Transition and crustal properties in neutron stars

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Neutron stars: the equation of state, superconductivity/superfluidity and transport coefficients (PHAROS WG1+WG2 meeting)
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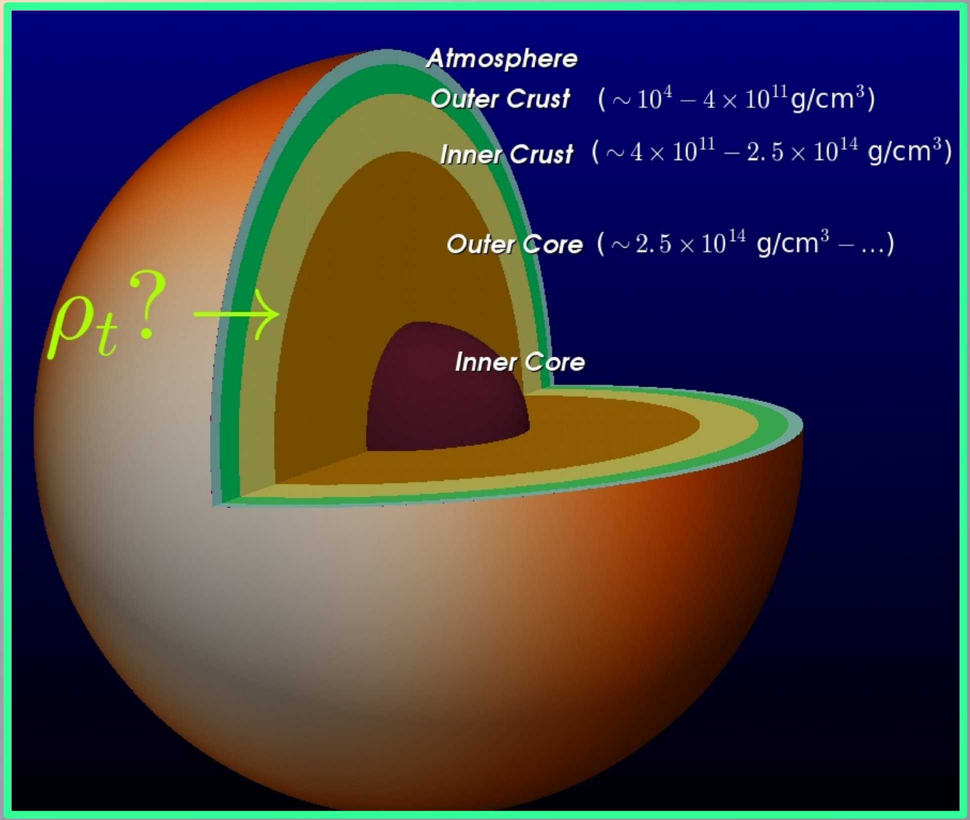
Universitat de Barcelona

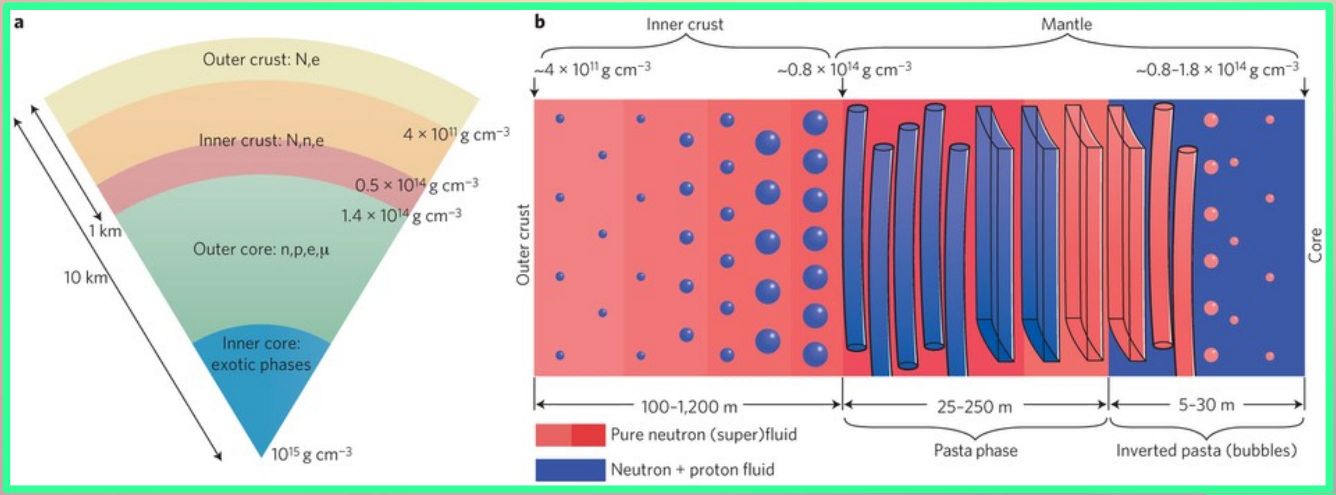
ICCUB
Excelencia María de Maeztu
Institute of Cosmos Sciences

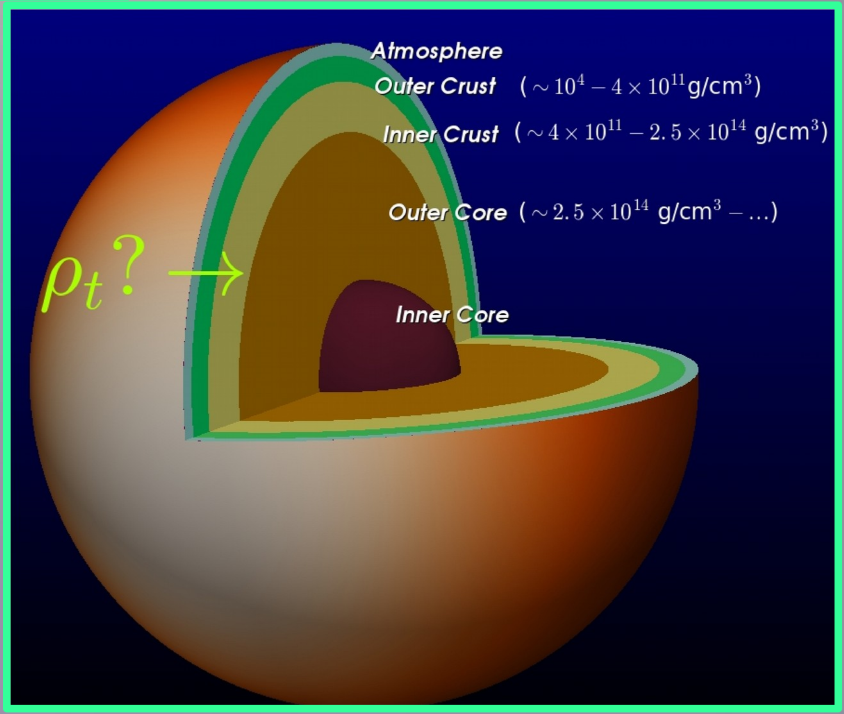
Importance of the limits between layers

- Crust goes from practically the surface to the core
- Thickness limited by the inner crust-core transition (aim of this work)
- Thickness of the crust may influence pulsar glitches.



Transition from the core to the crust





Gogny interactions

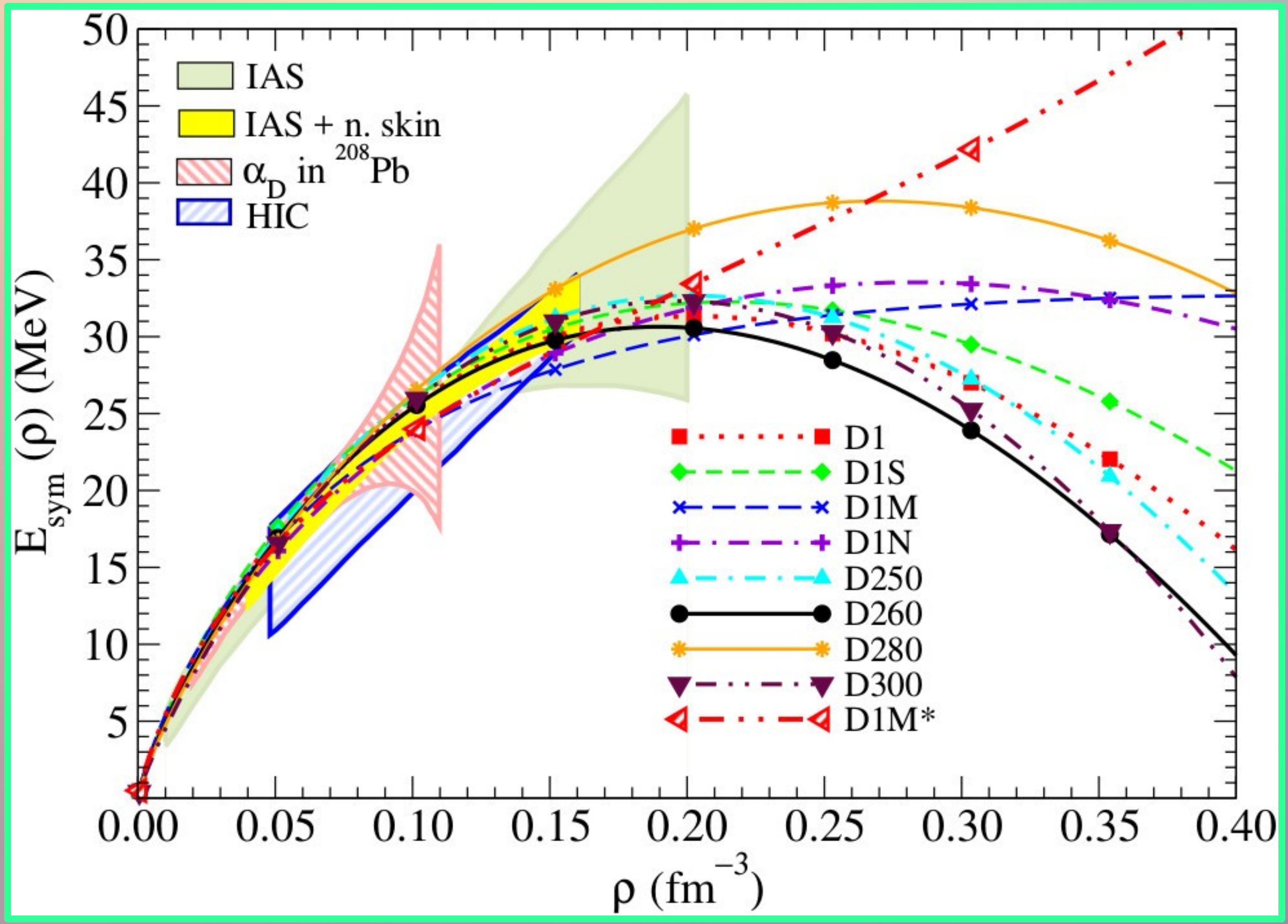
The standard Gogny two-body effective nuclear interaction in a homogeneous system reads as

$$V(\mathbf{r}_1, \mathbf{r}_2) = \sum_{i=1,2} \left(W_i + B_i P_\sigma - H_i P_\tau - M_i P_\sigma P_\tau\right) e^{-\frac{(r_1 - r_2)^2}{\mu_i^2}}$$

$$+t_3 \left(1 + x_3 P_\sigma\right) \rho^\alpha \left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}\right) \delta(\mathbf{r_1} - \mathbf{r}_2)$$

We have used different Gogny forces: D1, D1S, D1M, D1N, D250, D260, D280, D300, D1M*

Symmetry energy



*CGB, M. Centelles, X. Viñas, A. Rios, PR **C96**, 065806 (2017)*

*CGB, M. Centelles, X. Viñas, L.M. Robledo, PL **B779**, 195 (2018)*

Search of the core-crust transition: the dynamical method

- One imposes small variations of sinusiodal type to the density of neutrons and protons,

$$\delta\rho_q(\mathbf{r}) = \int d\mathbf{k}\,\delta n_q(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}}$$

- The total energy is expanded up to 2nd order in the variations of the densities, implying

$$E - E_0 = \frac{1}{2}\sum_{i,j}\int d\mathbf{k}\,\frac{\delta^2 E}{\delta n_i(\mathbf{k})\delta n_j^*(\mathbf{k})}\delta n_i(\mathbf{k})\delta n_j^*(\mathbf{k})$$

G. Baym, H.A. Bethe, C.J. Pethick. NP ***A175****, 225 (1971)*

C.J. Pethick, D.G. Ravenhall, C.P. LorenzNP ***A584****, 675 (1995)*

C. Ducoin, Ph. Chomaz, F. Gulminelli, NP ***A789****, 403 (2007)*

The Dynamical Method

$$E - E_0 = \frac{1}{2} \sum_{i,j} \int d\mathbf{k} \frac{\delta^2 E}{\delta n_i(\mathbf{k}) \delta n_j^*(\mathbf{k})} \delta n_i(\mathbf{k}) \delta n_j^*(\mathbf{k})$$

This curvature can be written in a matrix form as

$$\frac{\delta^2 E}{\delta n_i(\mathbf{k}) \delta n_j^*(\mathbf{k})} = \begin{pmatrix} \partial\mu_n/\partial\rho_n & \partial\mu_n/\partial\rho_p & 0 \\ \partial\mu_p/\partial\rho_n & \partial\mu_p/\partial\rho_p & 0 \\ 0 & 0 & \partial\mu_e/\partial\rho_e \end{pmatrix} \longleftarrow \text{BULK}$$

$$+k^2 \begin{pmatrix} D_{nn} & D_{np} & 0 \\ D_{pn} & D_{pp} & 0 \\ 0 & 0 & 0 \end{pmatrix} \longleftarrow \text{SURFACE}$$

$$+\frac{4\pi e^2}{k^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \longleftarrow \text{COULOMB}$$

J. Xu, C.M. Ko PR **C82**, 044311 (2000)

The Dynamical Method

A npe system is stable against fluctuations of the density if the dynamical potential is positive

$$V_{\rm dyn}(\rho, k) = \left(\frac{\partial \mu_p}{\partial \rho_p} + D_{pp}k^2 + \frac{4\pi e^2}{k^2}\right) - \frac{\left(\partial\mu_p/\partial\rho_n + D_{pn}k^2\right)^2}{\partial\mu_n/\partial\rho_n + D_{nn}k^2} - \frac{(4\pi e^2/k^2)^2}{\partial\mu_e/\partial\rho_e + 4\pi e^2/k^2} > 0$$

The coefficients in the surface term correspond to the terms of the nuclear energy density coming from the momentum dependent part of the interaction

$$\mathcal{H}^{\nabla} = D_{nn}\left(\nabla\rho_n\right)^2 + D_{pp}\left(\nabla\rho_p\right)^2 + 2D_{np}\nabla\rho_n\cdot\nabla\rho_p$$

$$f(k,k_{Fq},k_{Fq'}) = 1 + \frac{m}{\hbar^2 k}\frac{\partial V(k,k_{Fq},k_{Fq'})}{\partial k}$$

*J. Xu, C.M. Ko PR **C82**, 044311 (2000)*
*V.B. Soubbotin, X. Viñas, NP **A665**, 291 (2000)*

The Dynamical Method

$$\mathcal{H}^{\nabla} = D_{nn}\left(\nabla\rho_n\right)^2 + D_{pp}\left(\nabla\rho_p\right)^2 + 2D_{np}\nabla\rho_n\cdot\nabla\rho_p$$

Direct+ Exchange contributions

Exchange part → go further than nuclear matter.

Different ways for performing the density matrix expansion

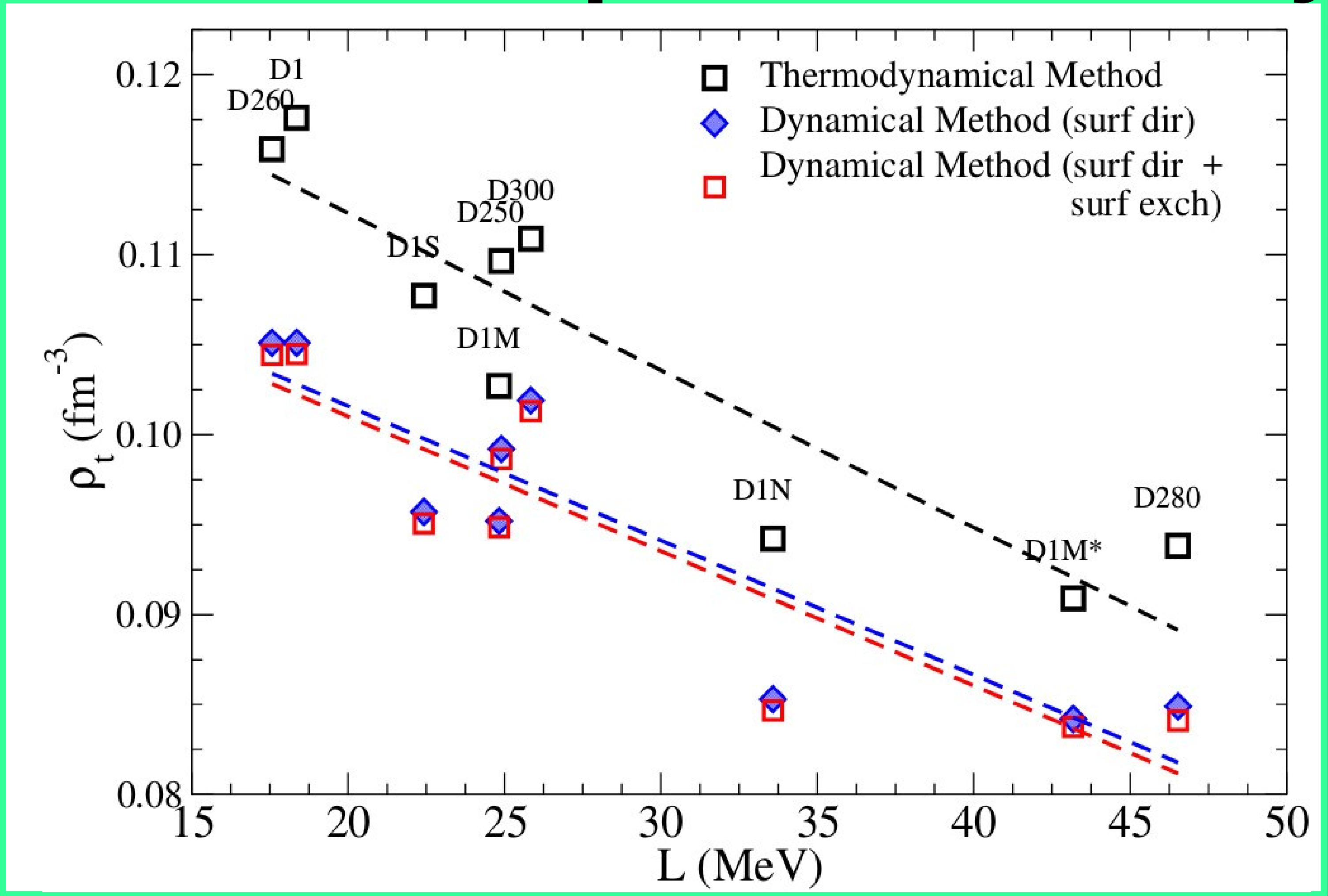
In our case: **Extended Thomas Fermi**

*V.B. Soubbotin, X. Viñas, NP **A665**, 291 (2000)*

$$\varepsilon_{\rm ex}^{\rm ETF}(\mathbf{R}) = \varepsilon_{\rm ex,0}^{\rm ETF}(\mathbf{R}) + \varepsilon_{\rm ex,2}^{\rm ETF}(\mathbf{R})$$
$$= -\tfrac{1}{2}\rho^2(\mathbf{R})\int d\mathbf{s}\, v(s)\frac{9j_1^2(k_F s)}{k_F^2 s^2} + \frac{\hbar^2}{2m}\left[(f-1)\left(\tau_{\rm ETF} - \tfrac{3}{5}k_F^2\rho - \tfrac{1}{4}\Delta\rho \quad + k_F f_k\left(\frac{1}{27}\frac{(\nabla\rho)^2}{\rho} - \frac{1}{36}\Delta\rho\right)\right]\right.$$

$$\tau_{\rm ETF}(\mathbf{R}) = \tau_{\rm ETF,0}(\mathbf{R}) + \tau_{\rm ETF,2}(\mathbf{R})$$
$$= \tfrac{3}{5}k_F^2\rho + \frac{1}{36}\frac{(\nabla\rho)^2}{\rho}\left[1 + \tfrac{2}{3}k_F\frac{f_k}{f} + \tfrac{2}{3}k_F^2\frac{f_{kk}}{f} - \tfrac{1}{3}k_F^2\frac{f_k^2}{f^2}\right]$$
$$+ \tfrac{1}{12}\Delta\rho\left[4 + \tfrac{2}{3}k_F\frac{f_k}{f}\right] + \frac{1}{6}\rho\frac{\Delta f}{f} + \frac{1}{6}\frac{\nabla\rho\cdot\nabla f}{f}\left[1 - \tfrac{1}{3}k_F\frac{f_k}{f}\right] + \frac{1}{9}\frac{\nabla\rho\cdot\nabla f_k}{f} - \frac{1}{12}\rho\frac{(\nabla f)^2}{f^2}$$

Transition Properties: density

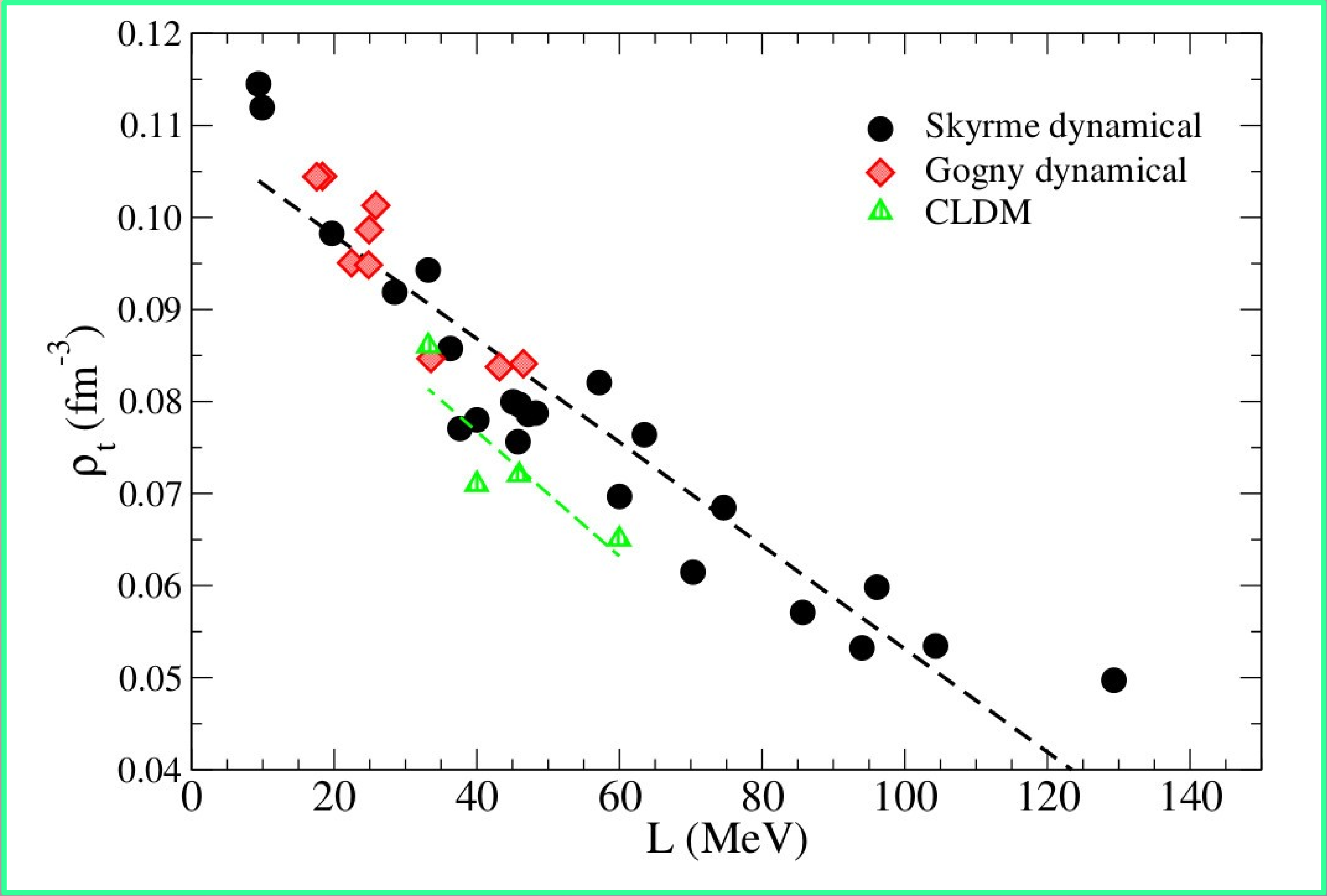


$$L = 3\rho_0 \frac{\partial E_{\mathrm{sym}}}{\partial \rho}\bigg|_{\rho_0}$$

CGB, M. Centelles, X. Viñas, work in progress

CGB, M. Centelles, X. Viñas, work in progress

Transition Properties: density



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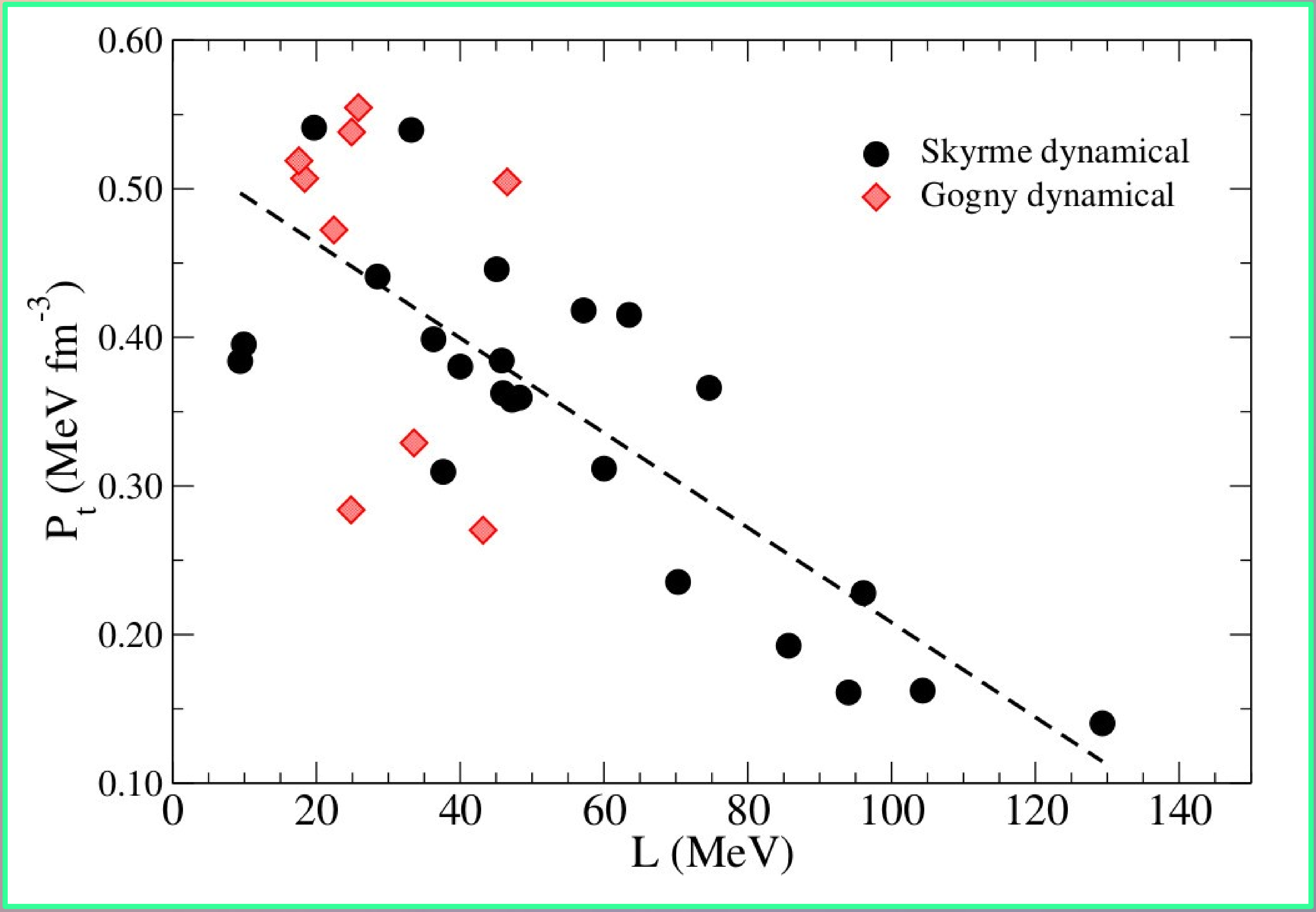
Transition Properties: pressure



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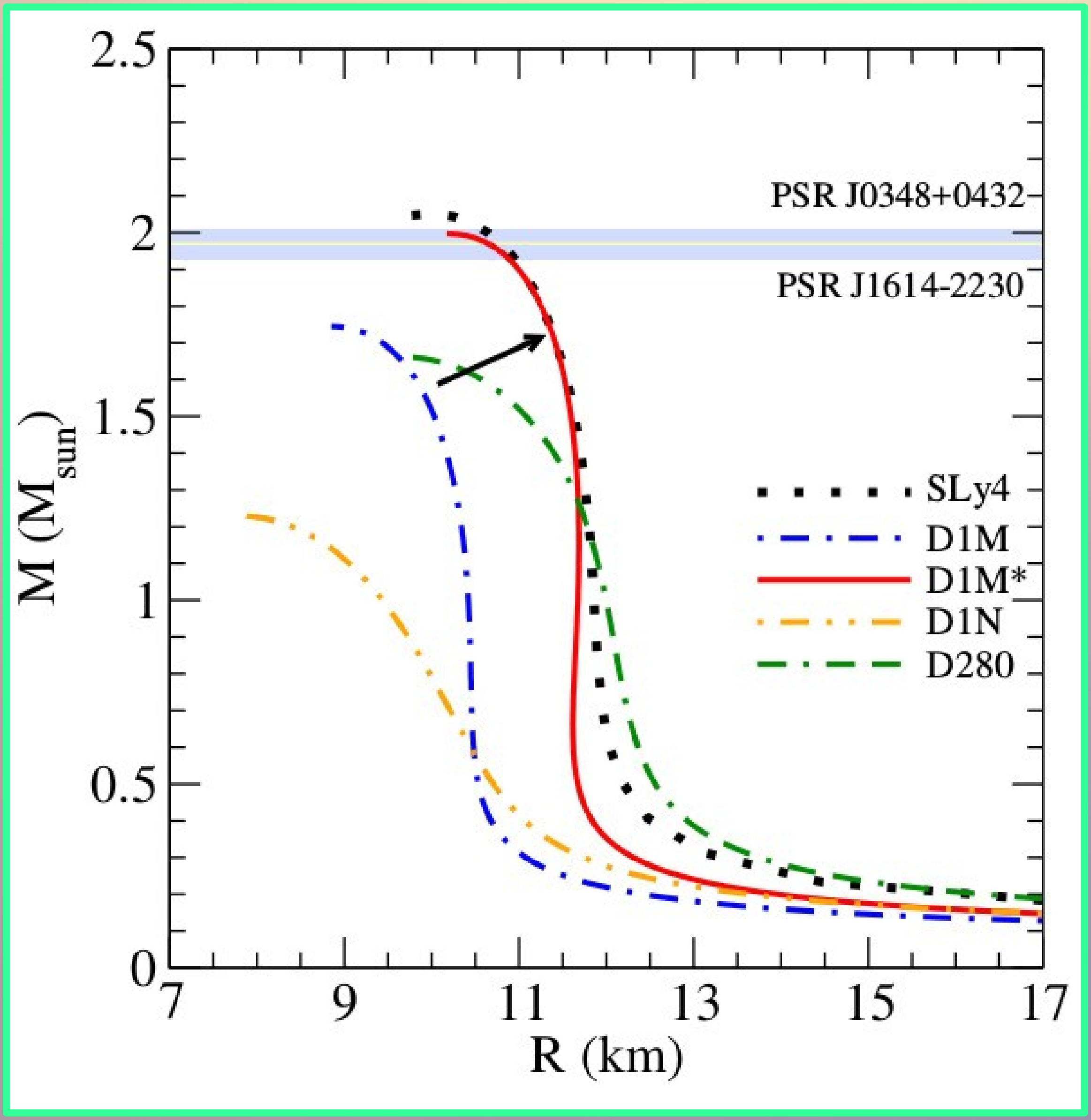
Mass-Radius Relation



- Edge of the crust determined by the transition density calculated before.
- Use of BBP EoS for the inner crust.
- Integration of the TOV equations.
- D1M* interaction can generate a 2 solar mass NS.

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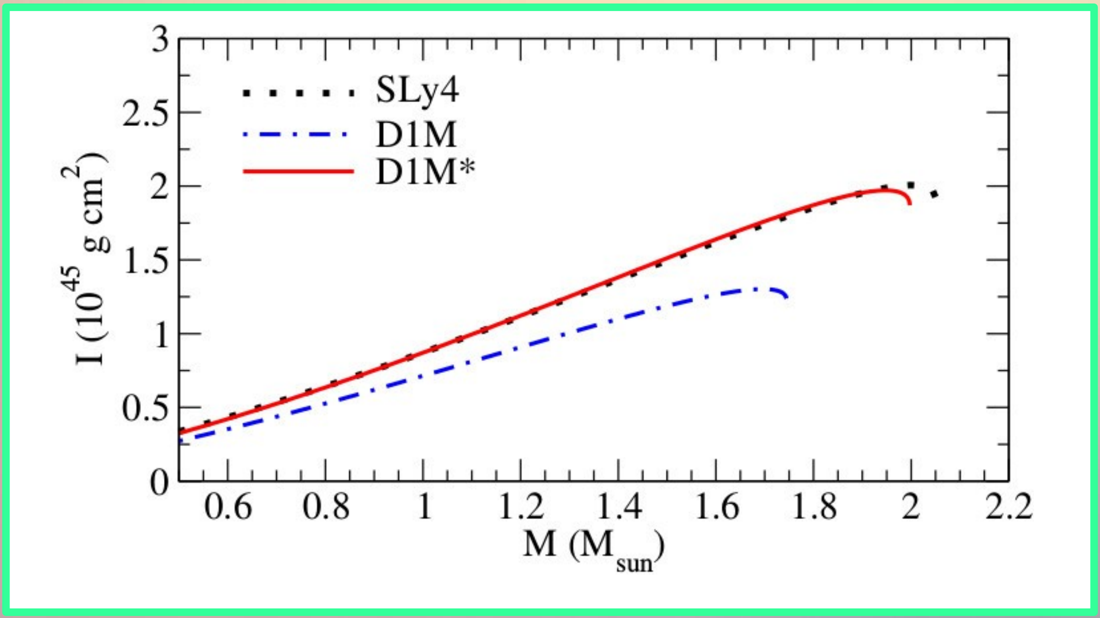
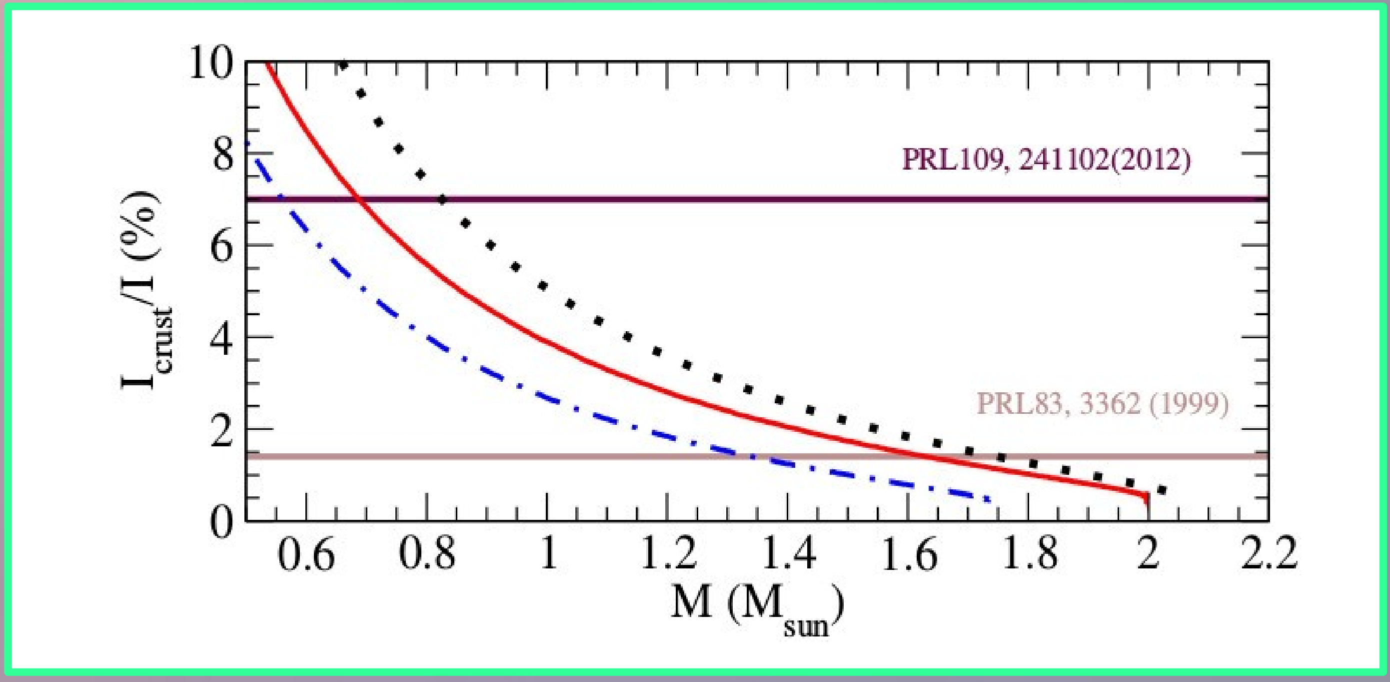
Moments of inertia



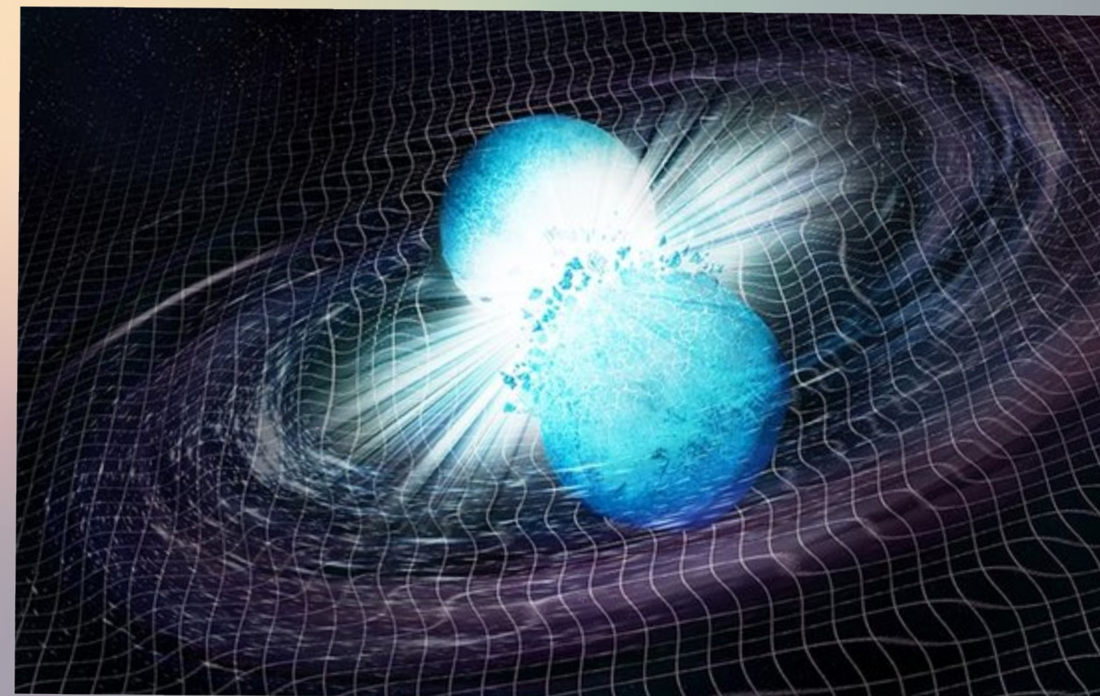


$$\mathcal{M} = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}}$$

Gravitational Waves

$$\Lambda = \frac{2}{3} \frac{k_2 R^5 c^{10}}{G^5 M^5}$$

- **GW170817:** Gravitational Waves from a Binary Neutron Star Inspiral
- Constraints on Tidal Deformability
  - B.P. Abbott *et al.* PRL **119**, 161101 (2017):
  - B.P. Abbott *et al.* ArXiv: 1805,11579(2018):

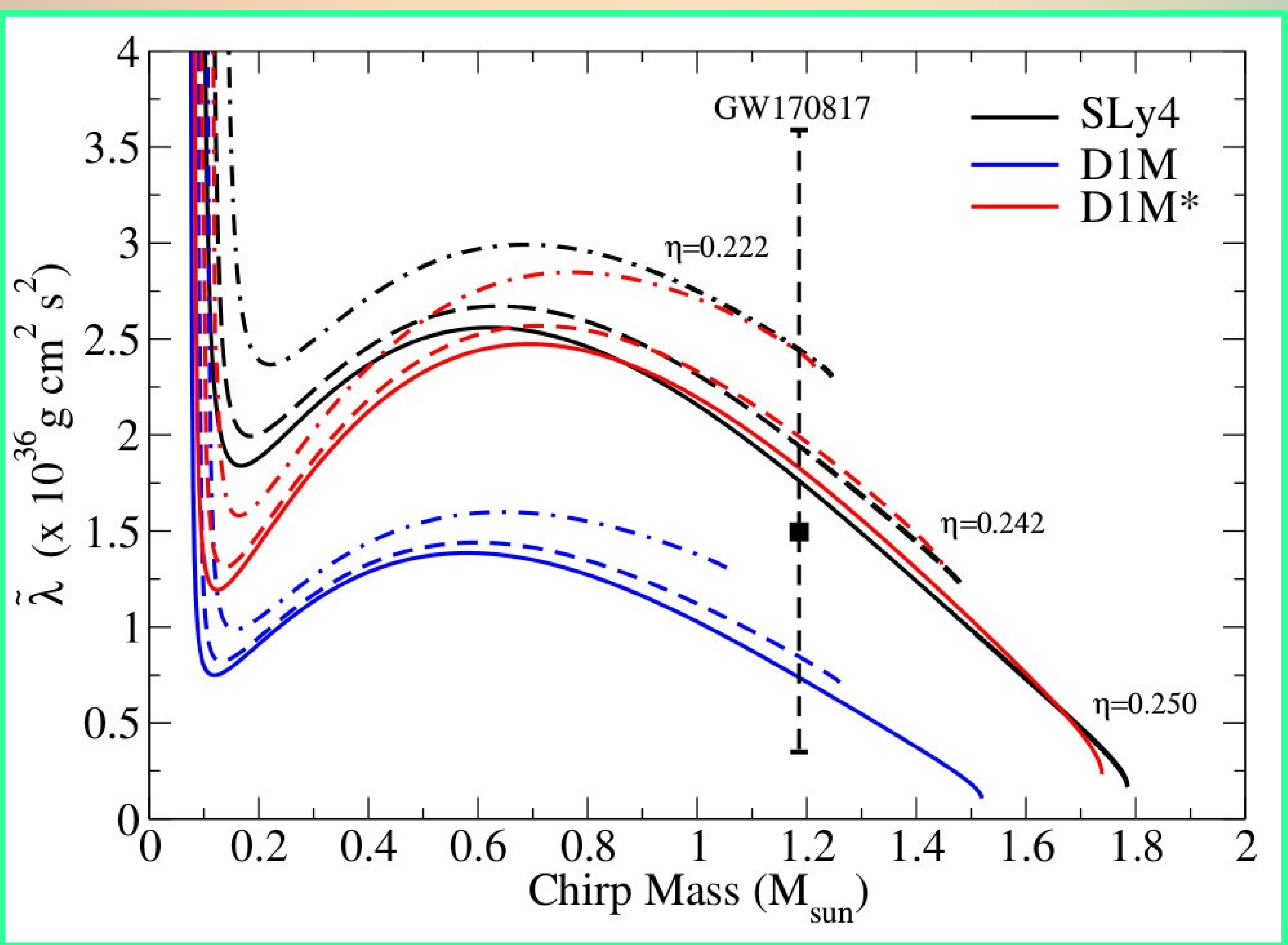


NASA, CXC, Trinity University and D. Pooley et al. Illustration: NASA, CXC and M. Weiss
http://chandra.si.edu/press/18_releases/press_053118.html

$$\tilde{\Lambda}(\mathcal{M} = 1.188 M_\odot) \leq 800$$

$$\tilde{\Lambda}(\mathcal{M} = 1.186 M_\odot) = 300^{+420}_{-230}$$

Tidal deformability



$$\mathcal{M} = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}}$$

$$\eta = \frac{M_1 M_2}{(M_1 + M_2)^2}$$

$$\lambda = \frac{2}{3} k_2 \frac{R^5}{G}$$

$$\tilde{\lambda} = \frac{1}{26} \left[\frac{M_1 + 12 M_2}{M_1} \lambda_1 + \frac{M_2 + 12 M_1}{M_2} \lambda_2 \right]$$

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Conclusions

- We have studied the core-crust transition of neutron stars going from the core to the crust, and using Gogny interactions and the dynamical method.
- To find the exchange part of the D_{ij} coefficients in the surface term of the dynamical method, one has to go further than nuclear matter. In our case, we have used a density matrix expansion using the extended Thomas-Fermi method.
- We find that Gogny interactions predict an anticorrelation of the transition density values with L, whereas the pressure do not present correlations with the slope parameter L.
- Few standard Gogny interactions provide numerically stable solutions of the TOV equations. The new D1M* force is able to provide a 2 solar mass neutron star.
- The results of the tidal deformability for D1M* are in agreement with the latest constraints coming from GW170817 detection.
- Future work: study of the inner crust with Gogny forces (extended Thomas Fermi) to be able to provide a unified EoS for them.

Thank you for your attention