Quark-Antiquark Excited Flux Tube

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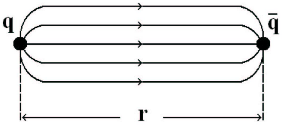
The vacuum of QCD is a magnetic (dual superconductor)[a]

[a] G. 'tHooft, Phys. Scripta 25 (1982) 133

The electric field is confined into flux tubes → QCD strings



$$V_{q\bar{q}} \to \sigma r$$

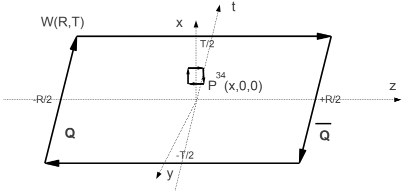
the dual Meissner effect causes the formation of chromoelectric flux tubes between chromoelectric charges leading to a linear rising potential

At zero temperature,

$$f_{\mu\nu} = \frac{\langle \mathrm{Tr}\, W \,\Box_{\mu\nu} \rangle}{\langle \mathrm{Tr}\, W \rangle} - \langle \Box_{\mu\nu} \rangle \to a^4 \left(\left\langle F^2_{\mu\nu} \right\rangle_{q\bar{q}} - \left\langle F^2_{\mu\nu} \right\rangle_{\mathrm{vac}} \right)$$

where W is the Wilson loop and $\Box_{\mu\nu}$ is the plaquette in the (μ, ν) plane,

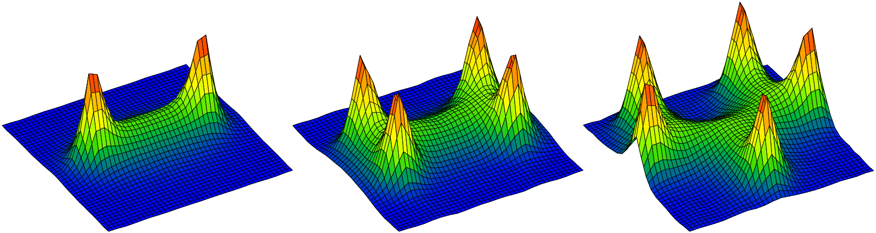
$$\Box_{\mu\nu} = 1 - \frac{1}{N_c} \mathrm{Tr} \left[U_\mu(s) U_\nu(s+\mu) U^\dagger_\mu(s+\nu) U^\dagger_\nu(s) \right]$$



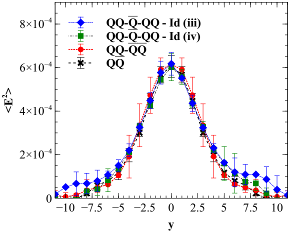
$$\left\langle E_i^2 \right\rangle = -f_{i,0} \qquad \text{and} \qquad \left\langle B_i^2 \right\rangle = f_{j,k}$$

and the Lagrangian ($\mathcal{L}$) density is given by

$$\mathcal{L} = \frac{1}{2} \left(\left\langle E^2 \right\rangle - \left\langle B^2 \right\rangle \right)$$



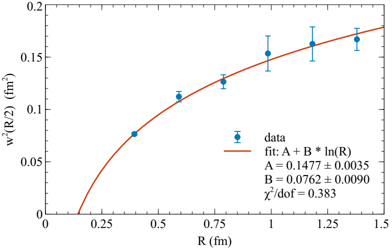
Flux tube profile:



Results in lattice spacing units, $a = 0.07261(85)$ *fm* or $a^{-1} = 2718(32)$ *MeV*
$24^3 \times 48$ lattice volume with $\beta = 6.2$

Widening in the mediator plane

Square of the width of the flux tube in the mediator plane.
Fit of the flux tube width to the leading order one-loop computation in effective string theory[a]



The B parameter can be compared with the theoretical leading order[a] value for the factor of the logarithmic term,

$$B = \frac{D-2}{2\pi\sigma} = 0.0640028\,\mathrm{fm}^2$$

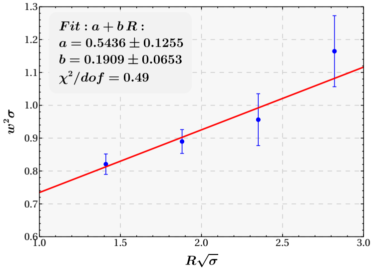
obtained using a string tension of $\sqrt{\sigma} = 0.44\,\mathrm{GeV}$.
The width complies, almost within one standard deviation, with the logarithmic widening obtained at leading order in the Nambu-Gotto effective string theory.

[a] F. Gliozzi et al. JHEP 1011, 053 (2010), arXiv:1006.2252.

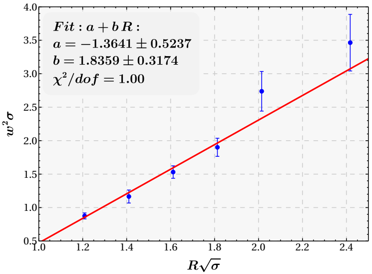
Widening in the mediator plane

Square of the width of the flux tube in the mediator plane.
Fit of the flux tube width to the leading order one-loop computation in effective string theory

- $T = 0.845\ T_c$



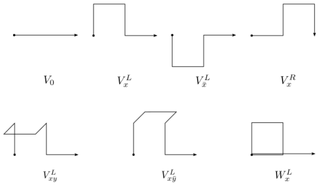
- $T = 0.986\ T_c$



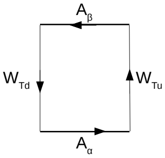
Bicudo et al., "Pure gauge QCD flux tubes and their widths at finite temperature", arXiv:1702.03454 [hep-lat], 2017.

- Gluon field in presence of static quark-antiquark pair can be excited
- In the study of hybrid excitations Lacock et al. Phys. Rev. D54 (1996), we follow the same quantum number notation as in Juge, Kuti and Morningstar, Phys. Rev. Lett. 90 (2003) 161601.
- We adopt the standard notation from the physics of diatomic molecules and use Λ to denote the magnitude of the eigenvalue of the projection $\mathbf{J}_g \cdot \hat{\mathbf{R}}$ of the total angular momentum $\mathbf{J}_g$ of the gluon field onto the molecular axis with unit vector $\hat{\mathbf{R}}$.
- The capital Greek letters $\Sigma, \Pi, \Delta, \Phi, \ldots$ are used to indicate states with $\Lambda = 0, 1, 2, 3, \ldots,$ respectively.
  - magnitude of glue spin projected onto molecular axis
- The combined operations of charge conjugation and spatial inversion about the midpoint between the quark and the antiquark is also a symmetry and its eigenvalue is denoted by η_{CP}.
  - States with $\eta_{CP} = 1(-1)$ are denoted by the subscripts g (u).
- There is an additional label for the Σ states; Σ states which are even (odd) under a reflection in a plane containing the molecular axis are denoted by a superscript $+$ $(-)$.
- Hence, the low-lying levels are labeled Σ_g^+, Σ_g^-, Σ_u^+, Σ_u^-, Π_g, Π_u, Δ_g, Δ_u, and so on.

- Gluonic terms extracted from generalized Wilson loops
- Large set of gluonic operators → correlation matrix
- we use the following basis used in space Wilson lines:



- Therefore, the Wilson loop is constructed as:

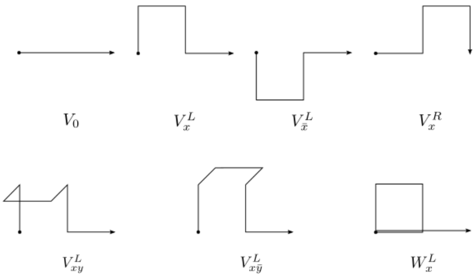


where the W_{Tu} and W_{Td} are temporal Wilson lines and A_α and A_β are the above basis inserted in the space Wilson lines.

- Finally, Wilson loop for the above operators are given by:

$$W_{\alpha\beta} = A_\alpha W_{Tu} A_\beta^\dagger W_{Td}^\dagger$$

- Paths from the quark to the antiquark used to construct the gauge field operators



$\rightarrow$ Gives a total of 33 different paths

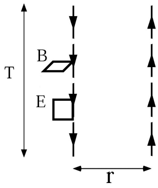
This basis is composed by four kinds of operators:

- The direct operator V_0.
- The eight open-staple operators V_x^L , V_y^L, $V_{\bar{x}}^L$, $V_{\bar{y}}^L$, V_x^R, V_y^R, $V_{\bar{x}}^R$ and $V_{\bar{y}}^R$.
- The sixteen open-staple two-direction operators V_{xy}^L, $V_{x\bar{y}}^L$, $V_{\bar{x}y}^L$, $V_{\bar{x}\bar{y}}^L$, V_{yx}^L, $V_{y\bar{x}}^L$, $V_{\bar{y}x}^L$, $V_{\bar{y}\bar{x}}^L$, V_{xy}^R, $V_{x\bar{y}}^R$, $V_{\bar{x}y}^R$, $V_{\bar{x}\bar{y}}^R$, V_{yx}^R, $V_{y\bar{x}}^R$, $V_{\bar{y}x}^R$ and $V_{\bar{y}\bar{x}}^R$.
- The eight closed-staple operators similar to the open-staple ones W_x^L, etc.

The bar, means that there is displacement in the negative axis direction. The L and R labels indicate whether the staple is on the left or on the right.

The central observables that govern the event in the flux tube can be extracted from the correlation of a plaquette, $\Box_{\mu\nu}$, with the Wilson loop, W,

$$f_{\mu\nu}(x,R) = \left[\frac{\langle \mathrm{Tr}\, W(x,R)\, \Box_{\mu\nu}(x) \rangle}{\langle \mathrm{Tr}\, W(x,R) \rangle} - \langle \Box_{\mu\nu}(x) \rangle \right]$$



where W is the quark-antiquark Wilson loop, x denotes the distance of the plaquette from the line connecting quark sources, R is the quark-antiquark separation.

Therefore, using the plaquette orientation $(\mu,\nu) = (2,3),(1,3),(1,2),(1,4),(2,4),(3,4)$, we can relate the six components to the components of the chromoelectric and chromomagnetic fields,

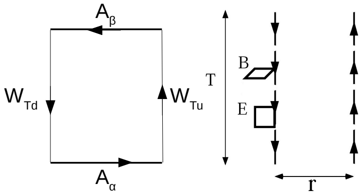
$$f_{\mu\nu} \to \left(-\left\langle B_x^2 \right\rangle, -\left\langle B_y^2 \right\rangle, -\left\langle B_z^2 \right\rangle, \left\langle E_x^2 \right\rangle, \left\langle E_y^2 \right\rangle, \left\langle E_z^2 \right\rangle \right)$$

and also calculate the total action (Lagrangian) density, $\langle \mathcal{L} \rangle = \frac{1}{2}\left(\left\langle E^2 \right\rangle - \left\langle B^2 \right\rangle \right)$.

In order to improve the signal over noise ratio, we use multihit technique in the temporal Wilson lines and the APE smearing spatial Wilson lines.

All the computation is entirely done in NVIDIA GPUs.
This is computationally intensive

- a lot of atomic reductions
- 2 planes (XY and XZ), each one with $24 \times 24 \times 12$ at each lattice point.
  - The quark and antiquark are placed at $(0, 0, -R/2)$ and $(0, 0, R/2)$



The Wilson loop for the above operators are given by:

$$W_{\alpha\beta} = A_\alpha W_{Tu} A_\beta^\dagger W_{Td}^\dagger$$

The Wilson lines $A_{\alpha/\beta}$ are pre-calculated.
90% of the computational time is spent in computing:

$$f_{\mu\nu}(x, R) = \left[\frac{\langle \mathrm{Tr}\, W(x, R)\, \Box_{\mu\nu}(x) \rangle}{\langle \mathrm{Tr}\, W(x, R) \rangle} - \langle \Box_{\mu\nu}(x) \rangle \right]$$

33 different operators $\rightarrow$ gives a 33×33 sets to calculate!

We need to reduce this otherwise the calculation of the color fields can take forever!

With these, we reconstruct operators for the following quantum numbers:

- $\boldsymbol{\Sigma_g^+}$
  For this quantum number, we have four operators:

$$\begin{aligned}
\mathcal{A}_0 &= V_0 \\
\mathcal{A}_1 &= \frac{1}{2\sqrt{2}}\left(V_x^L + V_y^L + V_{\bar{x}}^L + V_{\bar{y}}^L + V_x^R + V_y^R + V_{\bar{x}}^R + V_{\bar{y}}^R\right) \\
\mathcal{A}_2 &= \frac{1}{4}\left(V_{xy}^L + V_{x\bar{y}}^L + V_{\bar{x}y}^L + V_{\bar{x}\bar{y}}^L + V_{yx}^L + V_{y\bar{x}}^L + V_{\bar{y}x}^L + V_{\bar{y}\bar{x}}^L\right. \\
&\quad \left. + V_{xy}^R + V_{x\bar{y}}^R + V_{\bar{x}y}^R + V_{\bar{x}\bar{y}}^R + V_{yx}^R + V_{y\bar{x}}^R + V_{\bar{y}x}^R + V_{\bar{y}\bar{x}}^R\right) \\
\mathcal{A}_3 &= \frac{1}{2\sqrt{2}}\left(W_x^L + W_y^L + W_{\bar{x}}^L + W_{\bar{y}}^L + W_x^R + W_y^R + W_{\bar{x}}^R + W_{\bar{y}}^R\right)
\end{aligned}$$

Therefore the Wilson loop for $\boldsymbol{\Sigma_g^+}$ is

$$\begin{bmatrix} W_{00} & W_{01} & W_{02} & W_{03} \\ W_{10} & W_{11} & W_{12} & W_{13} \\ W_{20} & W_{21} & W_{22} & W_{23} \\ W_{30} & W_{31} & W_{32} & W_{33} \end{bmatrix}$$

which means, we have to calculate the $f_{\mu\nu}(x, R)$ for each element.

- Σ_u^+

For this quantum number, we have three operators:

$$\mathcal{A}_0 = \frac{1}{2\sqrt{2}}(V_x^L + V_y^L + V_{\bar{x}}^L + V_{\bar{y}}^L - (V_x^R + V_y^R + V_{\bar{x}}^R + V_{\bar{y}}^R))$$

$$\mathcal{A}_1 = \frac{1}{4}(V_{xy}^L + V_{x\bar{y}}^L + V_{\bar{x}y}^L + V_{\bar{x}\bar{y}}^L + V_{yx}^L + V_{y\bar{x}}^L + V_{\bar{y}x}^L + V_{\bar{y}\bar{x}}^L$$
$$-(V_{xy}^R + V_{x\bar{y}}^R + V_{\bar{x}y}^R + V_{\bar{x}\bar{y}}^R + V_{yx}^R + V_{y\bar{x}}^R + V_{\bar{y}x}^R + V_{\bar{y}\bar{x}}^R))$$

$$\mathcal{A}_2 = \frac{1}{2\sqrt{2}}(W_x^L + W_y^L + W_{\bar{x}}^L + W_{\bar{y}}^L - (W_x^R + W_y^R + W_{\bar{x}}^R + W_{\bar{y}}^R))$$

- Π_u

Four operators for these quantum numbers:

$$\mathcal{A}_0 = \frac{1}{2\sqrt{2}}(V_x^L + iV_y^L - V_{\bar{x}}^L - iV_{\bar{y}}^L + V_x^R + iV_y^R - V_{\bar{x}}^R - iV_{\bar{y}}^R)$$

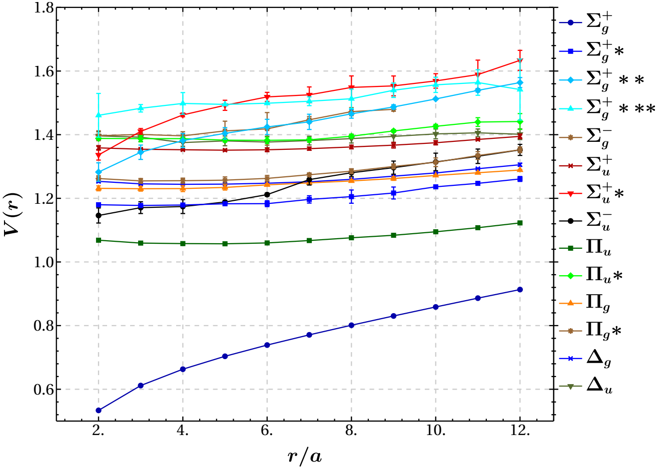
$$\mathcal{A}_1 = \frac{1}{4}(V_{xy}^L + V_{x\bar{y}}^L - V_{\bar{x}y}^L - V_{\bar{x}\bar{y}}^L + iV_{yx}^L + iV_{y\bar{x}}^L - iV_{\bar{y}x}^L - iV_{\bar{y}\bar{x}}^L$$
$$+V_{xy}^R + V_{x\bar{y}}^R - V_{\bar{x}y}^R - V_{\bar{x}\bar{y}}^R + iV_{yx}^R + iV_{y\bar{x}}^R - iV_{\bar{y}x}^R - iV_{\bar{y}\bar{x}}^R)$$

$$\mathcal{A}_2 = \frac{1}{4}(V_{xy}^L - V_{x\bar{y}}^L + V_{\bar{x}y}^L - V_{\bar{x}\bar{y}}^L - iV_{yx}^L + iV_{y\bar{x}}^L - iV_{\bar{y}x}^L + iV_{\bar{y}\bar{x}}^L$$
$$+V_{xy}^R - V_{x\bar{y}}^R + V_{\bar{x}y}^R - V_{\bar{x}\bar{y}}^R - iV_{yx}^R + iV_{y\bar{x}}^R - iV_{\bar{y}x}^R + iV_{\bar{y}\bar{x}}^R)$$

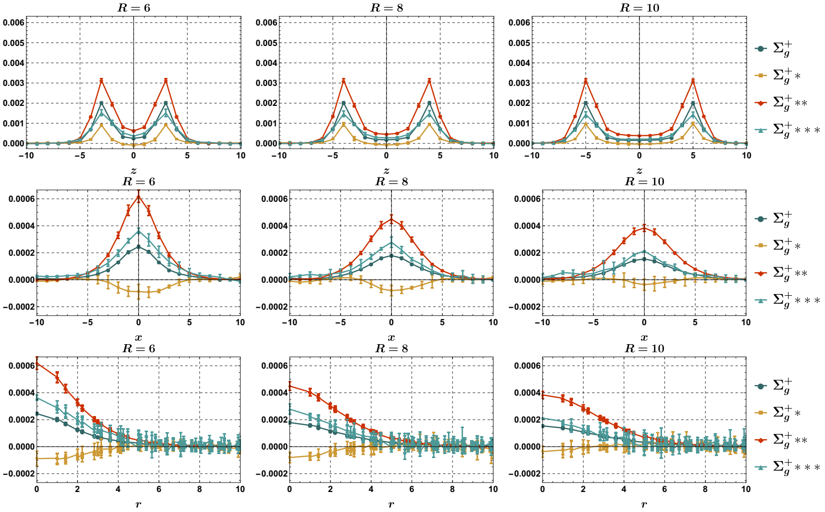
$$\mathcal{A}_3 = \frac{1}{2\sqrt{2}}(W_x^L + iW_y^L - W_{\bar{x}}^L - iW_{\bar{y}}^L + W_x^R + iW_y^R - W_{\bar{x}}^R - iW_{\bar{y}}^R)$$

- 1199 configurations
- Lattice volume of $24^3 \times 48$
- $\beta = 6.2$
- $a = 0.07261(85)\,\text{fm}$ or $a^{-1} = 2718(32)\,\text{MeV}$
- The quark and antiquark are placed at $(0, 0, -R/2)$ and $(0, 0, R/2)$ for R between 6, 8 and 10 in lattice spacing units.

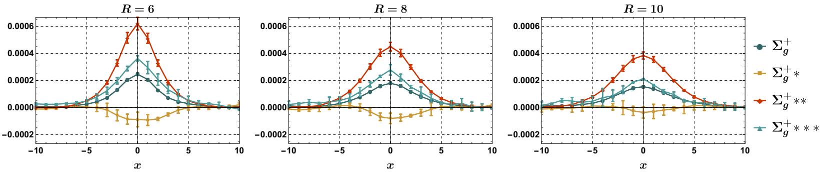
Results


1.8
1.6
1.4
1.2
1.0
0.8
0.6
$V(r)$
2.
4.
6.
8.
10.
12.
r/a
Σ_g^+
Σ_g^+*
Σ_g^+**
Σ_g^+***
Σ_g^-
Σ_u^+
Σ_u^+*
Σ_u^-
Π_u
Π_u*
Π_g
Π_g*
Δ_g
Δ_u

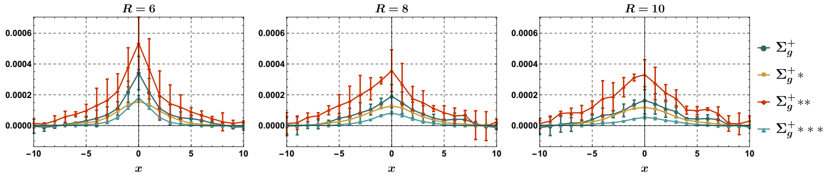
- Σ_g^+, Lagrangian density $\mathcal{L}$



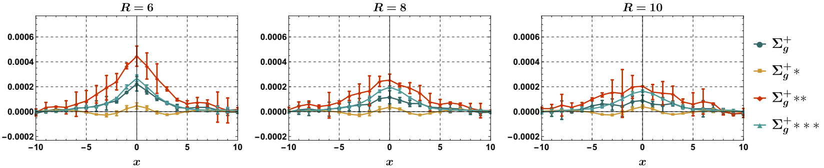
- $\mathcal{L}$



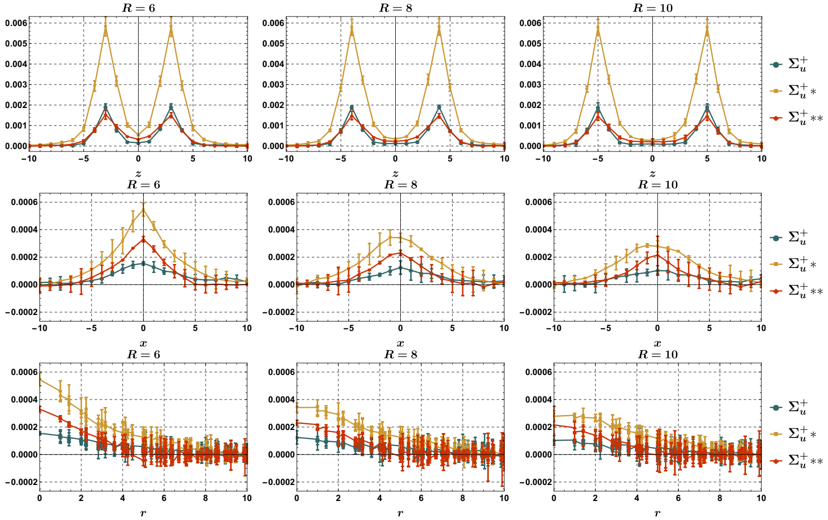
- $\langle E^2 \rangle$



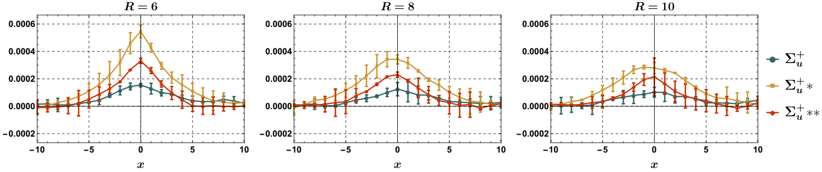
- $-\langle B^2 \rangle$



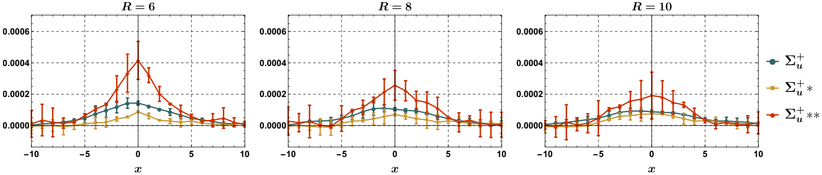
- Σ_u^+, Lagrangian density $\mathcal{L}$



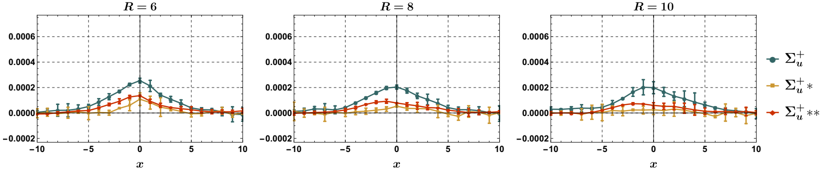
- $\mathcal{L}$



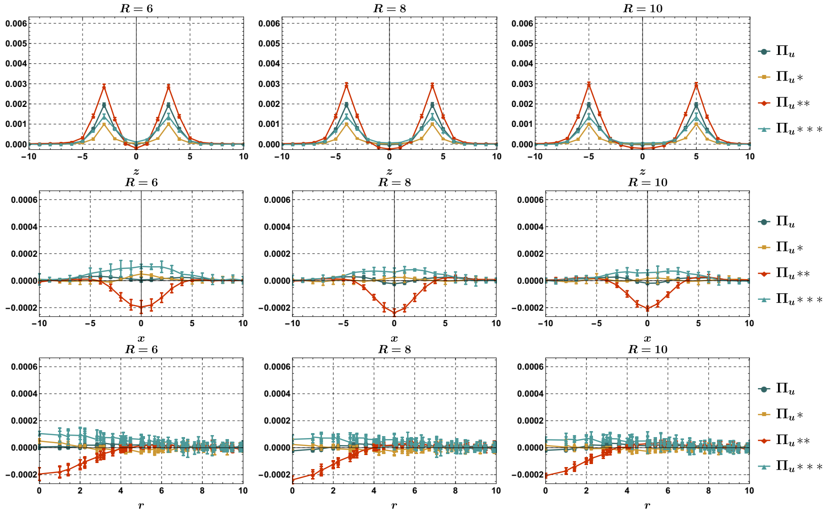
- $\langle E^2 \rangle$



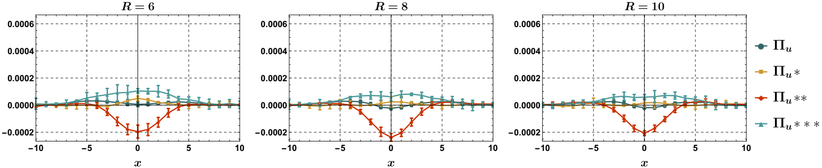
- $-\langle B^2 \rangle$



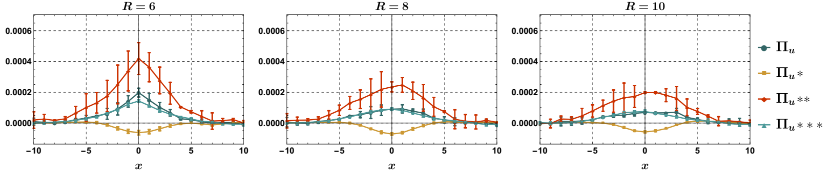
- $\mathbf{\Pi_u}$, Lagrangian density $\mathcal{L}$



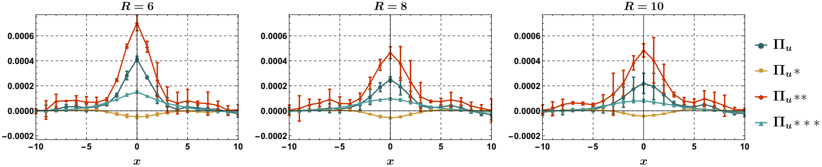
- $\mathcal{L}$



- $\langle E^2 \rangle$



- $-\langle B^2 \rangle$



- Work still in progress!
- We compute the potentials for several excitations of the flux tube, but for the colour field square densities we only select the main excited states in each quantum number.
- We consider radial excitations of the groundstate Σ_g^+, the first angular excitation Π_u and the first axial parity excitation Σ_u^+.
- As a preliminary result, we find states consistent with transverse excitations (the type of excitations considered in the Nambu-Goto string model) but we also find states who may correspond to other excitations. We leave the interpretation of the results to the theoretical community.

Thanks