Macro$\leftrightarrow$micro: improving magnetic-field models

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Coimbra
26th September 2018

Aim of this talk

This talk will hopefully not be of the/my conventional form:

- 'This is what I've done...
- ...and this is why you should all care'

Instead the aim is to look at some places where:

- Micro input can improve macro models
- Macro input can improve micro models

of neutron-star magnetic fields.

Apologies in advance for my (mis)understanding of the micro side...

Micro→macro: the crust-core boundary (1)

Models of NS magnetic fields broadly fall into two categories:

Steady-state/evolution in elastic crust alone

- Ions locked in place (crust assumed able to absorb *any* stress)
- Only electrons move
- $\mathbf{B} = 0$ 'type-I superconducting' (!) inner boundary
- Papers by Geppert, Pons, Vigano, Cumming, Gourgouliatos, ...

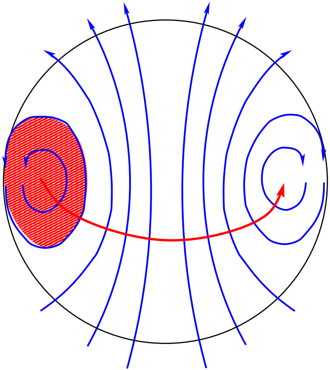
Steady-state in core and (fluid) crust

- Entire star is effectively fluid
- Can have a crust, but must be *unstressed* (equilibrium state is fluid)
- Force balance at crust-core boundary can lead to sharp features
- Papers by me, Ciolfi, Fujisawa, Eriguchi, ...

Crust-core boundary treatment important for both – will focus on latter case.

Micro→macro: the crust-core boundary (2)

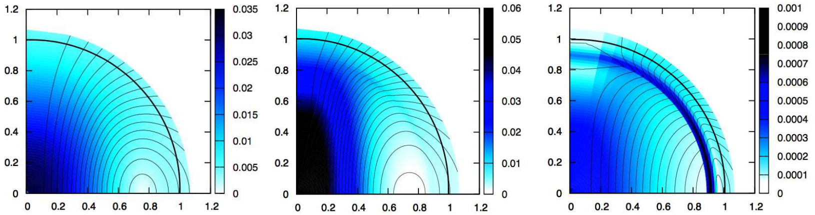
Typical global model of NS magnetic field geometry is a *twisted-torus*: toroidal field fills region of closed poloidal field lines.



Vector sum of poloidal+toroidal → coiled equatorial field lines.
Field geometry is quite general and follows from:

- $\nabla \cdot \mathbf{B} = 0$
- no exterior electric currents
- axisymmetry.

Micro⟶macro: the crust-core boundary (3)



normal core | sc core, $B \sim H$ | sc core, $B \ll H$

(Lander 2013, 2014; Palapanidis, Stergioulas, Lander 2015)

- at crust-core boundary, impose force balance
- this implies magnetic-force balance if everything else smooth
- crust is always normal, magnetic force $\propto B^2$
- if core normal, force $\propto B^2$ and field smooth
- if core superconducting, force $\propto HB$ where $H \sim 10^{15}$ G

So, in the latter case transition can be abrupt for $B \ll 10^{15}$ G.

Micro→macro: the crust-core boundary (4)

Question: what happens to the global **B** with a better, microphysical, treatment of the crust-core boundary?

- conductivity in pasta phases
- anchoring fluxtubes at the boundary
- current sheets?
- symmetry energy, localised instabilities etc (Coimbra group...)

Macro→micro: how strong can the field be?

Many studies concern microphysics at $B \gtrsim 10^{17}$ G. Makes theoretical sense to probe effects in extreme limits, but...

Question: how strong can **B** really be? What is a ‘realistic’ geometry?

Upper limits

- hard upper limit: a mythical ‘mega-magnetar’.
  $P = P_{mag} \sim B^2$ balances gravity $\implies$ $B \sim 10^{18}$ G
- for $B \gtrsim 10^{16}$ G, superconductivity* broken (Glampedakis+ 2011, Sinha&Sedrakian 2015)
- for $B \gtrsim 10^{15}$ G, elastic crust* readily fails (Lander+ 2015,Lander 2016)
- for $B \gtrsim 10^{16}$ G, field generation mechanisms at birth saturate

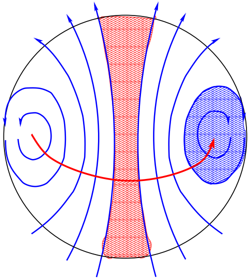
*possible key field stabilisation mechanism - see next

Macro→micro: what geometry can the field have?

Ignore the genesis of **B**. What allows for a stable magnetic field?

Main villain: the Tayler instability

- plasma kink instability in spherical star (Tayler, Markey&Tayler, Wright, 1973)
- pure-poloidal fields (e.g. those from the Lorene code) unstable in blue shaded region
- pure-toroidal fields unstable in red shaded region
- vigorous insuppressible dynamical instability, causes global field rearrangement
- timescale ~ 0.01 s at 10^{15} G (10^{-5} s at 10^{18} G!)



Above $\sim 10^{16}$ G, no stabilisation mechanisms work. Need a stable hydromagnetic equilibrium. May well not exist! (Lander&Jones 2012, Mitchell+ 2015).

→ **room for improvement** in microscopic models...

Summary

- two places for fruitful micro-macro collaboration:
  - microphysics of crust-core boundary likely very important in global eqm, especially for superconducting cores
  - more realistic macrophysical field geometries likely important for microphysics (beyond normal matter, poloidal fields, etc)
- plenty of other issues: fluxtubes at $T \ll T_c$, fluxtubes at $B \sim H$, NS ocean/surface, ...
- bigger goals: interpret magnetar QPOs, understand X-ray burst and flare energy reservoir, model large glitches, ...

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Appendix: equilibrium equations

No dynamo in mature NSs $\implies$ field regeneration not possible.
Natural assumption: **B** is in a dynamical equilibrium. Will also assume:

- core neutrons are superfluid (reasonable after a few hundred years)
- the elastic crust is unstressed (not so reasonable) $\implies$ crust is 'fluid'
- rigid rotation is a trivial extension: $\Phi \mapsto \Phi_{grav} + \Phi_{rot}$

One Euler equation per fluid:

$$\nabla\tilde{\mu}_p + \nabla\Phi - \mathbf{F}_{mag}/\rho_p = 0,$$
$$\nabla\tilde{\mu}_n + \nabla\Phi = 0.$$

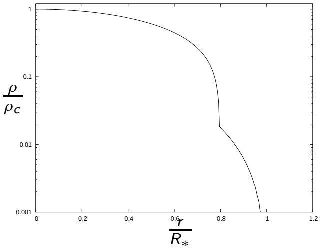
Equation of state is a double polytrope:

$$\tilde{\mu}_p = \tilde{\mu}_p(\rho_p), \tilde{\mu}_n = \tilde{\mu}_n(\rho_n).$$

The two fluids only couple through gravity:

$$\nabla^2\Phi = 4\pi G(\rho_p + \rho_n),$$

and we always need to satisfy $\nabla \cdot \mathbf{B} = 0$. The magnetic force $\mathbf{F}_{mag}$ will change though...



$x_p(r = 0) = 0.15$,
$\rho/\rho_c = 0.03$ at crust base,
$\Gamma_{core} \approx 2.4$,
$\Gamma_{crust} \approx 1.6$.

Appendix: magnetic force

- Since the lattice of fluxtubes is microscopic and regular, we can take a sensible macroscopic average
- This yields the supercon magnetic force, physically a fluxtube tension (Easson & Pethick 1977; Glampedakis, Andersson, Samuelsson 2011)

Normal

In normal MHD, $\mathbf{F}_{mag}$ is the familiar Lorentz force:

$$\mathbf{F}_{mag} = \frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B}.$$

Some algebraic tricks lead to a single PDE, which is fairly convenient to solve.

Superconducting

For superconducting matter we have instead:

$$\mathbf{F}_{mag} = \frac{1}{4\pi}\left((\nabla \times \mathbf{H}_{c1}) \times \mathbf{B} - \rho_p \nabla \left(B \frac{\partial H_{c1}}{\partial \rho_p} \right) \right).$$

There are now two magnetic fields: a 'global' one $\mathbf{B}$ and a 'local' one $\mathbf{H}_{c1}$.

Appendix: equilibrium equation (normal protons)

Now assume star is axisymmetric → magnetic field may now be rewritten in terms of a streamfunction u, so that $\nabla \cdot \mathbf{B} = 0$ is automatically satisfied:

$$\mathbf{B} = \frac{1}{\varpi}\nabla u \times \mathbf{e}_\phi + B_\phi \mathbf{e}_\phi .$$

The Grad-Shafranov equation (Grad & Rubin 1958, Shafranov 1958)

After some algebra we arrive at a single PDE for the magnetic field:

$$\frac{\partial^2 u}{\partial \varpi^2} + \frac{\partial^2 u}{\partial z^2} - \frac{1}{\varpi}\frac{\partial u}{\partial \varpi} = -4\pi\varpi^2 \rho_p \frac{dM}{du} - f_N \frac{df_N}{du} \ ,$$

$M(u)$ is related to the magnetic force through $\mathbf{F}_{mag} = \rho_p \nabla M$ and $f_N(u)$ to the toroidal component.

Note one peculiarity: u appears on both sides of the equation!
→ natural to solve with iterative methods

Appendix: equilibrium equation (superconducting protons)

Even when protons form a type-II superconductor, can perform a similar derivation as for Grad-Shafranov equation.

One key step fails from the normal-matter derivation; the magnetic-force function M is no longer a function of u. In addition, factors of the magnetic-field magnitude B appear. The result is:

GS-type equation for a superconductor (Lander 2013)

$$\frac{\partial^2 u}{\partial \varpi^2} + \frac{\partial^2 u}{\partial z^2} - \frac{1}{\varpi}\frac{\partial u}{\partial \varpi} = \frac{\nabla \Pi \cdot \nabla u}{\Pi} - \varpi^2 \rho_p \Pi \frac{dy}{du} - \Pi^2 f \frac{df}{du} \ ,$$

where $\Pi \equiv B/\rho_p$; $y(u) \sim M + B$ is related to the magnetic force and $f(u)$ is related to the toroidal component.