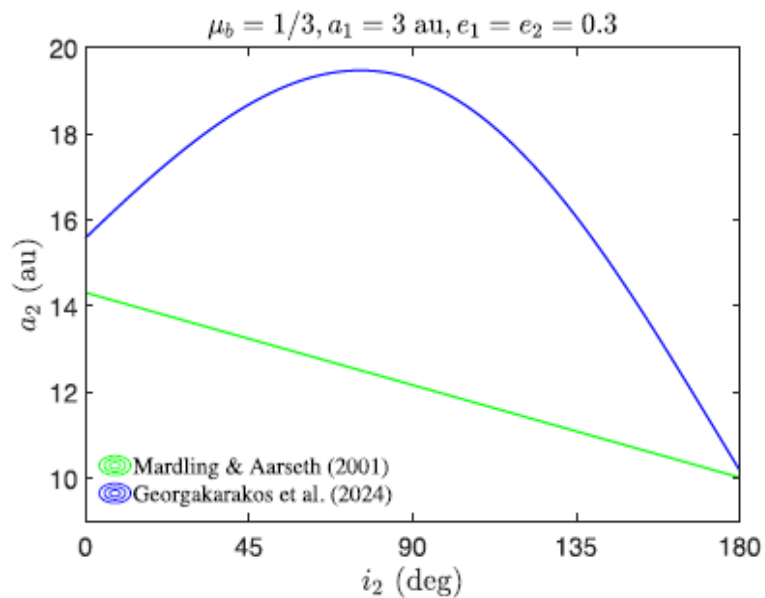
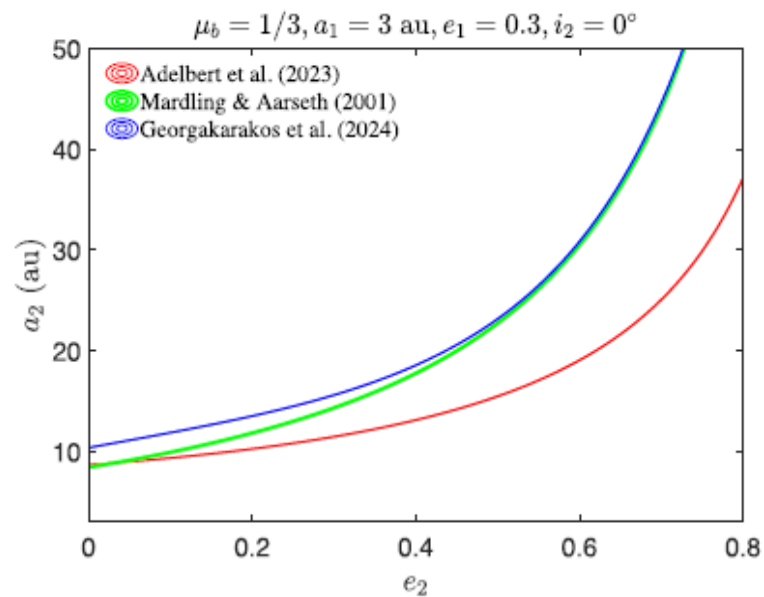
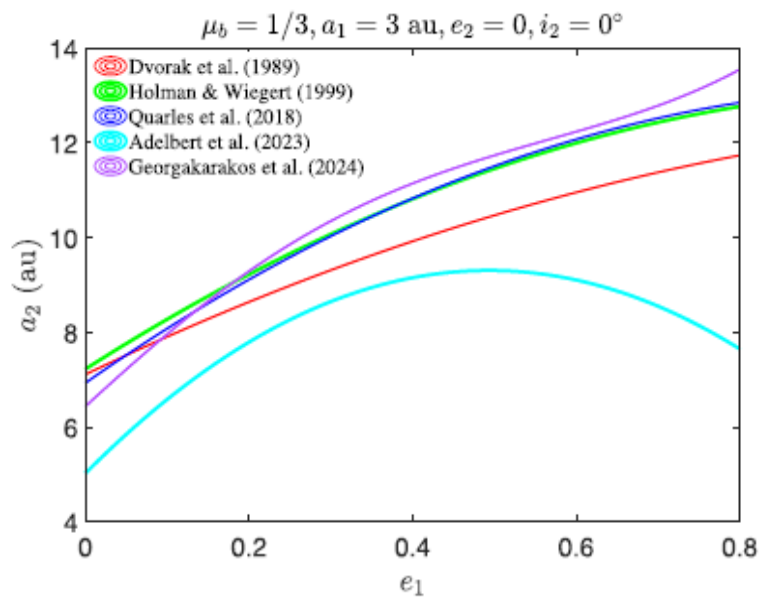


Figure 11. Comparison between different stability fits. Geo stands for the fits of the present work, Adel denotes the work by S. Adelbert et al. (2023), HW denotes the classification formula given in M. J. Holman & P. A. Wiegert (1999), and MA stands for the criterion developed by R. Mardling & S. Aarseth (1999) and R. A. Mardling & S. J. Aarseth (2001). As previously, the black color denotes the outer critical border, while the orange color represents the inner critical border. The circles are the results from our numerical simulations.



We have also trained a Machine Learning model with our simulation data (XGBRegressor model – Chen & Guestrin 2016).

The screenshot shows a Zenodo record page. At the top, there is a blue header with the Zenodo logo, a search bar, and links for 'Communities' and 'My dashboard'. Below the header, the record title is 'Stability of circumbinary planets -- ML model' by 'Ali-Dib, Mohamad'. The page includes a 'Published July 2, 2024 | Version v1' label and a 'Software' tag with an 'Open' button. The main content area contains a description of the model, its training data, and a link to the paper on arXiv.

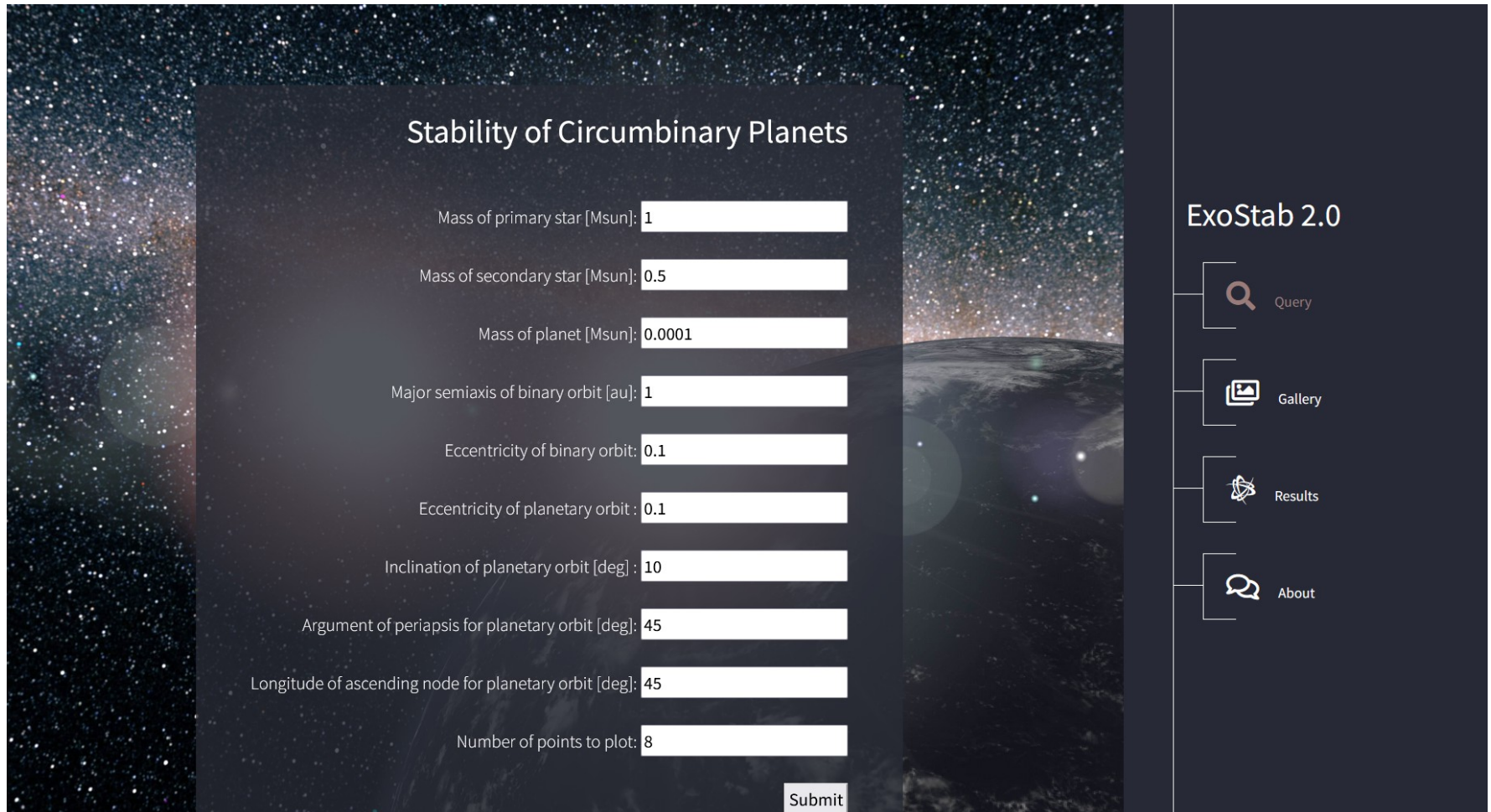
Classification of the Results of Random Simulation Using the Machine Learning Model ($e_p \leq 0.8$)

Inner Border Distribution				
N.U.I.P.	Stable	Quantity per Zone Mixed	Unstable	Total
0	12,658 (57.0%)	1312 (5.9%)	5 (0.0%)	13,975 (62.9%)
1–15	382 (1.7%)	1671 (7.5%)	20 (0.1%)	2073 (9.3%)
16	133 (0.6%)	2560 (11.5%)	3483 (15.7%)	6176 (27.8%)
Total	13,173 (59.3%)	5543 (24.9%)	3508 (15.8%)	22,224 (100.0%)
Outer Border Distribution				
N.U.I.P.	Stable	Quantity per Zone Mixed	Unstable	Total
0	15,415 (69.1%)	1203 (5.4%)	1 (0.0%)	16,619 (74.5%)
1–15	300 (1.3%)	1344 (6.0%)	19 (0.1%)	1663 (7.4%)
16	99 (0.5%)	1884 (8.4%)	2059 (9.2%)	4042 (18.1%)
Total	15,814 (70.9%)	4431 (19.8%)	2079 (9.3%)	22,324 (100.0%)

The ML model was tested against the 50000 randomly chosen systems with very good results.

Online portal and Application Programmng Interface (API)

A software interface designed to facilitate interaction with large catalogs of numerical stability simulations such as constructed in this Work.



Stability of Circumbinary Planets

Mass of primary star [Msun]:

Mass of secondary star [Msun]:

Mass of planet [Msun]:

Major semiaxis of binary orbit [au]:

Eccentricity of binary orbit:

Eccentricity of planetary orbit :





Inclination of planetary orbit [deg] :

Argument of periapsis for planetary orbit [deg]:

Longitude of ascending node for planetary orbit [deg]:

Number of points to plot:

ExoStab 2.0

-  Query
-  Gallery
-  Results
-  About

Summary

- We investigated the dynamical stability of circumbinary planets by carrying out a very large number of numerical simulations covering almost completely the parameter space
- We derived empirical formulae for the critical planetary distances
- We trained a Machine Learning model as an additional predictive tool
- We tested our tools against real and synthetic systems, as well as against older stability formulae with excellent results.
- We provide an online portal and application programming interface for accessing our simulation dataset.

● **More information can be found in Georgakarakos et al. (2024), AJ, 168, id.224.**

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Empirical Stability Criteria for 3D Hierarchical Triple Systems. I. Circumbinary Planets

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Abstract

In this work we revisit the problem of the dynamical stability of hierarchical triple systems with applications to circumbinary planetary orbits. We derive critical semimajor axes based on simulating and analyzing the dynamical behavior of 3×10^5 binary star-planet configurations. For the first time, three-dimensional and eccentric planetary orbits are considered. We explore systems with a variety of binary and planetary mass ratios, binary and planetary eccentricities from 0 to 0.9, and orbital mutual inclinations ranging from 0° to 180° . Planetary masses range between the size of Mercury and the lower fusion boundary (approximately 13 Jupiter masses). The stability of each system is monitored over 10^5 planetary orbital periods. We provide empirical expressions in the form of multidimensional, parameterized fits for two borders that separate dynamically stable, unstable, and mixed zones. In addition, we offer a machine learning model trained on our data set as an alternative tool for predicting the stability of circumbinary planets. Both the empirical fits and the machine learning model are tested for their predictive capabilities against randomly generated circumbinary systems with very good results. The empirical formulae are also applied to the Kepler and TESS circumbinary systems, confirming that many planets orbit their host stars close to the stability limit of those systems. Finally, we present a REST application programming interface with a web-based application for convenient access to our simulation data set.

Unified Astronomy Thesaurus concepts: Celestial mechanics (211)