

# Phenomenology from the three-gluon vertex in general kinematics.

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In collaboration with F. Pinto-Gómez, J. Rodríguez-Quintero, J. Papavassiliou, A.C. Aguilar, M.N. Ferreira.

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Coimbra University, 2023.

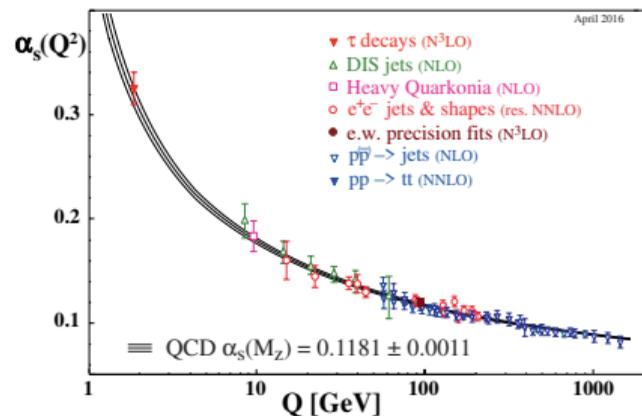
# Quantum Chromo-Dynamics

QCD Lagrangian depends on a few parameters: one coupling,  $\alpha_s$ , and quark masses ( $m_u$ ,  $m_d$ ,  $m_s$ ,  $m_c$ ,  $m_b$  and  $m_t$ ).

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + \sum_{f=u,\dots,t} \bar{\psi}_f (i\not{D} - m_f) \psi_f$$

$\alpha_s$  acquires a **renormalization scheme** dependent running with the momentum.

The running of  $\alpha_s(\mu^2) = \frac{g^2(\mu^2)}{4\pi}$  is controlled by its RGE,  $\frac{d\alpha_s}{d\ln\mu^2} = \beta(\alpha_s)$



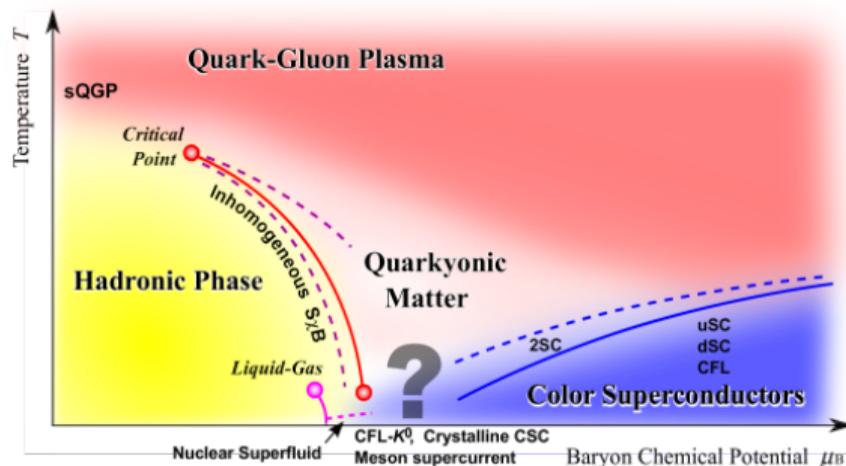
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## Emergent phenomena:

- Confinement (Hadron masses).
- Dynamically generated gluon-mass.
- Spontaneous chiral symmetry breaking.



# Exploring the phase diagram of QCD

At  $T = 0$  there are both  $\chi$ SB and confinement, and both disappear at large  $T$  or  $\mu$ . Current studies at physical quark masses suggest a continuous crossover at low chemical potential ending in a Critical Endpoint, being first order at larger  $\mu$ 's.

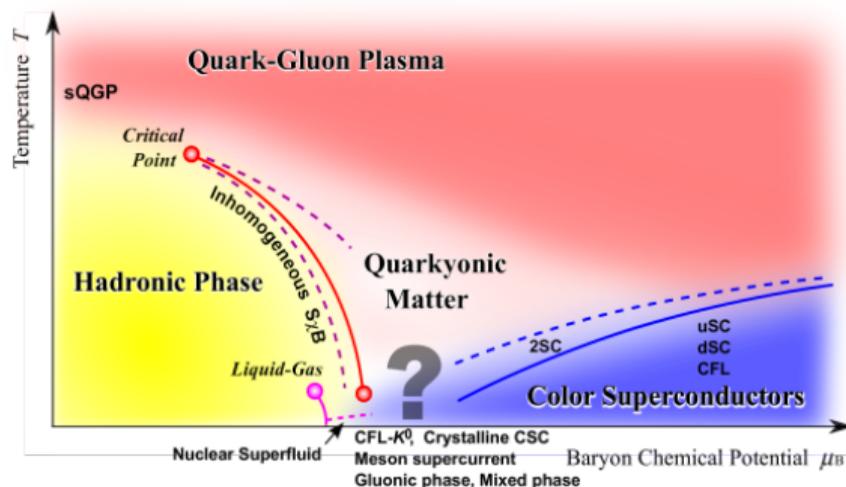
Current lattice QCD studies:

- Finite  $T$

$$S = \int_0^{1/T} dx_0 \int d^3x \mathcal{L}_{\text{QCD}}$$

- Finite  $\mu$  (Sign problems!)

$$\mathcal{L}_{\text{QCD}} \rightarrow \mathcal{L}_{\text{QCD}} - \sum_f \mu_f \bar{\psi}_f \gamma_0 \psi_f$$



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- S. Borsanyi *et al.*, PRL125 (2020) 5, 052001.
- A. Bazavov *et al.*, PLB795 (2019) 15 (HotQCD).
- C Bonati *et al.*, Phys.Rev.D 98 (2018) 5.
- M. Cheng *et al.*, PRD77 (2008) 014511.
- O. Philipsen, Eur.Phys.J.ST 152 (2007) 29
- Y. Aoki *et al.*, Nature 443 (2006) 675.

At small  $\mu$ :

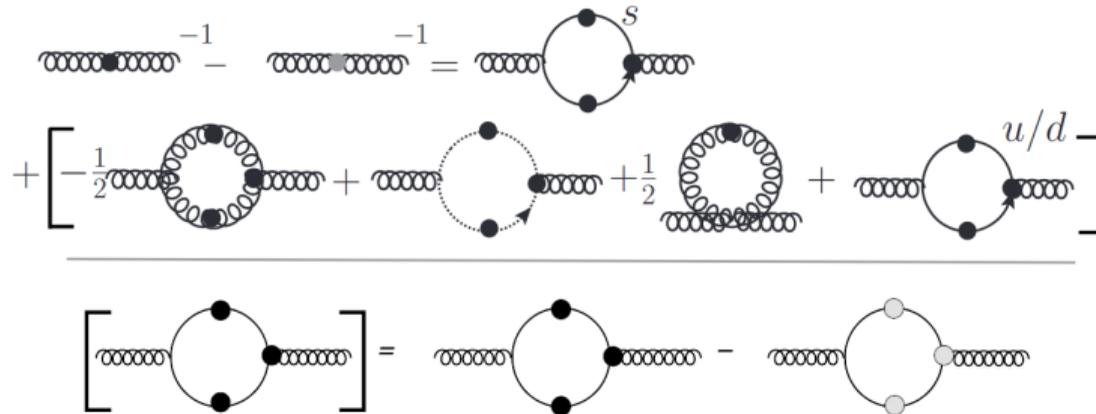
$$\frac{T_c(\mu)}{T_c(0)} \approx 1 - \kappa_2 \left(\frac{\mu}{T}\right)^2 - \kappa_4 \left(\frac{\mu}{T}\right)^4$$

# Exploring the phase diagram of QCD

Noticeable advances in functional methods in the last years:

- **Functional renormalization group (fRG)**, N. Dupuis *et al.*, *Phys.Rept.* 910 (2021), F. Gao, J. M. Pawłowski, *Phys. Rev. D* 102 (3) (2020) 034027.

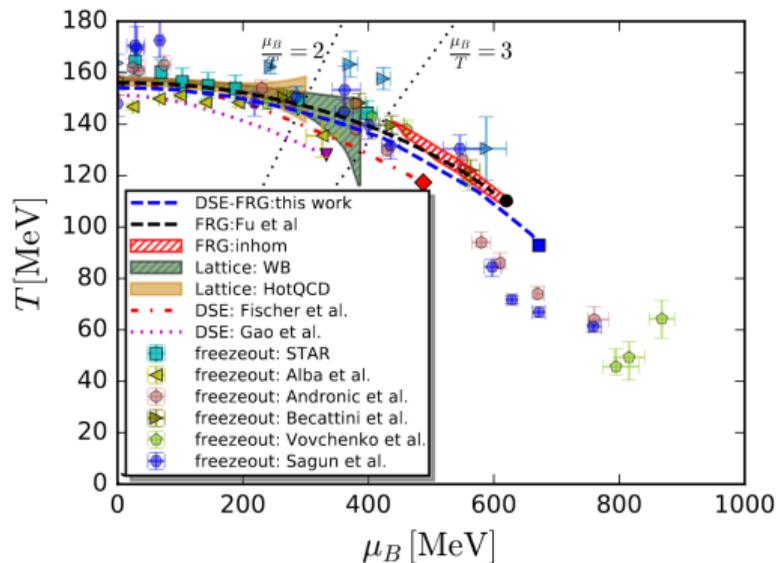
$$\Gamma_{T,\mu,N_f} = \Gamma_{0,0,N_f} + \Delta\Gamma$$



# Exploring the phase diagram of QCD

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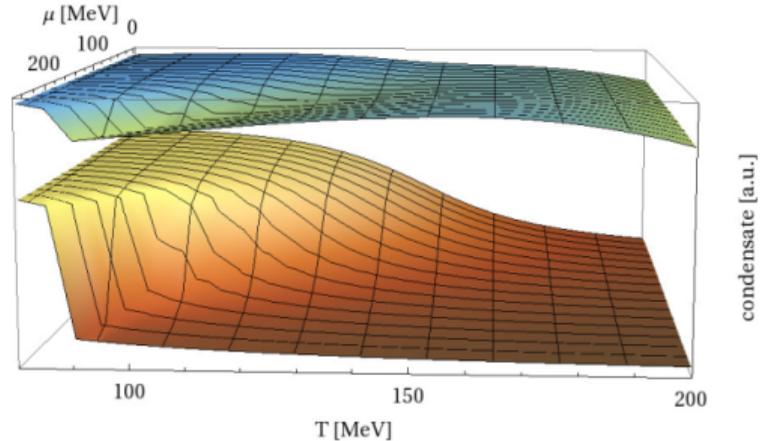
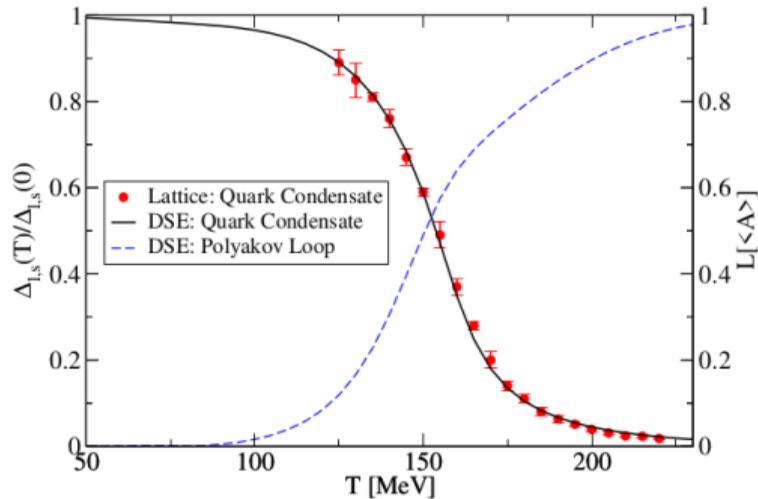
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# Exploring the phase diagram of QCD

Noticeable advances in functional methods in the last years:

- **Truncated DSE's + model propagators**, C. Fischer, Prog.Part.Nucl.Phys. 105 (2019) 1.



# Interplay between lattice & functional methods

## Lattice-QCD + functional methods

One of the major recent advances in our understanding of non-perturbative QCD comes from the interplay between lattice and DSE-based methods. In the context of the QCD phase diagram, it amounts to compute fundamental QCD Green functions from QCD and use them in functional methods (DSE).

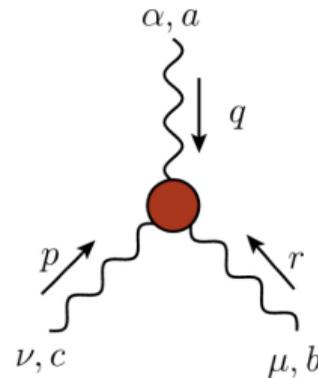
### Input: QCD Green functions

- Gluon(ghost) propagator
- Quark propagator
- Quark-gluon vertex
- Three-gluon vertex
- ...

# Gluon self-coupling

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a ; \quad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$$

- Three-gluon coupling responsible for the main differences between gluon and photon dynamics.
- It is itself a non-perturbative object which can be computed from the lattice or SDE.
- Key ingredient in SDE of quark-gluon or ghost-gluon vertices, for example.

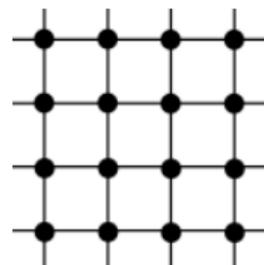


# Lattice formulation

Path integral in imaginary time:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int [dU d\psi d\bar{\psi}] \mathcal{O}(U, \psi, \bar{\psi}) e^{-S(U, \psi, \bar{\psi})} \rightarrow \frac{1}{N} \sum_{i=1}^N O_i$$

dimensionless; lattice spacing  $a$  fixed *a posteriori*.



## Pros

- **Just QCD.**
- Regularized per se ( $\Lambda \sim a^{-1}$ ).

## Cons

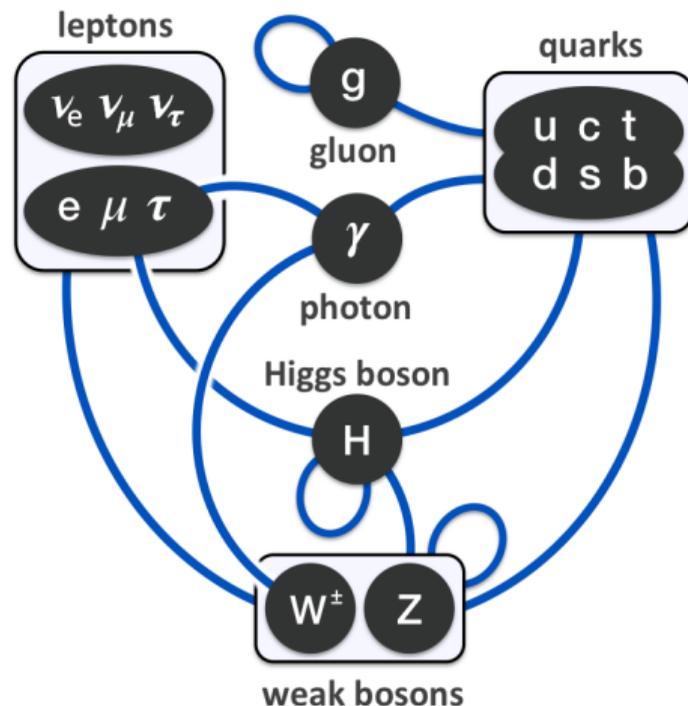
- Finite volume and discretization errors.
- Broken rotational symmetry!
- Expensive chiral fermions.

# Quenched approximation

The role of fermion loops in the path integral appears as the determinant of Dirac operator  $D$ :

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int [dU] \mathcal{O}(U, \psi, \bar{\psi}) e^{-S(U)} \det(D)$$

Yang-Mills theory already has a rich IR phenomenology!



# Lattice setups

Exploited quenched gauge field configurations with:

$\beta$	$L^4/a^4$	a (fm)	confs
5.6	$32^4$	0.236	2000
	$48^4$	0.236	2000
5.7	$32^4$	0.182	2000
5.8	$32^4$	0.144	2000
	$48^4$	0.144	500
6.0	$32^4$	0.096	2000
6.2	$32^4$	0.070	2000
6.4	$32^4$	0.054	2000

- Absolute calibration for  $\beta = 5.8$  taken from [S. Necco and R. Sommer, Nucl. Phys. B622, 328 (2002)].
- Relative calibrations based in gluon propagator scaling [Phys. Rev. D 98, 114515 (2018)]

# Computing three-gluon vertex in Landau gauge

## Landau gauge

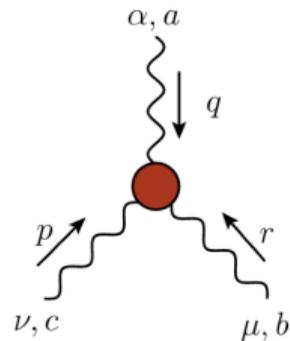
Landau gauge  $\partial_\mu A_\mu^a = 0$  fixed numerically, allowing to compute gauge dependent quantities.

- Gluon propagator:

$$\Delta_{\mu\nu}^{ab}(q^2) = \langle A_\mu^a(q) A_\nu^b(-q) \rangle = \delta^{ab} \Delta(q^2) P_{\mu\nu}(q)$$

- Three-gluon vertex:

$$f^{abc} \mathcal{G}_{\alpha\mu\nu}(q, r, p) = \langle A_\alpha^a(q) A_\mu^b(r) A_\nu^c(p) \rangle, \quad q + r + p = 0$$



## Extracting the transversely projected vertex

From the lattice data, we compute the transversely projected vertex,  $\bar{\Gamma}^{\alpha\mu\nu}(q, r, p)$ :

$$\mathcal{G}^{\alpha\mu\nu}(q, r, p) = g \bar{\Gamma}^{\alpha\mu\nu}(q, r, p) \Delta(q^2) \Delta(r^2) \Delta(p^2)$$

which corresponds to the transverse projection of the 1PI vertex:

$$\bar{\Gamma}^{\alpha\mu\nu}(q, r, p) = \Gamma^{\alpha'\mu'\nu'}(q, r, p) P_{\alpha'}^{\alpha}(q) P_{\mu'}^{\mu}(r) P_{\nu'}^{\nu}(p)$$

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No access to longitudinally coupled terms  $V^{\alpha\mu\nu}(q, r, p) = q^{\alpha}(\dots) + r^{\mu}(\dots) + p^{\nu}(\dots)$

If the 1PI vertex,  $\Gamma^{\alpha\mu\nu}(q, r, p)$  has longitudinally coupled term  $V^{\alpha\mu\nu}(q, r, p)$ :

$$\Gamma^{\alpha\mu\nu}(q, r, p) = \bar{\Gamma}^{\alpha\mu\nu}(q, r, p) + V^{\alpha\mu\nu}(q, r, p)$$

we will only access the transverse projection of  $\Gamma^{\alpha\mu\nu}(q, r, p)$ !

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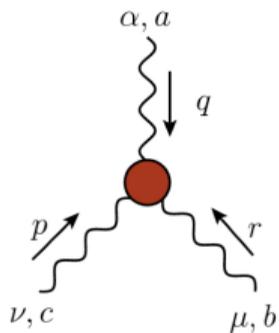
$$\bar{\Gamma}^{\alpha\mu\nu}(q, r, p) = \Gamma^{\alpha'\mu'\nu'}(q, r, p) P_{\alpha'}^{\alpha}(q) P_{\mu'}^{\mu}(r) P_{\nu'}^{\nu}(p)$$

The transversely projected tensor  $\bar{\Gamma}^{\alpha\mu\nu}(q, r, p)$  will have at most the contribution of *four* independent tensors:

$$\bar{\Gamma}^{\alpha\mu\nu}(q, r, p) = \bar{\Gamma}_1 \lambda_1^{\alpha\mu\nu} + \bar{\Gamma}_2 \lambda_2^{\alpha\mu\nu} + \bar{\Gamma}_3 \lambda_3^{\alpha\mu\nu} + \bar{\Gamma}_4 \lambda_4^{\alpha\mu\nu}$$

# Kinematics of the three-gluon vertex

$\bar{\Gamma}^{\alpha\mu\nu}(q, r, p)$  depends on three momenta, with  $q + r + p = 0$ . The scalar form factors can be cast in terms of the three squared momenta.

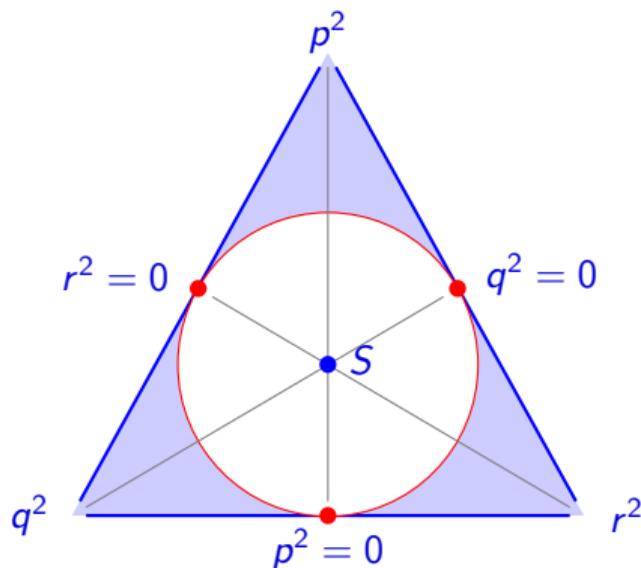
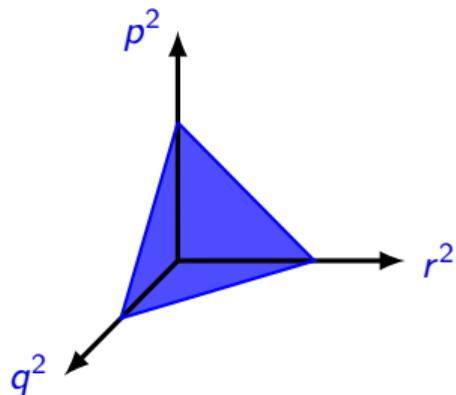


We will write them in terms of  $q^2$ ,  $r^2$ ,  $p^2$ , with the angles given by:

$$\cos\theta_{qr} = \frac{p^2 - q^2 - r^2}{2\sqrt{q^2 r^2}}, \dots$$

# Kinematics of the three-gluon vertex

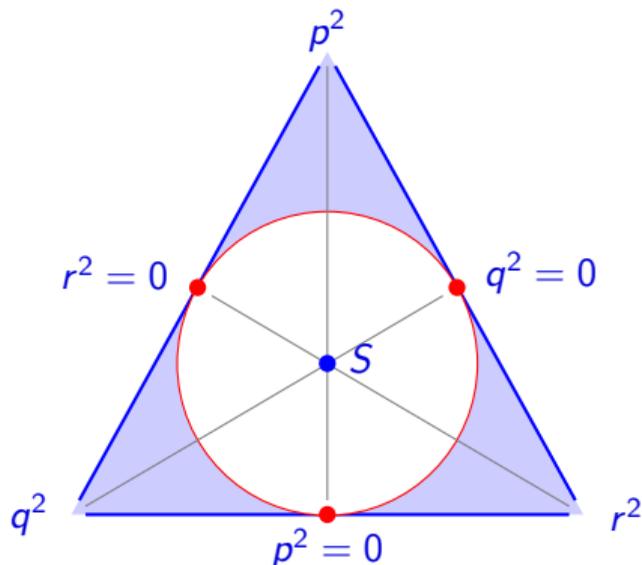
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## Kinematics of the three-gluon vertex

Particular cases:

Case	Def.	$\hat{q}r$	Tensors
Soft gluon	$p = 0$	$\pi$	$\lambda_3^{sg}$
Sym.	$q^2 = r^2 = p^2$	$\frac{2\pi}{3}$	$\lambda_{1,2}^{sym}$
Bisectoral	$q^2 = r^2$	$(0, \pi)$	3
General		–	4



Symmetric and soft-gluon cases already studied in [Phys.Lett.B 818 (2021) 136352]

# Tensor basis

We chose the following basis:

$$\begin{aligned}\lambda_1^{\alpha\mu\nu} &= \bar{\Gamma}_0^{\alpha\mu\nu} = \left( g^{\alpha'\mu'}(q-r)^{\nu'} + g^{\mu'\nu'}(r-p)^{\alpha'} + g^{\alpha'\nu'}(p-q)^{\mu'} \right) P_{\alpha'}^{\alpha}(q) P_{\mu'}^{\mu}(r) P_{\nu'}^{\nu}(p) \\ &= \left( \ell_1^{\alpha'\mu'\nu'} + \ell_4^{\alpha'\mu'\nu'} + \ell_7^{\alpha'\mu'\nu'} \right) P_{\alpha'}^{\alpha}(q) P_{\mu'}^{\mu}(r) P_{\nu'}^{\nu}(p) \quad \rightarrow \lambda_1^{sym}, \lambda_3^{s.g.}\end{aligned}$$

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 &= \left( \ell_1^{\alpha'\mu'\nu'} + \ell_4^{\alpha'\mu'\nu'} + \ell_7^{\alpha'\mu'\nu'} \right) P_{\alpha'}^{\alpha}(q) P_{\mu'}^{\mu}(r) P_{\nu'}^{\nu}(p) \quad \rightarrow \lambda_1^{\text{sym}}, \lambda_3^{\text{s.g.}} \\
 \lambda_2^{\alpha\mu\nu} &= 3 \frac{(r-p)^{\alpha'}(p-q)^{\mu'}(q-r)^{\nu'}}{q^2 + r^2 + p^2} P_{\alpha'}^{\alpha}(q) P_{\mu'}^{\mu}(r) P_{\nu'}^{\nu}(p) \quad \rightarrow \lambda_2^{\text{sym.}}
 \end{aligned}$$

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$$\lambda_2^{\alpha\mu\nu} = 3 \frac{(r-p)^{\alpha'}(p-q)^{\mu'}(q-r)^{\nu'}}{q^2 + r^2 + p^2} P_{\alpha'}^{\alpha}(q) P_{\mu'}^{\mu}(r) P_{\nu'}^{\nu}(p) \quad \rightarrow \lambda_2^{\text{sym.}}$$

$$\lambda_3^{\alpha\mu\nu} = \frac{3}{q^2 + r^2 + p^2} \left( \ell_3^{\alpha'\mu'\nu'} + \ell_6^{\alpha'\mu'\nu'} + \ell_9^{\alpha'\mu'\nu'} \right) P_{\alpha'}^{\alpha}(q) P_{\mu'}^{\mu}(r) P_{\nu'}^{\nu}(p)$$

$$\lambda_4^{\alpha\mu\nu} = \left( \frac{3}{q^2 + r^2 + p^2} \right)^2 (t_1^{\alpha\mu\nu} + t_2^{\alpha\mu\nu} + t_3^{\alpha\mu\nu})$$

## Tensor basis

We have chosen the tensor basis *antisymmetric* under two-gluon permutation, i.e.  $\{q, \alpha\} \leftrightarrow \{r, \mu\}$ :

$$\lambda_i \rightarrow -\lambda_i$$

Recall

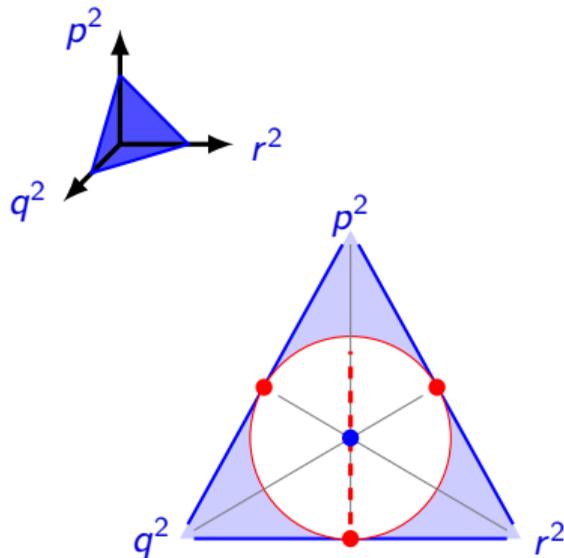
$$\langle A_\alpha^a(q) A_\mu^b(r) A_\nu^c(p) \rangle = f^{abc} g \bar{\Gamma}_{\alpha\mu\nu}(q, r, p) \Delta(q^2) \Delta(r^2) \Delta(p^2)$$

and

$$g \bar{\Gamma}^{\alpha\mu\nu}(q, r, p) = \sum_i \bar{\Gamma}_i(q^2, r^2, p^2) \lambda_i^{\alpha\mu\nu}(q, r, p)$$

### Bose symmetry

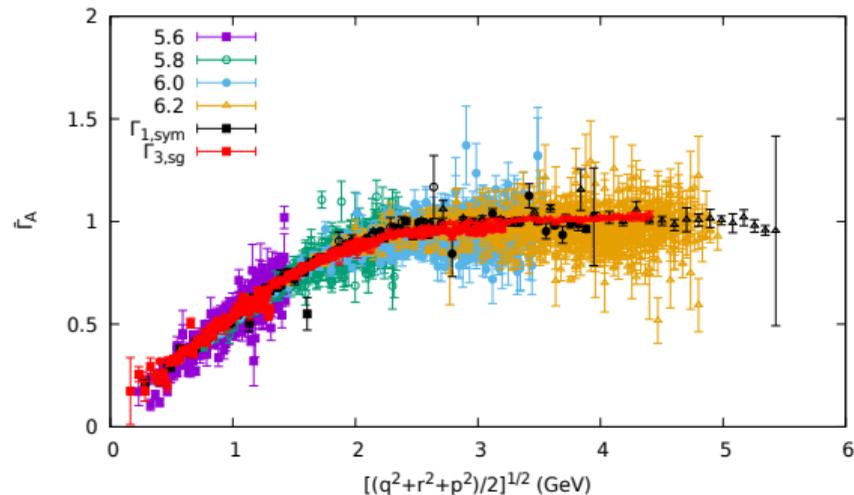
The form-factors  $\bar{\Gamma}_i(q^2, r^2, p^2)$  can only depend on symmetric combination of the momenta.

Results for the bisectoral case  $q^2 = r^2$ .

The scalar form factors can only depend on symmetric momentum variables [G. Eichmann *et al*, PRD89 (2014) 105014]:

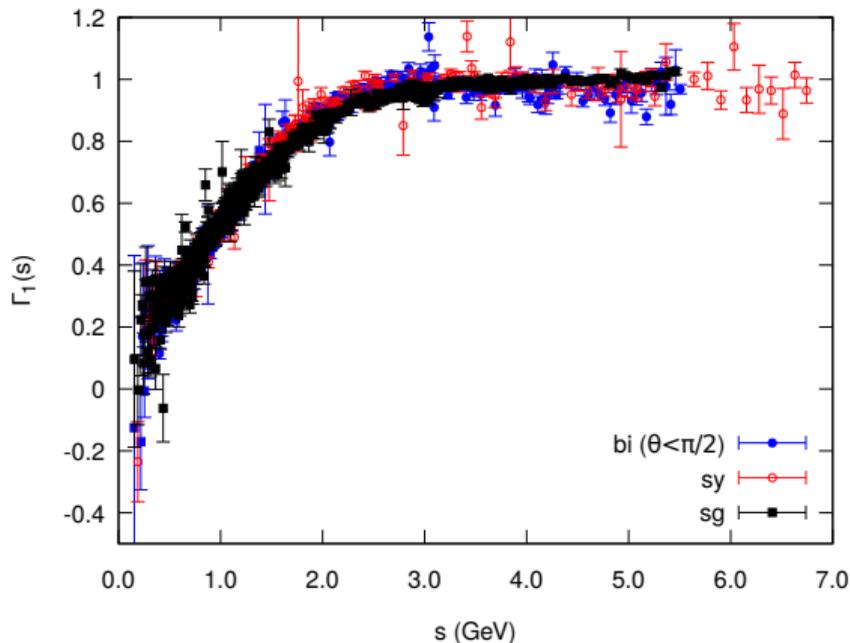
- $s^2 = \frac{q^2+r^2+p^2}{2}$  (plane)
- $(q^2 - r^2)^2 + (r^2 - p^2)^2 + (p^2 - q^2)^2$  (radius)
- $(q^2 + r^2 - 2p^2)(r^2 + p^2 - 2q^2)(p^2 + q^2 - 2r^2)$  (phase)

Alternatively, we will use  $s$  and  $\theta_{qr}$  for the bisectoral case.

Results for the bisectoral case  $q^2 = r^2$ :  $\bar{\Gamma}_1$ 

[F. Pinto-Gómez, FS, *et al* PLB838 (2023) 137737]

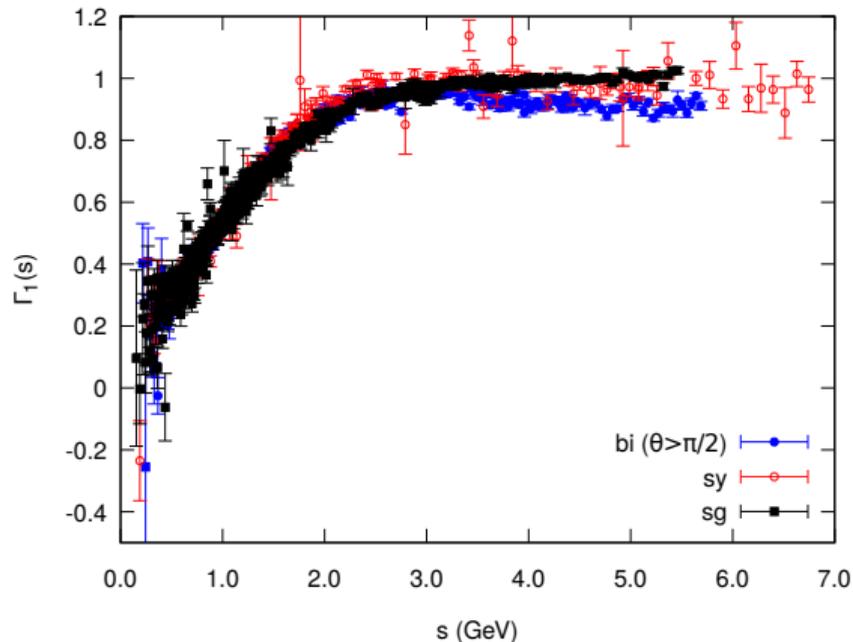
Represented in terms of  $s$ , there is a nice overlap between the already published *symmetric* and *soft-gluon* cases, but also with the bisectoral one.

Results for the bisectoral case  $q^2 = r^2: \bar{\Gamma}_1$ 

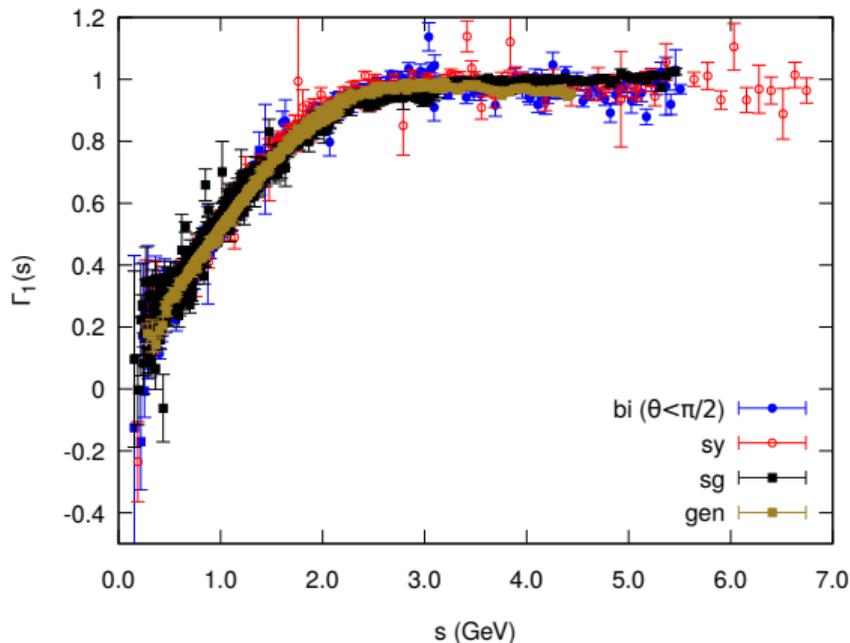
Represented in terms of  $s$ , there is a nice overlap between the already published *symmetric* and *soft-gluon* cases, but also with the bisectoral one.

There is an excellent overlap for the deep IR (below  $s \sim 1.5 - 2$  GeV).

The bisectoral case separates from the soft-gluon one at  $s \sim 3$  GeV.

Results for the bisectoral case  $q^2 = r^2: \bar{\Gamma}_1$ 

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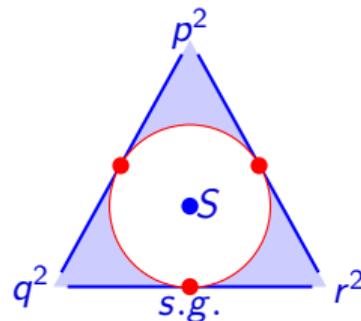
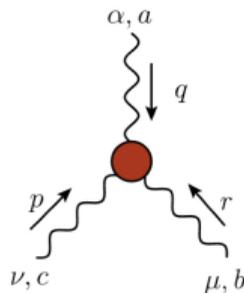
Results for the general case  $q^2 \neq r^2 \neq p^2$ .

The tree-level form-factor for general kinematics,  $q^2 \neq r^2 \neq p^2$ , overlaps with the rest of cases for the deep IR (below  $s \sim 1.5 - 2$  GeV).

The different kinematics separate from the soft-gluon one at  $s \sim 3$  GeV.

## Results for the bisectoral case $q^2 = r^2$ .

- $\bar{\Gamma}_1$  dominates.
- Quantitative agreement among different kinematics for  $s^2 = \frac{q^2+r^2+p^2}{2} \lesssim 3 \text{ GeV}$
- For  $q^2 = r^2$  (bisectoral)  $\bar{\Gamma}_1$  depends on  $\theta_{qr}$  for large  $s^2$ .
- Preliminary data for the general case  $q^2 \neq r^2 \neq p^2$  confirm the latter results.

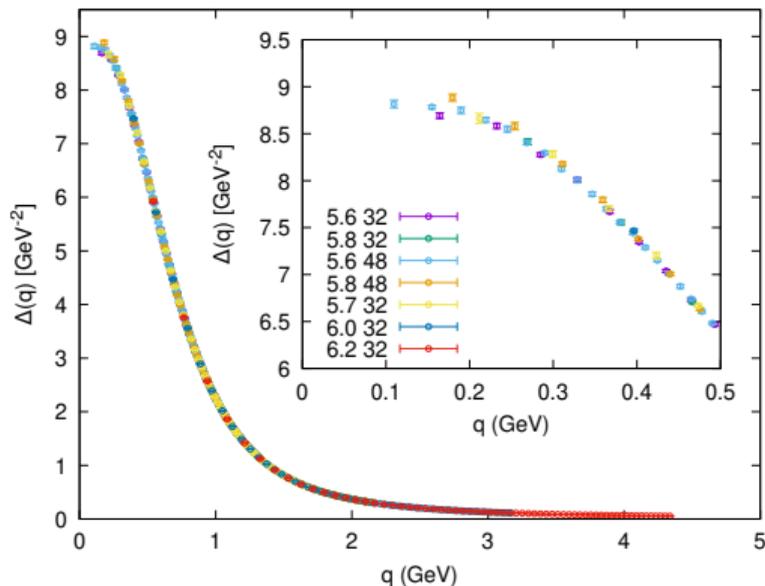


The full vertex seems to be well described by:

$$\bar{\Gamma}^{\alpha\mu\nu}(q, r, p) \approx \bar{\Gamma}^{sg}(s^2) \Big|_{s^2 = \frac{q^2+r^2+p^2}{2}} \bar{\Gamma}_0^{\alpha\mu\nu}(q, r, p)$$

# Gluon propagator

Lattice data finite at  $p \rightarrow 0$  (mass):



Gluon propagator:

$$\Delta^{-1}(q^2) = q^2 J(q^2) + m^2(q^2)$$

PT-BFM [D. Binosi, *et al.* PRD86 (2012) 085033]

- mass:  $m_{gluon} = \lim_{q \rightarrow 0} m(q^2)$
- kinetic term presents a logarithmic divergence:

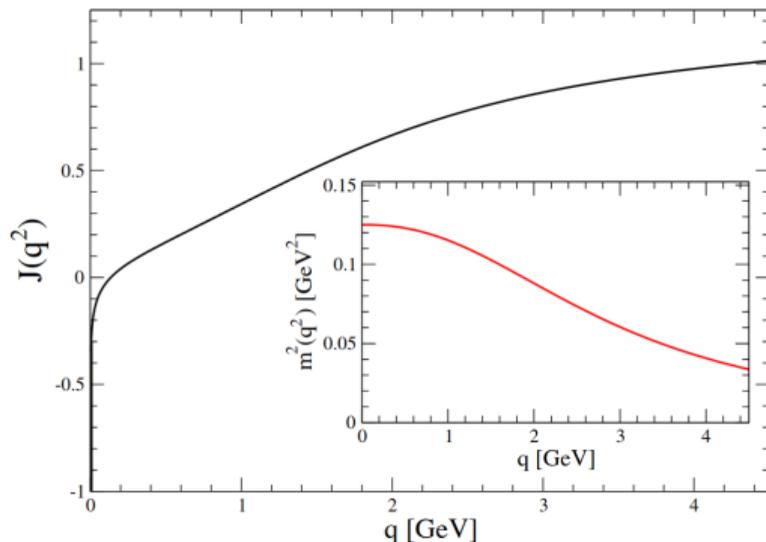
$$J(q^2)|_{q \rightarrow 0} \sim a \log\left(\frac{q^2}{\mu^2}\right) + b$$

related to the masslessness of the ghost

[A.C. Aguilar, *et al.* PRD89 (2014) 085008].

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[A.C. Aguilar, FS, *et al*, PLB818 (2021) 136352]

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- mass:  $m_{gluon} = \lim_{q \rightarrow 0} m(q^2)$
- kinetic term presents a logarithmic divergence:

$$J(q^2)|_{q \rightarrow 0} \sim a \log\left(\frac{q^2}{\mu^2}\right) + b$$

related to the masslessness of the ghost

[A.C. Aguilar, *et al*, PRD89 (2014) 085008].

# Zero crossing

If we write the full three-gluon vertex as:

$$\Gamma_{\alpha\mu\nu}^{abc}(q, r, p) \xrightarrow{q \rightarrow 0} \underbrace{\Gamma_{\alpha\mu\nu}^{abc}(q, r, p)}_{\text{pole-free}} + V_{\alpha\mu\nu}^{abc}(q, r, p)$$

and assume a separation of the STI satisfied by  $\Gamma$  into two *partial* STI's matching  $\Gamma \leftrightarrow J$  and  $V \leftrightarrow m^2$ , then:

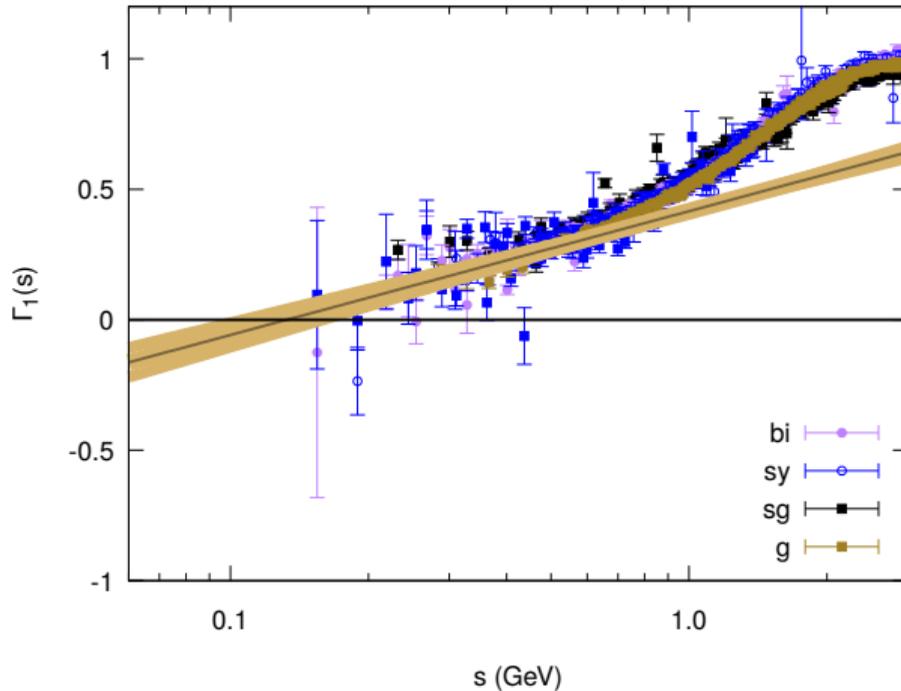
$$X_1(s^2) \xrightarrow{s \rightarrow 0} \alpha \log(s^2/\mu^2) + \beta$$

while  $s^2 X_3(s^2)$  is finite.

The form-factor  $\bar{\Gamma}_1(s^2)$  is logarithmically divergent.

[A.C. Aguilar, FS, *et al*, PLB818 (2021) 136352].

# Zero crossing

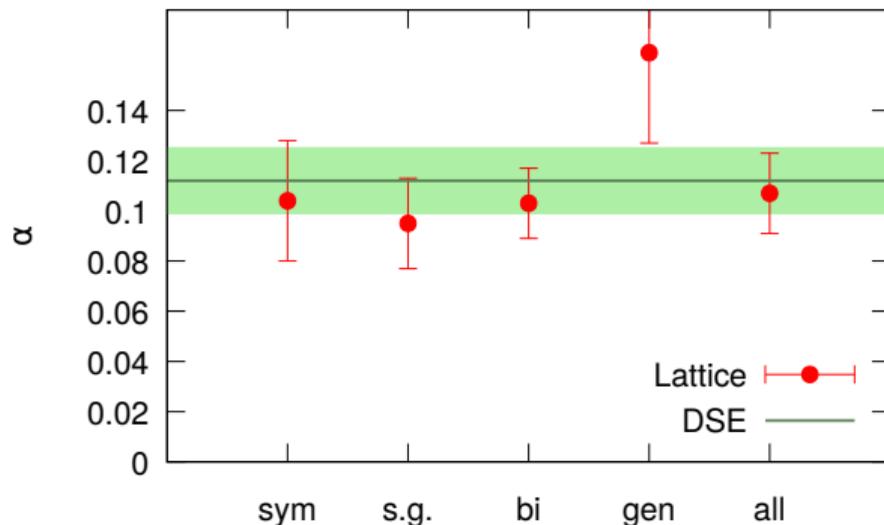


Fitting all data with  $s \leq 0.5$  GeV to  $\bar{\Gamma}_1(s^2) = \alpha \ln(s^2/\mu^2) + \beta$

The logarithmic slope obtained is  $\alpha \approx 0.107(16)$ , while the SDE prediction is  $0.112(10)$ !

A zero crossing appears at  $s \sim 130(20)$  MeV.

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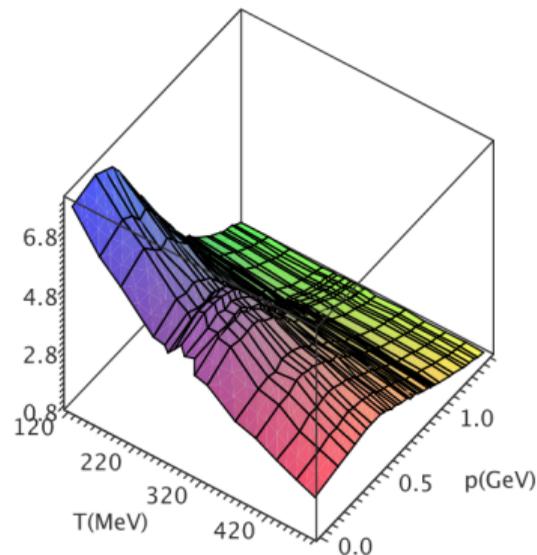
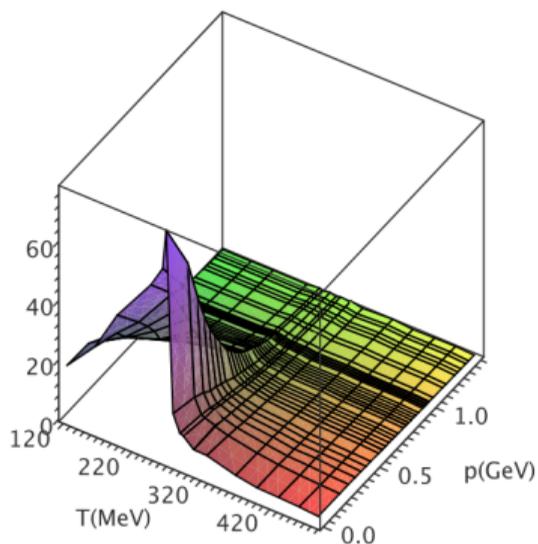
A zero crossing appears at  $s \sim 130(20) \text{ MeV}$ .

**There is a zero crossing for the tree-level form factor in nice agreement with SDE prediction!**

# Finite- $T$ and $\mu$ modifications: propagators.

Landau-gauge gluon propagator:

$$\Delta_{\mu\nu}(p) = \Delta_{\mu\nu}^T(p) + \Delta_{\mu\nu}^L(p)$$



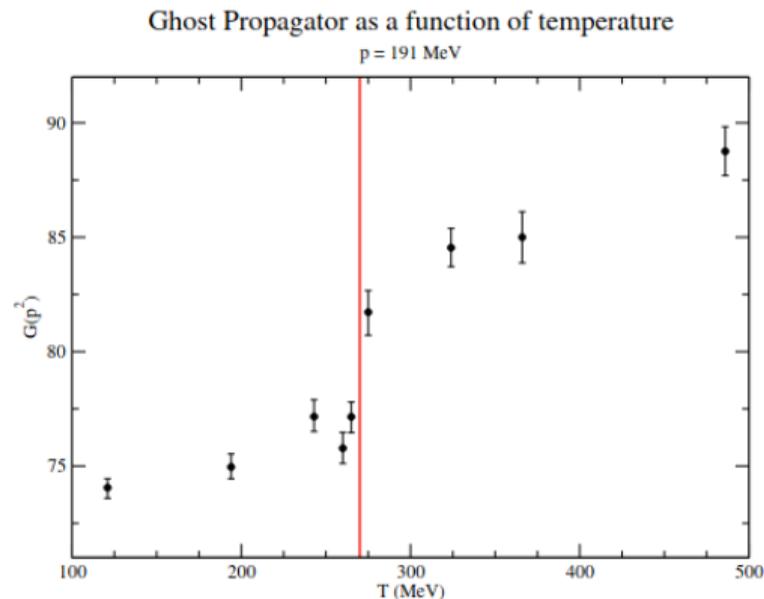
P. Bicudo *et al.*, PoS LATTICE2013 (2014) 368.

# Finite- $T$ and $\mu$ modifications: propagators.

Landau-gauge ghost propagator:

$$G^{ab}(p) = \delta^{ab} G(p^2)$$

- The ghost remains massless
- There is an enhancement of the ghost form factor  $G(p^2)$  above the critical temperature  $T_c$ ...



O. Oliveira *et al.*, EPJ Web Conf. 274 (2022) 05008.

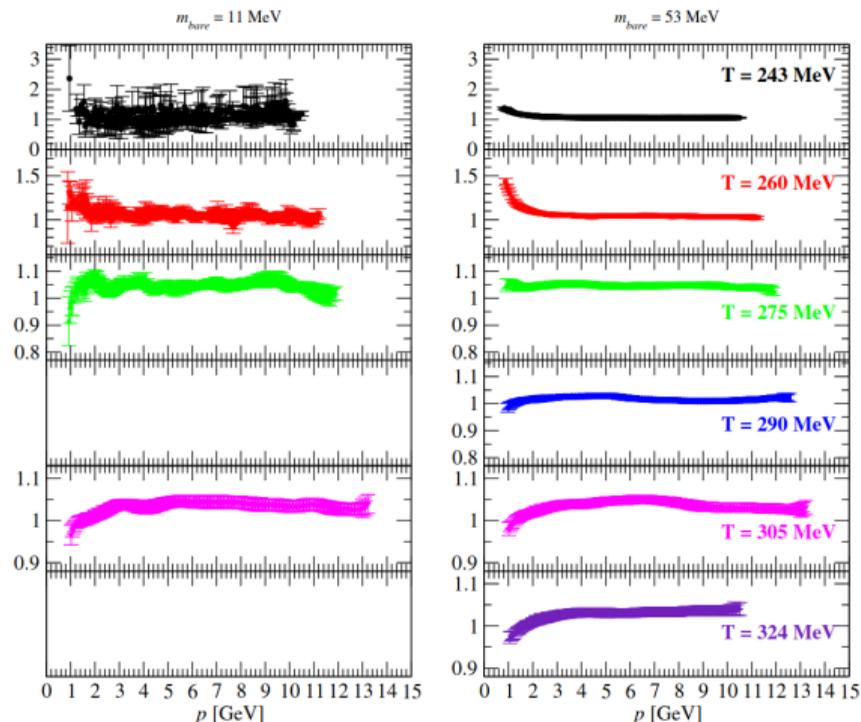
Finite- $T$  and  $\mu$  modifications: propagators.

Quark propagator:

$$S^{-1}(p) = i\gamma_0 p_0 \omega(p) + i\vec{\gamma} \cdot \vec{p} Z(p) + \sigma(p)$$

Above  $T_c$  there is an IR suppression of the quark mass function as compared to  $T < T_c$ .

O. Oliveira, P. Silva Eur.Phys.J.C 79 (2019) 793



## Finite- $T$ and $\mu$ modifications: vertices.

Ongoing effort to determine the finite- $T$  quark-gluon vertex J. Marques *et al.* PoS LATTICE2022 (2023)

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### Vertices

There are (almost) no lattice results about three-point Green functions modifications at either  $T$  or  $\mu$  finite!

A preliminary study of the  $T$  and  $\mu$  dependency of two- and three-point Green functions in QC<sub>2</sub>D has been made in T. Boz, O. Hajizadeh, A. Maas, J.I. Skullerud, Phys. Rev. D 99, 074514 (2019)

### Future

A bigger effort should be devoted to determine how QCD three-point Green functions are modified at finite  $T$  (and  $\mu$ ) and their impact on the phase-diagram!

# Conclusions

## Conclusions

- Transversely projected Landau-gauge 3g vertex dominated by the tree-level contribution.
- Form-factors dependence can be encoded in the symmetric combination  $s^2 = (q^2 + r^2 + p^2)/2$  below  $\sim 3 \text{ GeV}$  (**planar degeneracy**).
- A rather simple parametrization of the full vertex emerges:

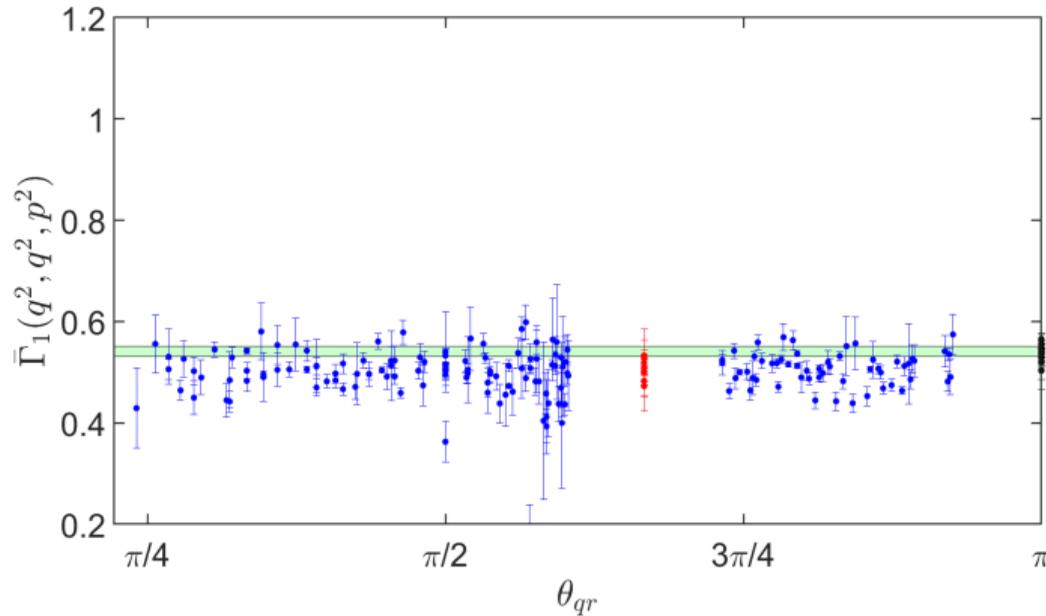
$$\bar{\Gamma}^{\alpha\mu\nu}(q, r, p) = \bar{\Gamma}_1(s) \Big|_{s^2 = \frac{q^2+r^2+p^2}{2}} \lambda_{\alpha\mu\nu}^{\text{t.l.}}(q, r, p)$$

- Log-slope at small momenta compatible with a zero crossing at  $s \lesssim 130(20) \text{ MeV}$ .

## Outlook

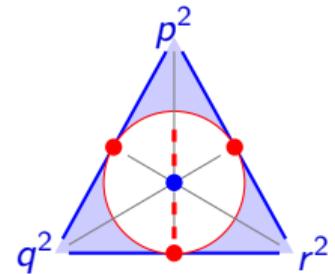
- Impact over the phase-diagram of QCD.
- Vertex modification at finite  $T$  or  $\mu$ .

# Results for the bisectoral case $q^2 = r^2$ : $\bar{\Gamma}_1$

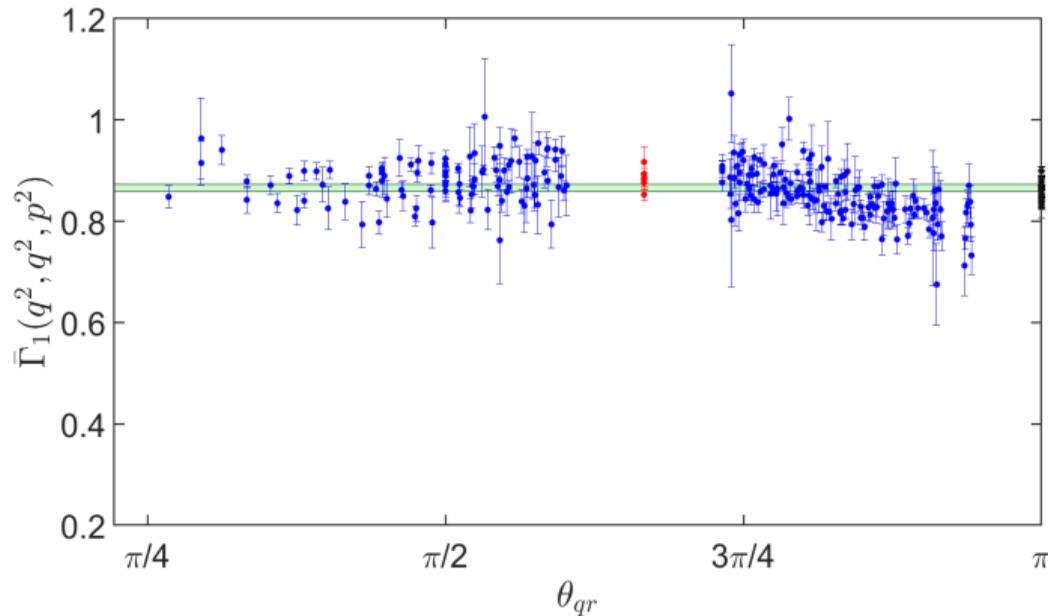


$s = 1 \text{ GeV}$

For small momenta, there is a negligible effect of the angle  $\theta_{qr}$

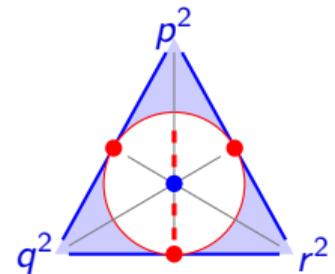


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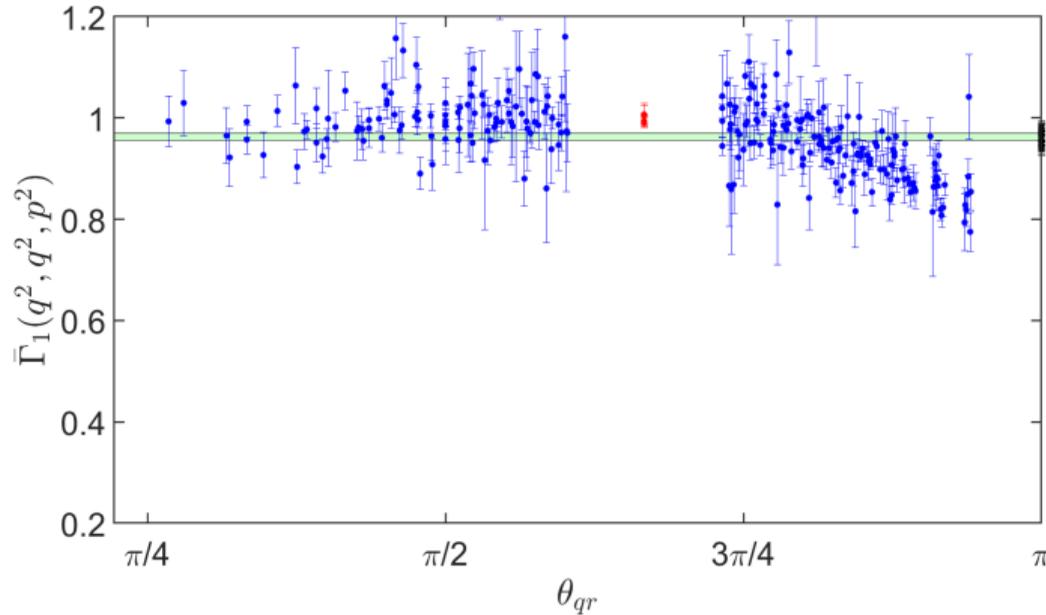


$s = 2 \text{ GeV}$

For small momenta, there is a negligible effect of the angle  $\theta_{qr}$

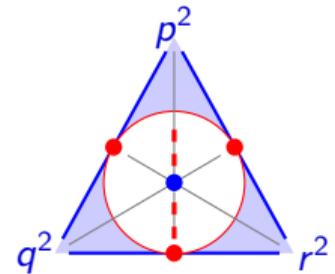


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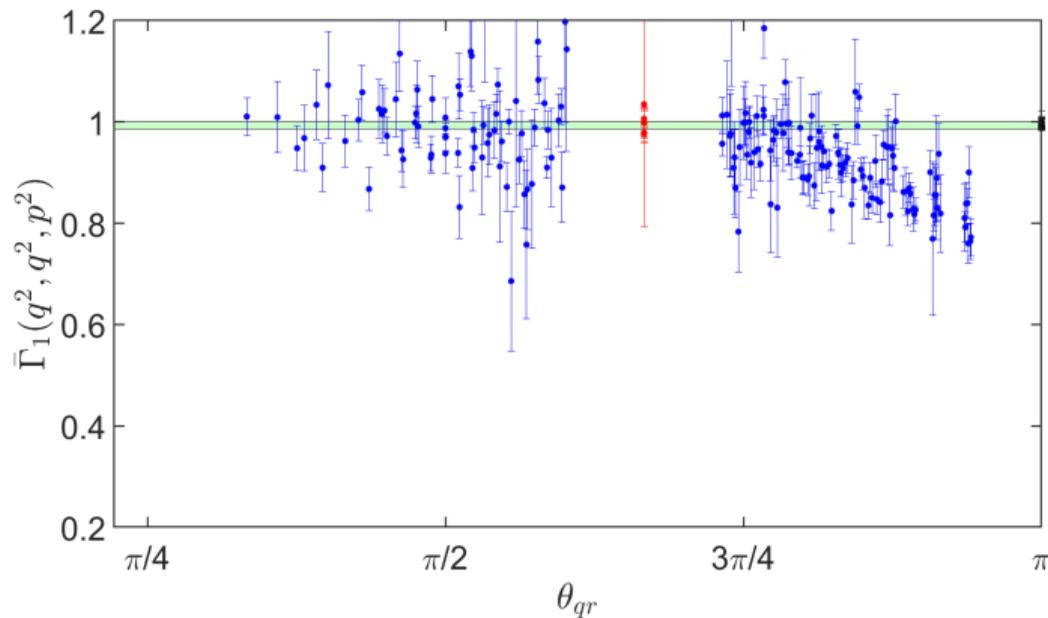


$s = 3 \text{ GeV}$

For larger momenta, it gets smaller values for  $\theta_{qr} \rightarrow \pi$

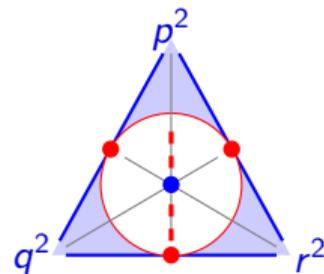


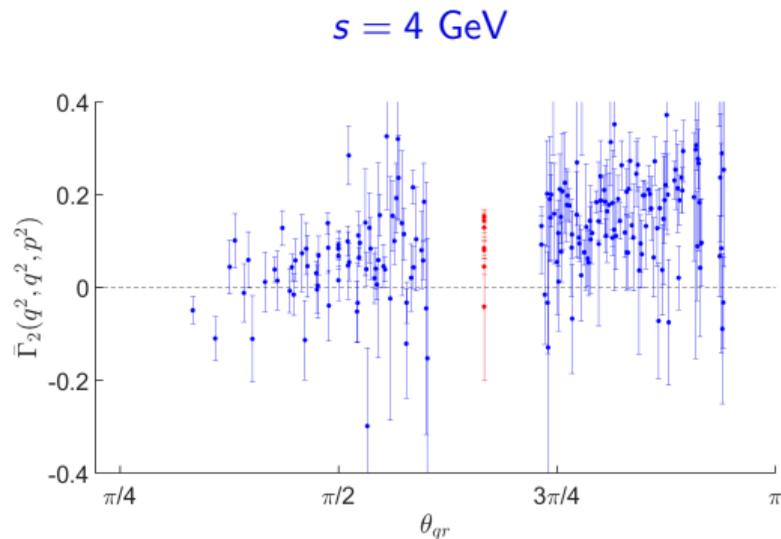
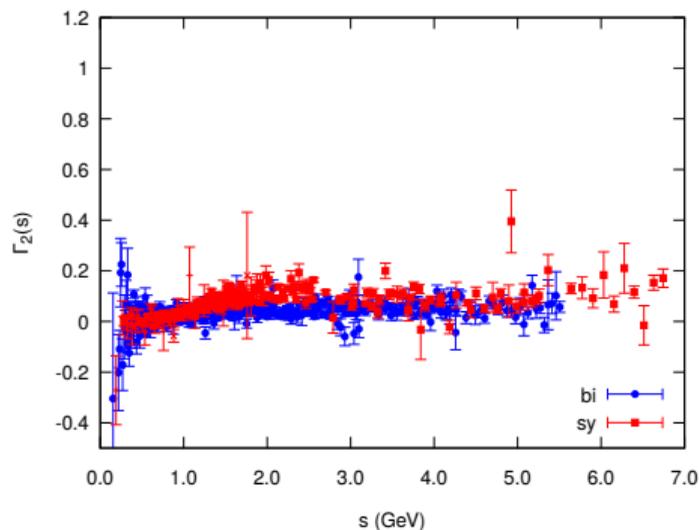
# Results for the bisectoral case $q^2 = r^2$ : $\bar{\Gamma}_1$



$s = 4 \text{ GeV}$

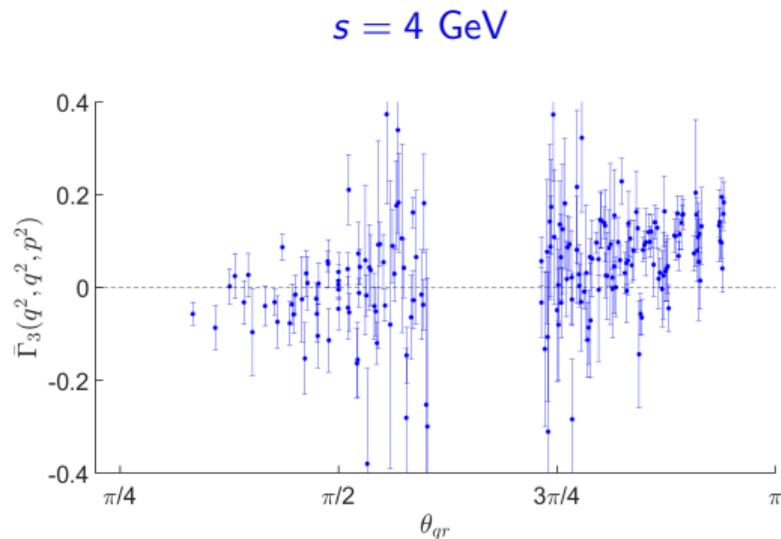
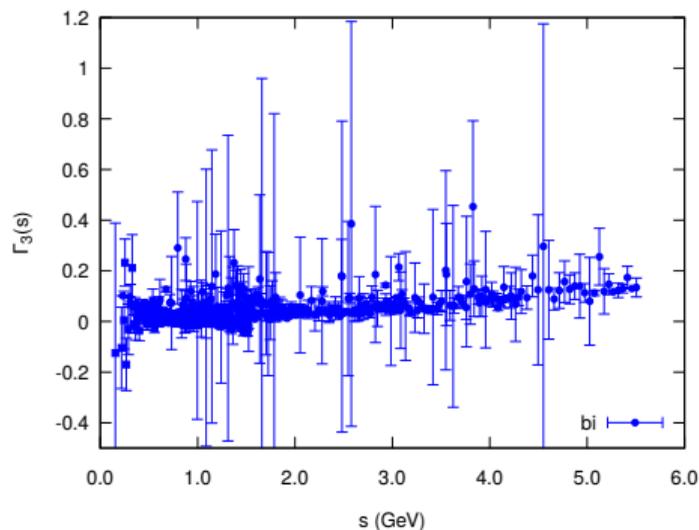
For larger momenta, it gets smaller values for  $\theta_{qr} \rightarrow \pi$



Results for the bisectoral case  $q^2 = r^2: \bar{\Gamma}_2$ 

Qualitatively compatible with recent SDE results [2305.05704]

# Results for the bisectoral case $q^2 = r^2: \bar{\Gamma}_3$



Qualitatively compatible with recent SDE results [2305.05704]