Stability and dynamics of the compact planetary system K2-72

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Ural Federal University named after the first President of Russia B.N.Yeltsin

Introduction

Study of the dynamic evolution of the compact four-planet system K2-72 began with this fact.

- When simulated orbital evolution with nominal orbital elements, the system disintegrated after 3 Myr.

Problems

1. Search for conditions for stable orbital evolution. No collisions or planetary ejections on interval 100 Myr.
2. Study of resonance properties. Are resonances necessary for the stability of a compact planetary system?

Resonant angles

Two planets in MMR (Huang, Ormel, 2022)

- Planets i and $i+1$ in MMR $k_i/(k_i - l_i)$

$$\varphi_{i,\,i+1,\,i+s} = (k_i - l_i)\lambda_i - k_i\lambda_{i+1} + l_i\varpi_{i+s}$$

- Planets $i+1$ and $i+2$ in MMR $k_{i+1}/(k_{i+1} - l_{i+1})$

$$\varphi_{i+1,\,i+2,\,i+1+s} = (k_{i+1} - l_{i+1})\lambda_{i+1} - k_{i+1}\lambda_{i+2} + l_{i+1}\varpi_{i+1+s}$$

l_i is the resonance order,

λ_i, λ_{i+1} are the mean longitudes of planets i and $i+1$,

ϖ_{i+s} is the longitude of the periapsis of planet i ($s=0$) or $i+1$ ($s=1$) orbit.

Resonant angles

Three-body resonance

- Three-body resonance as a chain of two two-body resonances $k_i/(k_i - l_i)$ and $k_{i+1}/(k_{i+1} - l_{i+1})$

$$\Phi^{p,\,p+q,\,q}_{i,\,i+1,\,i+2} = p\lambda_i - (p+q)\lambda_{i+1} + q\lambda_{i+2}$$

$p = l_{i+1}(k_i - l_i)$ and $q = l_i k_{i+1}$

- Notation for three-body resonance is $(p,\, -(p+q),\, q)$

Four-planetary system K2-72 (Dressing et al., 2017)

K2-72 is an M2V dwarf

Age is ? Gyr $m_\star = 0.27^{+0.08}_{-0.09}\ M_\odot$ $R_\star = 0.33^{+0.03}_{-0.03}\ R_\odot$

Parameter	K2-72 b	K2-72 c	K2-72 d	K2-72 e
$R\ [R_\oplus]$	1.08 ± 0.11	1.16 ± 0.13	1.01 ± 0.12	$1.29^{+0.14}_{-0.13}$
$m\ [M_\oplus]$	$1.39^{+0.58}_{-0.44}$	$1.80^{+0.85}_{-0.63}$	$1.09^{+0.55}_{-0.39}$	$2.65^{+1.18}_{-0.85}$
P [day]	5.577212 ± 0.00042	15.189034 ± 0.0031	7.760178 $^{+0.001496}_{-0.001496}$	24.158868 $^{+0.003726}_{-0.00385}$
e	$0.11^{+0.02}_{-0.09}$	$0.11^{+0.02}_{-0.09}$	$0.11^{+0.02}_{-0.09}$	$0.11^{+0.02}_{-0.09}$
i [deg]	$89.15^{+0.59}_{-0.86}$	$89.54^{+0.32}_{-0.44}$	$89.26^{+0.5}_{-0.7}$	$89.68^{+0.22}_{-0.32}$
g [deg]	7.49^{+120}_{-134}	16.83^{+113}_{-138}	14.28^{+114}_{-137}	11.39^{+117}_{-136}

Density of a planet $\varrho = 2.43 + 3.39(R/R_\oplus)$ g cm^{-3} for $R < 1.5R_\oplus$ (Weiss, Marcy, 2014)

Possible resonances in the K2-72 system

1 — K2-72 b, 2 — K2-72 d, 3 — K2-72 c, 4 — K2-72 e

Resonance offsets for K2-72

Planets		MMR	Δ
K2-72 b — K2-72 d	(1–2)	7/5	0.0086
K2-72 d — K2-72 c	(2–3)	2/1	*0.0427*
K2-72 c — K2-72 e	(3–4)	8/5	0.0095

For numerator and denominator values of 10 or less.

Resonance offset $\Delta_{ij} = \frac{P_j}{P_i} - \frac{k_i}{k_i - l_i}$

Posidonius software

Posidonius (Blanco-Cuaresma & Bolmont, 2017)

- N-body simulator written in Rust
- Symplectic integrator WHFAST (Rein & Tamayo, 2015) without correctors (since tidal forces are velocity dependent)
- Spin evolution is computed for all the bodies by using a midpoint integrator (Bulirsch, 2013)
- Tidal forces, rotational-flattening effects and general relativity corrections are implemented (Bolmont et al., 2015)

Variation of masses and orbital elements

Variation of masses

- Star mass is minimal, nominal, and maximal: $m_\star = 0.18$, 0.27, and $0.35\ M_\odot$
- Masses of planets are minimal, nominal, and maximal

Orbital elements

- Initial periods P (and semi-major axes a) and inclinations i are minimal, nominal, and maximal
- Initial eccentricities are $e \in \{0.02, 0.03, \ldots, 0.11\}$
- Initial arguments of periapsis are $g \in \{g_{min}, \ldots, g_{max}\}$ with step $\Delta g = 10^\circ$
- Initial longitudes of ascending nodes are $\Omega = 0^\circ$
- Simulation interval is 100 Myr

Variation of eccentricities

Intersection of the orbits of planets K2-72 b and K2-72 d

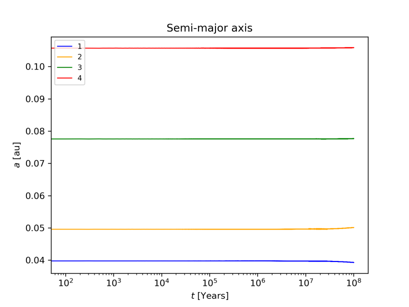
- $Q_b > q_d$
- $e_b = 0.085 - 0.130$ and $e_d = 0.129 - 0.093$, respectively

Initial values of orbital eccentricities, at which the planetary system does not disintegrate

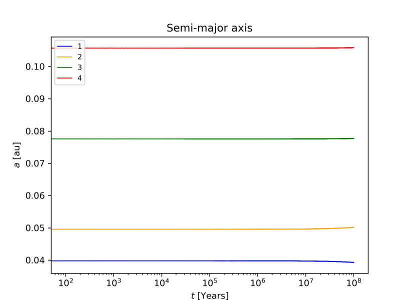
K2-72 b	K2-72 d	K2-72 c	K2-72 e
0.04	0.04	0.04	0.03
0.04	0.04	0.03	0.04
≤ 0.03	≤ 0.03	≤ 0.03	≤ 0.03

Stable solutions

Semi-major axis ($e_0 = 0.03$)



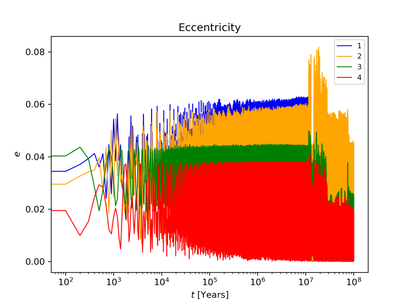
Semi-major axis ($e_0 = 0.02$)



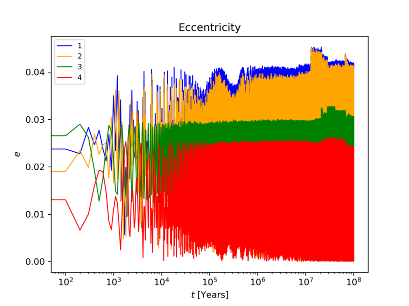
1 — K2-72 b, 2 — K2-72 d, 3 — K2-72 c, 4 — K2-72 e

Stable solutions

Eccentricity ($e_0 = 0.03$)



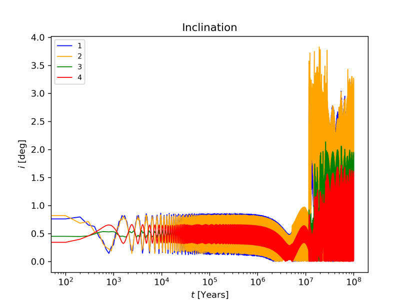
Eccentricity ($e_0 = 0.02$)



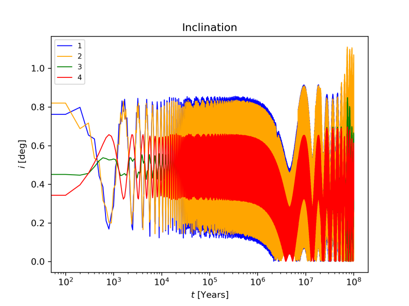
1 — K2-72 b, 2 — K2-72 d, 3 — K2-72 c, 4 — K2-72 e

Stable solutions

Inclination ($e_0 = 0.03$)



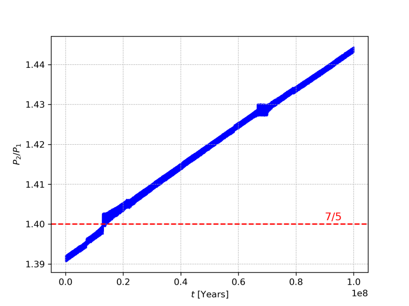
Inclination ($e_0 = 0.02$)



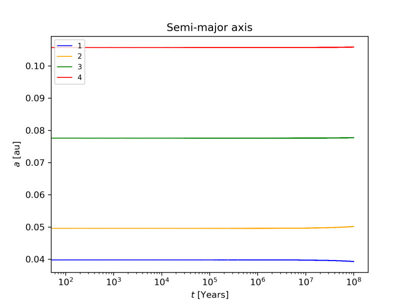
1 — K2-72 b, 2 — K2-72 d, 3 — K2-72 c, 4 — K2-72 e

Resonant conditions

K2-72 b and K2-72 d (1–2)



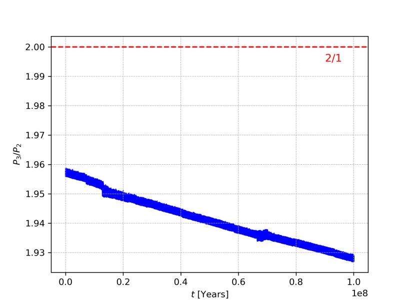
Semi-major axis ($e_0 = 0.02$)



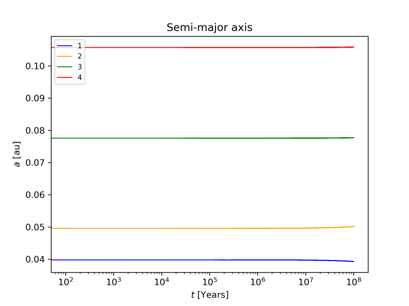
1 — K2-72 b, 2 — K2-72 d, 3 — K2-72 c, 4 — K2-72 e

Resonant conditions

K2-72 d and K2-72 c (2–3)



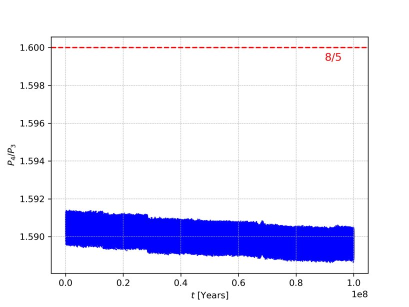
Semi-major axis ($e_0 = 0.02$)



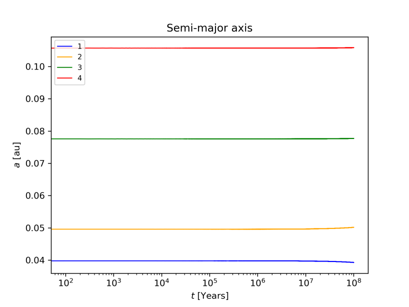
1 — K2-72 b, 2 — K2-72 d, 3 — K2-72 c, 4 — K2-72 e

Resonant conditions

K2-72 c and K2-72 e (3–4)



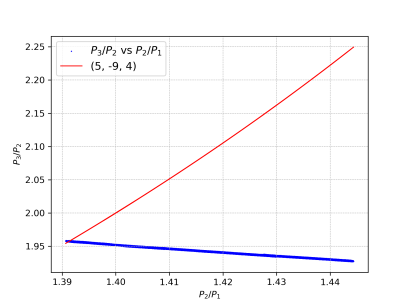
Semi-major axis ($e_0 = 0.02$)



1 — K2-72 b, 2 — K2-72 d, 3 — K2-72 c, 4 — K2-72 e

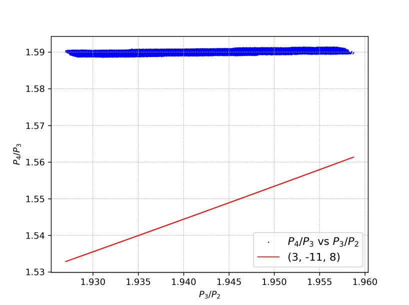
Resonant conditions

K2-72 d and K2-72 c (2–3) vs K2-72 b and K2-72 d (1–2)



The system evolves from left to right.

K2-72 c and K2-72 e (3–4) vs K2-72 d and K2-72 c (2–3)



The system evolves from right to left.

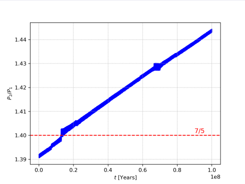
1 — K2-72 b, 2 — K2-72 d, 3 — K2-72 c, 4 — K2-72 e

Future work

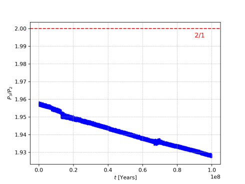
"*Resonant configurations are expected to be more common in younger systems*" (Dai et al., 2024).

About 120 Myr ago,

- the planets K2-72 b and K2-72 d were near the 4/3 resonance,
- the planets K2-72 d and K2-72 c were near the 2/1 resonance
- and planets K2-72 b, d, c could be in three-body resonance (3, -5, 2).
- We plan to test this assumption using numerical simulation.



K2-72 b and K2-72 d (1–2)



K2-72 d and K2-72 c (2–3)

Conclusions

- Currently, the compact planetary system K2-72 likely evolves beyond low-order resonances.
- A significant change in the large semi-major axes of the orbits of the K2-72 b and K2-72 d planets leads to the moving of the adjacent planets b–d and d–c out of the 7/5 and 8/5 resonance regions, respectively.
- The adjacent planets K2-72 d and K2-72 c are located far from the 2/1 resonance, which excludes the possibility of forming chains of MMRs and, hence, 3-planet MMRs.
- If the orbital eccentricities do not exceed 0.03, the evolution of the compact planetary system K2-72 over 100 Myr remains stable even in the presence of tidal perturbations.

Thank you for your attention!