Nonperturbative QCD in Euclidean and Minkowski metric

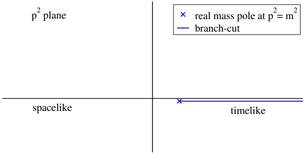
Pieter Maris

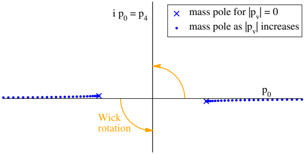
Dept. of Physics and Astronomy
Iowa State University
Ames, IA 50011

Nonperturbative QFT in Euclidean and Minkowski space,
Sept 2019, Coimbra, Portugal

Perturbative QFT

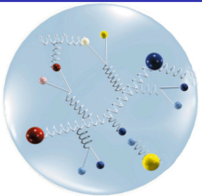
- Starting point: Lagrangian
- Green's functions or *n*-point functions
- Wick rotation between Minkowski and Euclidean metric
- Tree level: propagators have mass poles in timelike region
- Perturbation theory
  - for particles with mass m, interacting by exchange of particle with mass μ branch-cuts in timelike starting at $p^2 = (m + n\mu)^2$ corresponding to particle emission
  - massless exchange particles: series of branch-points collapse to logarithmic branch-cut starting at mass pole $p^2 = m^2$





- Propagators can be represented by Källen–Lehmann representations

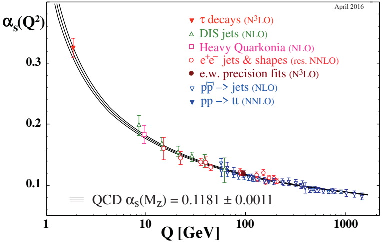
Hadron Physics



- Asymptotic States: Hadrons
  - Mesons
  - Baryons
- Fundamental Degrees of Freedom: Quarks and Gluons
  - Non-abelian gauge theory
  - Running coupling:

    strong coupling
    at low momenta
    (long distance)

    weak coupling
    at high momenta
    (short distance)



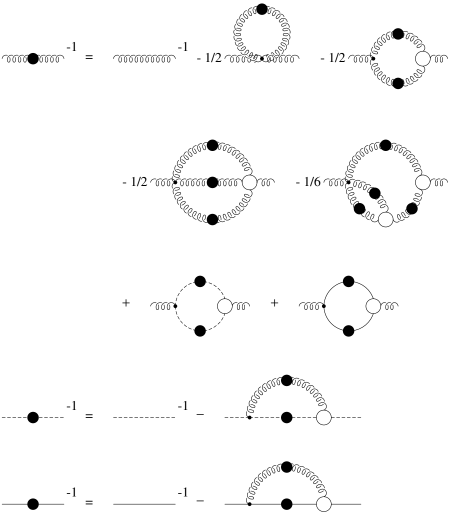
- Nonperturbative phenomena
  - Dynamical Chiral Symmetry breaking
  - Confinement

Nonperturbative QCD

$$\mathcal{L}(\psi,\bar\psi,A) = \bar\psi^i\Big(i\gamma^\mu\big(\partial_\mu + ig\frac{\lambda^{(a)}_{ij}}{2}A^{(a)}_\mu\big) - m\Big)\psi^j - \frac{1}{4}F^{(a)}_{\mu\nu}F^{(a)\mu\nu} + \text{gauge fixing}$$

- Lattice simulations
  - based on Euclidean formulation of QCD
- Dyson–Schwinger Equations
  - typically in Euclidean metric
  - can also be done in Minkowski metric,
    at least for weak coupling / perturbative regime
- Renormalization Group Methods
- Hamiltonian Methods based on Minkowski formulation
  - Lightfront
  - Equal-time
- Effective Field Theory
- ...

Dyson–Schwinger Equations

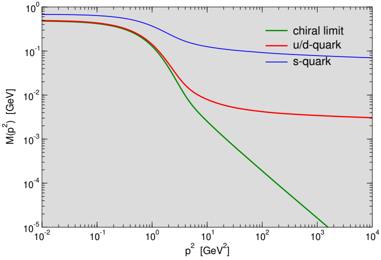


- Infinite hierarchy of coupled integral eqns for Green's functions of QCD
- Reduce to pQCD in weak coupling limit
- Nonperturbative
- Truncations needed
- Constraints on truncations
  - preserve symmetries
  - self-consistency

Nonperturbative quark mass function

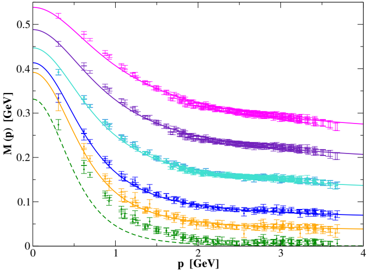
Rainbow truncation for quark DSE

$$Z_1^g\, g^2 D_{\mu\nu}(q)\, \Gamma_\nu(k,p) \longrightarrow 4\pi\alpha_{\rm model}(q^2)\, D_{\mu\nu}^{\rm free}(q)\, \gamma_\nu$$



- Evolution from constituent to current quark mass
- Absence of mass pole on real axis: confinement?
- Complex-conjugate singularities: Wick rotation??

Fig. adapted from Maris & Roberts, PRC56, 3369 (1997)



- Qualitative agreement with lattice data in spacelike region

Lattice-inspired DSE model:
Bhagwat, Pichowsky, Roberts, Tandy, PRC68, 015203 (2003)
Quenched lattice data: Bowman, Heller, Leinweber, Williams, NP Proc.Suppl.119, 323 (2003)

published 43 years ago: Nucl. Phys. B **117**, 250 (1976)

SCHWINGER-DYSON EQUATION FOR MASSLESS VECTOR THEORY AND THE ABSENCE OF A FERMION POLE

Reijiro FUKUDA
Research Institute for Fundamental Physics, Kyoto University, Kyoto, 606 Japan

Taichiro KUGO
Department of Physics, Kyoto University, Kyoto, 606 Japan

Received 2 April 1976
(Revised 23 August 1976)

The Schwinger-Dyson equation of the fermion propagator in the massless vector theory is discussed. It is found that the Baker-Johnson-Willey solution in lowest approximation is in fact a confining solution: the Fermion propagator has no pole or cut in the time-like region. Discussions of homogeneous and inhomogeneous equations with momentum integration cut-off are also given in some detail.

published 40 years ago: Nucl. Phys. B **151**, 342 (1979)

DETERMINATION OF THE SINGULARITIES OF THE ELECTRON PROPAGATOR

D. ATKINSON * and D.W.E. BLATT
Department of Mathematics, University of Newcastle, New South Wales, 2308, Australia

Received 2 November 1978

It is shown, by means of the Runge-Kutta method of numerical integration, that the electron propagator, in the first approximation of the Johnson-Baker-Willey scheme, has complex branch-points in the momentum variable, instead of the real branch-point that physics requires.

Open Questions

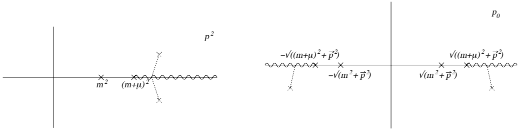
Is there a fundamental difference between
the analytic structure of propagators of confined particles
and that of propagators of asymptotically observable particles?

If the answer is yes, then the following questions arise

- What is the analytic structure of confined propagators?
- More specific: what is the analytic structure of confined quark and gluon propagators?
- More general: what is the analytic structure of n-point functions describing confined fields?
- Could the analytic structure be dependent on the approach or gauge or renormalization scheme?

Analytic structure of propagators

1. have one (or more) singularities on the timelike axis
2. have one (or more) singularities on the timelike axis, in combination with singularities at complex momenta on a second Riemann sheet, corresponding to resonances and/or virtual states

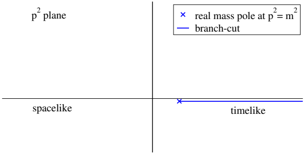


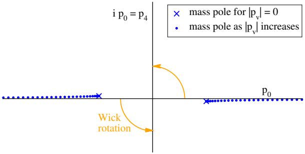
3. have one (or more) singularities at complex momenta p^2 on the first Riemann sheet
   - e.g. a pair of complex-conjugate singularities
4. are entire functions (no singularities)
   - constant or (sum of) exponential(s)
5. are, mathematically speaking, not analytic functions
   - distributions

Possibilities (3), (4), and (5) would invalidate the naïve Wick rotation from Minkowski space to Euclidean space

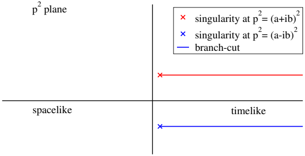
Pair of complex-conjugate singularities

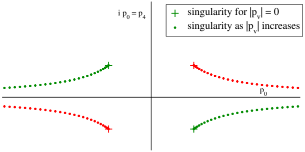
- Singularity on real timelike axis





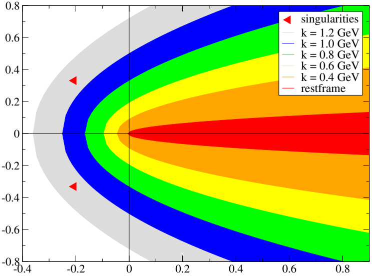
- Pair of complex-conjugate singularities

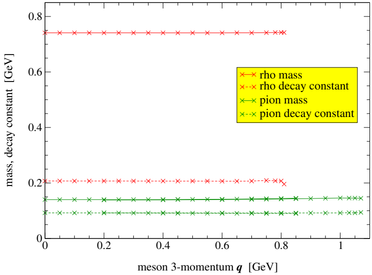




  - invalidates the naïve Wick rotation from Minkowski space to Euclidean space
- Possible interpretation, analogous to mass and width of resonances
  - real part: mass
  - imaginary part: hadronization scale

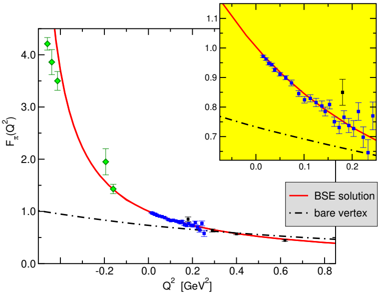
Hadron observables – Moving frames



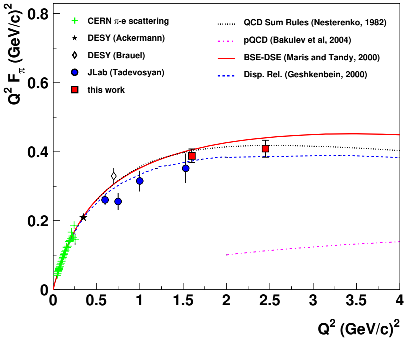


- for e.g. form factors and scattering amplitudes we need hadron bound state amplitudes (BSA) in moving frames
- singularities in the propagator limit the range over which we can obtain these BSAs without the need for nontrivial deformations of integration contours

Hadron observables – Form factors



Maris and Tandy, PRC62,055204 (2000) [nucl-th/0005015]

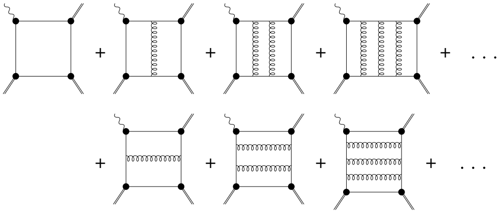


Tadevosyan *et al.* [Fpi2 Collaboration], nucl-ex/0607007;
Horn *et al.* [Fpi2 Collaboration], nucl-ex/0607005

- Pion elastic form factor can be calculated up to about $Q^2 = 4$ GeV^2 within the Maris-Tandy model using consistently dressed propagators and vertices without nontrivial deformations of integration contours

Hadron observables – Scattering

- Use ladder kernel not only for propagators and vertices, but also inside box diagrams in order to preserve symmetries



- Results for $\pi\pi$ scattering agree with dynamical χSB

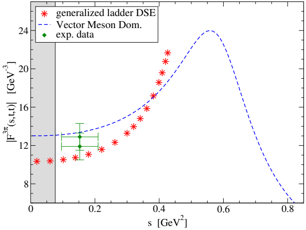
  Bicudo, Cotanch, Llanes-Estrada, Maris, Ribeiro and Szczepaniak, PRD65, 076008 (2002)

- Results for $\gamma 3\pi$ agree with χSB and current conservation

  Cotanch and Maris, PRD68, 036006 (2003)

- New data from JLAB for $0.27\ \text{GeV}^2 < s < 0.72\ \text{GeV}^2$?

  (private comm. 2003 ?)



Is QCD defined in Euclidean space?

published almost 30 years ago: Phys. Lett. A **146**, 467 (1990)

IS SPACE–TIME EUCLIDEAN "INSIDE" HADRONS?

S.J. STAINSBY and R.T. CAHILL

School of Physical Sciences, The Flinders University of South Australia, GPO Box 2100, Adelaide SA 5001, Australia

Received 12 March 1990; accepted for publication 5 April 1990
Communicated by J.P. Vigier

Results from a numerical study of the QCD Schwinger–Dyson (SD) equation indicate that the Wick rotation may be disallowed due to the presence of complex branch points in the quark propagator. Atkinson and Blatt obtained a similar result in a study of massless QED. This leads us to suggest that the preferred defining metric for such confining theories is Euclidean, as has also been suggested for quantum gravity.

- How do we go from quarks and gluons in Euclidean space to hadrons in Minkowski space?
- How can we define 'light-cone' observables (e.g. quark and gluon pdf's) from a purely Euclidean formulation?

Goals of the Workshop

- Discuss these (and related!) open questions
- Consider 'all' possibilities without prejudice
- Compare and contrast different approaches and seemingly contradictory results
- Exchange information between experts in different methods

In order to achieve (some of) these goals

- Questions during talks are encouraged
- Every day we have a discussion session for follow-up questions and in-depth discussion
- Potentially lengthy discussions during the sessions can be postponed to the discussion session
- Friday is available for additional in-depth discussions