ASYMMETRY OF THE NEUTRINO MEAN FREE PATH IN HOT NEUTRON MATTER UNDER STRONG MAGNETIC FIELDS

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PHAROS WG1-WG2 Meeting

Coimbra, September 26th-28th 2018

The talk in few words

- ✧ Study of the neutrino mean free path in hot neutron matter under the presence of strong magnetic fields.

- ✧ Polarized neutron matter described within the non-relativistic BHF approach using the Av18 NN + UIX NNN. Explicit expressions of the σ/V for the scattering of a neutrino from spin up and spin down neutrons are derived from Fermi Golden rule.

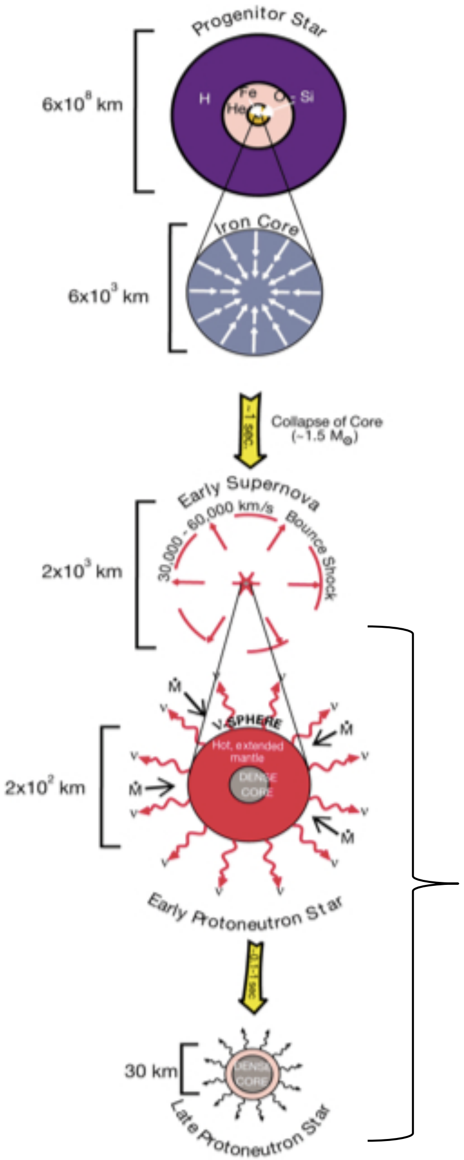
- ✧ Strong dependence of the mean free path on the angle of the incoming neutrino.

In collaboration with: Julio Torres Patiño & Eduardo Bauer (La Plata, Argentina)

For details see:

arXiv: 1809.00688

Neutrinos, SN & NS

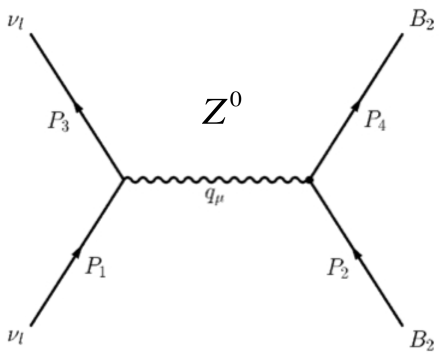


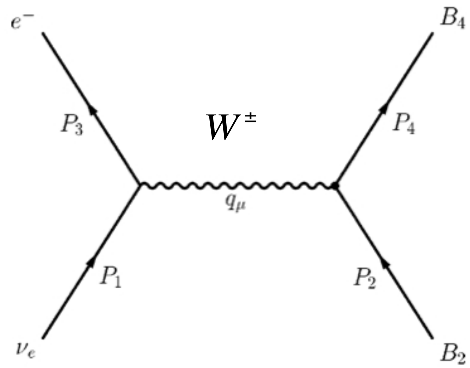
Neutrinos play a crucial role in the physics of supernova, in the early evolution of neutron stars & binary merger of compact objects

- ✧ Large number of neutrinos produced by e-capture during the collapse of the pre-supernova core. Most of the initial gravitational binding energy is stored in neutrinos
- ✧ λ_ν decreases as the radius of the neutron star shrinks from ~ 100 km to ~ 10 km becoming smaller than the NS radius ⟶ *neutrino trapping* ⟶ strong influence on the overall properties of hot & lepton-rich newborn neutron star, substantially different from the cold & deleptonized one.
- ✧ Cooling of newly born NS driven first by neutrino emission from the interior
- ✧ Neutrino cross sections & emissivities fundamental inputs for SN simulations and cooling calculations can be affected by the presence of magnetic fields (e.g., asymmetric emission)

Neutrino Interactions with Matter

During their propagation in matter neutrinos can be:





Scattered via weak coupling with **baryon neutral currents**

$$L_{NC} = \frac{G_F}{\sqrt{2}} l_\mu^\nu j_Z^\mu$$

$$l_\mu^\nu = \frac{1}{2} \bar{\psi}_\nu \gamma_\mu (1-\gamma_5) \psi_\nu, \quad j_Z^\mu = \bar{\psi}_4 \gamma^\mu (c_\nu - c_A \gamma_5) \psi_2$$

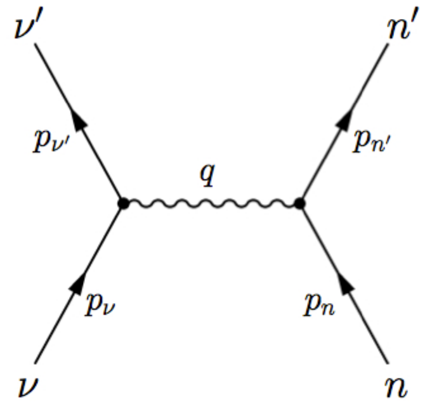
Absorbed via weak coupling with **baryon charged currents**

$$L_{CC} = \frac{G_F \cos\theta_c}{\sqrt{2}} l_\mu j_W^\mu$$

$$l_\mu = \bar{\psi}_l \gamma_\mu (1-\gamma_5) \psi_\nu, \quad j_W^\mu = \bar{\psi}_4 \gamma^\mu (g_\nu - g_A \gamma_5) \psi_2$$

Scattering Cross Section: Non-polarized Case

Here we consider only the ν-MFP in pure neutron matter ⟶ only scattering processes contribute



Using the Fermi Golden rule:

$$\frac{\sigma(p_\nu)}{V} = \int \frac{d\vec{p}_{\nu'}}{(2\pi)^3} \int \frac{d\vec{p}_n}{(2\pi)^3} \int \frac{d\vec{p}_{n'}}{(2\pi)^3} (2\pi)^4 \delta^{(4)}\left(p_\nu + p_n - p_{\nu'} - p_n\right) f_n(\vec{p}_n, T)(1 - f_{n'}(\vec{p}_{n'}, T)) \frac{\left|M_{\nu' n', \nu n}\right|^2}{16 E_\nu E_{\nu'} E_n E_{n'}}$$

where the square of the transition matrix is

$$\left|M_{\nu' n', \nu n}\right|^2 = \frac{1}{2} G_F l^{\mu\alpha} H_{\mu\alpha}$$

with

$$l^{\mu\alpha} = \left(\bar{\psi}_\nu \gamma^\mu \frac{1}{2}(1-\gamma_5)\psi_{\nu'}\right)\left(\bar{\psi}_{\nu'} \gamma^\alpha \frac{1}{2}(1-\gamma_5)\psi_\nu\right)$$

$$H_{\mu\alpha} = \left(\bar{\psi}_n (C_V + C_A\gamma_5)\gamma_\mu \psi_{n'}\right)\left(\bar{\psi}_{n'}\gamma_\alpha (C_V - C_A\gamma_5)\psi_n\right)$$

Scattering Cross Section: Spin polarized Case

In the presence of a magnetic field matter is polarized. In this case the hadronic tensor must be written as sum of spin up & down contributions

$$H_{\mu\alpha} = \frac{1-A}{2} H^{-}_{\mu\alpha} + \frac{1+A}{2} H^{+}_{\mu\alpha}, \quad A = \frac{\rho_+ - \rho_-}{\rho_+ + \rho_-}$$

with

$$H^{\pm}_{\mu\alpha} = \left(\bar{\psi}_n \frac{1}{2}(1+\gamma_5 \not{a}_{\pm})(C_V + C_A\gamma_5)\gamma_\mu \psi_{n'} \right)\left(\bar{\psi}_{n'}\gamma_\alpha (C_V - C_A\gamma_5)\frac{1}{2}(1+\gamma_5 \not{a}_{\pm})\psi_n \right)$$

Consequently:

$$\left| M_{\nu' n', \nu n} \right|^2 = \left| M^{-}_{\nu' n', \nu n} \right|^2 + \left| M^{+}_{\nu' n', \nu n} \right|^2$$

$$\frac{\sigma(p_\nu)}{V} = \frac{\sigma^-(p_\nu)}{V} + \frac{\sigma^+(p_\nu)}{V}$$

$$\frac{1}{\lambda(p_\nu)} = \frac{1}{\lambda^-(p_\nu)} + \frac{1}{\lambda^+(p_\nu)} \longrightarrow \lambda(p_\nu) = \frac{\lambda_-(p_\nu)\lambda_+(p_\nu)}{\lambda_-(p_\nu) + \lambda_+(p_\nu)}$$

Scattering Cross Section: Spin polarized case (non-relativistic limit)

Neutron matter is described here within the non-relativistic BHF approach. For consistency, neutrino scattering cross sections are evaluated in the non-relativistic limit

$$\left|M_{\nu'n',\nu n}\right|^2 = 16G_F^2(m_\pm^*)^2 E_\nu E_{\nu'}\Big((C_V^2+C_A^2)+(C_V^2-C_A^2)\cos\theta_{\nu\nu'}$$
$$\pm 2C_A\big((C_V+C_A)\cos\theta_\nu+(C_V-C_A)\cos\theta_{\nu'}\big)\Big)$$

we obtain

$$\frac{\sigma^\pm(p_\nu)}{V} = G_F^2\frac{1\pm A}{2}\int\frac{d\vec{p}_{\nu'}}{(2\pi)^3}\Big((C_V^2+C_A^2)+(C_V^2-C_A^2)\cos\theta_{\nu\nu'}$$
$$\pm 2C_A\big((C_V+C_A)\cos\theta_\nu+(C_V-C_A)\cos\theta_{\nu'}\big)\Big)S_\pm^0(q_0,\vec{q},T)$$

with

$$S_\pm^0(q_0,\vec{q},T)=\frac{1}{(2\pi)^2}\int d\vec{p}_n f_n^\pm(\vec{p}_n,T)(1-f_{n'}^\pm(\vec{p}_n+\vec{q},T))\delta\Big(q_0+E_n^\pm(\vec{p}_n,T)-E_{n'}^\pm(\vec{p}_n+\vec{q},T)\Big)$$

the structure function describing the response of neutron matter to the excitations induced by neutrinos

A few remarks

Note that in absence of the magnetic field we have $S^0_- = S^0_+ = S^0$ & $\sigma^- = \sigma^+$ so:

$$\frac{\sigma(p_\nu)}{V} = G_F^2 \int \frac{d\vec{p}_{\nu'}}{(2\pi)^3} \left(C_V^2 (1 + \cos\theta_{\nu\nu'}) + C_A^2 \left(3 - \cos\theta_{\nu\nu'}\right)\right) S^0(q_0, \vec{q}, T)$$

we recover the expression frequently found in the literature. Comparing it with that for the polarized case we see:

- The new terms due to spin polarization are those propotional to $\cos\theta_\nu$ and $\cos\theta_{\nu'}$
- Since the integral is done over $\mathbf{p}_{\nu'}$ the contribution from the term proportional $\cos\theta_{\nu'}$ is almost negligible but not zero because $S^0_{+/-}$ depends implicitly on $\cos\theta_{\nu'}$
- If the momentum of the incoming neutrino is perpendicular to the magnetic field then $\cos\theta_\nu = 0$ and one expects no appreciable difference with respect to the non-polarized case.

Analytical Expression of the Structure Function

An analytical expression of the structure function can be obtained is the momentum dependence of the s.p. energy is quadratic. This is not the case of the BHF approach, however, if we approximate it by

$$E^{\pm}(\vec{p},T) \approx \frac{|\vec{p}|^2}{2m^*_{\pm}} + U^{\pm}(\vec{p}=\vec{0},T), \quad \frac{m^*_{\pm}}{m} = \frac{|\vec{p}|}{m}\left(\frac{dE^{\pm}(\vec{p},T)}{dp}\right)^{-1}\Bigg|_{|\vec{p}|=p_{F_{\pm}}}$$

then

$$S^0_{\pm}(q_0,\vec{q},T) = \frac{1}{\pi}\frac{1}{1-e^{-q_0/T}}\frac{(m^2_{\pm})^2 T}{4\pi q}\ln\left(\frac{1+e^{(A_{\pm}+q_0/2)T}}{1+e^{(A_{\pm}-q_0/2)T}}\right)$$

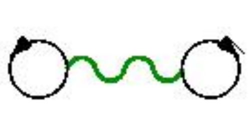
with $$A_{\pm} = \mu_{\pm} - \frac{m^*_{\pm} q_0^2}{2q^2} - \frac{q^2}{8m^*_{\pm}}$$

BHF approach of spin polarized neutron matter in a Magnetic Field

- Energy density

$$\varepsilon = \frac{1}{\Omega}\sum_{\sigma} f_{\sigma}(\vec{k},T)\left(\frac{\hbar^2 k^2}{2m} + \frac{1}{2}\mathrm{Re}\left[U_{\sigma}(\vec{k})\right]\right) - \kappa WB$$

$\kappa = -1.913\mu_N$ neutron anomalous magnetic moment



Infinite sumation of two-hole line diagrams

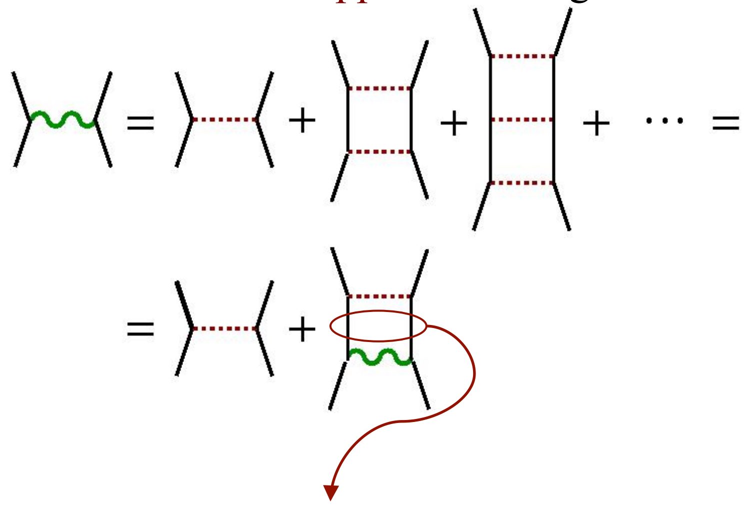
- Bethe-Goldstone Equation

$$G(\omega) = V + V\frac{Q}{\omega - E - E' + i\eta}G(\omega)$$

$$E_{\sigma}(k) = \frac{\hbar^2 k^2}{2m} + \mathrm{Re}\left[U_{\sigma}(k)\right] \mp \kappa B$$

$$U_{\sigma}(k) = \sum_{\sigma'} f_{\sigma'}(\vec{k},T)\left\langle \vec{k}\vec{k}'\right| G\left(\omega = E_{\sigma}(k) + E_{\sigma'}(k')\right)\left|\vec{k}\vec{k}'\right\rangle_{\mathcal{A}}$$

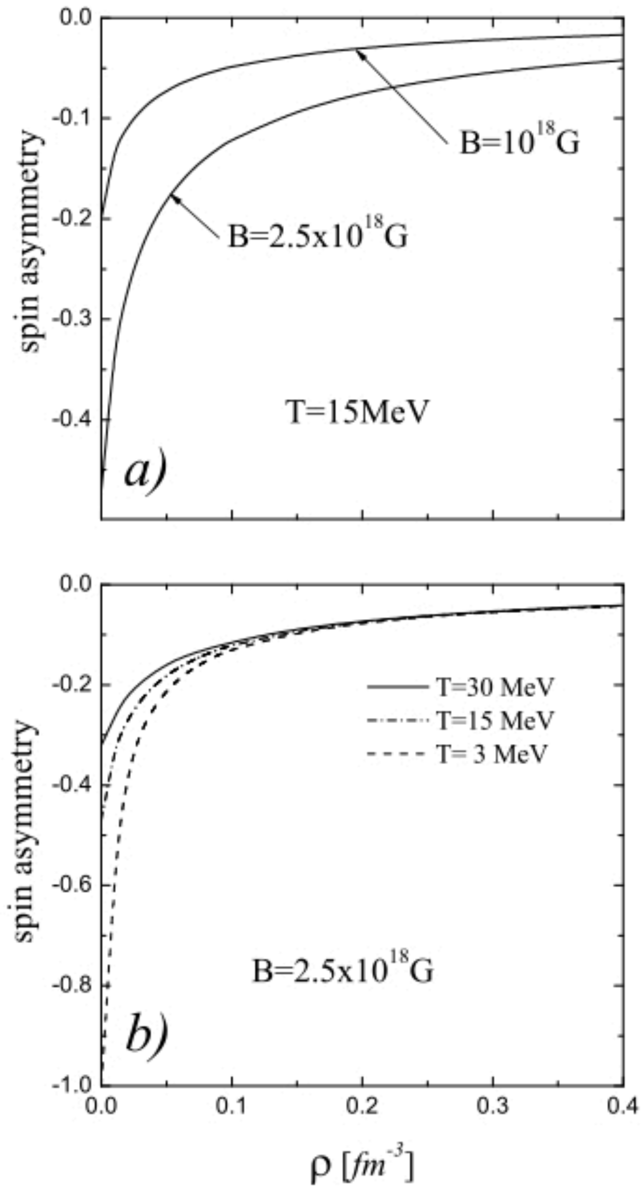
Partial sumation of pp ladder diagrams



✓ Pauli blocking

✓ Nucleon dressing

Spin Asymmetry



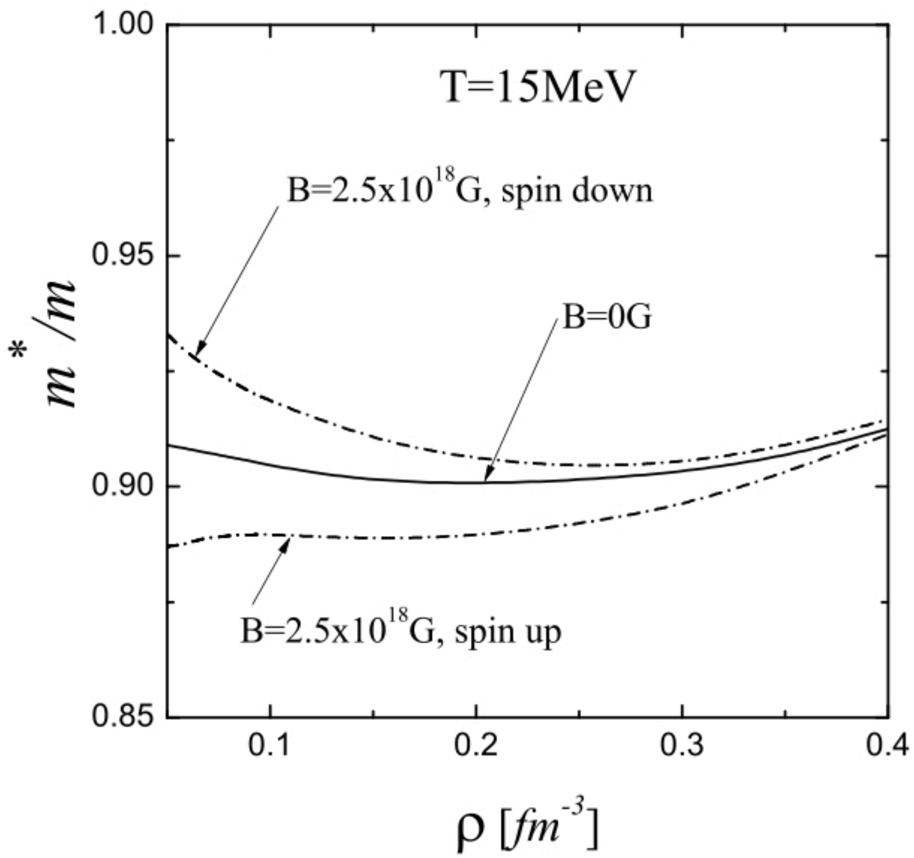
Polarization state of the system $\Rightarrow \frac{\partial(\varepsilon - TS)}{\partial A} = 0, \quad A = \frac{n_\uparrow - n_\downarrow}{n_\uparrow + n_\downarrow}$

- If B=0 the system is non-polarized (A = 0)
- For low densities & temperatures the system is expected to be totally polarized (A = -1) up to a given density & partially polarized above it with predominance of spin-down states
- A grows monotonously with density and would reach the non-polarized state asymptotically at high densities
- As one intuitively expects the increase of B (T) makes the system more (less) polarized

Neutron Effective Mass

$$\frac{m^*_\pm}{m} = \frac{|\vec{p}|}{m}\left(\frac{dE^\pm(\vec{p},T)}{dp}\right)^{-1}\Bigg|_{|\vec{p}|=p_{F_\pm}}$$

Evaluated here at the Fermi momentum of spin up & down neutrons

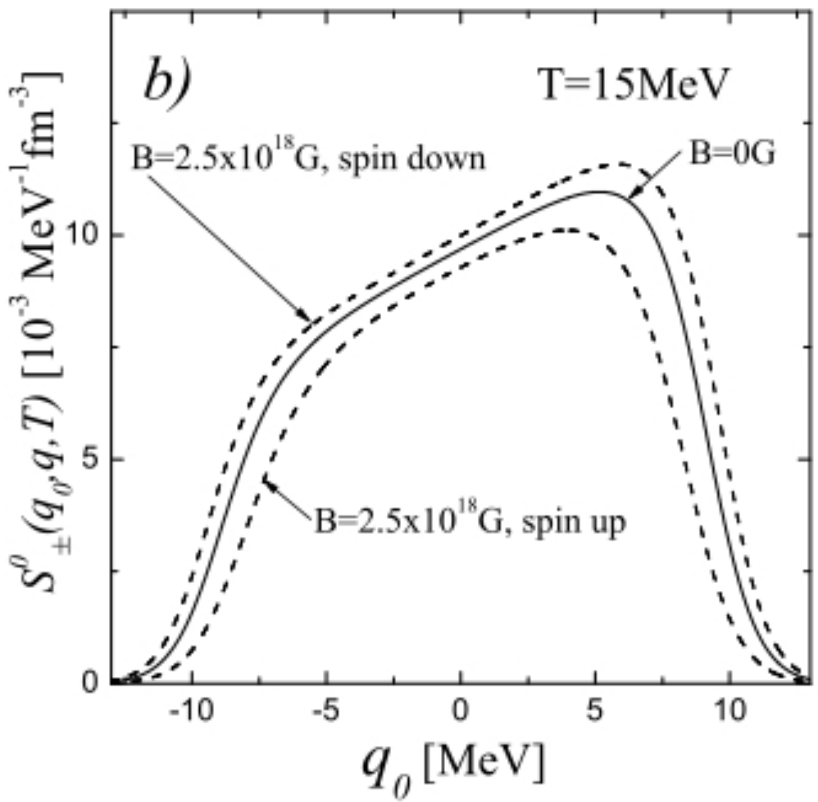


- As expected if B=0 $m_+ = m_-$
- B induces a splitting between m_+ & m_- with $m_+ < m_-$. This can be trace back to:
  - ✧ spin polarization dependence of the s.p. potential
  - ✧ the general issue that in an imbalanced two component fermionic system the most abundant one is less correlated

Structure Function

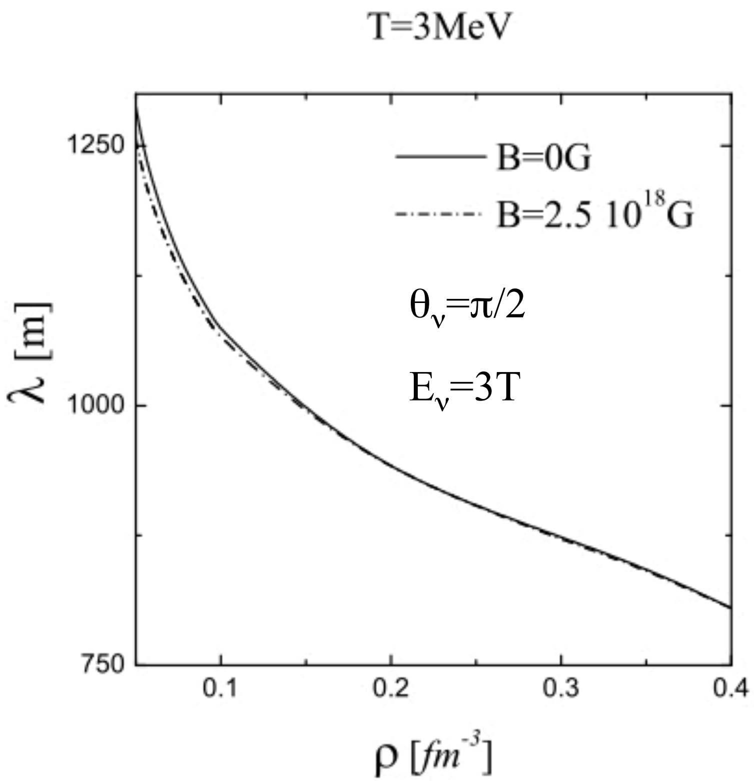
B induces a splitting between S_+ and S_- with $S_+ < S_-$. This can be understood from

$$S^0_\pm(q_0,\vec{q},T) = \frac{1}{\pi}\frac{1}{1-e^{-q_0/T}}\frac{(m^2_\pm)^2 T}{4\pi q}\ln\left(\frac{1+e^{(A_\pm+q_0/2)T}}{1+e^{(A_\pm-q_0/2)T}}\right)$$



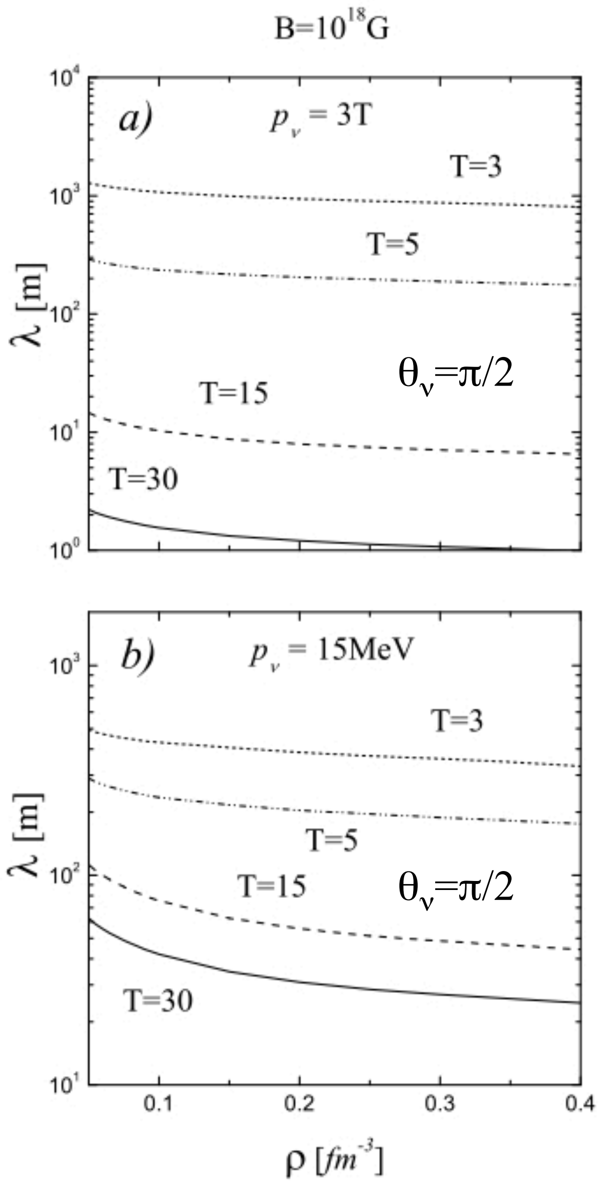
- Since B $\longrightarrow$ $m_+ < m_-$ clearly $S_+ < S_-$
- When B increases A becomes more negative $\longrightarrow$ the factor (1+A) [(1-A)] in the expression of σ_+ (σ_-) decrease (increase)
- An increase of B $\longrightarrow$ σ_+ (σ_-) decrease (increase). Since σ_- dominates the total cross section (ν mean free path) increases (decreases)

A first general remark on the ν mean free path



- If B=0 λ does not depend on the direction of the incoming neutrino
- The presence of B establishes a preferred direction & λ depends on the angle between the momentum of the incoming neutrino $\mathbf{p}_\nu$ and **B**
- However if $\theta_\nu=\pi/2$ λ is expected to be quite insensitive to B (see figure). The term proportional to $\cos\theta_\nu$ vanishes & remains only an small implicit dependence through the structure function
- The only remaining dependence on B is that of the structure function which is mostly appreciable in the low/medium density region where the spin asymmetry A is larger in absolute value

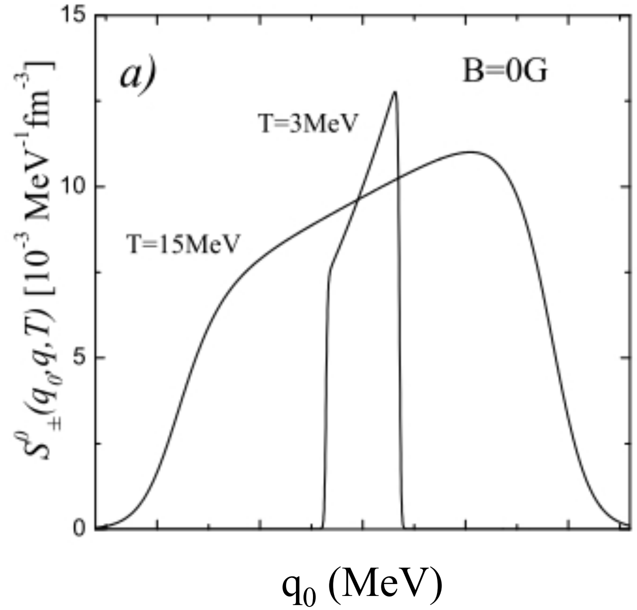
Temperature dependence of the ν mean free path



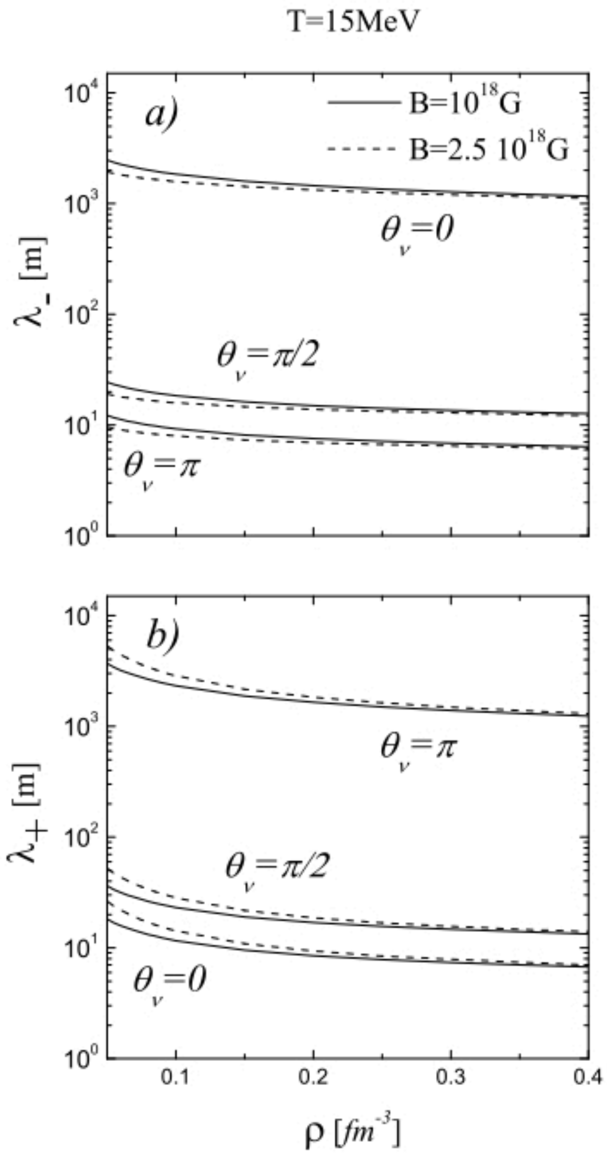
- For fixed T, the larger p_ν the smaller λ because the response of the system to the excitations induced by neutrinos is larger for larger values of p_ν.

- λ decreases dramatically when incresing T due to the temperature dependence of the structure function.

An increase of T leads to a much broader structure function (due to the increase of the phase space) and consequently to a larger (smaller) cross section (ν mean free path)



Partial contributions to the ν mean free path



- Both contributions vary by more than 2 orders of magnitude with θ_ν due to a combine effect of the explicit angular factors in

$$\frac{\sigma^{\pm}(p_\nu)}{V} = G_F^2 \frac{1 \pm A}{2} \int \frac{d\vec{p}_{\nu'}}{(2\pi)^3}\Big((C_V^2 + C_A^2) + (C_V^2 - C_A^2)\cos\theta_{\nu\nu'}$$
$$\pm 2C_A\big((C_V + C_A)\cos\theta_\nu + (C_V - C_A)\cos\theta_{\nu'}\big)\Big) S_{\pm}^0(q_0, \vec{q}, T)$$

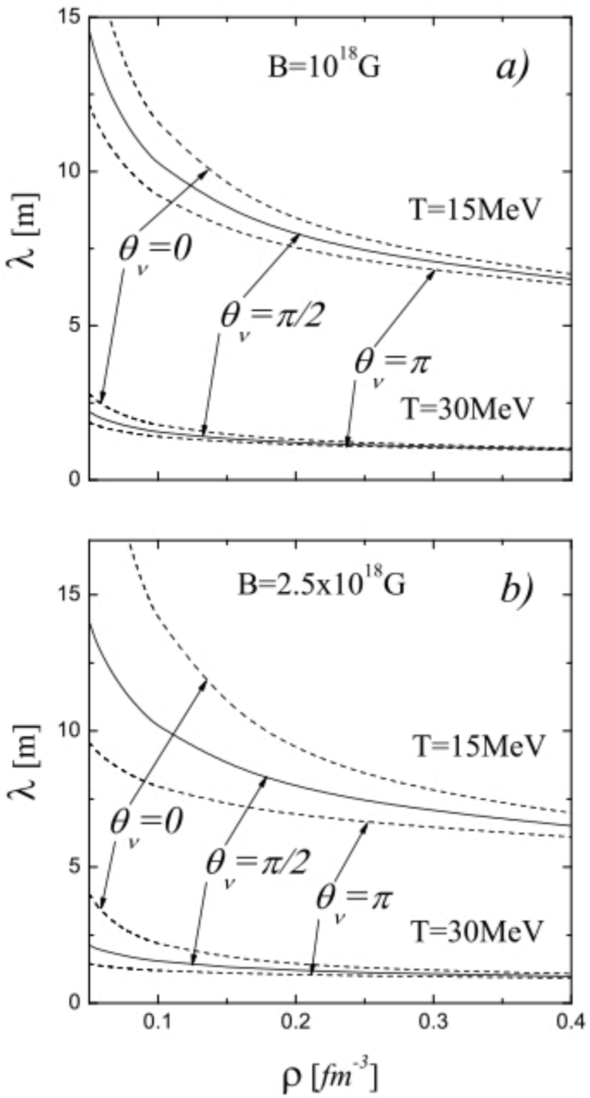
  and the implicit angular dependence of the structure function

- In polarized neutron matter:

  - ✓ Neutrons with spin down (up) are almost transparent to the neutrinos if the incoming angle is θ_ν=0 (π)
  - ✓ λ_- (λ_+) is shorter for $\theta_\nu=\pi$ (0)

Dependence of the ν mean free path on the angle θ_ν

$$\lambda(p_\nu) = \frac{\lambda_-(p_\nu)\lambda_+(p_\nu)}{\lambda_-(p_\nu)+\lambda_+(p_\nu)}$$



- Polarized matter is more transparent to neutrinos when they move parallel to the magnetic field (θ_ν=0) & more opaque when they move anti-parallel to it (θ_ν=π)

- We define a "mean free path asymmetry"

$$\chi_\lambda = \frac{\lambda(\theta_\nu = 0) - \lambda(\theta_\lambda = \pi)}{\lambda(\theta_\nu = \pi/2)}$$

ρ [fm^{-3}]	$\chi_\lambda(B = 10^{18}G)$	$\chi_\lambda(B = 2.5 \times 10^{18}G)$
0.050	0.40	1.17
0.100	0.23	0.60
0.150	0.16	0.40
0.200	0.12	0.29
0.250	0.09	0.23
0.400	0.05	0.14

- ✓ larger for higher fields
- ✓ relevant for low & medium densities
- ✓ as density increases nuclear interaction overcomes the coupling of neutrons with B

The Message (again) of this Talk

- ✧ Study of the neutrino mean free path in hot neutron matter under the presence of strong magnetic fields.

- ✧ Polarized neutron matter described within the non-relativistic BHF approach using the Av18 NN + UIX NNN. Explicit expressions of the σ/V for the scattering of a neutrino from spin up and spin down neutrons are derived from Fermi Golden rule.

- ✧ Strong dependence of the mean free path on the angle of the incoming neutrino.

- My collaborators: Julio Torres Patiño & Eduardo Bauer

- You for your time & attention

- Constança, Pedro, Helena & Márcio for their invitation & support