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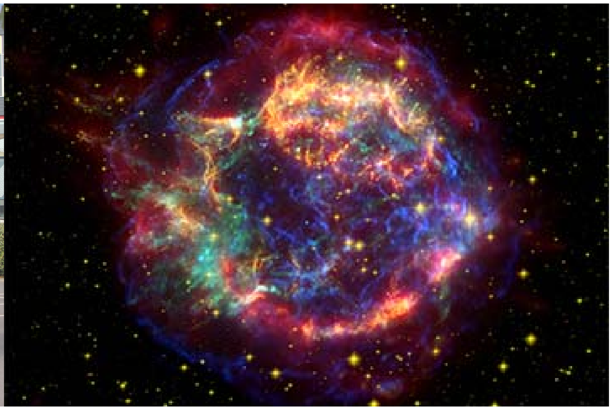
Transport in neutron stars



see recent review: A. Schmitt and P. Shternin, arXiv:1711.06520 [astro-ph.HE]

other reviews: N. Chamel and P. Haensel, Living Reviews in Relativity 11, 10 (2008) (crust)
D. Page and S. Reddy, in "Neutron Star Crust", 1201.5602 (inner crust)
D. G. Yakovlev *et al.*, Phys. Rep. 354, 1 (2001) (neutrino emissivity)
A. Sedrakian, Prog. Part. Nucl. Phys. 58, 168 (2007) (many-body techniques)
M. G. Alford *et al.*, Rev. Mod. Phys. 80, 1455 (2008) (quark matter)
A. Y. Potekhin, J. A. Pons, and D. Page, Space Sci. Rev. 191, 239 (2015) (cooling)

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Transport in neutron stars

- What is transport? Why are transport properties interesting?
- Specific examples: shear viscosity & bulk viscosity of dense matter in neutron star cores
- Open questions and future directions

General remarks

”Transport”: transfer of conserved quantities
(energy, momentum, particle number, electric charge, ...)
from one region to another due to non-equilibrium
(temperature gradient, non-uniform chemical composition, ...)

- transport properties usually computed from particle collisions: energy & momentum exchange or flavor-changing processes (strong coupling: quasiparticle picture not valid)
- transport at small T typically dominated by fermions (electrons, muons, neutrinos, neutrons, protons, hyperons, quarks)
- if fermion contribution suppressed (e.g., Cooper pairing), bosons may dominate (pions, kaons, lattice phonons, superfluid phonons)

What do we learn from calculating transport properties of neutron star matter?

- Phenomenology of neutron stars

  → see next page

- Theoretical aspects
  - analogies/differences to heavy-ion transport (”hot QCD vs. dense QCD”)
  - conceptual questions of (general) relativistic hydrodynamics (e.g., second order hydro)
  - strong coupling effects:
    of practical relevance (e.g., modified Urca rate) and
    of theoretical interest (beyond quasiparticle picture?
    compare to AdS/CFT, see heavy ions?)
  - superfluid (multifluid) transport,
    transport in the presence of vortices (quantum turbulence?)

Transport and phenomenology

Phenomenon	**Transport properties**
oscillatory modes (r-modes)	shear & bulk viscosity
pulsar glitches	superfluid transport (vortex pinning)
thermal radiation	heat transport in outermost layers
cooling	neutrino emissivity, heat conductivity
magnetic field evolution	magnetohydrodynamics electrical & thermal conductivities
crust disruption (accretion, magnetar flares)	transport properties of the crust nuclear reactions (”deep crustal heating”)
core-collapse supernovae	neutrino transport, neutrino-nucleus reactions
neutron star mergers	high-temperature transport (viscous) magnetohydrodynamics

Basic transport coefficients (page 1/2)

- start from Boltzmann equation

$$\frac{\partial f}{\partial t}+\boldsymbol{u}\cdot\frac{\partial f}{\partial \boldsymbol{x}}+\boldsymbol{F}\cdot\frac{\partial f}{\partial \boldsymbol{p}}=-\int_{\boldsymbol{p}_1}\int_{\boldsymbol{p}'}\int_{\boldsymbol{p}_1'}W(\boldsymbol{p},\boldsymbol{p}_1;\boldsymbol{p}',\boldsymbol{p}_1')[ff_1(1-f')(1-f_1')-(1-f)(1-f_1)f'f_1']$$

- multiply by 1, $\mathbf{p}$, $\frac{p^2}{2m}$ and integrate over $\mathbf{p}$ → hydrodynamic eqs

$$\frac{\partial \rho}{\partial t}+\nabla\cdot(\rho\boldsymbol{v}) = 0$$

continuity equation

$$\frac{\partial s}{\partial t}+\nabla\cdot\left(s\boldsymbol{v}+\frac{\boldsymbol{j}_T}{T}\right) = -\frac{\pi_{ji}\partial_j v_i+\boldsymbol{j}_T\cdot\nabla T/T}{T}$$

entropy production

$$\frac{\partial v_i}{\partial t}+(\boldsymbol{v}\cdot\nabla)v_i = -\frac{\partial_i P}{\rho}+\frac{F_i}{m}-\frac{\partial_j\pi_{ji}}{\rho}$$

Navier-Stokes equation

- dissipative terms

$$\begin{aligned}\boldsymbol{j}_T &= -\kappa\nabla T\\ \pi_{ij} &= -\eta\left(\partial_i v_j+\partial_j v_i-\frac{2}{3}\delta_{ij}\nabla\cdot\boldsymbol{v}\right)-\zeta\delta_{ij}\nabla\cdot\boldsymbol{v}\end{aligned}$$

with thermal conductivity κ, shear viscosity η, bulk viscosity ζ

Basic transport coefficients (page 2/2)

- more generally:
  add dissipative terms systematically order by order in derivatives

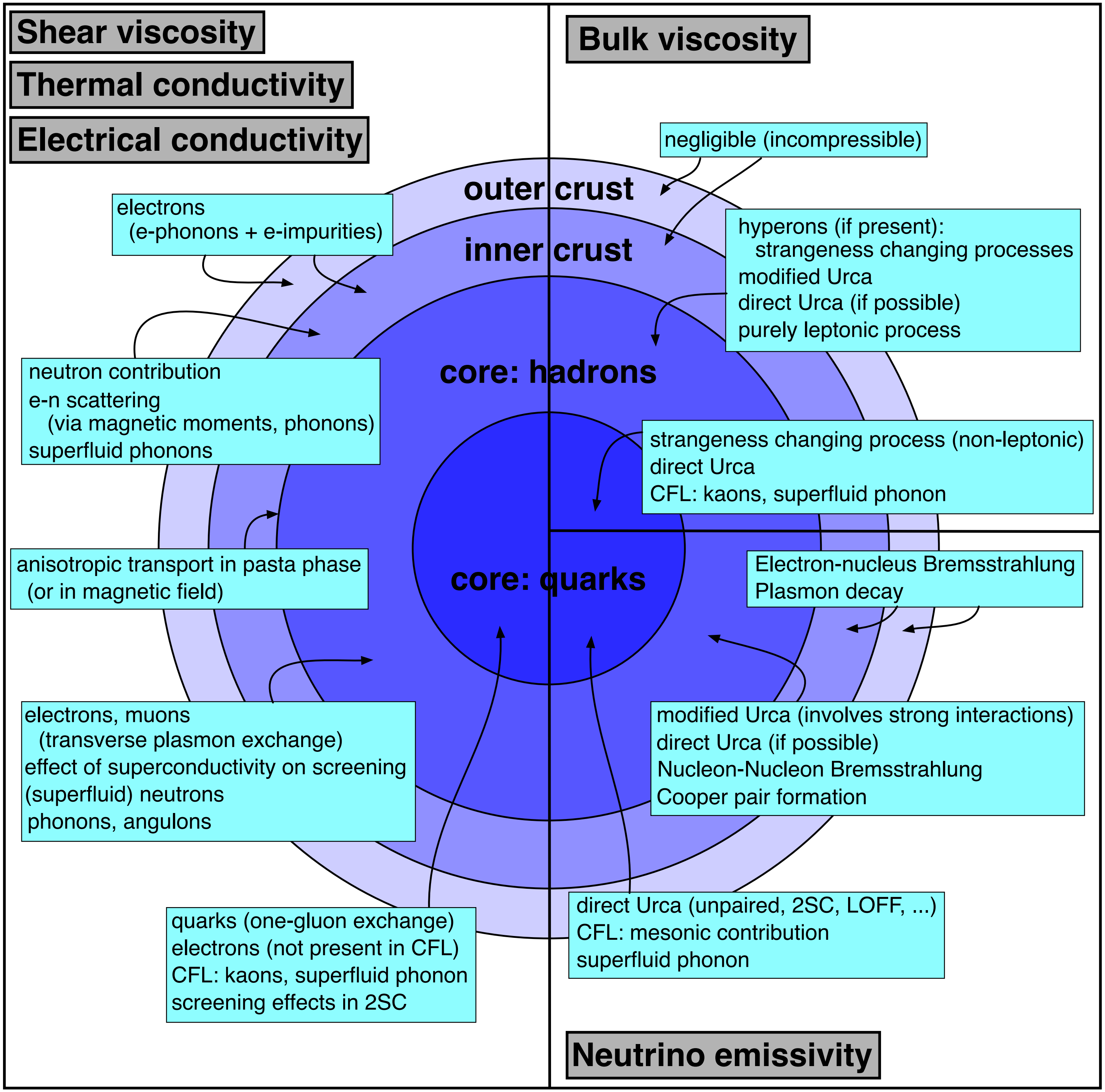
  $$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - Pg^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu + \tau^{\mu\nu} + \ldots , \quad j^\mu = nu^\mu + J^\mu + \ldots$$

  where q^μ, $\tau^{\mu\nu}$, $J^\mu \sim \mathcal{O}(\partial)$, and consider conservation eqs.

  $$\partial_\mu T^{\mu\nu} = \partial_\mu j^\mu = 0$$

- complications in neutron star context:
  - (general) relativistic effects
  - external force → Lorentz force (magnetic field)
  - two-fluid (multi-fluid) transport
    (electron-ion in the crust, *npe* matter in the core)
  - superfluid (two-fluid) transport → more transport coefficients

- general recipe: compute transport coefficients from some microscopic theory (e.g., Boltzmann eq) and insert into hydro eqs (if sufficiently close to equilibrium)



Shear viscosity

Thermal conductivity

Electrical conductivity

Bulk viscosity

Neutrino emissivity

electrons
(e-phonons + e-impurities)

neutron contribution
e-n scattering
(via magnetic moments, phonons)
superfluid phonons

anisotropic transport in pasta phase
(or in magnetic field)

electrons, muons
(transverse plasmon exchange)
effect of superconductivity on screening
(superfluid) neutrons
phonons, angulons

quarks (one-gluon exchange)
electrons (not present in CFL)
CFL: kaons, superfluid phonon
screening effects in 2SC

negligible (incompressible)

hyperons (if present):
strangeness changing processes
modified Urca
direct Urca (if possible)
purely leptonic process

strangeness changing process (non-leptonic)
direct Urca
CFL: kaons, superfluid phonon

Electron-nucleus Bremsstrahlung
Plasmon decay

modified Urca (involves strong interactions)
direct Urca (if possible)
Nucleon-Nucleon Bremsstrahlung
Cooper pair formation

direct Urca (unpaired, 2SC, LOFF, ...)
CFL: mesonic contribution
superfluid phonon

Transport in the core: two examples

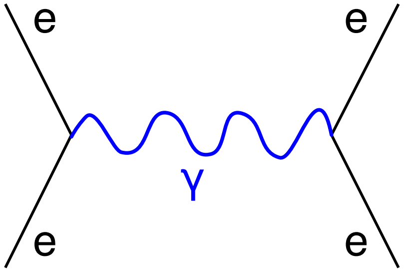
- Shear viscosity η: lepton contribution in nuclear matter
  unpaired and 2SC quark matter

- Bulk viscosity ζ: resonance phenomenon
  results for nuclear & quark matter

Shear viscosity (page 1/2)

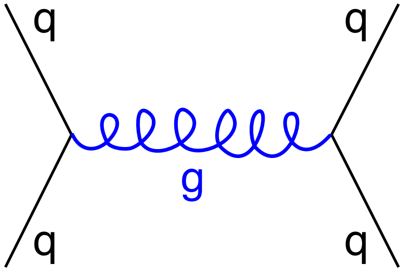
Nuclear matter:

ee (or $\mu\mu$) scattering (also nn)



Quark matter:

qq scattering (also ee)



- need photon (gluon) propagator in medium
  photon in dense nuclear matter: S. Stetina, E. Rrapaj and S. Reddy, PRC 97, 045801 (2018)
- scattering dominated by transverse plasmon exchange
  H. Heiselberg, G. Baym, C. J. Pethick and J. Popp, NPA 544, 569C (1992)
- transverse plasmons dynamically screened (Landau damping)

$$\eta_{ee} \propto \frac{1}{T^{5/3}} \sim 10^{20} \frac{\mathrm{g}}{\mathrm{cm\,s}}$$

Shternin, Yakovlev, PRD 78, 063006 (2008)

$$\eta_{qq} \propto \frac{1}{T^{5/3}} \sim 10^{15} \frac{\mathrm{g}}{\mathrm{cm\,s}}$$

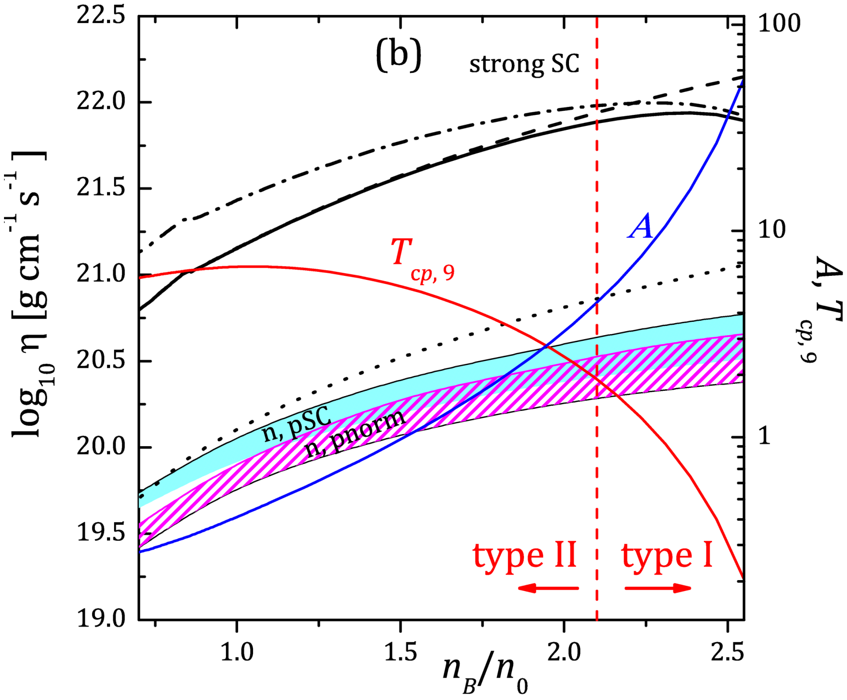
Heiselberg, Pethick, PRD 48, 2916 (1993)

→ different behavior from Fermi liquid $\eta \propto T^{-2}$

Shear viscosity (page 2/2)

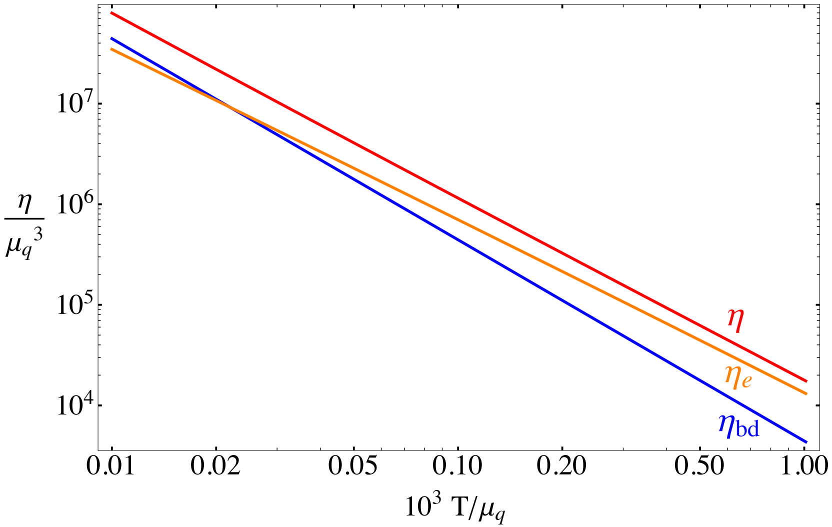
- Superconductivity affects η, even when scatterers remain unpaired

- Nuclear matter: e scatter via γ properties of γ changed by $\langle pp \rangle$
- Meissner effect: static magnetic screening (dynamic screening relevant for larger T)



P. Shternin, PRD 98, 063015 (2018)

- 2SC quark matter: blue q scatter via g properties of g changed by red, green $\langle qq \rangle$
- g and γ mix
A. Schmitt *et al.*, PRD 69, 094017 (2004)
- small T: unpaired bd dominates (interacts only via screened gauge bosons)



M. G. Alford, H. Nishimura, A. Sedrakian, PRC 90, 055205 (2014)
$[\mu_q^3 \sim 10^{13}\,\mathrm{g\,cm^{-1}s^{-1}}]$

Bulk viscosity (page 1/3): definition

- bulk viscosity $\zeta(\omega)$ = dissipative response to compression and expansion $V(t) = V_0 + \delta V \cos \omega t$,

$$\zeta(\omega) = 2\langle \dot{E} \rangle \left(\frac{V_0}{\delta V} \right)^2 \frac{1}{\omega^2}$$

with the dissipated power in an oscillation period $\tau = 2\pi/\omega$

$$\langle \dot{E} \rangle = -\frac{1}{\tau V_0} \int_0^\tau dt\, P(t) \frac{dV}{dt} = \frac{B}{C} \gamma \langle \delta\mu(t) \delta V(t) \rangle$$

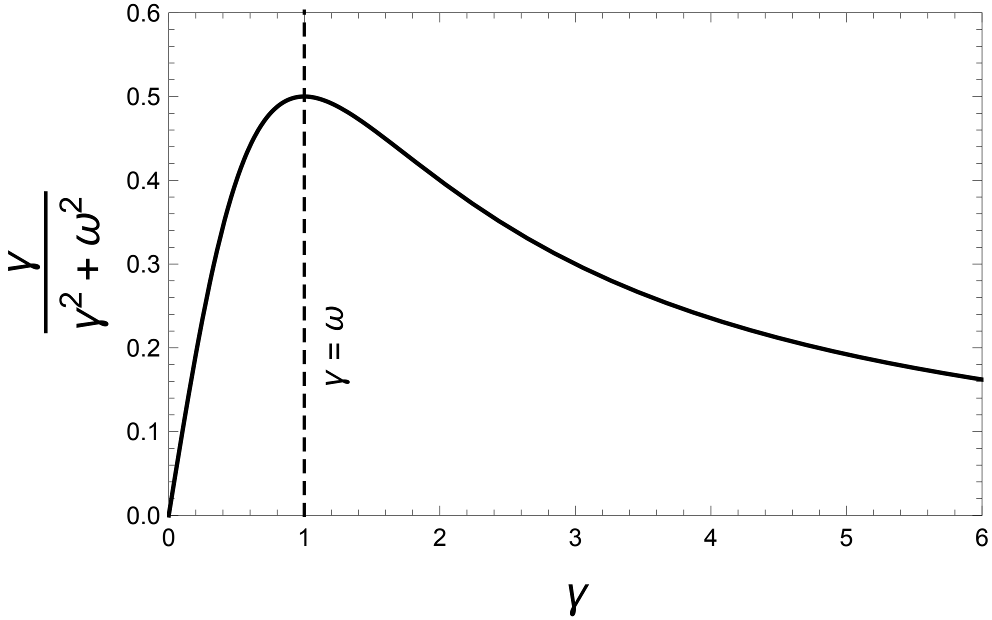
- B, C susceptibilities in equilibrium, and, e.g., $\delta\mu = \mu_p + \mu_e - \mu_n$
- neutrons produced per time and volume in $p+e \to n+\nu_e$ approximately $\gamma\, \delta\mu/C$

$$\boxed{\zeta(\omega) = \frac{B^2}{C} \frac{\gamma}{\gamma^2 + \omega^2}}$$

[in above hydro equations, $\zeta = \zeta(\omega = 0)$]

Bulk viscosity (page 2/3): resonance phenomenon

- ζ maximal for $\gamma = \omega$

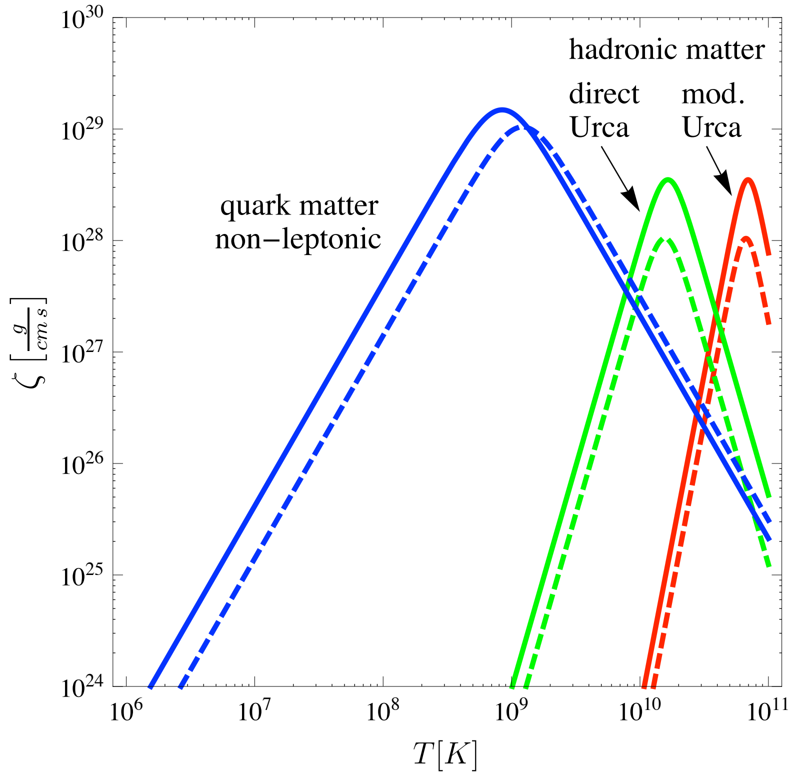


- need microscopic rate γ to be of the order of star oscillations ω
  $\rightarrow$ ζ dominated by electroweak interactions
- γ is typically monotonically increasing with T
  $\rightarrow$ maximum of ζ at a certain T

Bulk viscosity (page 3/3): hadronic and quark matter

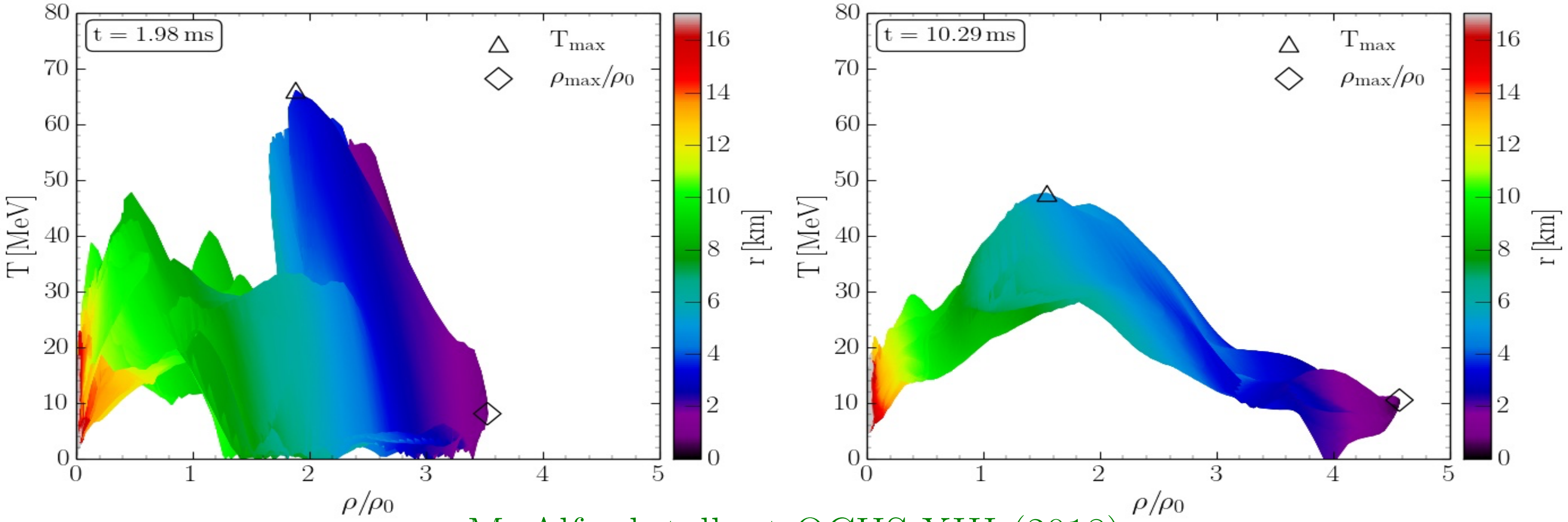
hadronic matter	quark matter
direct Urca, $p+e\rightarrow n+\nu_e$ **modified Urca**	**non-leptonic process** $u+d\leftrightarrow u+s$
hyperons, e.g.: $n+n\leftrightarrow p+\Sigma^-$	Urca, $u+e\rightarrow d+\nu_e$
nn, pp pairing: rates suppressed	CFL: $K^0\leftrightarrow\phi+\phi$

- nonleptonic processes: larger phase space
- uncertainties from strong interactions in susceptibilities



M. G. Alford, *et al.*, J. Phys. G 37, 125202 (2010)

Open questions/future directions (page 1/3): transport in neutron star mergers



M. Alford, talk at QCHS XIII (2018)

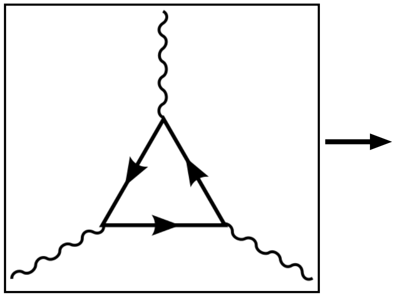
- transport at relatively large T → neutrino trapping?
- importance of dissipative coefficients?

  viscosity, heat flow: M. G. Alford *et al.*, PRL 120, 041101 (2018)

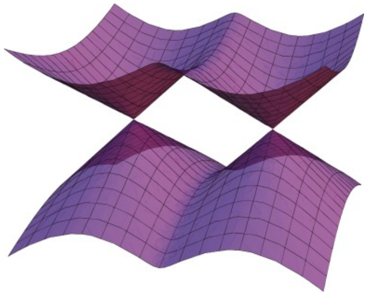
  electrical conductivity: A. Harutyunyan *et al.*, arXiv:1803.09215 [astro-ph.HE]

Open questions/future directions (page 2/3): anomalous transport

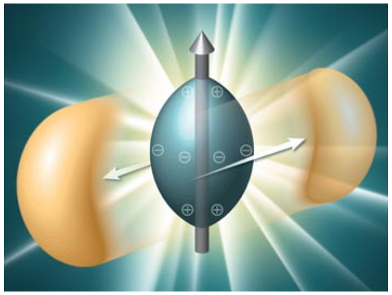
- anomalous transport for systems with chiral imbalance ($N_R > N_L$)
  → macroscopic manifestation of quantum anomaly ("chiral anomaly")



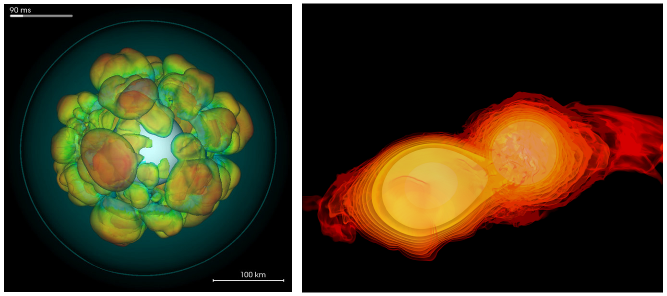
quantum anomaly



Weyl semi-metal: CME



heavy-ion collision: CME (?)



chiral transport in supernovae, NS mergers?

- chiral magnetic effect (CME)
  Weyl semi-metals Q. Li *et al.*, Nature Physics 12, 550 (2016)
  heavy-ion collisions D. E. Kharzeev *et al.*, Prog. Part. Nucl. Phys. 88, 1 (2016)

- chiral magnetohydrodynamics for electrons in core-collapse supernovae due to $p + e_L \to n + \nu_L$: chiral magnetic instability & inverse energy cascade
  P. Pavlović, N. Leite and G. Sigl, PRD 96, 023504 (2017)
  Y. Masada, K. Kotake, T. Takiwaki and N. Yamamoto, arXiv:1805.10419 [astro-ph.HE]

→ chiral hydrodynamics for neutrinos ("chiral vortical effect")
→ relevant for neutron star mergers?

Open questions/future directions (page 3/3)

- uncertainties in input from strongly coupled physics (modified Urca, shear viscosity involving nucleons, ...)
- anisotropic transport: transport in a magnetic field,
  transport in "nuclear pasta"
- superfluid transport: vortices, magnetic flux tubes,
  nonzero temperature
- cross-disciplinary approaches
  - critical velocities in two-component (super)fluids
    A. Haber, A. Schmitt and S. Stetina, PRD 93, 025011 (2016)
    cold atoms: M. Delehaye *et al.*, PRL 115, 265303 (2015)
  - simulation of pulsar glitches with cold atoms?
  - explore analogies to superfluid helium (e.g., vortex interfaces)
    V. Graber, N. Andersson, and M. Hogg, Int. J. Mod. Phys. D26, 1730015 (2017)
  - transport in heavy-ion collisions
    → future experiments at large μ at FAIR and NICA