Nakanishi representation for propagators

Vladimír Šauli
OTF UJF Rez
sauli@ujf.cas.cz

Coimra,10-14.9. 2019

Outline

- what is NIR -properties
- NIR in field theories without confinement
- NIR in QCD....
- quark gap equation in Minkowski space
- quark-photon vertex
- VHP and PTF applications

What is NIR

NIR is IR for GF's of the theory

time ago NIR was PTIR (Perturbation Theory Integral representation)

N.Nakanishi workout PTIR for scalar QFT Nakanishi- Graph Theory and Feynman Integrals, 1971

Examples of PTIR for scalars:

- propagator

$$G(k) = \int_0^\infty dx \frac{g(x)}{k^2 - x + i\varepsilon} \tag{1}$$

The inverse propagator (selfenergy, polarization, etc..) DR

$$
\begin{aligned}
G^{-1} &= k^2 - m^2 + \Sigma(k) \\
\Sigma(k) &= \int_0^\infty dx \frac{g_\sigma(x)}{k^2 - x + i\varepsilon}
\end{aligned}
$$

- 3-legs vertex function $\Gamma(p_1, p_2, p_3)$ analysis of contributing Feynman diagrams leads to the PTIR

$$
\Gamma(p_1, p_2, p_3) = \int_0^\infty d\alpha \prod_{i=1}^{3} \int_0^1 dz_i \, \delta(1 - \sum_{i=1}^{3} z_i) \, \frac{\rho_3(\alpha, \vec{z})}{\alpha - (z_1 p_1^2 + z_2 p_2^2 + z_3 p_3^2) - i\epsilon} \, ,
$$

in most cases the topology of considered diagrams is much simple, allowing to use simplified NIR

simpler physics is , more dirac deltas we have in N.w. ρ

$$\Gamma(q,Q) = \int_{\alpha_{min}}^{\infty} d\alpha \int_{-1}^{1} dz \frac{\rho(\alpha, z; Q^2)}{q^2 + zq\cdot Q + \frac{Q^2}{4} - \alpha + i\epsilon},$$

NIR + DSEs

NIR gradually moved to non-perturbative and strong coupling regime of QFT

The DSEs are the equations of motion for Green's functions of the theory and NIR is useful way to express the Green's functions in specific manner

V.S. J. Adam NPA 2001, ficube; V. S. Non-perturbative solution of metastable scalar models, JPA36 (2003).

DSE for N.w.s of 3theory

$$
\begin{aligned}
\rho(o) &= g_\sigma(o) + (m^2 - o) P \int_t^\infty dx \frac{\rho(x) g_\sigma(o) + \rho(o) g_\sigma(x)}{x - o} \\
g_\sigma(o) &= const \frac{\sqrt{1 - 4m^2/o}}{(o - m^2)^2} + \int K_1 \rho + \int \int K_2 \rho \rho
\end{aligned}
$$

Review of applications to SDEs, BSE: V. Sauli, FBS.39:45,(2006)

NIR + DSE/BSE formalism: properties

++

- since the momentum is very explicit in the expression for NIR, all required space-time transformation, including Lorentz boost of vertices and meson wave functions turns out to be an easy ask.

- When NIR is used for evaluating of form factor, one can integrate over the loop momenta analytically. Consequently, the analytical continuation of calculated form factors can be easily achieved and the result can be obtained in the entire domain of Minkowski space.

To appreciate two above points, one obviously has to know NIR. For this purpose I should point out the following desired property of NIR

- When NIR is used properly in the tower of DSEs, it allows the analytical integration in DSEs and it automatically provides analytical continuation of the solution to the entire domain of Minkowski space.

- Regarding the solution of DSEs, NIR should be self-consistently self-reproducing. In other words: when NIR is used to express propagator and vertices inside the DSE for n-point vertex, the NIR for this vertex must comes out as a result as well.

– –

converting DSEs and BSEs into NIR is complicated and time consuming (and impossible for some popular model/truncations of DSEs)

solutions for strong coupling QED, Yukawa theory propagators see

momentum DSE $\rightarrow$ new (soluble!) equation for Nakanishi weight g

2b- BSE and NIR in various QFT

, for more (see talk by Giovanni Salme) The vertex BSE

$$\Gamma = -\int V G_{BS} \Gamma$$

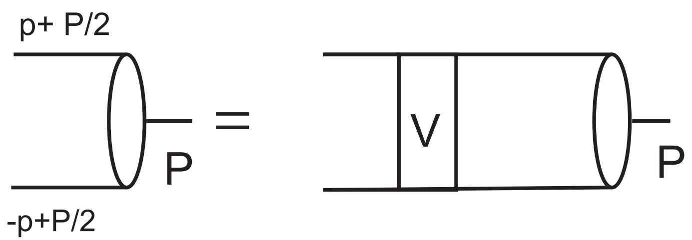

Figure 1: diag. BSE

$$g(\alpha',z')=\lambda\int_{-1}^{1}dz\int_{-\infty}^{\infty}d\alpha\;K(\alpha',z';\alpha,z)\,g(\alpha,z)$$

NIR BSE: massive Wick-Cutkosky model (K.K. AGW 1996 PRD) , other models including fields with higher spins , Karmanov, T. Frederico. G. Salme ,...

coupled DSE and BSE V.S. and J.A. 2003

attempts for mesons BSE exist V.S. JPG 2008

Curiozities:

- BSE for Higgsonium in Minkowski space

V. S.,arXiv:0808.1894, AIPConf.Proc.1030:274-279,2008 arXiv:0806.3454

- Regge trajetories for $M^2(n)$ in toy BSE model. conf. Ecxcited QCD , Trento

QCD

QCD DSE/BSEs not by NIR: (see talk by Gernot Eichmann)

YM part- The first suggestion to use the NIR in QCD is related with introduction of PT in pure Yang-Mills theory in 1982 by J. Cornwall

Confinement issues- no real poles? But where the singularities comes?

The functional QCD resisted against marriage with the NIR formalism for more many decades

Near critical solution:

Dynamical chiral symmetry breaking with Minkowski space integral representations V. Sauli, J. Adam, P. Bicudo PRD75,2007

SAB model

Dynamical chiral symmetry breaking with Minkowski space integral representations

$$M(p^2) = S_0^{-1}(p) - S^{-1}(p) = ig^2 \int \frac{d^4q}{(2\pi)^4} \gamma^\alpha S(q) \gamma^\beta$$

$$\left[-g^{\alpha\beta} + \frac{l_\alpha l_\beta}{l^2} \right] \frac{-\Lambda^2 + m_g^2}{(l^2 - m_g^2 + i\epsilon)(l^2 - \Lambda^2 + i\epsilon)} ,$$

$$m_g/\Lambda = 30/,/,; M(M) = 0.001(\alpha = 1.64); M(M) \simeq m_g(\alpha = 2)$$

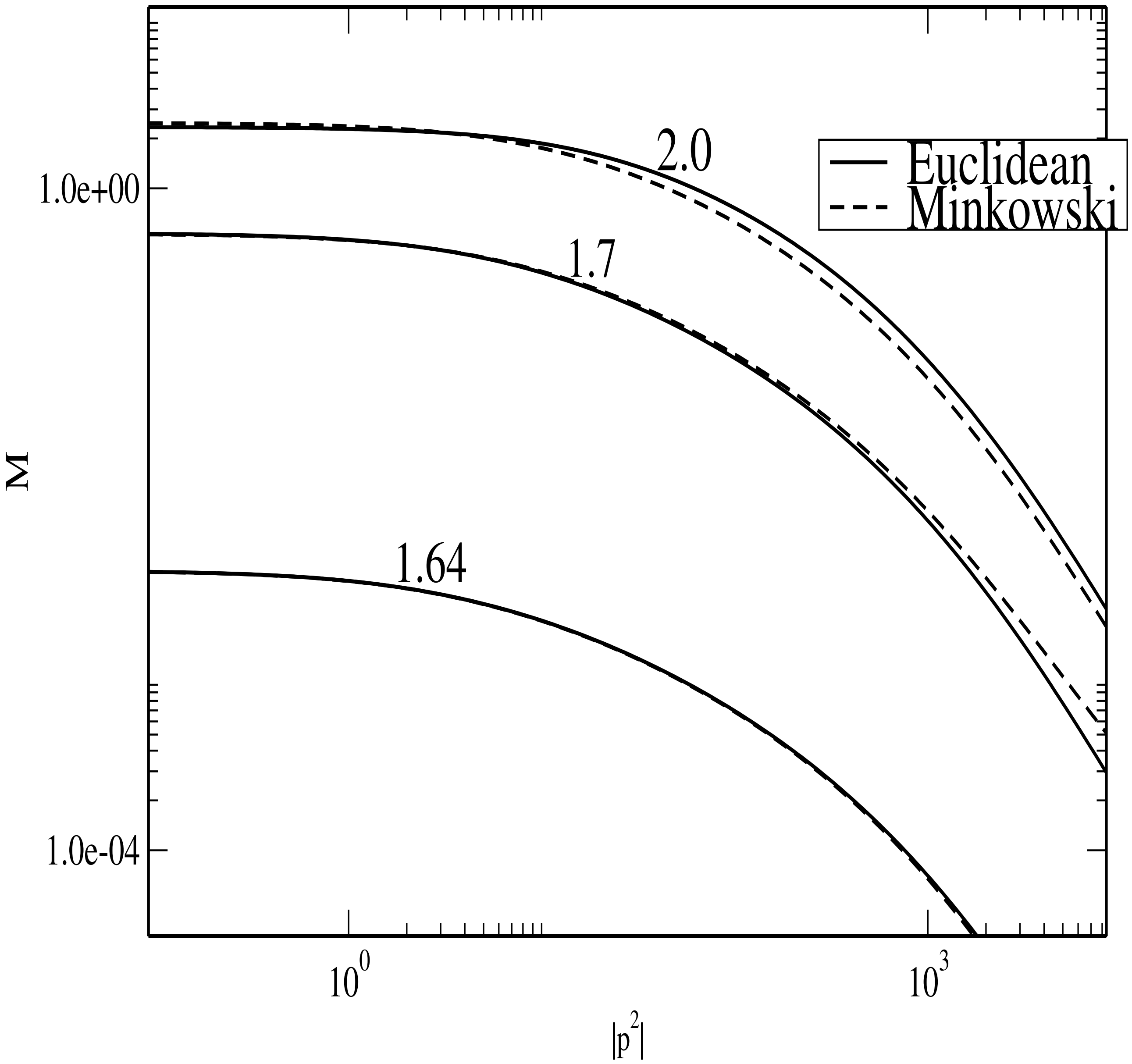

Figure 2: M for $\alpha = 1.64;\ 1.7,\ 2.0$ and spacelike P Solid (dashed)

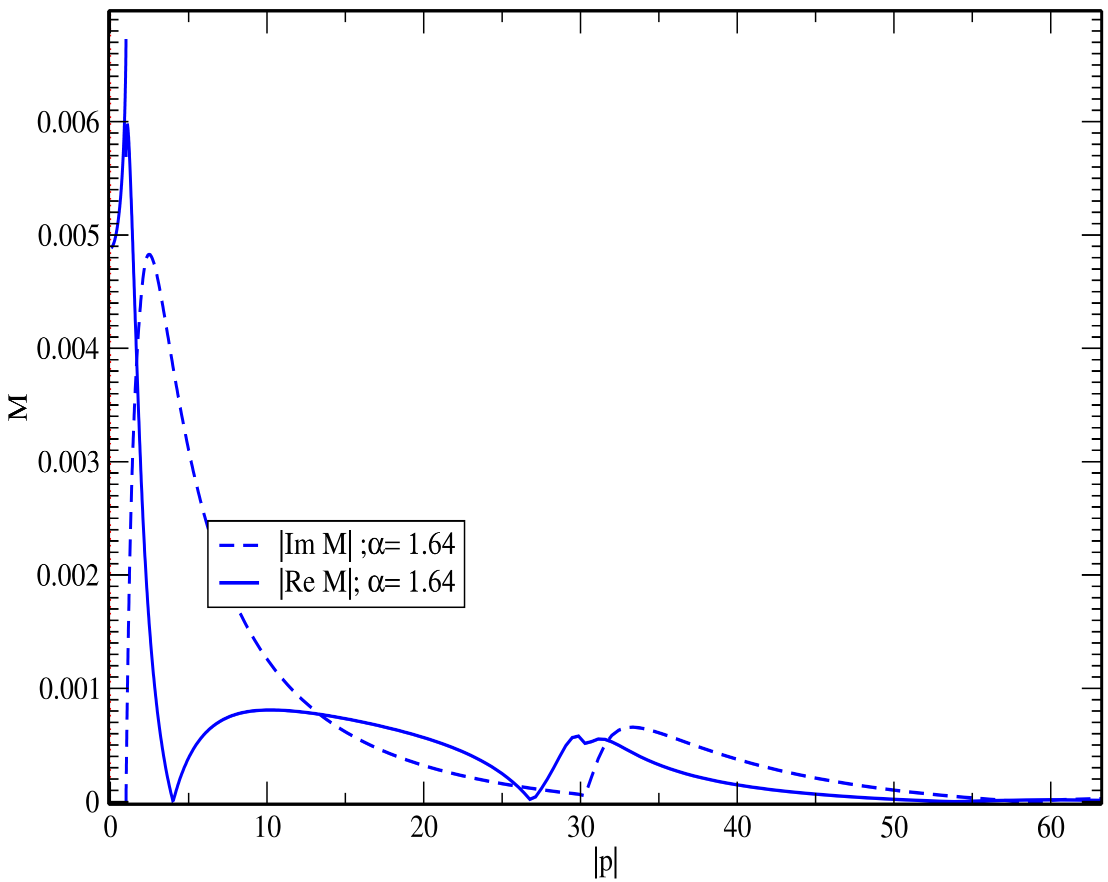

Figure 3: M for timelike p

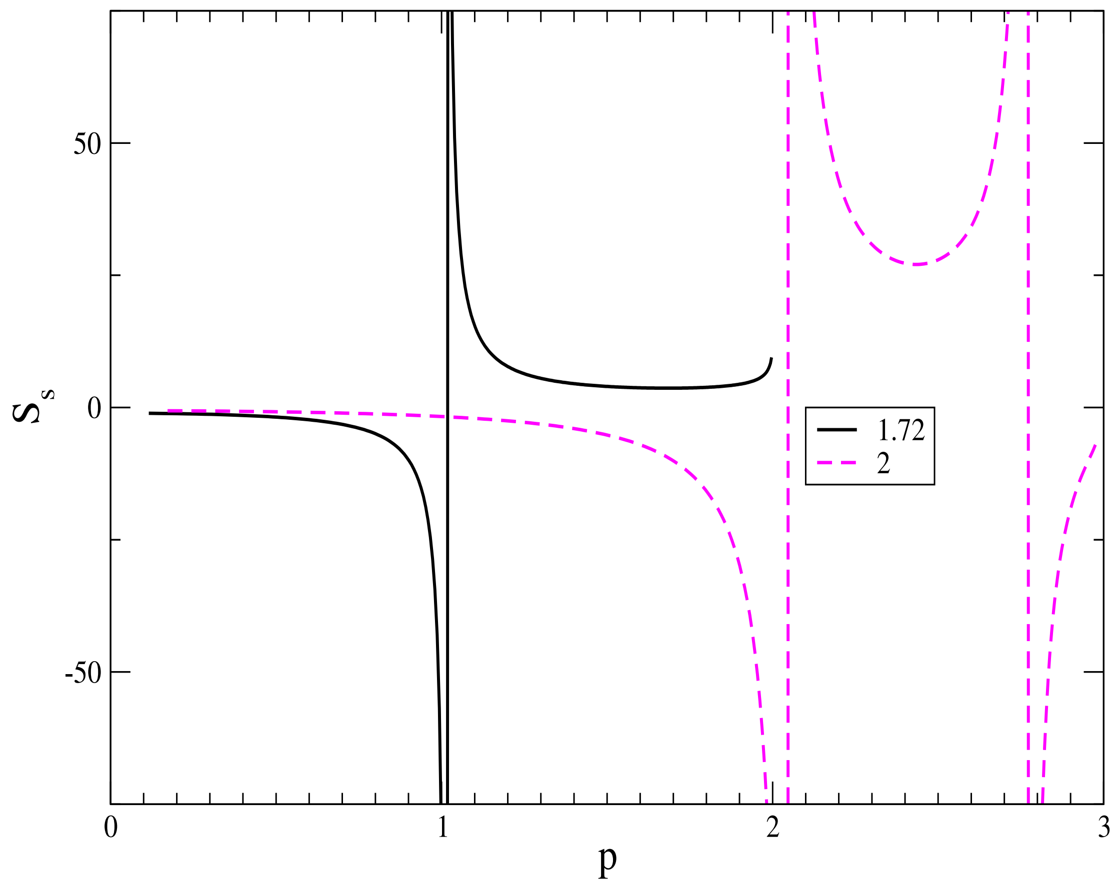

Figure 4: (Color online) Propagators...

ABS model was not yet QCD since the coupling was too small, the slope of M was to small. No conf. = in-consistent with QCD

NIR in QCD&QED NIR

-Understanding of confinemet of light quarks in NIR formalism...

-colorless form factor (production of mesons)

first suggestionJ. Cornwall 1982 pure YM

NO DSE for quarks was solved till 2018, gluodynamics not yet solved

V.S. Hadron Vacuum Polarization from application of DSEs and analytical confinement , ArXiv:1809.07644

V.S. nakanishi integral representation for $qq\gamma$ vertex ArXiv

spectral representation does not work in QCD...

This problem has been circumvented only very recently and the QCD DSEs were solved in the entire Minkowski space providing the light pion, correct pion decay constant, correct width of neutral pion $pi_0 \to \gamma\gamma$ as well as a very simple explanation of quark confinement.

- Formalism of IR for untruncated vertices in DSEs (generalization Gauge Technique Salam 63, Strathdee 67, Delbourgho , West 67)

- solution rely on tricks: departure from standards and arrival back in certain limit ... trick with contours

- model was tuned on VHP (in addition to m_π)

$$S(p) = \int_\Gamma dx \frac{\rho(x)}{(\not p - x)}$$

$qq\gamma$ vertex

$$G^{\mu}(q;Q) \quad = \quad \int_{\Gamma} dx \frac{1}{\not{q}_{-} - x} \left(\gamma^{\mu} \rho(x) + \frac{\rho_1(x) T_1^{\mu}}{q^2 + Q^2/4 - x^2 + \varepsilon} \right) \frac{1}{\not{q}_{+} - x}$$

1. Gauge Technique & Nakanishi IR ? Y -easy
2. QCD DSEs transormable into NIR? Equation for Nakanishi weights?
3. solution ? Y - nontrivial new approach needed beacuse of analyticity

$$S_q(p) = \int_\Gamma dx \frac{\rho(x)}{(\not p - x)} \,.$$

$$G^\mu(q;Q) \quad = \quad \int_\Gamma dx \frac{1}{\not q_- - x} \left(\gamma^\mu \rho(x) + \frac{\rho_1(x) T_1^\mu}{q^2 + Q^2/4 - x^2 + \varepsilon} \right) \frac{1}{\not q_+ - x} \,,$$

2. QCD and NIR? ? Yes, at least in some truncation of the system!

2a) shown for the S_q in LRA trunc. if the kernel satisfies NIR

already solved numerically V.S. hep-ph

2b) vertex study $G^\mu(q;Q)$ DSE converted for the N.w. for GT $+\gamma_\mu$

ad 2) Quark gap equation in NIR formalism

$$S_q(p) = \int_{\Gamma} dx \frac{\rho(x)}{(\not p - x)} \, . \tag{2}$$

How to solve it ? 2steps precede (2)

Specify Γ (subtleties due to the unitarity of physical form factors) , convert DSE

solve DSE + fit it form BSE for the pion

ad 2) QCD & NIR -the model:

$$
\begin{aligned}
V(l) &= \gamma_\mu \times \gamma_\nu \left(-g^{\mu\nu} V_V(l) - \frac{4g^2}{3} \xi \frac{L^{\mu\nu}(l)}{l^2} \right) \ , \\
V_V(l) &= \frac{c_V (m_g^2 - \Lambda_g^2)}{(l^2 - m_g^2 + i\varepsilon)(l^2 - \Lambda_g^2 + i\varepsilon)} \ , \\
L^{\mu\nu}(l) &= l^\mu l^\nu / l^2 \ ,
\end{aligned}
$$

for which the quark Dyson-Schwinger equation can be written as

$$
\begin{aligned}
S^{-1}(q) &= \not{q} - m_q - \Sigma(q) \, , \\
\Sigma(q) &= i \int \frac{d^4 k}{(2\pi)^4} S(k) V(k-q)
\end{aligned}
\qquad (3)
$$

parameters consistent with pion:

$$
[\alpha = \frac{g_v^2}{4\pi} = 22.62, \frac{g^2 \xi}{4\pi} = 2.13; \frac{m_g}{\Lambda_g} = 0.516; m_g = \sqrt{2} m_\pi \simeq m_\pi]
$$

Confinement of quarks

No real in pole S , but bump. position $M \simeq 265 MeV$ and $M(0) \simeq 235 MeV$

remind the ABS result/fit completely different $\frac{m_g^2}{\Lambda_g^2} = 1/900$

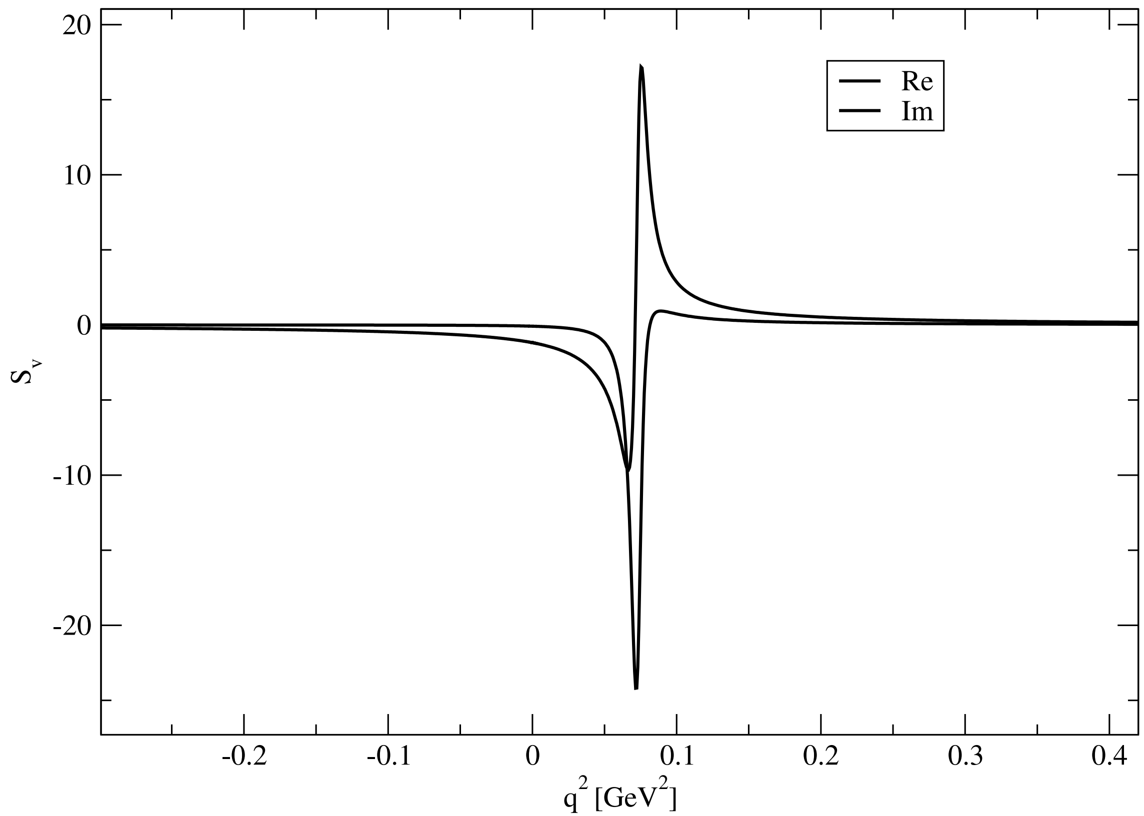

Figure 5: The quark propagator function S_v

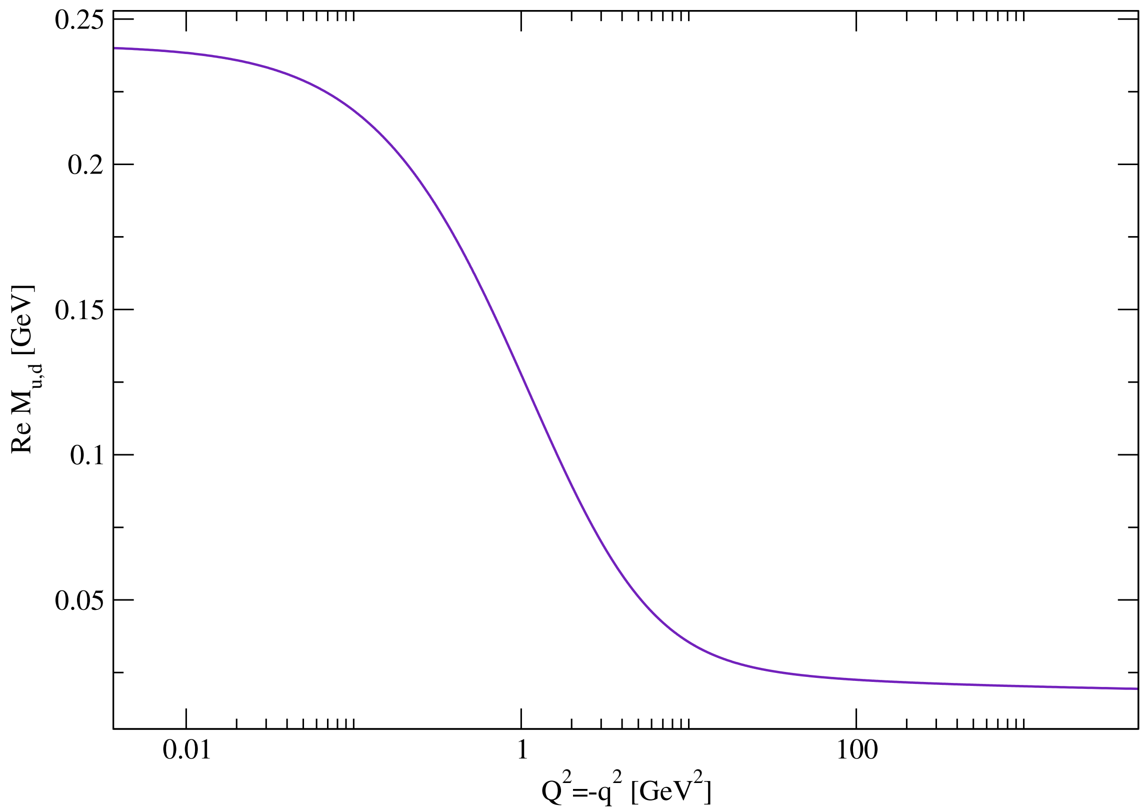

Figure 6: Typical look known from the E studies but solved in M

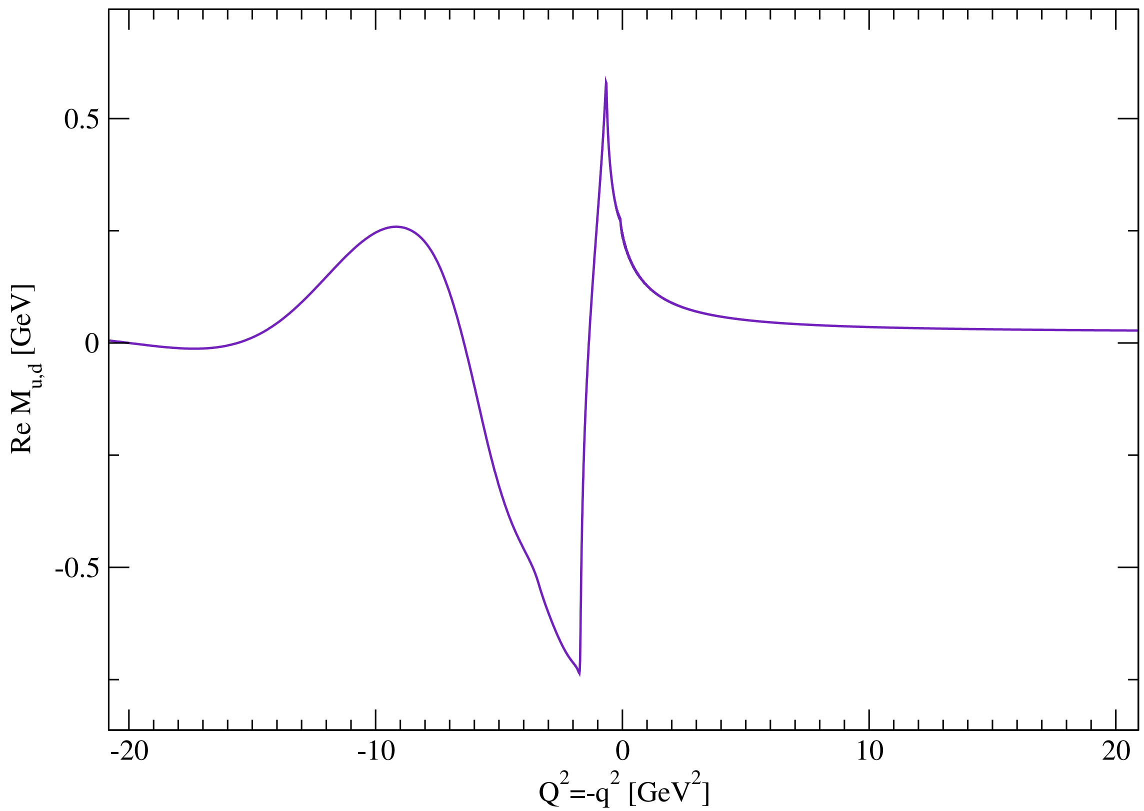

Figure 7: The same as in previous figure, but larger piece of Minkowski space shown.

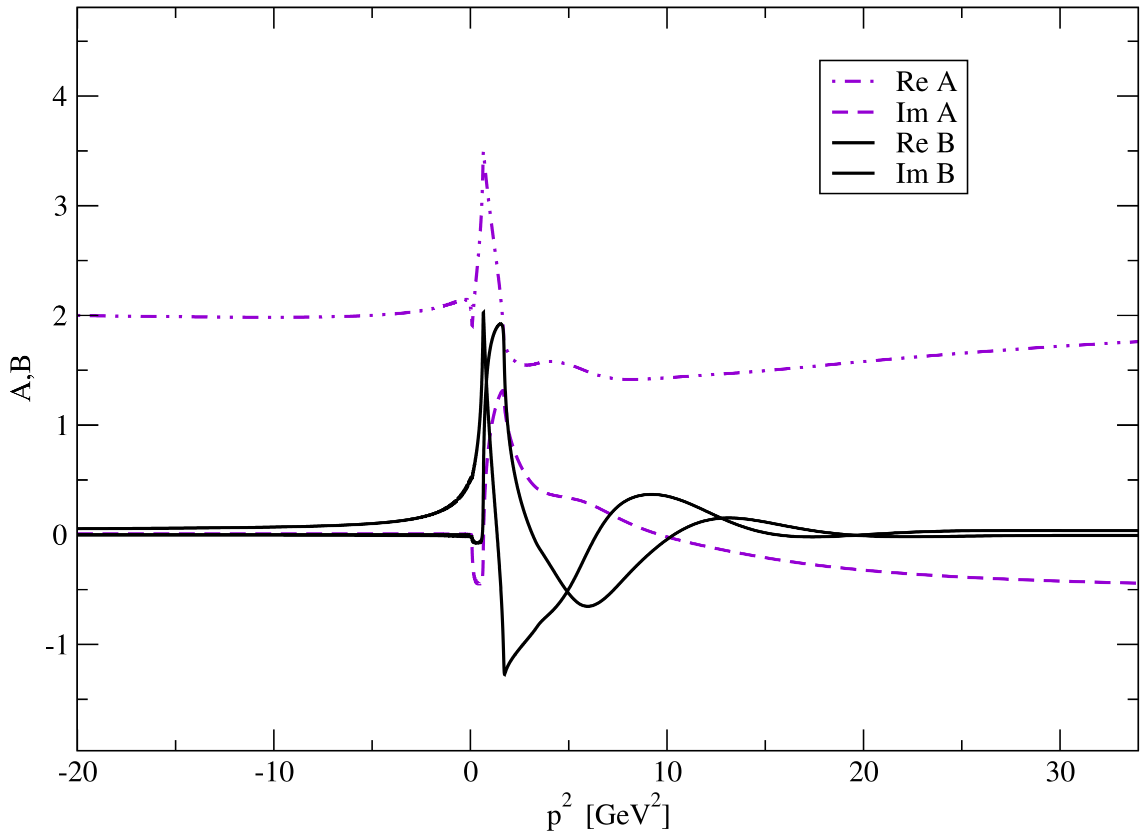

Figure 8: Conventional look of the quark function A and B

Application to Hadron Vacuum Polarization (just GT Ansatz, T not yet incorporated)

$$\Pi_h^{\mu\nu}(s) = -ie^2 N_c \sum_q e_q Tr \int \frac{d^4k}{(2\pi)^4} \Gamma_q^{\mu}(k-q,k) S_q(k) \gamma^{\nu} S_q(k-q) \quad (4)$$

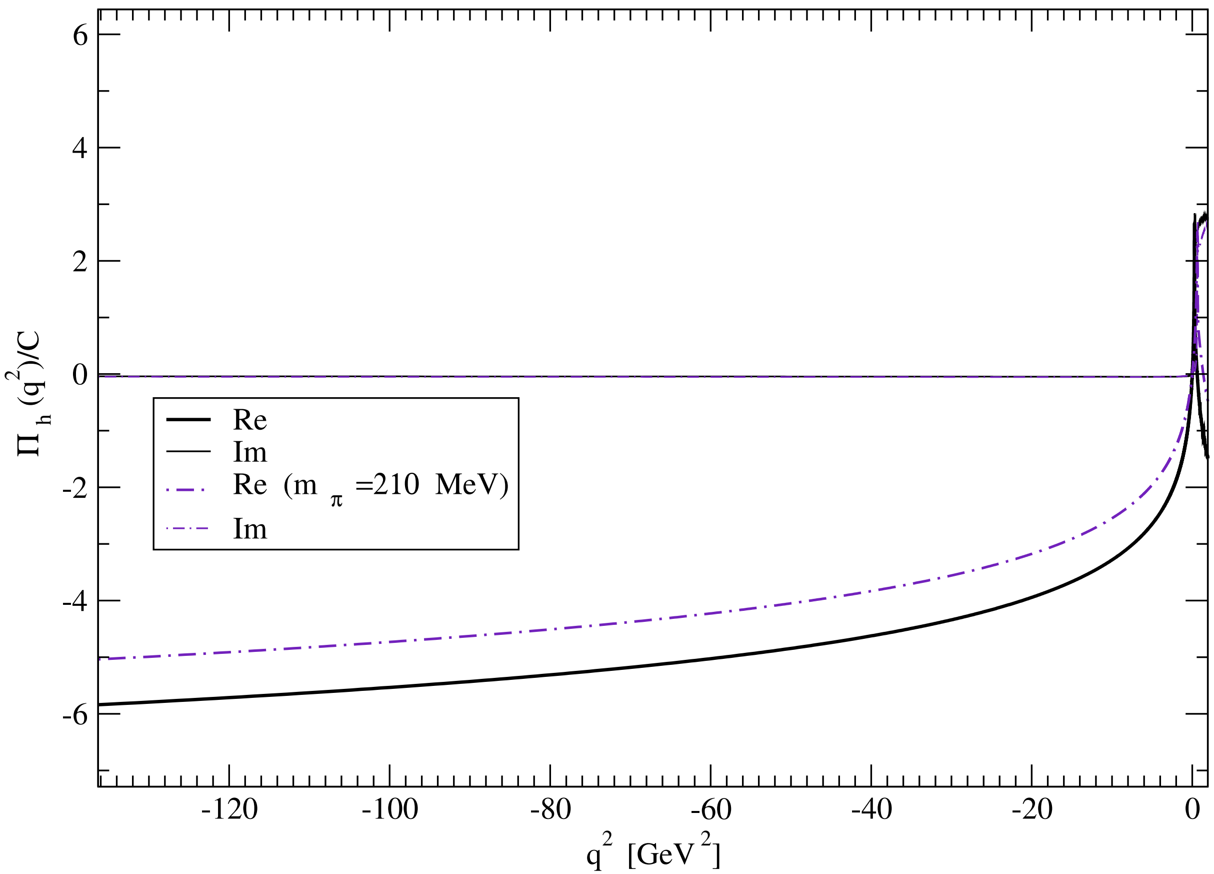

Figure 9: VHP zoomed in the spacelike domain of momenta. The one has $m_\pi = 140 MeV$ (solid one) and rescaled one corresponds with $m_\pi = 210 MeV$

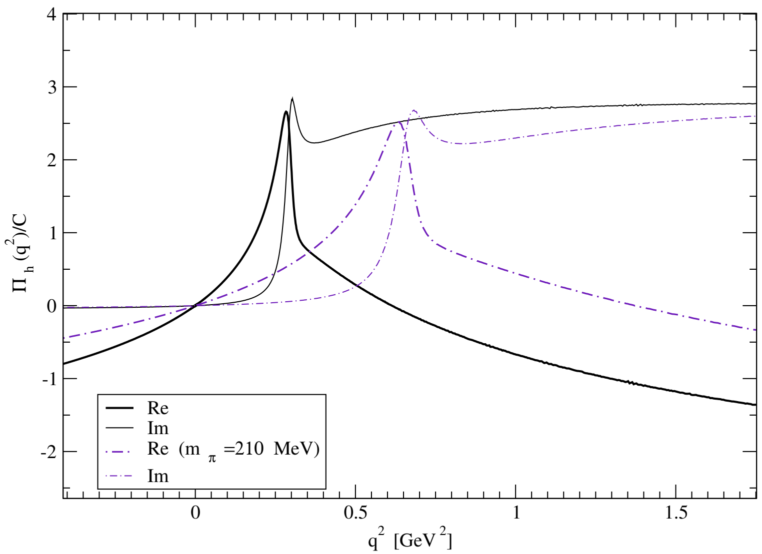

Figure 10: The function Π_h/C obtained via Gauge Technique with the constant C defined as $C = -40\alpha/(9\pi)$.

Point 3- Tuning the Analyticity !

reminder:

1. Gauge Technique & Nakanishi IR ? Y -easy

2. ARE QCD DSEs transormable into NIR? Equation for Nakanishi weights? Y- easy for S, hard for Γ

3. solution ? Y - nontrivial new approach required because of Unitarity/analyticity

Ad3:

getting solution in practice:

Empirical observation: QCD does not like spectral representation, when use it to find N.w. numerical convergence dies

NO GO for Lehmann R.

$$S_q(p) = \int_T^{\infty} do \frac{\not{p} g_v(o) - g_s(o)}{p^2 - o + i\varepsilon} \, .$$

Introduce un-physical auxiliary cut in the NIR for GFs (in all), which makes DSEs for NWs convergent and minimize it during numerical solution.

The appropriate contour is represented by infinite axis cross in the complex plane of variable x, i.e. $\Gamma = \Gamma_1 + \Gamma_2$ where $\Gamma_1 : Rex$, and $\Gamma_2 : Imx$. Making the substitution $o = x^2$ leads to the following appearance of two common functions

$$
\begin{aligned}
g_v(o) &= \frac{\rho(\sqrt{o}) + \rho(-\sqrt{o})}{2\sqrt{o}} \\
g_s(o) &= \frac{\rho(\sqrt{o}) - \rho(-\sqrt{o})}{2}
\end{aligned}
$$

defined in the R^+ domain of variable o, which gives plus part of the quark propagator

$$S_+(p) = \int_0^\infty do \frac{\not{p} g_v(o) + g_s(o)}{p^2 - o + i\varepsilon}$$

Considering the contribution from "unphysical " contour Γ_2 one gets the above functions defined for $o < 0$ as superposition of ρ defined at two branches of square root function of x:

$$g_v(o) = i\frac{\rho(\sqrt{-o}) - \rho(-\sqrt{-o})}{2\sqrt{-o}}$$
$$g_s(o) = -\frac{\rho(\sqrt{-o}) + \rho(-\sqrt{-o})}{2},$$

which gives us the auxiliary function S_- and completes the quark propagator $S = S_- + S_+$

$$S(p) = \int_{-\infty}^{\infty} do \frac{\not{p} g_v(o) + g_s(o)}{p^2 - o + i\varepsilon}\,. \tag{5}$$

the introduction of S_- modes should be regarded as an auxiliary step and this function is subject of minimization when the system is solved numerically.

VHP was calculated simultaneously with DSE and BSE

Minimize S_- by using free parameters associated with truncation of DSEs system

If minimization is not used, one gets stable solutions, which violates Unitarity of S-matrix

Results were obtained by fitting the pion BSE with simultaneous Minimization of S_- modes.

Conclusion of QCD part

Desired analytical property (Relations od Unitarity between amplitudes and cross sections) of physical form factor becomes a part of solution of DSEs system in QCD in Minkowski space. Tuning parameters, which determine details of functions which were truncated away from DSEs system, was used to recover standard analytical property of VHP, which has cut at timelike axis of p^2

"Pion properties determine the VHP and vice versa"

Improvement in future is expected.