Wydział Fizyki

POLITECHNIKA WARSZAWSKA

From quantum theory of nuclear matter to hydrodynamics of neutron stars

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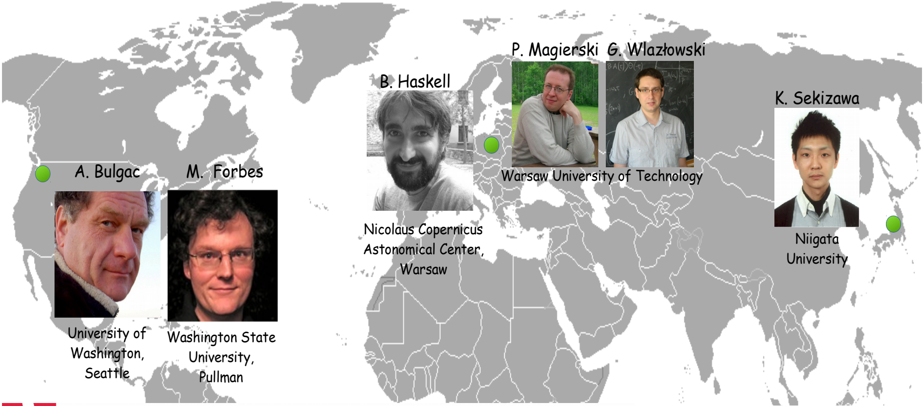
Nuclear theory group

PHAROS WG1+WG2 meeting, Coimbra, 27.09.2018

Colaboration



NATIONAL SCIENCE CENTRE
POLAND

PHAROS
THE MULTI-MESSENGER PHYSICS AND ASTROPHYSICS OF NEUTRON STARS

PRACE

Presentation plan

Open question: **How to perform transition from qunatum to macroscopic regime preserving "the physics"?**

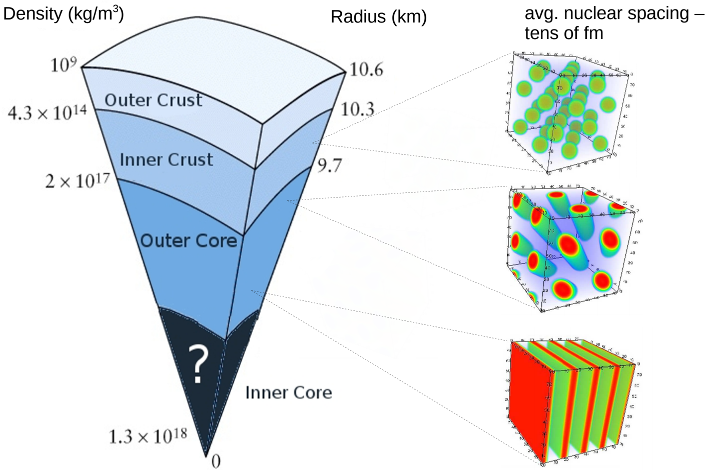
Presentation plan

Correction to M. Antonelli: **Transition from superfluid coherence lenght to intervortex distance scale.**

Plan of presentation

Star internal structure

Macroscopic - hydrodynamics **Microscopic - qunatum**

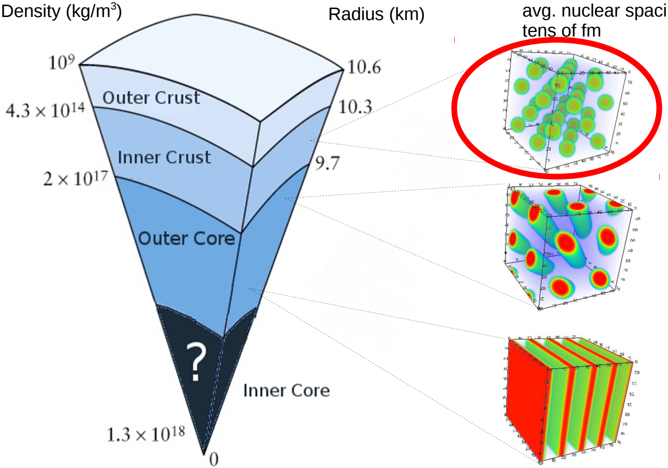


Source: https://apatruno.wordpress.com/neutron-stars/

M. Okamoto et al.,
Phys. Rev. C 88.025801 (2013)

Star internal structure

Macroscopic - hydrodynamics **Microscopic - qunatum**



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M. Okamoto et al.,
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Description on different lenght scales

Microscopic

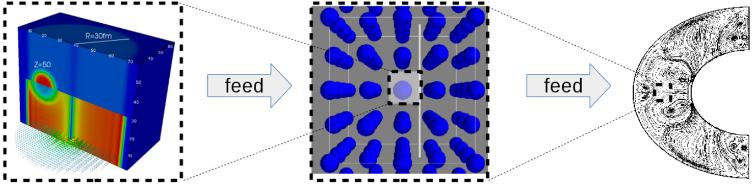
- Quantum many-body description
- Dynamics of neutrons and protons

Mesoscopic

- Semi-classical model
- **Distinguishable vortices and nuclei**

Macroscopic

- Relativistic hydrodynamics?
- Continous vorticity



Method: TDDFT
DoF: neutrons and protons.
Scale: ~10^{-13}m

Method: Vortex Filament Model
DoF: impurities and vortices
Scale: ~10^{-9}m

Method: Hydrodynamics
DoF: fluid elements
Scale: ~size of star

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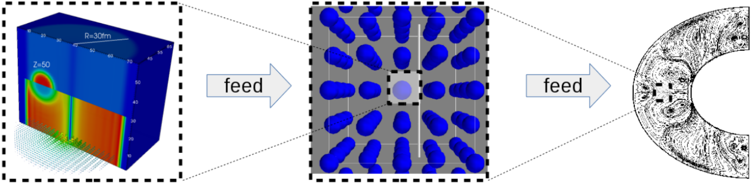

Method: TDDFT
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Scale: ~10^{-13}m

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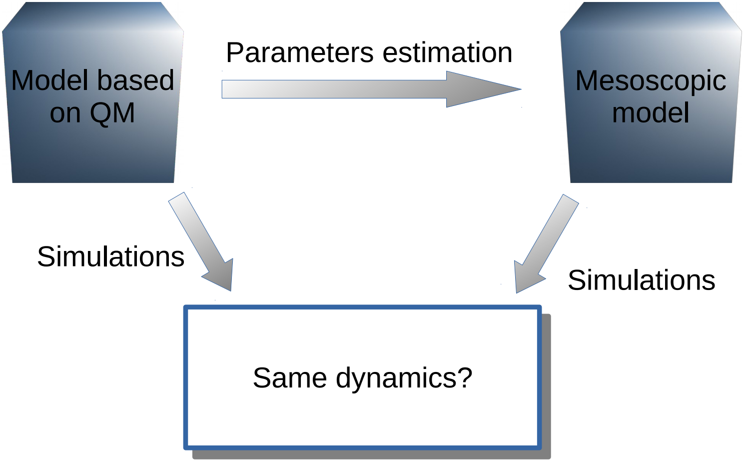
Scale: ~10^{-9}m

Method: Hydrodynamics
DoF: fluid elements
Scale: ~size of star

Collective effort required!

Our aim


Model based on QM
Parameters estimation
Mesoscopic model
Simulations
Simulations
Same dynamics?

Plan of presentation

Methods - our proposal

Microscopic

- ASLDA framewrok
- Explicit s-wave superfluidity
- Equivalent of Schrödinger eq.

Mesoscopic

- Relativistic?
- Vortex Filament Model
- Explicit dynamics of nuclei and vortices

Macroscopic

- **Relativistic**
- **Fluid of clusters**
- **Fluid of neutrons**
- **Continous vorticity**

- At macroscopic level we want to describe fluid of nuclei and neutronic superfluid
- How to estimate friction between them?
- Pinning force between nuclei and vortices in neutron superfluid

A. Melatos, C. Peralta
ApJ, 709, 77 (2010)

Methods

Microscopic

- ASLDA framewrok
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Mesoscopic

- **Relativistic?**
- **Vortex Filament Model**
- **Explicit dynamics of nuclei and vortices**

Macroscopic

- Relativistic
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- Fluid of neutrons
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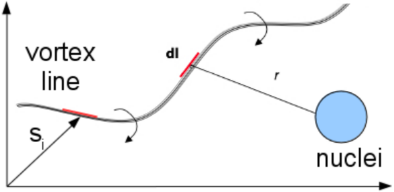
$$\rho\kappa \times (\dot{\mathbf{s}} - \mathbf{v}_{ind}(\mathbf{s}) - \mathbf{v}_{ext}) + \mathbf{f}^{VN}(\mathbf{s},\mathbf{r}) + \mathbf{f}^{D}(\dot{\mathbf{s}},\dot{\mathbf{r}}) = 0$$

$$\mathbf{s} = \mathbf{s}(\xi)$$

where ξ is a parameter describing position of line element along curve



Methods

Microscopic

- **ASLDA framewrok**
- **Explicit s-wave superfluidity**
- **Equivalent of Schrödinger eq.**

Mesoscopic

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Macroscopic

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Two sets HFB equations with density functional for protons and neutrons

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{n\uparrow}(\mathbf{r},t) \\ u_{n\downarrow}(\mathbf{r},t) \\ v_{n\uparrow}(\mathbf{r},t) \\ v_{n\downarrow}(\mathbf{r},t) \end{pmatrix} = \begin{pmatrix} h_{\uparrow,\uparrow}(\mathbf{r},t) & h_{\uparrow,\downarrow}(\mathbf{r},t) & 0 & \Delta(\mathbf{r},t) \\ h_{\downarrow,\uparrow}(\mathbf{r},t) & h_{\downarrow,\downarrow}(\mathbf{r},t) & -\Delta(\mathbf{r},t) & 0 \\ 0 & -\Delta^*(\mathbf{r},t) & -h^*_{\uparrow,\uparrow}(\mathbf{r},t) & -h^*_{\uparrow,\downarrow}(\mathbf{r},t) \\ \Delta^*(\mathbf{r},t) & 0 & -h^*_{\uparrow,\downarrow}(\mathbf{r},t) & -h^*_{\downarrow,\downarrow}(\mathbf{r},t) \end{pmatrix}\begin{pmatrix} u_{n\uparrow}(\mathbf{r},t) \\ u_{n\downarrow}(\mathbf{r},t) \\ v_{n\uparrow}(\mathbf{r},t) \\ v_{n\downarrow}(\mathbf{r},t) \end{pmatrix}$$

A. Bulgac, Phys. Rev. A 76, 040502 (2007)

SLDA method capabilities

Two sets HFB equations with density functional for protons and neutrons

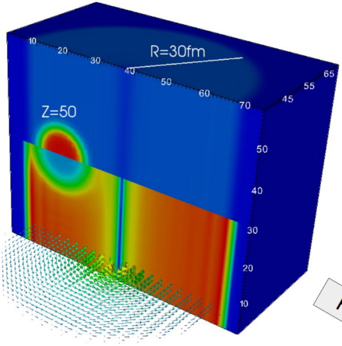
$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{n\uparrow}(\mathbf{r},t) \\ u_{n\downarrow}(\mathbf{r},t) \\ v_{n\uparrow}(\mathbf{r},t) \\ v_{n\downarrow}(\mathbf{r},t) \end{pmatrix} = \begin{pmatrix} h_{\uparrow,\uparrow}(\mathbf{r},t) & h_{\uparrow,\downarrow}(\mathbf{r},t) & 0 & \Delta(\mathbf{r},t) \\ h_{\downarrow,\uparrow}(\mathbf{r},t) & h_{\downarrow,\downarrow}(\mathbf{r},t) & -\Delta(\mathbf{r},t) & 0 \\ 0 & -\Delta^*(\mathbf{r},t) & -h^*_{\uparrow,\uparrow}(\mathbf{r},t) & -h^*_{\uparrow,\downarrow}(\mathbf{r},t) \\ \Delta^*(\mathbf{r},t) & 0 & -h^*_{\uparrow,\downarrow}(\mathbf{r},t) & -h^*_{\downarrow,\downarrow}(\mathbf{r},t) \end{pmatrix}\begin{pmatrix} u_{n\uparrow}(\mathbf{r},t) \\ u_{n\downarrow}(\mathbf{r},t) \\ v_{n\uparrow}(\mathbf{r},t) \\ v_{n\downarrow}(\mathbf{r},t) \end{pmatrix}$$

Present computing capabilities:

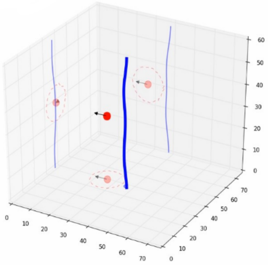
(for nuclear systems)

- full 3D (unconstrained) dynamics
- volumes up to 100^3 fm^3
- number of particles of order 10^4
- up to 10^6 time steps (for nuclear systems it gives trajectory length 10^{-19} sec)

Aim to construct mesoscopic model preserving low-energy physics



When exact dynamics of quantum system is known a semi-classical model preserving low-energy excitations dynamics can be constructed.

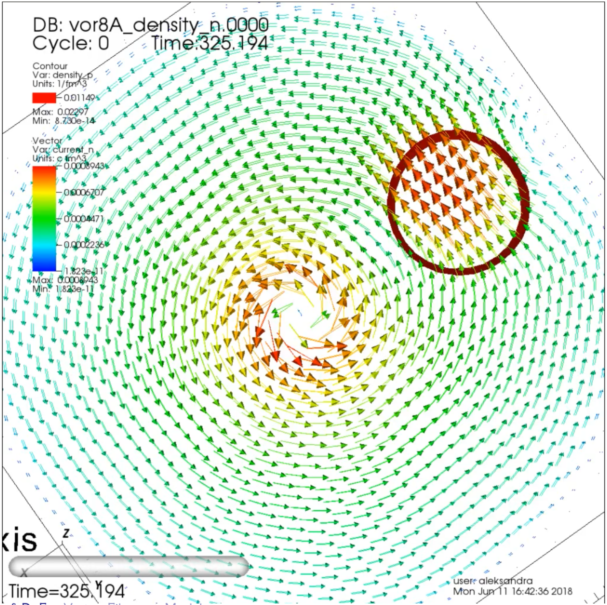


Exact dynamics of all fermionic DoFs

Computational cost limits size of system

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{n\uparrow}(\mathbf{r},t) \\ u_{n\downarrow}(\mathbf{r},t) \\ v_{n\uparrow}(\mathbf{r},t) \\ v_{n\downarrow}(\mathbf{r},t) \end{pmatrix} = \begin{pmatrix} h_{\uparrow,\uparrow}(\mathbf{r},t) & h_{\uparrow,\downarrow}(\mathbf{r},t) & 0 & \Delta(\mathbf{r},t) \\ h_{\downarrow,\uparrow}(\mathbf{r},t) & h_{\downarrow,\downarrow}(\mathbf{r},t) & -\Delta(\mathbf{r},t) & 0 \\ 0 & -\Delta^{*}(\mathbf{r},t) & -h^{*}_{\uparrow,\uparrow}(\mathbf{r},t) & -h^{*}_{\uparrow,\downarrow}(\mathbf{r},t) \\ \Delta^{*}(\mathbf{r},t) & 0 & -h^{*}_{\uparrow,\downarrow}(\mathbf{r},t) & -h^{*}_{\downarrow,\downarrow}(\mathbf{r},t) \end{pmatrix}\begin{pmatrix} u_{n\uparrow}(\mathbf{r},t) \\ u_{n\downarrow}(\mathbf{r},t) \\ v_{n\uparrow}(\mathbf{r},t) \\ v_{n\downarrow}(\mathbf{r},t) \end{pmatrix}$$

Example of transition between micro- and meso- scopic regime



Quantum mechanics vs hydrodynamics

$$i\hbar\partial_t\Psi = \left[\hat{H}_{one-body} + \cancel{\hat{H}_{int}} + V_{ext}\left(x,t\right)\right]\Psi, \quad \Psi = \sqrt{n}e^{i\hbar\theta}$$

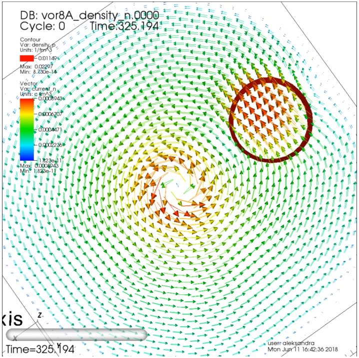
$$\downarrow$$

$$\frac{d\mathbf{v}}{dt} = \partial_t\mathbf{v} + \mathbf{v}\cdot\nabla\mathbf{v} = -\frac{1}{m}\nabla\left(-\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{n}}{\sqrt{n}} + V_{ext}\left(x,t\right)\right) \quad \partial_t n + \nabla\left(n\mathbf{v}\right) = 0$$

probability current

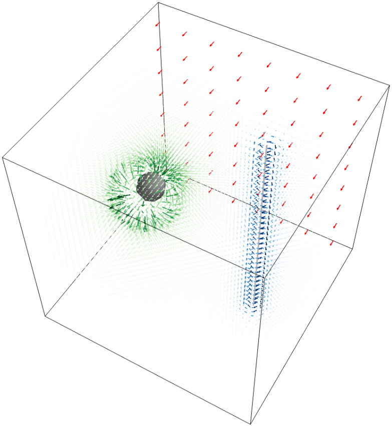
$$n\mathbf{v} = \mathbf{j} = \frac{\hbar}{2mi}\left[\Psi^*\left(\nabla\Psi\right) - \Psi\left(\nabla\Psi^*\right)\right]$$

Madelung, E. (1926). "Eine anschauliche Deutung der Gleichung von Schrödinger". Naturwissenschaften. 14 (45): 1004–1004



Helmholz-Hodge decomposition (HHD)

Compressible and incompressible velocity fields



$$\mathbf{v} = \underbrace{\mathbf{v}_{ext}}_{\approx \text{const}} + \underbrace{\mathbf{v}_{ind}}_{\nabla \times \mathbf{A}} + \underbrace{\mathbf{v}_{nucl}}_{\nabla \phi_N} + \mathbf{v}_{\text{boundary}}$$

How to estimate $\nabla \phi_N$?

Vortex line motion

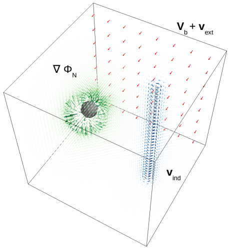
Description with fluid velocities

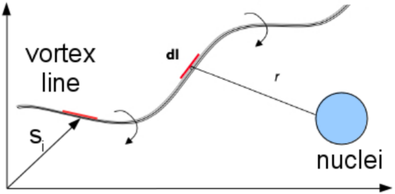
$$\dot{\mathbf{s}} = \mathbf{v}_{ind} + \mathbf{v}_b + \mathbf{v}_{ext} + \nabla\phi_N + \text{dissipative terms}$$

Description with forces acting on a vortex line

$$\rho_s \kappa \times \dot{\mathbf{s}} = \rho_s \kappa \times \left[\mathbf{v}_{ind} + \mathbf{v}_b + \mathbf{v}_{ext} + \nabla\phi_N\right] + \mathbf{f}^D$$

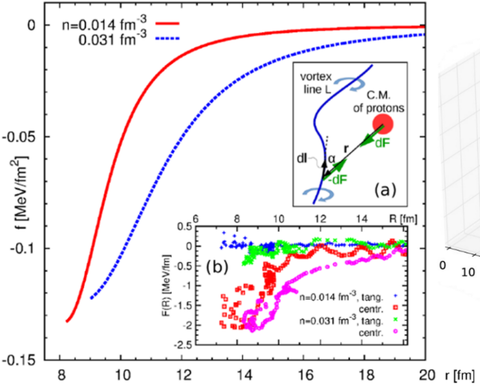
$$\rho_s \kappa \times \dot{\mathbf{s}} = \rho_s \kappa \times \left[\mathbf{v}_b + \mathbf{v}_{ext}\right] - \mathbf{f}^T - \mathbf{f}^{VN} - \mathbf{f}^D$$



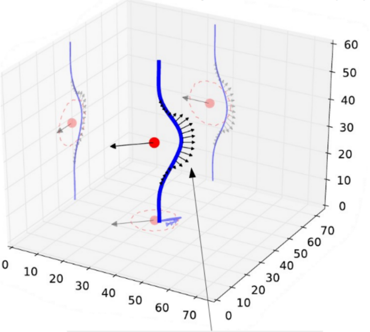


K. W. Schwarz, Phys. Rev. B 31, 5782 (1985)

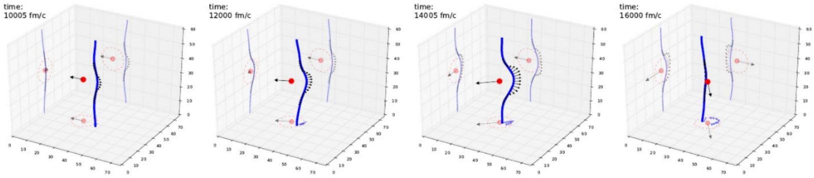
Vortex-nucleus interaction



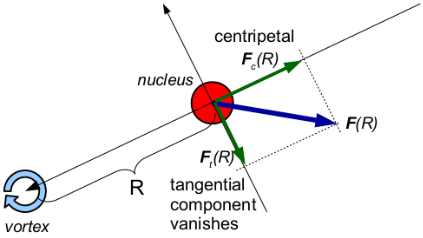
G. Wlazłowski et al., Phys. Rev. Lett. 117, 232701 (2016)

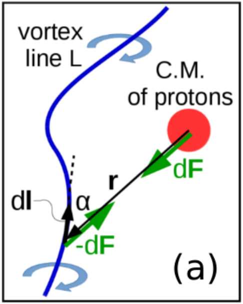


Nucleus is dragged by force adjusted to preserve uniform motion.
Interaction has to equilibrate system.

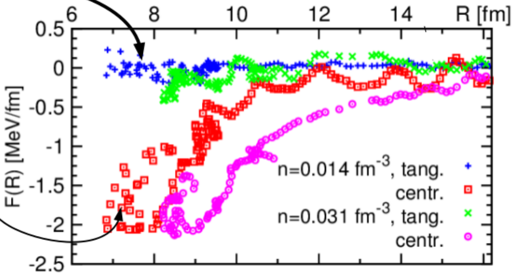


$$m_{nucl.}\frac{d}{dt}\mathbf{v}_{nucl.} = 0 = \mathbf{F}_{ext} + \hat{\mathbf{r}}\int_{\mathscr{L}} f(r)\sin\alpha dl$$





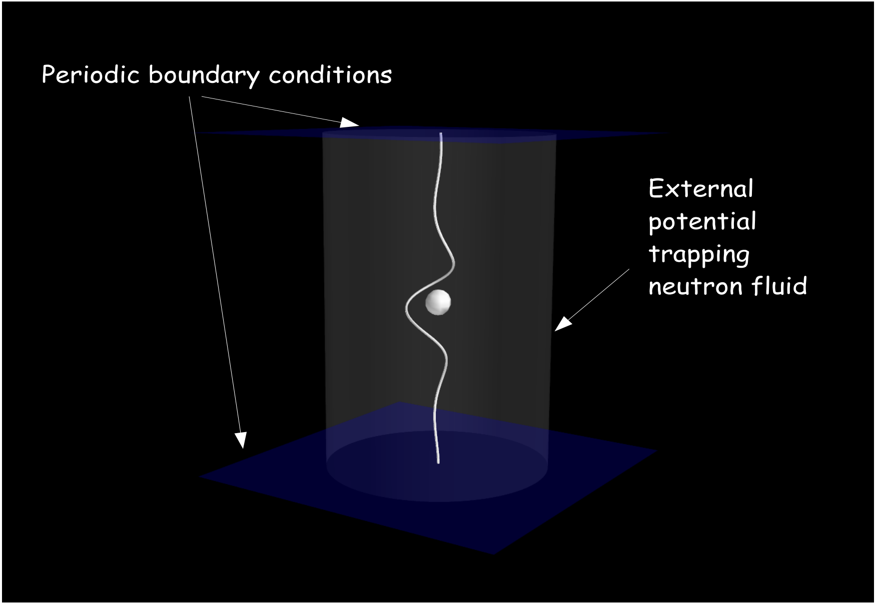
Force is **central**



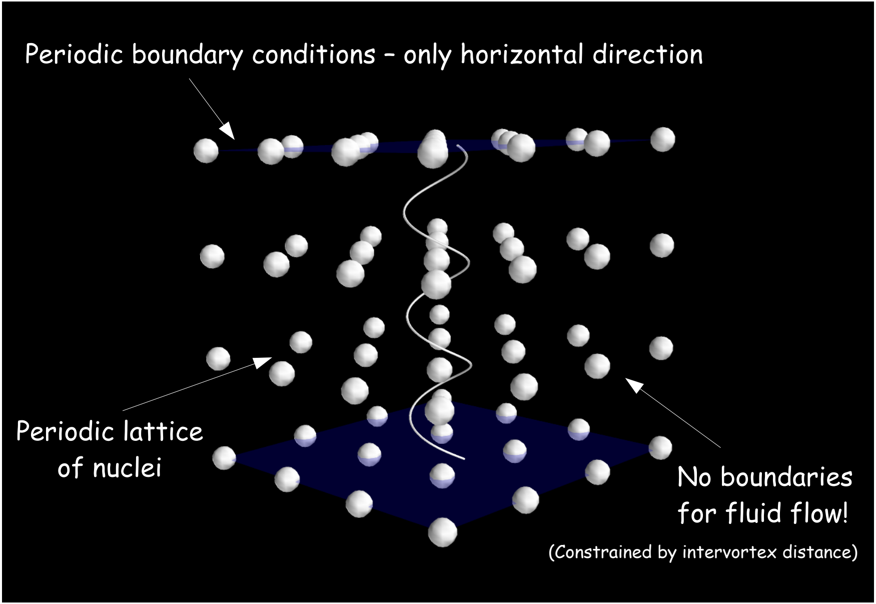
At close separation the force is not a function of distance R only

Plan of presentation

Setup of DFT simulations



Setup of realistic system for NS crust simulation



Self-induced velocity

$$\rho_s \kappa \times \dot{\mathbf{s}} = \rho_s \kappa \times \left[\mathbf{v}_b + \mathbf{v}_{ext} + \mathbf{v}_{ind} \right] - \mathbf{f}^{VN} - \mathbf{f}^{D}$$

$$\mathbf{v}_{ind}(\mathbf{r},t) = \frac{\kappa}{4\pi} \underbrace{\int_{\mathscr{L}}}_{\text{over vortex line}} dz \frac{(\mathbf{s}(z,t)-\mathbf{r}) \times \hat{\mathbf{t}}(z,t)}{|\mathbf{s}(z,t)-\mathbf{r}|^3}$$ is divergent when $\mathbf{r} \to \mathbf{s}(z)$

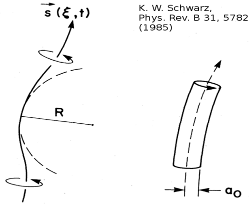
Local Induction Approximation (LIA) - a cannonical regularization

$$\mathbf{v}_{ind}(\mathbf{s},t) = \mathbf{v}_{LIA}(\mathbf{s},t) + \frac{\kappa}{4\pi} \underbrace{\int'_{\mathscr{L}}}_{\text{ommiting divergent point}} dz \frac{(\mathbf{s}(z,t)-\mathbf{s}) \times \hat{\mathbf{t}}(z,t)}{|\mathbf{s}(z,t)-\mathbf{s}|^3} \approx \mathbf{v}_{LIA}(\mathbf{r},t)$$

$$\mathbf{v}_{LIA}(\mathbf{s}(z),t) = \frac{\kappa}{4\pi R(z)} \ln\left(\frac{R(z)}{a_0}\right) \hat{\mathbf{b}}$$

a_0 - vortex core radius

$R(\mathbf{r})$ - radius of curvature



K. W. Schwarz, Phys. Rev. B 31, 5782 (1985)

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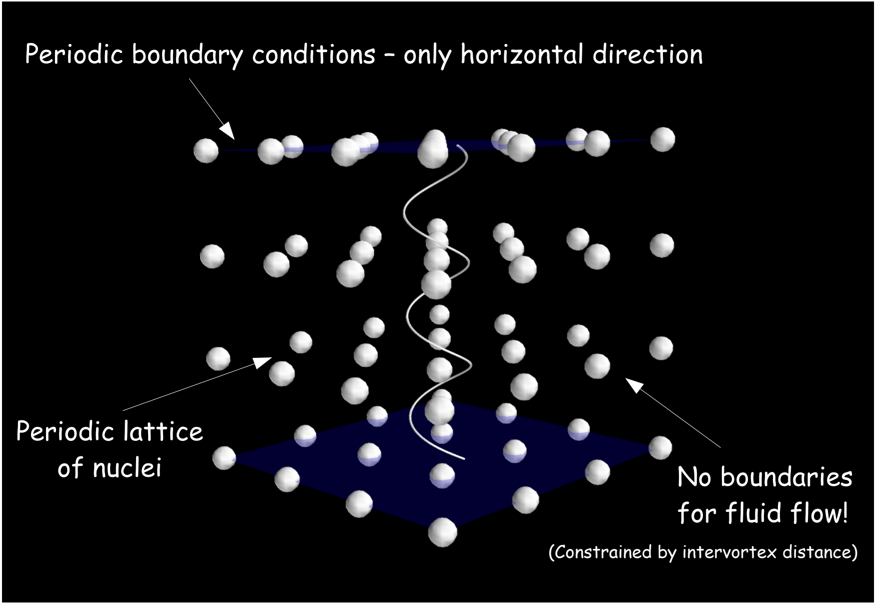

Self-induced velocity

$$\rho_s \kappa \times \dot{\mathbf{s}} = \rho_s \kappa \times \left[\mathbf{v}_b + \mathbf{v}_{ext} + \mathbf{v}_{ind}\right] - \mathbf{f}^{VN} - \mathbf{f}^{D}$$

$\omega[s] = \nabla \times \mathbf{v}[\mathbf{s}]$ - vorticity field, $\mathbf{s}$ - vortex line element position

$$\partial_t \omega[\mathbf{s}] = \nabla \times [\mathbf{v}[\mathbf{s}] \times \omega[\mathbf{s}]] \quad \Longrightarrow \quad \nabla^2 \mathbf{A} = -\omega[\mathbf{s}]$$

Effect of different domain

$$\mathbf{v}_{ind} = \nabla \times \mathbf{A}$$

$$\mathbf{A} = \sum_{n=-\infty}^{+\infty} e^{ik_n(z-z')} \int_0^L \frac{\kappa(z')}{2\pi} K_0 \left(|k_n| \sqrt{(x - s_x(z'))^2 + (y - s_y(z'))^2} \right) dz'$$

Also divergent when $\mathbf{r} \to \mathbf{s}(z)$

Vortex self-interaction

$$\rho_s \kappa \times \dot{\mathbf{s}} = \rho_s \kappa \times [\mathbf{v}_b + \mathbf{v}_{ext}] - \mathbf{f}^T - \mathbf{f}^{VN} - \mathbf{f}^D$$

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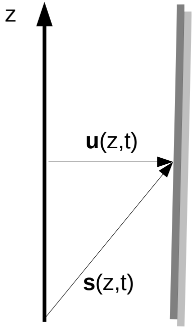


Energy flow can be rewritten as

$$E_V = \sum_n \omega(k_n)\,\widehat{\mathbf{u}}(k_n)\cdot\widehat{\mathbf{u}}^*(k_n)$$

Leading to

$$\mathbf{f}^T = \mathscr{F}^{-1}\left[\frac{\delta E_V(k_n)}{\delta \widehat{\mathbf{u}}^*(k_n)}\right]$$

$$\mathbf{f}^T = \mathscr{F}^{-1}\left[\omega(k_n)\,\widehat{\mathbf{u}}(k_n)\right]$$

A. Fetter, Phys. Rev. 162.143 (1967)
sir W. Thompson, Phil. Mag. 10, 155 (1880)

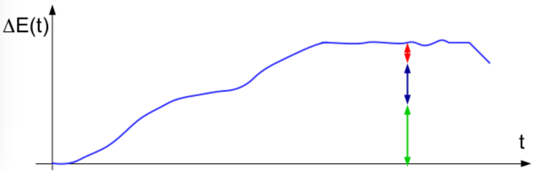
Dissipation force

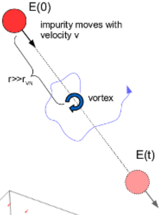
$$\rho_s \kappa \times \dot{\mathbf{s}} = \rho_s \kappa \times [\mathbf{v}_b + \mathbf{v}_{ext}] - \mathbf{f}^T - \mathbf{f}^{VN} - \mathbf{f}^D$$

- Dissipation through vortex-phonon interactions $\mathbf{f}_1^D = -\eta_1 (\dot{\mathbf{u}} - \mathbf{v}_{ext})$
- Dissipation through nuclei deformation $\mathbf{f}_2^D = -\eta_2 (\dot{\mathbf{u}} - \dot{\mathbf{r}}_N)$

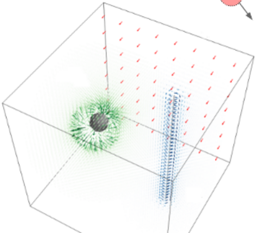
Estimation methodology

drag impurity with various velocities...
look at excitation energy...





$$\Delta E(t) = \Delta E_{vortex}(t) + \Delta E_{compressible}(t) + \Delta E_{internal}(t)$$



Dissipation force

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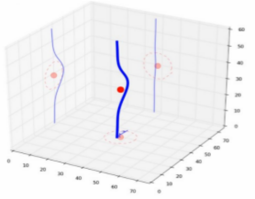
Estimation methodology



Use "ansatz" here for the dissipation force, and try to reproduce $W_d(t)$...

$$W_d(t) = \int_{\mathcal{L}} \left[\int_0^t f^D(t') \cdot \dot{s}_l(t')\, dt' \right] dl$$

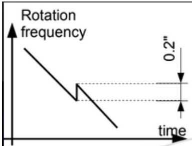
Reflected by the vortex line deformation in VFM

$W_d(t)$ = Dissipated energy (from point of view of VFM)

Known from simulations...

$$\Delta E(t) = \Delta E_{vortex}(t) + \Delta E_{compressible}(t) + \Delta E_{internal}(t)$$

Glitch size estimation



Observed relative sizes of glitches: $\Delta\nu/\nu \sim 10^{-11} - 10^{-5}$

maximum glitch amplitude

$$\Delta\Omega_{\max} = \frac{\pi^2}{\kappa\, I} \int_0^{R_d} dr\, r^3\, f_P(r)$$

quantum of circulation

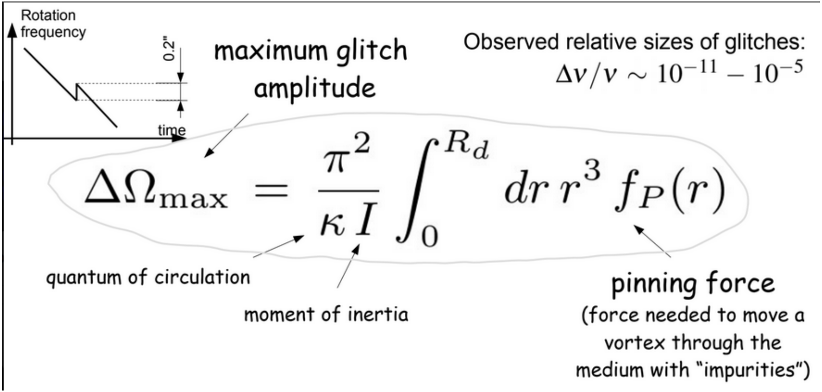
moment of inertia

pinning force (force needed to move a vortex through the medium with "impurities")

M. Antonelli, A. Montoli, P. M. Pizzochero, MNRAS 475, 5403 (2018)

Test for crustal vs flux tubes pinning!

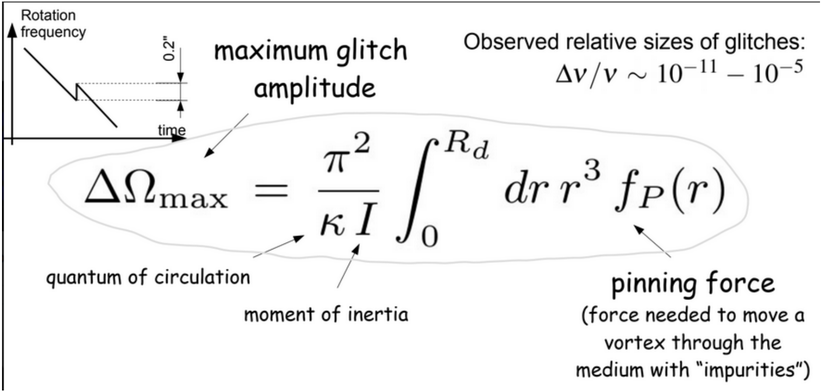
Glitch size estimation



M. Antonelli, A. Montoli, P. M. Pizzochero, MNRAS 475, 5403 (2018)

Could provide test for microscopic models?

Glitch size estimation



M. Antonelli, A. Montoli, P. M. Pizzochero, MNRAS 475, 5403 (2018)

Plan of presentation

Future work objectives

- Development of nuclear SLDA
  1. More accurate density functionals - BSk
  2. Nuclei lattice - structure and dynamics
- Vortex Filament Model validation
  1. Comparison between LIA and small deflections approach
  2. Extraction for Kelivn Waves dispersion relation
  3. Inclusion of nuclei dynamics
  4. Extraction of dissipation coefficients
- Large-scale VFM simulations
  1. Extraction of pinning force
  2. Connection to relativistic hydrodynamics of NS
- Rigorious testing of density functionals?

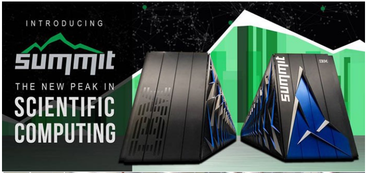
High performance computing

Rank	Site	System	Cores	Rmax (TFlop/s)	Rpeak (TFlop/s)	Power (kW)
1	DOE/SC/Oak Ridge National Laboratory United States	**Summit** - IBM Power System AC922, IBM POWER9 22C 3.07GHz, NVIDIA Volta GV100, Dual-rail Mellanox EDR Infiniband IBM	2,282,544	122,300.0	187,659.3	8,806
2	National Supercomputing Center in Wuxi China	**Sunway TaihuLight** - Sunway MPP, Sunway SW26010 260C 1.45GHz, Sunway NRCPC	10,649,600	93,014.6	125,435.9	15,371
3	DOE/NNSA/LLNL United States	**Sierra** - IBM Power System S922LC, IBM POWER9 22C 3.1GHz, NVIDIA Volta GV100, Dual-rail Mellanox EDR Infiniband IBM	1,572,480	71,610.0	119,193.6	
4	National Super Computer Center in Guangzhou China	**Tianhe-2A** - TH-IVB-FEP Cluster, Intel Xeon E5-2692v2 12C 2.2GHz, TH Express-2, Matrix-2000 NUDT	4,981,760	61,444.5	100,678.7	18,482
5	National Institute of Advanced Industrial Science and Technology (AIST) Japan	**AI Bridging Cloud Infrastructure (ABCI)** - PRIMERGY CX2550 M4, Xeon Gold 6148 20C 2.4GHz, NVIDIA Tesla V100 SXM2, Infiniband EDR Fujitsu	391,680	19,880.0	32,576.6	1,649
6	Swiss National Supercomputing Centre (CSCS) Switzerland	**Piz Daint** - Cray XC50, Xeon E5-2690v3 12C 2.6GHz, Aries interconnect , NVIDIA Tesla P100 Cray Inc.	361,760	19,590.0	25,326.3	2,272
7	DOE/SC/Oak Ridge National Laboratory United States	**Titan** - Cray XK7, Opteron 6274 16C 2.200GHz, Cray Gemini interconnect, NVIDIA K20x Cray Inc.	560,640	17,590.0	27,112.5	8,209
19	GSIC Center, Tokyo Institute of Technology Japan	**TSUBAME3.0** - SGI ICE XA, IP139-SXM2, Xeon E5-2680v4 14C 2.4GHz, Intel Omni-Path, NVIDIA Tesla P100 SXM2 HPE	135,828	8,125.0	12,127.1	792

?

TOP 500 The List.

June 2018



Thank you!

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Effects of boundaries

$$\rho_s \kappa \times \dot{\mathbf{s}} = \rho_s \kappa \times \left[\mathbf{v}_b + \mathbf{v}_{ext} \right] - \mathbf{f}^T - \mathbf{f}^{VN} - \mathbf{f}^D$$

Flow past boundaries have to be supressed $\implies$ $\mathbf{v}^\infty \cdot \hat{\mathbf{n}} = \mathbf{v}_b \cdot \hat{\mathbf{n}}$