The role of mass, equation of state and superfluid reservoir in pulsar glitches

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Neutron stars: the equation of state, superconductivity/superfluidity and transport coefficients (PHAROS WG1 + WG2 meeting)

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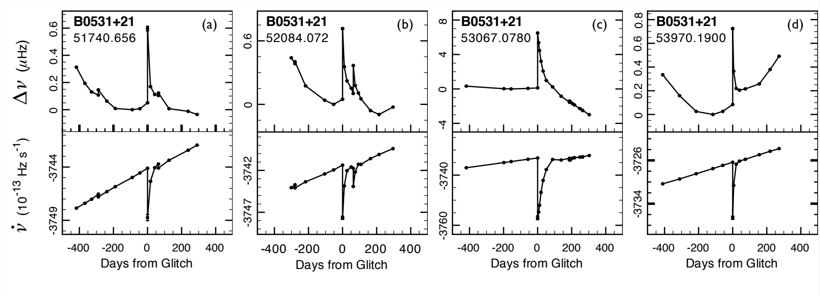
Pulsar glitches



Two components: a normal charged one (visible) and a superfluid one.

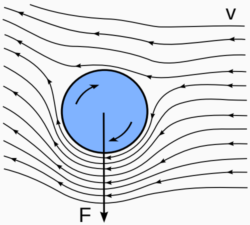
Long recoveries ⇒ Effect due to superfluid components.

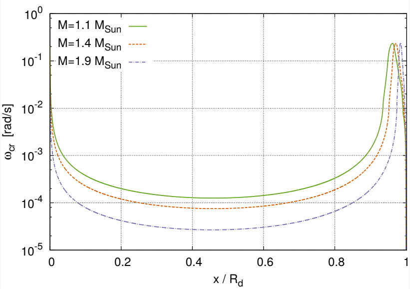
Diverse phenomenology:

- Mostly radiopulsars, but also magnetars and millisecond.
- Periodic glitchers vs single glitchers.
- Similar size vs. different size.

Glitch mechanism

The relative motion between the vortices and the superfluid itself causes a Magnus force. If the Magnus force is strong enough to overcome the pinning force, vortices detach and a glitch occurs.





$$\int_{\mathrm{vortex}} f_P \, \mathrm{d}l = \int_{\mathrm{vortex}} f_M \, \mathrm{d}l$$

Critical lag for vortices extended to the whole star with SLy4 EoS (Antonelli & Pizzochero, 2017)

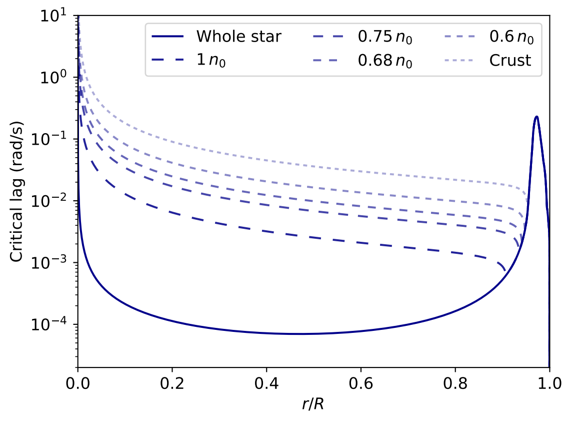
Superfluid reservoir

In literature many different regions for the neutron superfluid have been considered:

1. Crust-only superfluid (Datta & Alpar 1993, Link et al. 1999, etc.)
2. Superfluid extended to the whole star (Pizzochero 2011, Antonelli & Pizzochero 2017, etc.)
3. Superfluid involved in the glitch limited to $^1S_0$ pairing gap (Ho et al. 2015)

Question addressed here: what is the dependence of glitch mechanism on the superfluid reservoir extension?
In particular, I will recall the model of Pizzochero et al. (2017) for the mass estimate of a glitcher.

Critical lag



$$\int_{\text{vortex}} f_P \, \mathrm{d}l = \int_{\text{vortex}} f_M \, \mathrm{d}l$$

Same as before, but this time the vortex is limited within n_{drip} and n_{cutoff}, using the BSk20 EoS and for $1.4 M_{\odot}$.

Maximal glitch

Maximal glitch as a function of time:

$$\Delta\Omega_{\max}(t) = \frac{\Delta L[\Omega_{np}(t)]}{I}$$

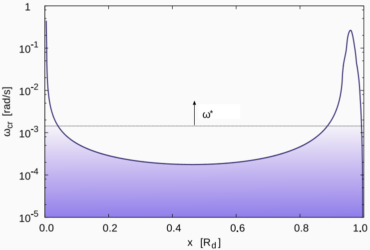
In the Newtonian framework:

$$\Delta L[\Omega_{np}] = \int \mathrm{d}^3x\, x^2 \rho_n(r)\, \Omega_{np}$$

We can use a simplified (and unified) prescription that allow us to study the quantitative trend of the lag:

$$\Omega_{vp}(x, \omega^*) = \min\,[\Omega_{vp}^{\mathrm{cr}}, \omega^*],$$

where $\Omega_{vp} \equiv (1 - \varepsilon_n)\Omega_{np}$ and $\omega^* \equiv |\dot{\Omega}_\infty|t$.



Pulsar activity and ω^*_{act}

The absolute pulsar activity is defined as:

$$\mathcal{A}_a = \frac{1}{T_{\mathrm{obs}}} \sum_{i=1}^{N} \Delta\Omega_i.$$

We can also define a timescale for a pulsar that defines the mean waiting time for a fictitious pulsar that exhibits only glitches of size $\Delta\Omega_{obs}$ and has activity $\mathcal{A}_a$:

$$t_{\mathrm{act}} = \frac{\Delta\Omega_{\mathrm{obs}}}{\mathcal{A}_a}.$$

Finally, we can find the nominal lag associated to this timescale:

$$\omega^*_{\mathrm{act}} = t_{\mathrm{act}} |\dot{\Omega}_\infty|.$$

Mass estimate $M_{\rm act}$

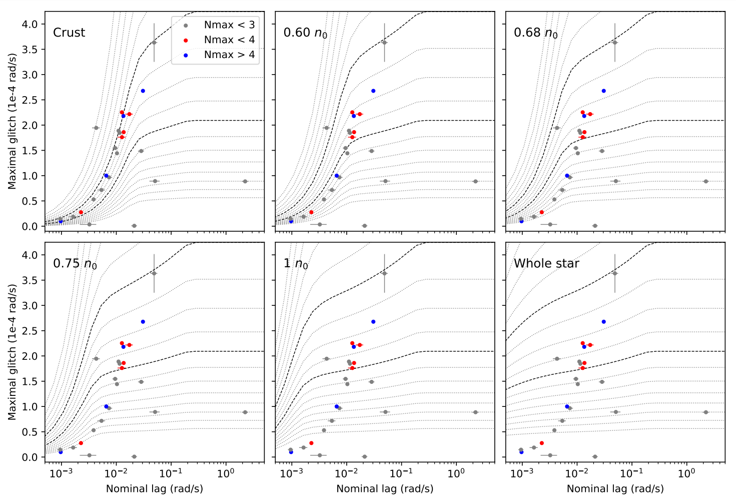
The maximal glitch $\Delta\Omega(\omega^*, M)$ depends only on the mass once the microphysical parameters (EoS, pinning and entrainment) are fixed.

Measuring a maximum glitch $\Delta\Omega_{\rm obs}$ and calculating the activity, we can estimate a mass for the pulsar inverting the relation:

$$\Delta\Omega(\omega^*_{\rm act}, M_{\rm act}) = \Delta\Omega_{\rm obs}.$$

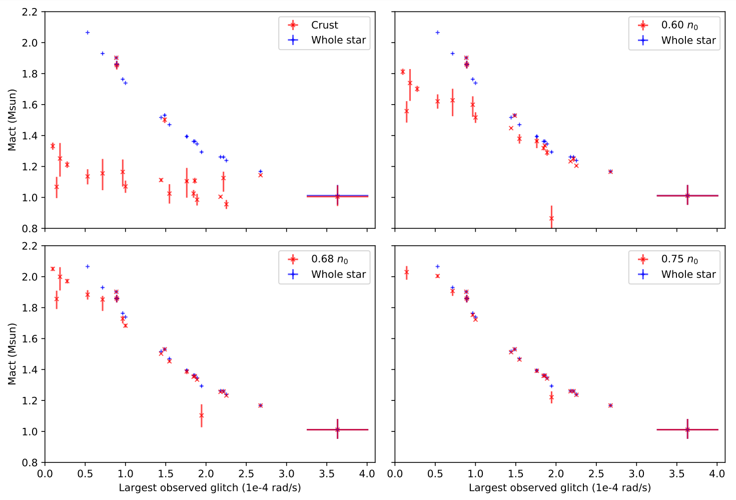
This mass estimate *depends* on the extension of the superfluid reservoir.

Maximal glitch



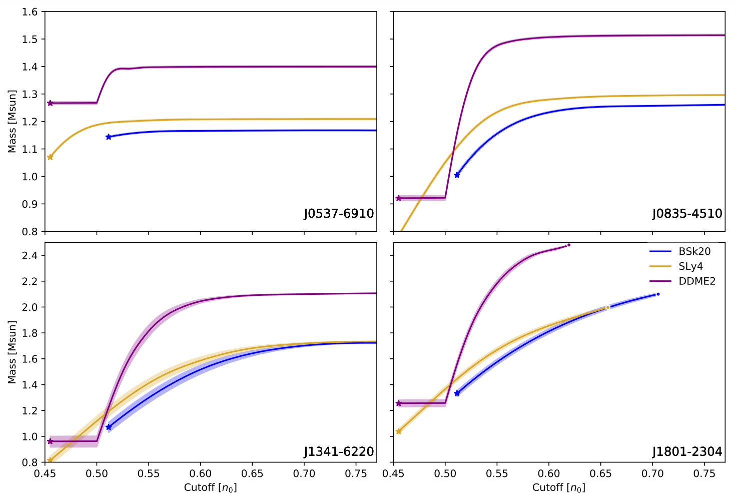
Pulsars have been chosen with $N_{\rm gl} > 2$, $N_{\rm max} > 1.1$, $T_{\rm obs}\,|\dot{\Omega}_\infty| > 10^{-3}$ rad/s.

Mass dependence on cuts



Montoli, Antonelli & Pizzochero, `arXiv:1809.07834`

Mass dependence on the EoS



Montoli, Antonelli & Pizzochero, arXiv:1809.07834

Conclusions

- The mass estimate M_{act} depends on the extension of the superfluid region.
- M_{act} is generally lower with a smaller superfluid reservoir.
- Like in other models (Andersson et al. 2012, Chamel 2013, Delsate et al. 2016), also here there is too little angular momentum stored in the crust for glitches.
- Pulsars with high ω_{act} (e.g. Crab) are independent from the cuts performed.

Open question: Where actually resides the superfluid involved in the glitch?

Thank you!

Maximal glitch

Angular momentum of the star:

$$L = I\Omega_p + \Delta L[\Omega_{np}]$$

Time derivative of the angular momentum:

$$I\dot{\Omega}_p + \Delta L[\dot{\Omega}_{np}] = -I|\dot{\Omega}_\infty|$$

Integration between a time before and one after the glitch:

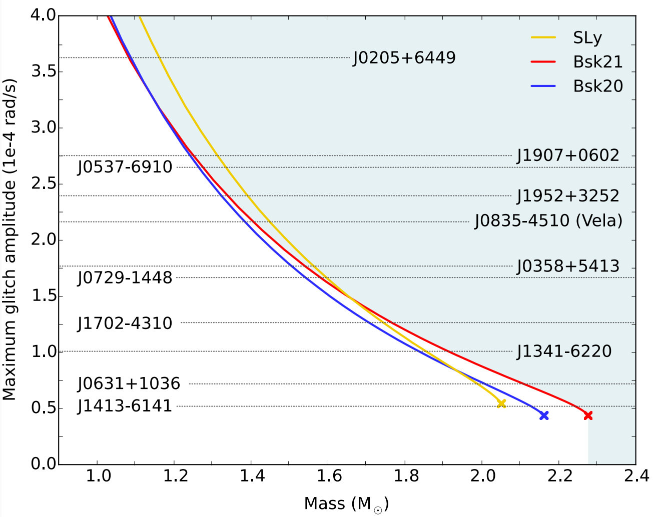
$$\Rightarrow \Delta\Omega_{\max}(t) = \frac{\Delta L[\Omega_{np}(t)]}{I}$$

In the Newtonian framework:

$$\Delta L[\Omega_{np}] = \int \mathrm{d}^3x\, x^2 \rho_n(r)\, \Omega_{np}$$

In General Relativity this splitting is not always possible, but it is in the slow rotation approximation!

Maximum glitch amplitude

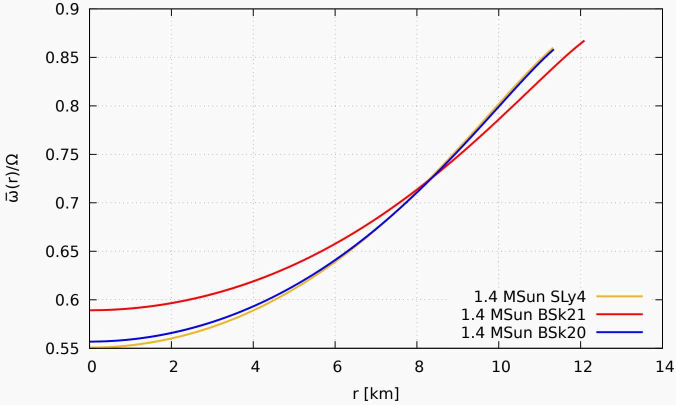


Metric of a slowly rotating star

$$\mathrm{d}s^2 = -e^{2\Phi(r)}\mathrm{d}t^2 + e^{2\Lambda(r)}\mathrm{d}r^2 + r^2\left[\mathrm{d}\vartheta^2 + \sin^2\vartheta(\mathrm{d}\varphi - \omega(r)\mathrm{d}t)^2\right]$$

The radial profiles are still given by the TOV equations + EoS. $\omega(r)$ is given by an additional differential equation.

Drag of the local inertial frames



The relevant quantity is $\overline{\omega}(r) = \Omega - \omega(r)$.

Due to the linearity of the equation giving $\omega(r)$, $\overline{\omega}(r)/\Omega$ does not depend on Ω.

Moments of inertia

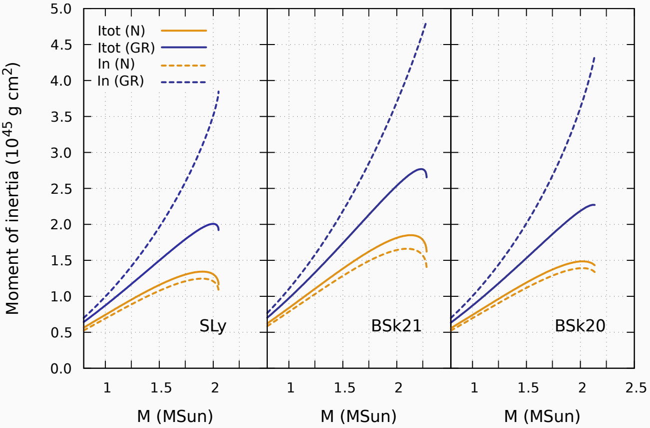
Total moment of inertia:

$$I = \frac{8\pi}{3} \int_0^R \mathrm{d}r \; r^4 e^{-\Phi(r)+\Lambda(r)} \left(\rho(r) + \frac{P(r)}{c^2} \right) \frac{\overline{\omega}(r)}{\Omega}$$

Moment of inertia of the superfluid component:

$$I_v = \frac{8\pi}{3} \int_0^R \mathrm{d}r \; r^4 e^{-\Phi(r)+\Lambda(r)} x_n(r) \left(\rho(r) + \frac{P(r)}{c^2} \right) \frac{m_n}{m_n^*(r)}$$

Moments of inertia



- The total moment of inertia varies up to 50%.
- In GR the moment of inertia of the superfluid reservoir exceeds the total one.

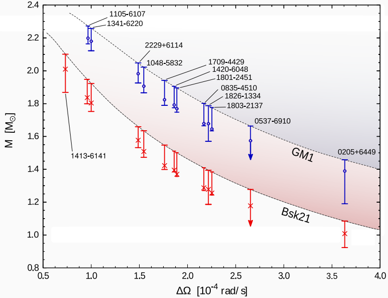
Mass upper limit M_{max}

Maximum glitch = maximal glitch in the case of the critical lag.

$$\Delta\Omega_{\mathrm{max}} = \frac{\pi^2}{I\kappa} \int_0^{R_d} \mathrm{d}r\, r^3\, f_P(r).$$

As for M_{act}, we can find a mass corresponding to the maximum glitch, M_{max}. This is analytically *independent* on the extent of the superfluid region - as long as this extends at least in the pinning region - and entrainment paramters.

Already discussed in Pizzochero, Antonelli, Haskell & Seveso (2017) for the Newtonian case.



Slow rotation approximation

$$\frac{R^3\Omega^2}{GM} \ll 1$$

Slow rotation condition, Hartle (1967)

Extreme cases (millisecond pulsars, $M = 1.4M_\odot$, $R = 10km$):

- J1748-2446ad, $\Omega = 4501$ rad s^{-1} (not seen glitching):

$$R^3\Omega^2/(GM) \approx 0.11$$

- J1824-2452A, $\Omega = 2057$ rad s^{-1} (seen glitching, Cognard & Backer 2004):

$$R^3\Omega^2/(GM) \approx 0.023$$

Mass upper limit $M_{\max}$, relativistic corrections

The calculation of the maximum glitch has been remade in the slow rotation framework in Antonelli, Montoli & Pizzochero (2018).

The dependence of $M_{\max}$ on relativistic corrections is weak.