

# THE LIQUID-GAS PHASE TRANSITION WITHIN THE TEMPERATURE DEPENDENT DD-NLD MODEL

**Sofija Antić<sup>1</sup>**, Helena Pais<sup>2</sup>, Stefan Typel<sup>1</sup>, Constanca Providencia<sup>2</sup>

<sup>1</sup>*GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt, Germany,*

<sup>2</sup>*CFisUC, Department of Physics, University of Coimbra, Coimbra, Portugal*

# THE LIQUID-GAS PHASE TRANSITION WITHIN THE TEMPERATURE DEPENDENT DD-NLD MODEL

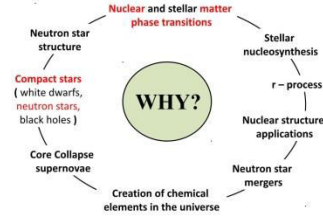
Sofija Antić<sup>1</sup>, Helena Pais<sup>2</sup>, Stefan Typel<sup>1</sup>, Constanca Providencia<sup>2</sup>  
<sup>1</sup>GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt, Germany,  
<sup>2</sup>CFisUC, Department of Physics, University of Coimbra, Coimbra, Portugal

# MY POSTER



## INTRODUCTION

We investigate the thermodynamic properties (equation of state) of nuclear matter in different conditions (different temperatures, density regions) and phase transitions between different states of nuclear matter.



Two different major phase transitions for nuclear matter:

PHASE TRANSITION	TEMP	DENSITY	STUDY
Liquid Gas phase transition	moderate (up to 15-20 MeV)	sub-saturation	Core collapse SN
Quark Hadron phase transition	high	high	Early Universe after Big Bang

From: N. Chamel and P. Haensel, Living Reviews in Relativity, vol. 11, no. 10, 2008

## NUCLEAR MATTER PHASE TRANSITIONS

- Below saturation density and T < 80 MeV: nuclear matter unstable to density fluctuations occurrence of LIQUID-GAS phase transition
- stability conditions for asymmetric nuclear matter with volume and temperature kept constant, are related to the condition  $\frac{\partial^2 \mathcal{F}}{\partial \rho_i^2} > 0$
- SPINODALS: determined by the values of  $T$ ,  $\rho$  and  $Y_p$  for which  $\det(\mathcal{F}_{ij})$  goes to zero
- Eigenvalues:  $\lambda_{\pm} = \frac{1}{2} [\text{Tr}(\mathcal{F}) \pm \sqrt{\text{Tr}(\mathcal{F})^2 - 4 \text{Det}(\mathcal{F})}]$
- Looking for solution:  $\lambda_{\pm} = 0$
- It happens in two points in space
- Stability condition:  $\lambda_{\pm} > 0$  (stable region)
- $\lambda_{\pm}$  can become < 0: system is thermodynamically unstable
- Looking for solution:  $\lambda_{\pm} = 0$
- It happens in two points in  $\rho_0$  vs  $\rho_0$  space - boundaries of the spinodal (unstable region)

## CONSTRAINTS

1) OBSERVATIONS: Neutron star mass-radius (M-R) diagram showing constraints from various observations.

2) EXPERIMENTS: Neutron star tidal deformability  $\Lambda$  vs mass-radius diagram.

3) SIMULATIONS: Phase diagrams showing the liquid-gas phase transition and quark-hadron phase transition.

## THE DD-NLD MODEL EOS

EXTENSION TO FINITE TEMPERATURES (with H. Pais and C. Providencia, University of Coimbra)

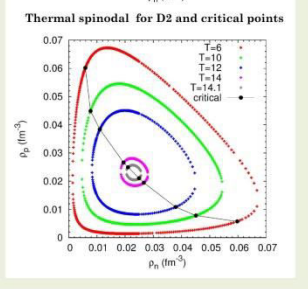
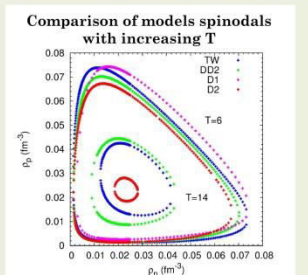
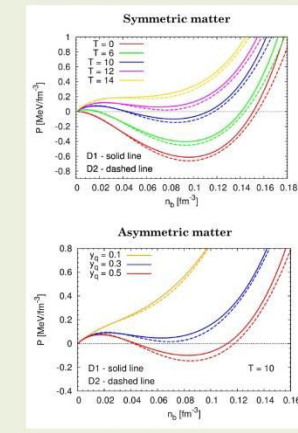
Temperature increasing  $\rightarrow$  increasing of kinetic energy of nucleons  $\rightarrow$  transformation of kinetic energy to excitation energy  $\rightarrow$  phase transitions between different forms of nuclear matter

## MODEL

DD - NLD MODEL

- Inclusion of higher order derivative couplings between nucleons and mesons - takes care of the high density EoS part
- the interaction Lagrangian is 
$$\mathcal{L}_{int}^{DD-NLD} = \frac{1}{2} \bar{\psi} (\gamma_0 + \gamma_0 \omega) \psi + \frac{1}{2} \bar{\psi} (\gamma_0 + \gamma_0 \omega) \psi + \frac{1}{2} \bar{\psi} (\gamma_0 + \gamma_0 \omega) \psi + \frac{1}{2} \bar{\psi} (\gamma_0 + \gamma_0 \omega) \psi$$
- where:  $\bar{\psi}_m = \sum_{k=0}^{\infty} C_k^{(m)} (\nabla^2)^k \bar{\psi}$
- Two forms:  $\beta = 1$  and  $D2 = \frac{1}{1 + (\frac{E-m}{\Lambda})^2}$
- Dirac equation is:  $(\beta_0 \not{\partial} - \Sigma_0 - \Sigma_2) \Psi = 0$
- Scalar self-energy:  $\Sigma_0 = \Gamma_0 \sigma \bar{\psi} \psi + \dots$
- Vector self-energy:  $\Sigma_2 = \Gamma_2 \omega^\mu \bar{\psi} \gamma_\mu \psi + \Gamma_4 \tau^3 \bar{\psi} \gamma_\mu \psi + \Sigma_2^p + \dots$
- Optical potential  $U_{opt}(E) = \frac{E}{m_{nuc}} \Sigma_0 - \Sigma_2 + \frac{E^2 - (m_{nuc})^2}{2m_{nuc}}$

## RESULTS: EoS for increasing T and p fraction / thermal spinodals



## CONCLUSION

- The pressure increases with increasing T and proton fraction for both (D1 and D2) models in same manner
- With increasing temperature, the envelope of the spinodals decreases for all the models considered
- For T=14 the D2 model with energy dependent self-energies shows the smallest spinodal compared to the other models

## REFERENCES

- S. Antić and S. Typel, Nucl. Phys. A 938 (2015) 92-108
- S.S. Avancini, L. Brito, Ph. Chomaz, D.P. Menezes, C. Providencia, Phys. Rev. C 74, 024217 (2006)
- J.M. Lattimer, M. Prakash Phys. Rev. Lett. 94 111101 (2005)
- J.M. Lattimer, A.W. Steiner, Eur. Phys. J. A (2014) 50
- T. Gaitanos, M. Kishimoto, Nucl. Phys. A 899 (2013) 133-169
- P. Danielewicz, R. Lacey, W.G. Lynch, Science, 2002 Nov 22; 298(5598)



# THE LIQUID-GAS PHASE TRANSITION WITHIN THE TEMPERATURE DEPENDENT DD-NLD MODEL

**Sofija Antić<sup>1</sup>**, Helena Pais<sup>2</sup>, Stefan Typel<sup>1</sup>, Constanca Providencia<sup>2</sup>

<sup>1</sup>*GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt, Germany,*

<sup>2</sup>*CFisUC, Department of Physics, University of Coimbra, Coimbra, Portugal*

THE LIQUID-GAS PHASE  
TRANSITION WITHIN THE  
TEMPERATURE DEPENDENT  
DD-NLD MODEL

**EoS model**

**Sofija Antić<sup>1</sup>**, Helena Pais<sup>2</sup>, Stefan Typel<sup>1</sup>, Constanca Providencia<sup>2</sup>

<sup>1</sup>*GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt, Germany,*

<sup>2</sup>*CFisUC, Department of Physics, University of Coimbra, Coimbra, Portugal*

**Liquid-gas PT**

**THE LIQUID-GAS PHASE  
TRANSITION WITHIN THE**

**TEMPERATURE DEPENDENT  
DD-NLD MODEL**

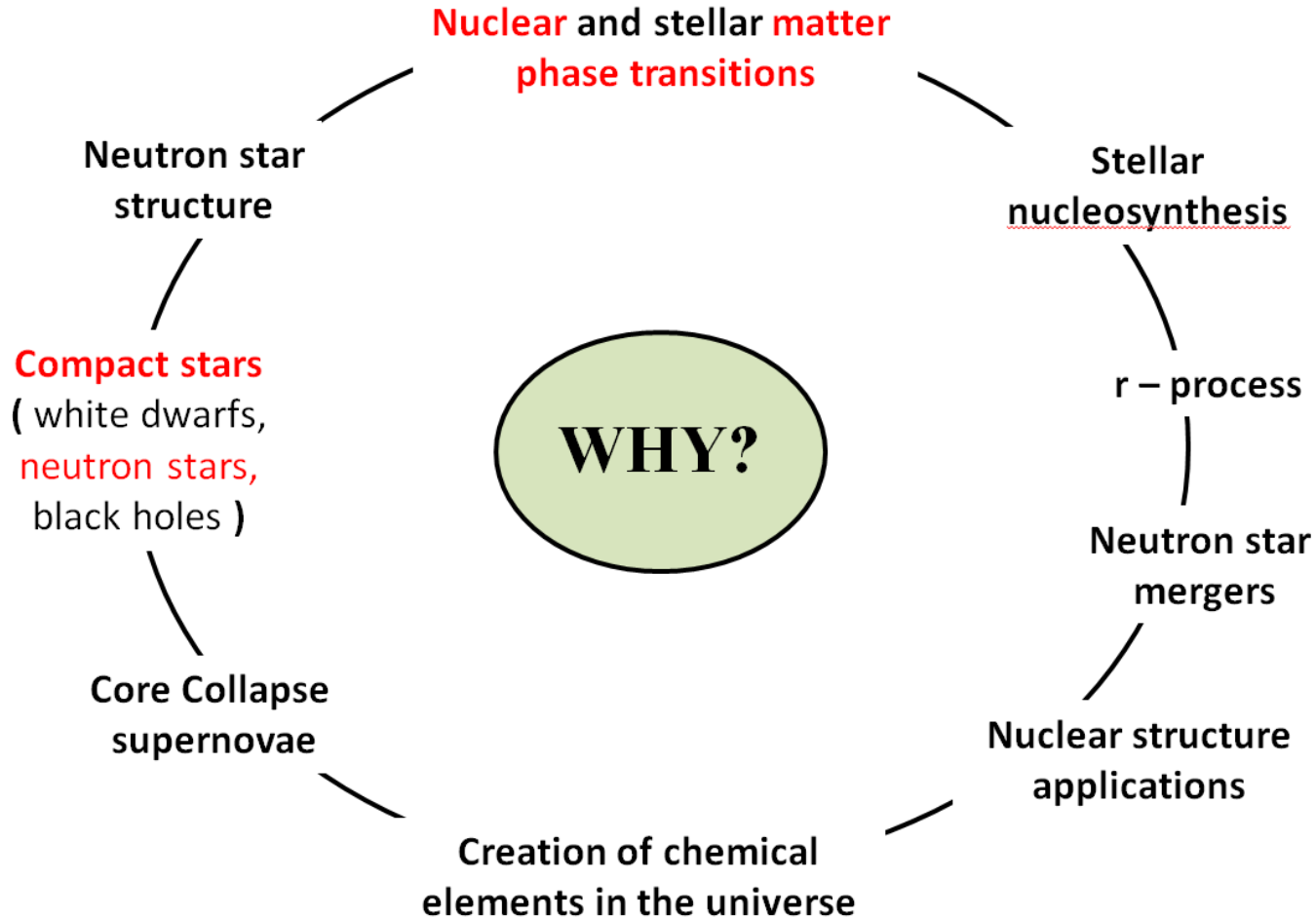
**EoS model**

**Sofija Antić<sup>1</sup>, Helena Pais<sup>2</sup>, Stefan Typel<sup>1</sup>, Constanca Providencia<sup>2</sup>**

*<sup>1</sup>GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt, Germany,*

*<sup>2</sup>CFisUC, Department of Physics, University of Coimbra, Coimbra, Portugal*

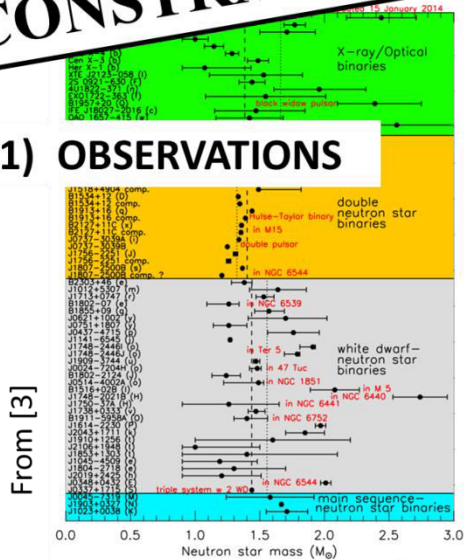
# EoS model



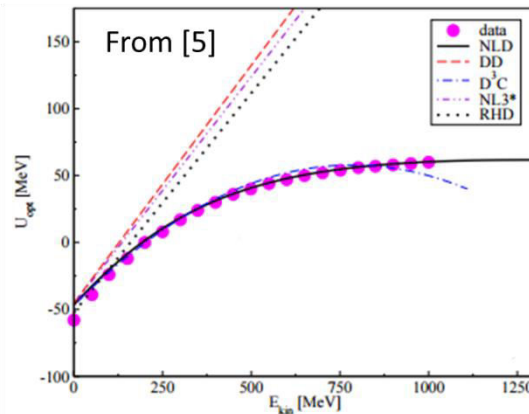
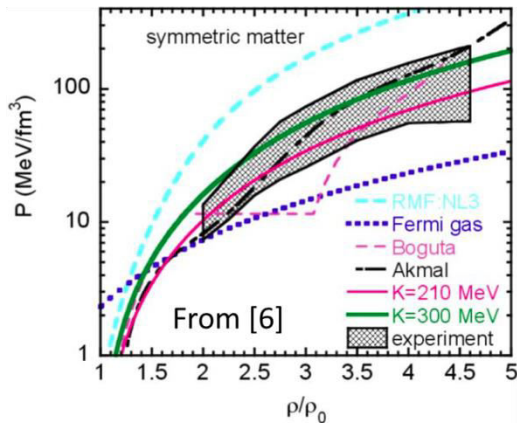
# EoS model

## CONSTRAINTS

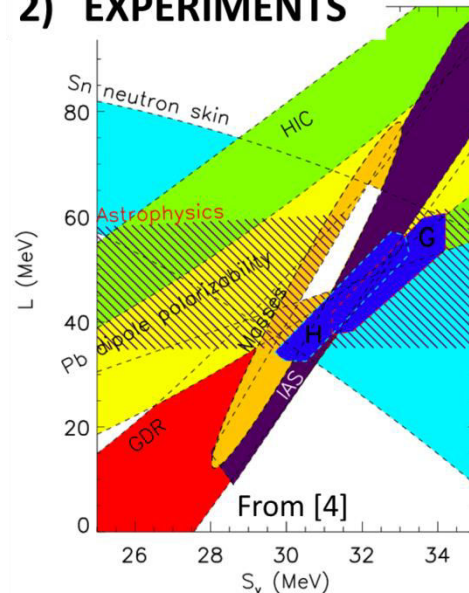
### 1) OBSERVATIONS



### 3) SIMULATIONS



### 2) EXPERIMENTS



## REFERENCES

- 3) *J.M. Lattimer, M. Prakash Phys. Rev, Let. 94 111101 (2005)*
- 4) *J.M. Lattimer, A.W. Steiner, Eur. Phys. J. A (2014) 50*
- 5) *T. Gaitanos, M. Kaskulov, Nucl. Phys. A 899 (2013) 133-169*
- 6) *P. Danielewicz, R. Lacey, W.G. Lynch, Science\_2002 Nov 22; 298(5598)*

# DD – NLD model

approaches

Phenomenological approaches

Based on effective density-dependent NN force with parameters fitted on nuclei properties.

- Liquid Drop models
  - ◇ BPS Baym et al., *ApJ* 170, 299 (1971)
  - ◇ BBP Baym et al., *NRA* 175, 225 (1971)
  - ◇ LS Lattimer&Swesty, *NRA* 535, 331 (1992)
  - ◇ DH Douchin&Haensel, *A&A* 380, 151 (2002)
- TF + RMF
  - ◇ Shen et al., *NRA* 637, 435 (1998)
- ETFSI + Eff. Skyrme force
  - ◇ BSk Goriely et al., *PRC* 82, 035804 (2010)
- Hartree-Fock
  - ◇ NY Nagels&Metherell, *NRA* 200, 200 (1971)
  - ◇ RMF Serot&Walecka, *Adv. NP* 16, 1 (1986)
  - ◇ RMF Boulay et al., *PRC* 50, 1732 (1995)
  - ◇ QMC Guichon et al., *NRA* 814, 66 (2008)
- Statistical models
  - ◇ NSE Raduta&Gulminelli, *PRC* 82, 065801 (2010)
  - ◇ HS Hempel&Schaffner-Bielich, *NRA* 837, 210 (2010)

Ab initio approaches

The nuclear problem is solved starting from the two- and three-body realistic nucleon interaction.

- Diagrammatic
  - ◇ BBG Day, *RMP* 39, 719 (1967)
  - ◇ SCGF Kadanoff&Baym, *Quantum Statistical Mechanics* (1962)
  - ◇ DBHF Ter Haar&Malfiet, *Phys. Rep.* 149, 207 (1987).
- Variational
  - ◇ APR Akmal et al., *PRC* 58, 1804 (1998)
  - ◇ FHNC Fantoni&Rosati, *Nuovo Cimento A* 20, 179 (1974)
  - ◇ CBF Fabrocini&Fantoni, *PLB* 298, 263 (1993)
  - ◇ LOCV Owen et al., *NRA* 277, 45 (1978)
- Monte Carlo
  - ◇ VMC Wiringa, *PRC* 43, 1585 (1991)
  - ◇ GFMC Carlson, *PRC* 68, 025802 (2003)
  - ◇ AFDMC Schmidt&Fantoni, *PLB* 446, 99 (1999)

13



# DD – NLD model

# DD – NLD model

- the **interaction Lagrangian** is

$$\mathcal{L}_{int}^{\sim \text{walecka}} = \frac{1}{2} \Gamma_{\sigma} \sigma (\bar{\Psi} \Psi + \bar{\Psi} \Psi) + \frac{1}{2} \Gamma_{\omega} \omega_{\mu} (\bar{\Psi} \gamma^{\mu} \Psi + \bar{\Psi} \gamma^{\mu} \Psi) + \frac{1}{2} \Gamma_{\rho} \rho_{\mu} (\bar{\Psi} \tau \gamma^{\mu} \Psi + \bar{\Psi} \tau \gamma^{\mu} \Psi)$$

# DD – NLD model

## Density-Dependent

- the **interaction Lagrangian** is

$$\mathcal{L}_{int}^{DD} = \frac{1}{2} \Gamma_{\sigma}(n_v) \sigma (\bar{\Psi} \Psi + \bar{\Psi} \Psi) + \frac{1}{2} \Gamma_{\omega}(n_v) \omega_{\mu} (\bar{\Psi} \gamma^{\mu} \Psi + \bar{\Psi} \gamma^{\mu} \Psi) + \frac{1}{2} \Gamma_{\rho}(n_v) \rho_{\mu} (\bar{\Psi} \tau \gamma^{\mu} \Psi + \bar{\Psi} \tau \gamma^{\mu} \Psi)$$

# DD – NLD model

Density-Dependent

Non-Linear Derivative

- the interaction Lagrangian is

$$\mathcal{L}_{int}^{DD-NLD} =$$

$$\frac{1}{2} \Gamma_{\sigma}(n_{\nu}) \sigma (\bar{\Psi} \overleftrightarrow{D} \Psi + \bar{\Psi} \overrightarrow{D} \Psi)$$

$$\frac{1}{2} \Gamma_{\omega}(n_{\nu}) \omega_{\mu} (\bar{\Psi} \overleftrightarrow{D} \gamma^{\mu} \Psi + \bar{\Psi} \gamma^{\mu} \overrightarrow{D} \Psi) +$$

$$\frac{1}{2} \Gamma_{\rho}(n_{\nu}) \rho_{\mu} (\bar{\Psi} \overleftrightarrow{D} \tau \gamma^{\mu} \Psi + \bar{\Psi} \tau \gamma^{\mu} \overrightarrow{D} \Psi)$$

# DD – NLD model

Density-Dependent

Non-Linear Derivative

- the interaction Lagrangian is

$$\mathcal{L}_{int}^{DD-NLD} = \frac{1}{2} \Gamma_{\sigma}(n_{\nu}) \sigma (\bar{\Psi} \overleftrightarrow{D} \Psi + \bar{\Psi} \overrightarrow{D} \Psi) + \frac{1}{2} \Gamma_{\omega}(n_{\nu}) \omega_{\mu} (\bar{\Psi} \overleftrightarrow{D} \gamma^{\mu} \Psi + \bar{\Psi} \gamma^{\mu} \overrightarrow{D} \Psi) + \frac{1}{2} \Gamma_{\rho}(n_{\nu}) \rho_{\mu} (\bar{\Psi} \overleftrightarrow{D} \tau \gamma^{\mu} \Psi + \bar{\Psi} \tau \gamma^{\mu} \overrightarrow{D} \Psi)$$

where : 
$$\vec{D}_m = \sum_{k=0}^{\infty} C_k^{(m)} (v^{\beta} i \vec{\partial}_{\beta})^k$$

- Two forms:  $D1 = 1$  and  $D2 = \frac{1}{1 + \left(\frac{E-m}{\Lambda}\right)^2}$

# DD – NLD model

Density-Dependent

Non-Linear Derivative

- the interaction Lagrangian is

$$\mathcal{L}_{int}^{DD-NLD} = \frac{1}{2} \Gamma_{\sigma}(n_{\nu}) \sigma (\bar{\Psi} \overleftrightarrow{D} \Psi + \bar{\Psi} \overrightarrow{D} \Psi) + \frac{1}{2} \Gamma_{\omega}(n_{\nu}) \omega_{\mu} (\bar{\Psi} \overleftrightarrow{D} \gamma^{\mu} \Psi + \bar{\Psi} \gamma^{\mu} \overrightarrow{D} \Psi) + \frac{1}{2} \Gamma_{\rho}(n_{\nu}) \rho_{\mu} (\bar{\Psi} \overleftrightarrow{D} \tau \gamma^{\mu} \Psi + \bar{\Psi} \tau \gamma^{\mu} \overrightarrow{D} \Psi)$$

where :  $\vec{D}_m = \sum_{k=0}^{\infty} C_k^{(m)} (v^{\beta} i \vec{\partial}_{\beta})^k$

- Two forms:  $D1 = 1$  and  $D2 = \frac{1}{1 + \left(\frac{E-m}{\Lambda}\right)^2}$

WHY?

# DD – NLD model

- Dirac equation is:

$$[\gamma_\mu (i\partial^\mu - \Sigma_V^\mu) - (m - \Sigma_S)]\Psi = 0$$

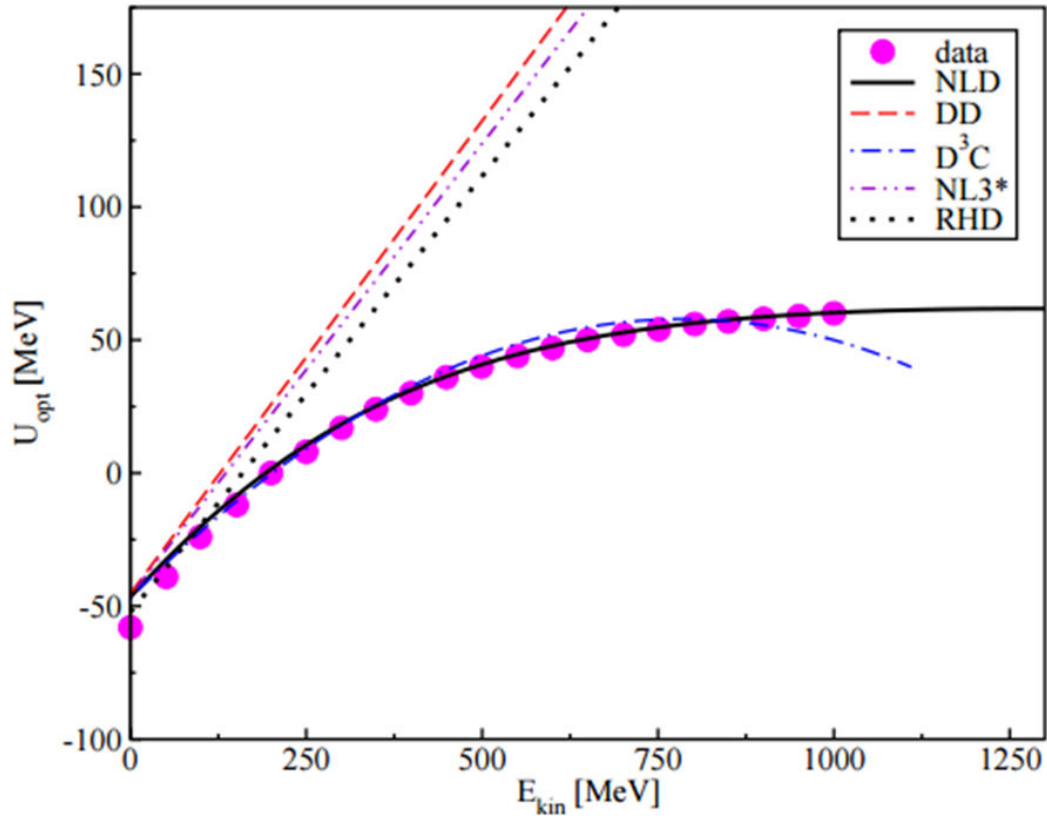
- Scalar self-energy:  $\Sigma_S = \Gamma_\sigma \sigma \vec{D}_\sigma + \dots$
- Vector self-energy:

$$\Sigma_V^\mu = \Gamma_\omega \omega^\mu \vec{D}_\omega + \Gamma_\rho \vec{t} \cdot \vec{\rho}^\mu \vec{D}_\rho + \Sigma_R^\mu + \dots$$

- Optical potential

$$U_{opt}(E) = \frac{E}{m_{nuc}} \Sigma_V^\mu - \Sigma_S + \frac{\Sigma_S^2 - (\Sigma_V^\mu)^2}{2m_{nuc}}$$

# DD – NLD model

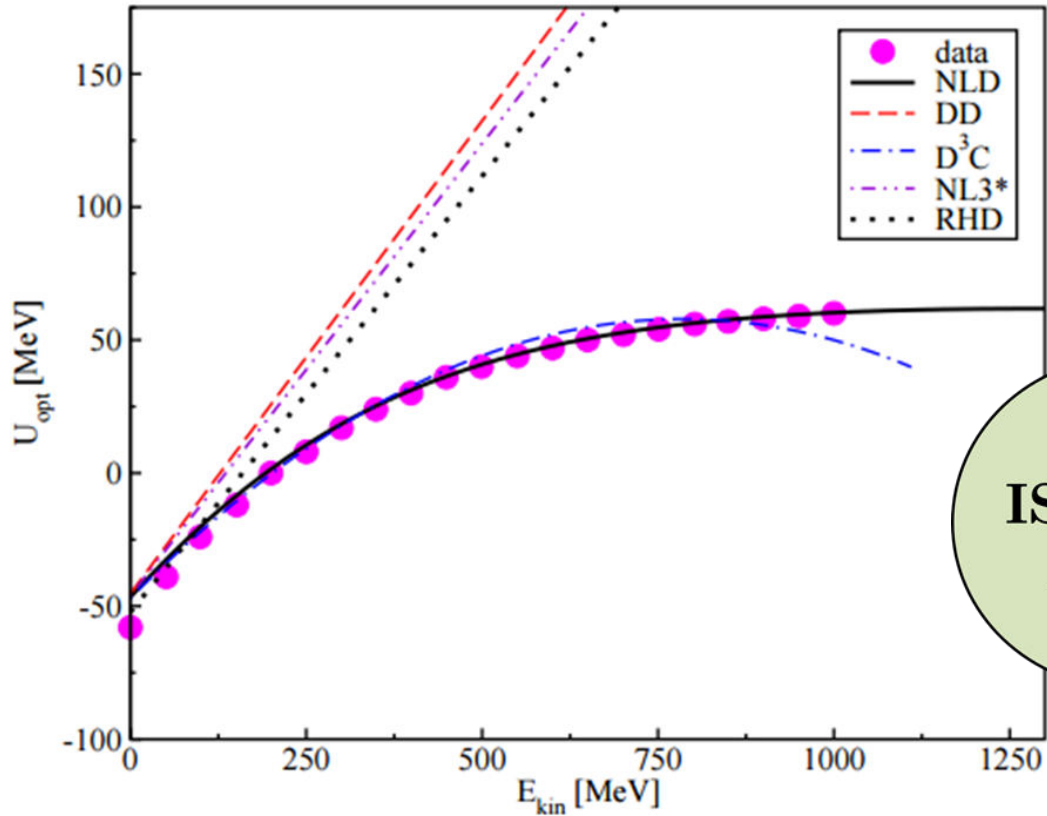


- **Optical potential**

$$U_{opt}(E) = \frac{E}{m_{nuc}} \Sigma_V^0 - \Sigma_S + \frac{\Sigma_S^2 - (\Sigma_V^0)^2}{2m_{nuc}}$$



# DD – NLD model



IS THAT ALL?

- **Optical potential**

$$U_{opt}(E) = \frac{E}{m_{nuc}} \Sigma_V^0 - \Sigma_S + \frac{\Sigma_S^2 - (\Sigma_V^0)^2}{2m_{nuc}}$$

# DD – NLD model

- Parameterization:
  - Fit to nuclear saturation properties
  - Fit to nuclei properties (binding en, radii...)
- Application:
  - infinite nuclear matter (SM, NM, PM)
  - NS at  $T = 0$  (to get M-R relation)

*S. Antić and S. Typel, Nucl. Phys. A 938 (2015) 92-108*

# DD – NLD model

- Parameterization:
  - Fit to nuclear saturation properties
  - Fit to nuclei properties (binding en, radii...)

**Liquid-gas phase transition**

THE LIQUID-GAS PHASE  
TRANSITION WITHIN THE

TEMPERATURE DEPENDENT  
DD-NLD MODEL

**EoS model**

# Liquid – Gas Phase Transition

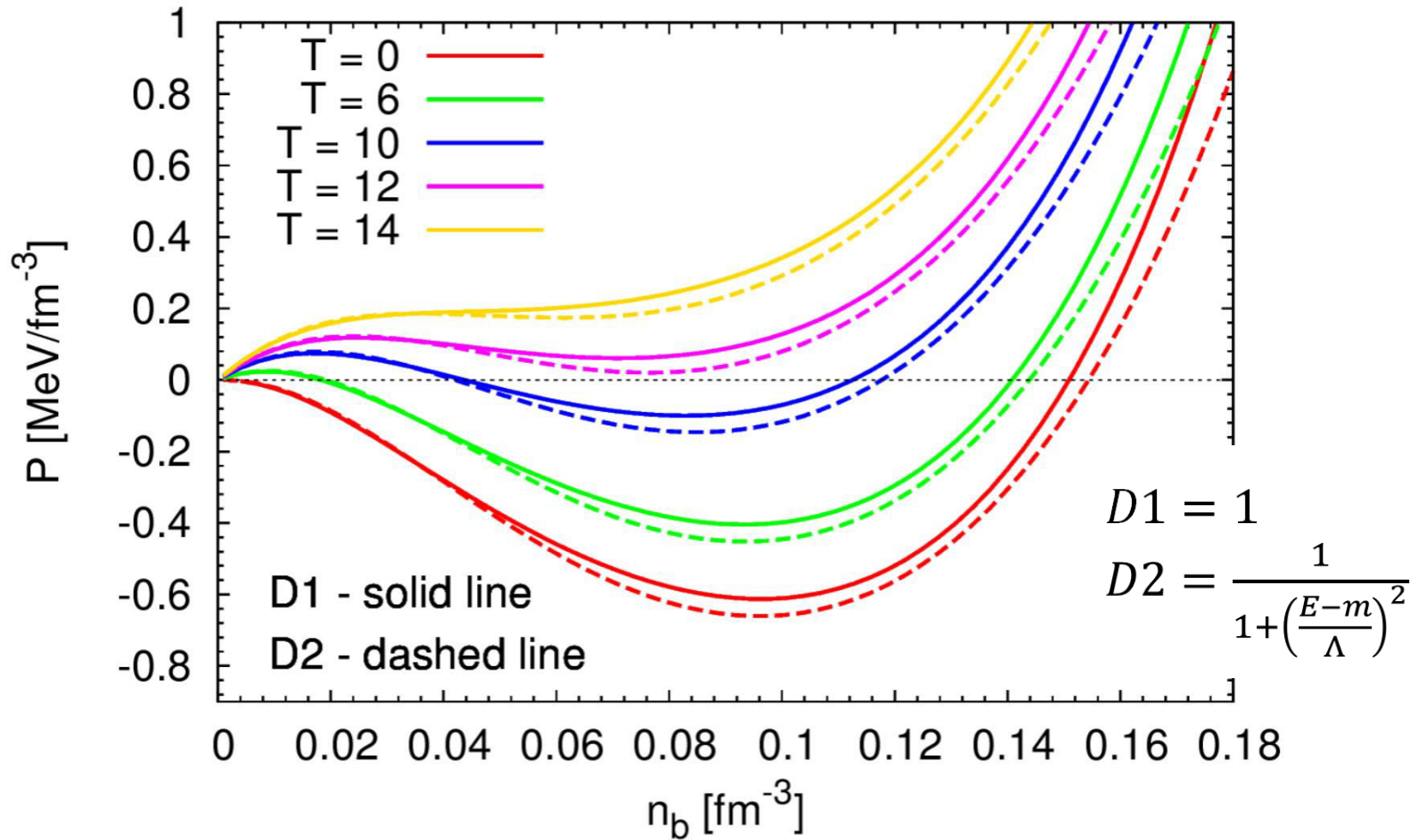
- To study LGPT :

model extension, implement T dependence

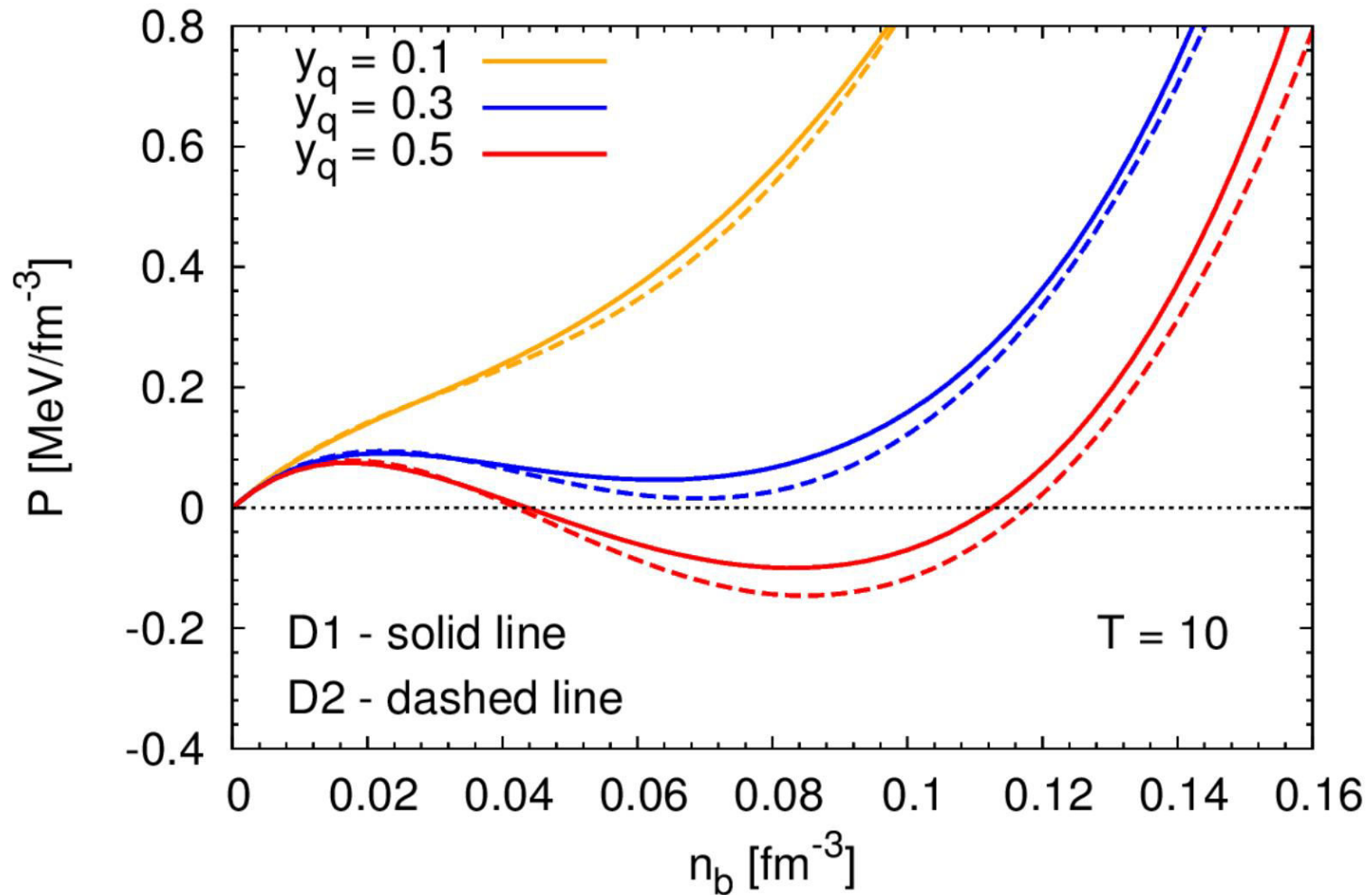
$$n_B \sim \int d^3p \quad \rightarrow \quad n_B \sim \int \frac{1}{1 + e^{\frac{(E-\mu)}{T}}} d^3p$$

- The finite temperature description necessary for general astrophysical applications (i.e. in order to provide the EoS tables for CCSN simulations)

# Symmetric matter with change of $T$



# SM to NM matter with change of $y_q$



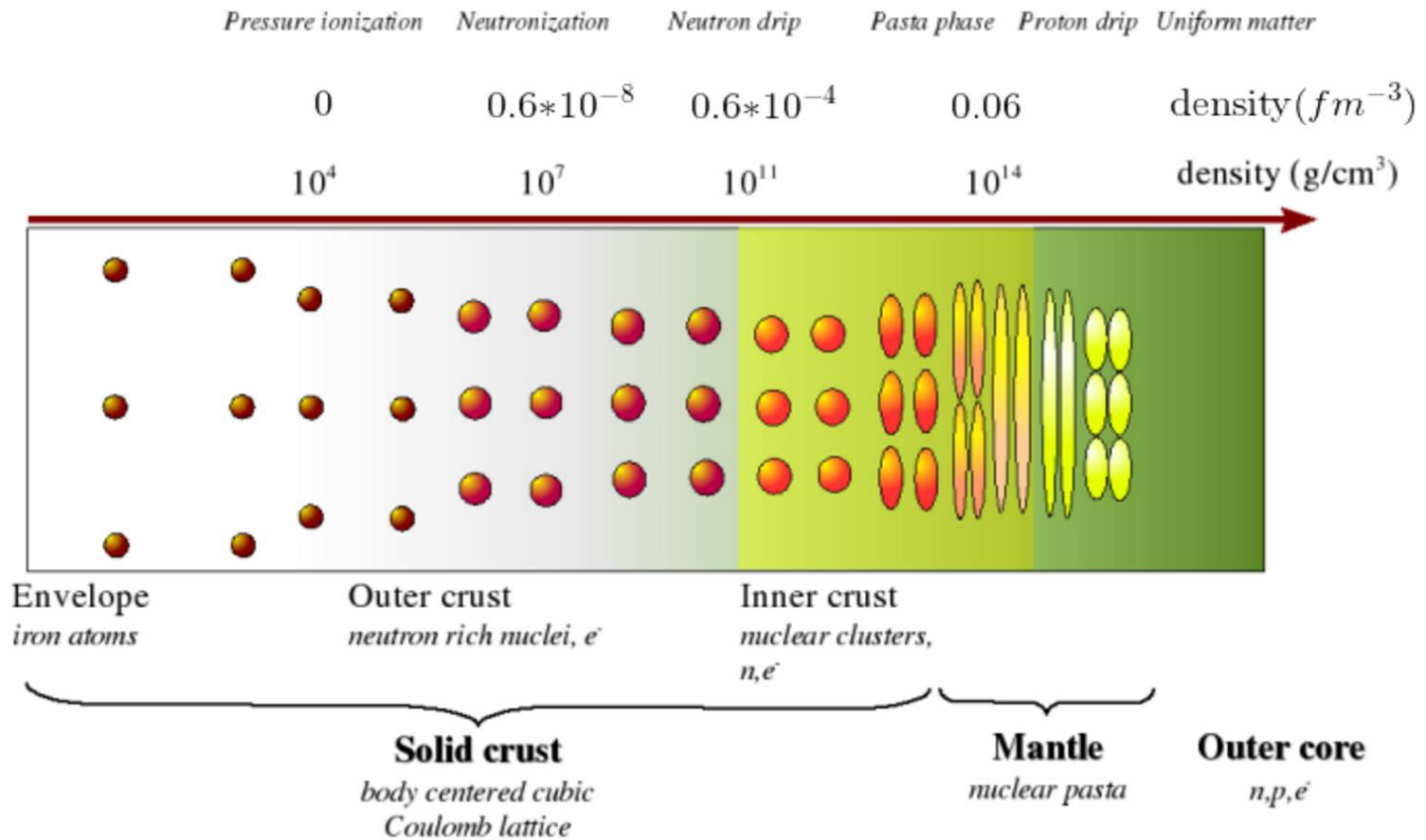
# Liquid – Gas Phase Transition

- Two different major phase transitions for nuclear matter:

PHASE TRANSITION	TEMP	DENSITY	STUDY
Liquid Gas phase transition	moderate (up to 15-20 MeV)	sub-saturation	Core collapse SN
Quark Hadron phase transition	high	high	Early Universe after Big Bang

- Below saturation density and  $T \lesssim 10$  MeV:
  - nuclear matter unstable to density fluctuation
  - occurrence of LIQUID-GAS phase transition

# Liquid – Gas Phase Transition





# Liquid – Gas Phase Transition

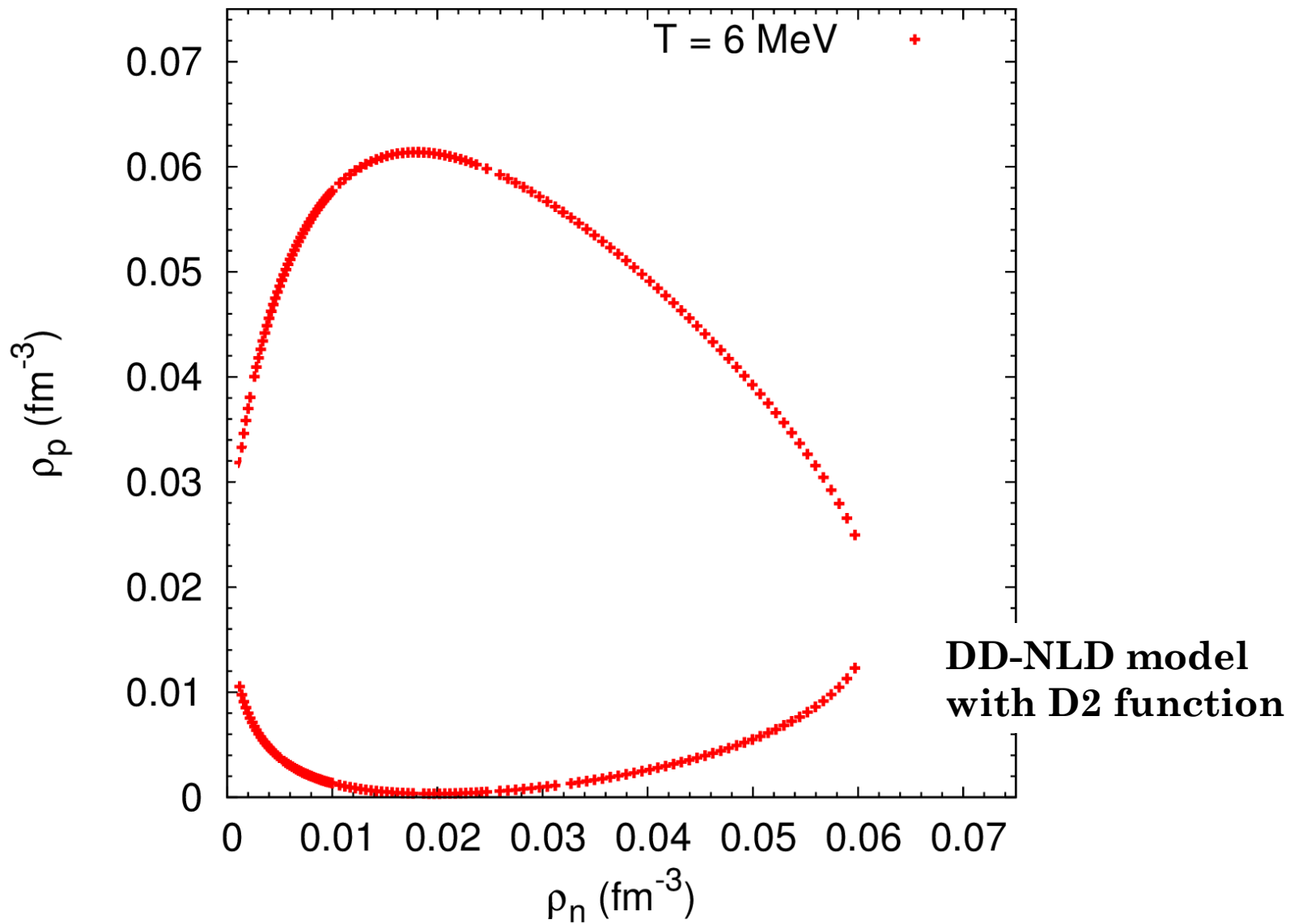
- Stability condition (T=const, V=const):

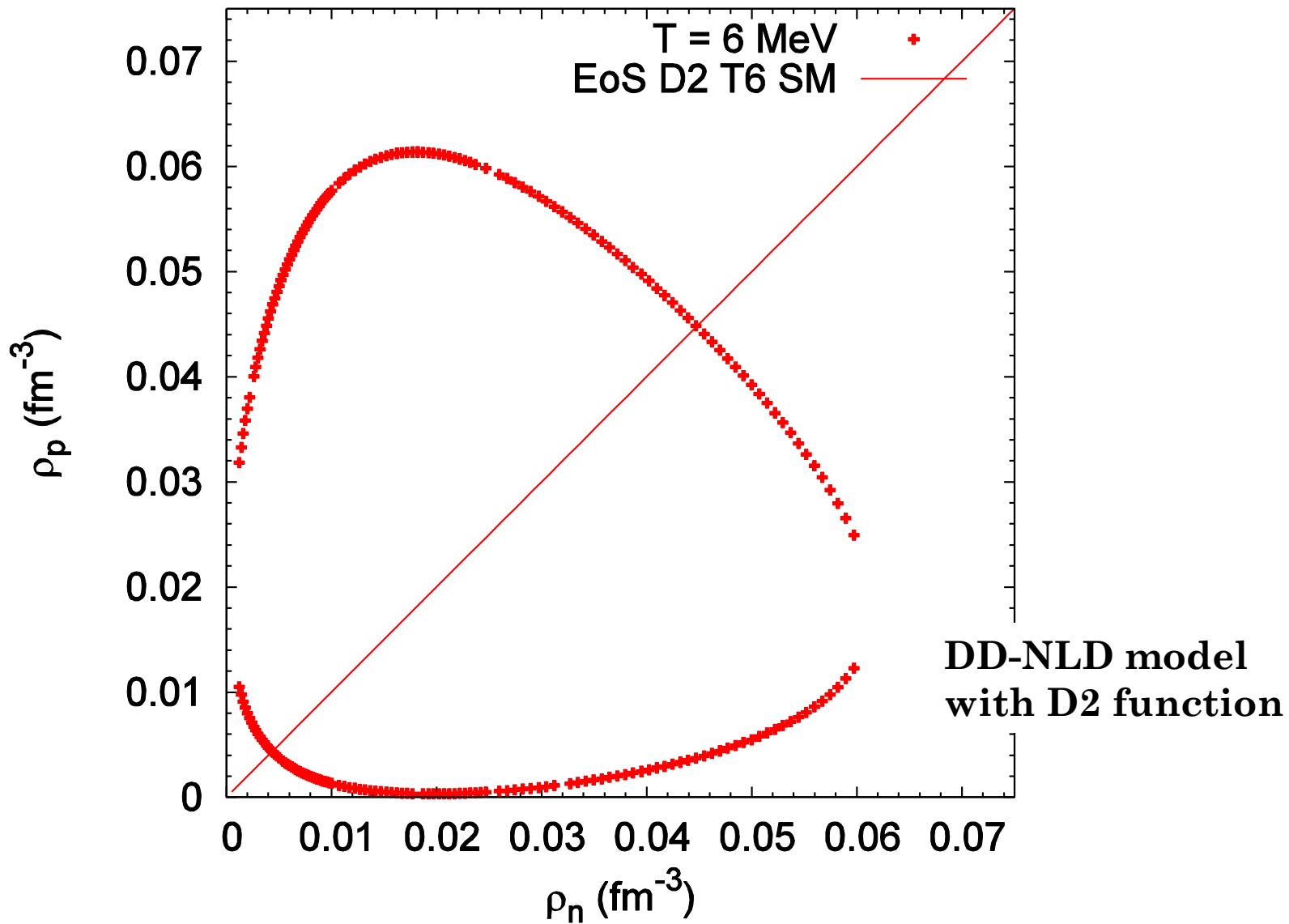
$$\mathcal{F}_{ij} = \left( \frac{\partial^2 \mathcal{F}}{\partial \rho_i \partial \rho_j} \right) = \frac{\partial}{\partial \rho_i} \left( \frac{\partial \mathcal{F}}{\partial \rho_j} \right) = \frac{\partial \mu_j}{\partial \rho_i} = \begin{pmatrix} \frac{\partial \mu_p}{\partial \rho_p} & \frac{\partial \mu_p}{\partial \rho_n} \\ \frac{\partial \mu_n}{\partial \rho_p} & \frac{\partial \mu_n}{\partial \rho_n} \end{pmatrix}_T > 0$$

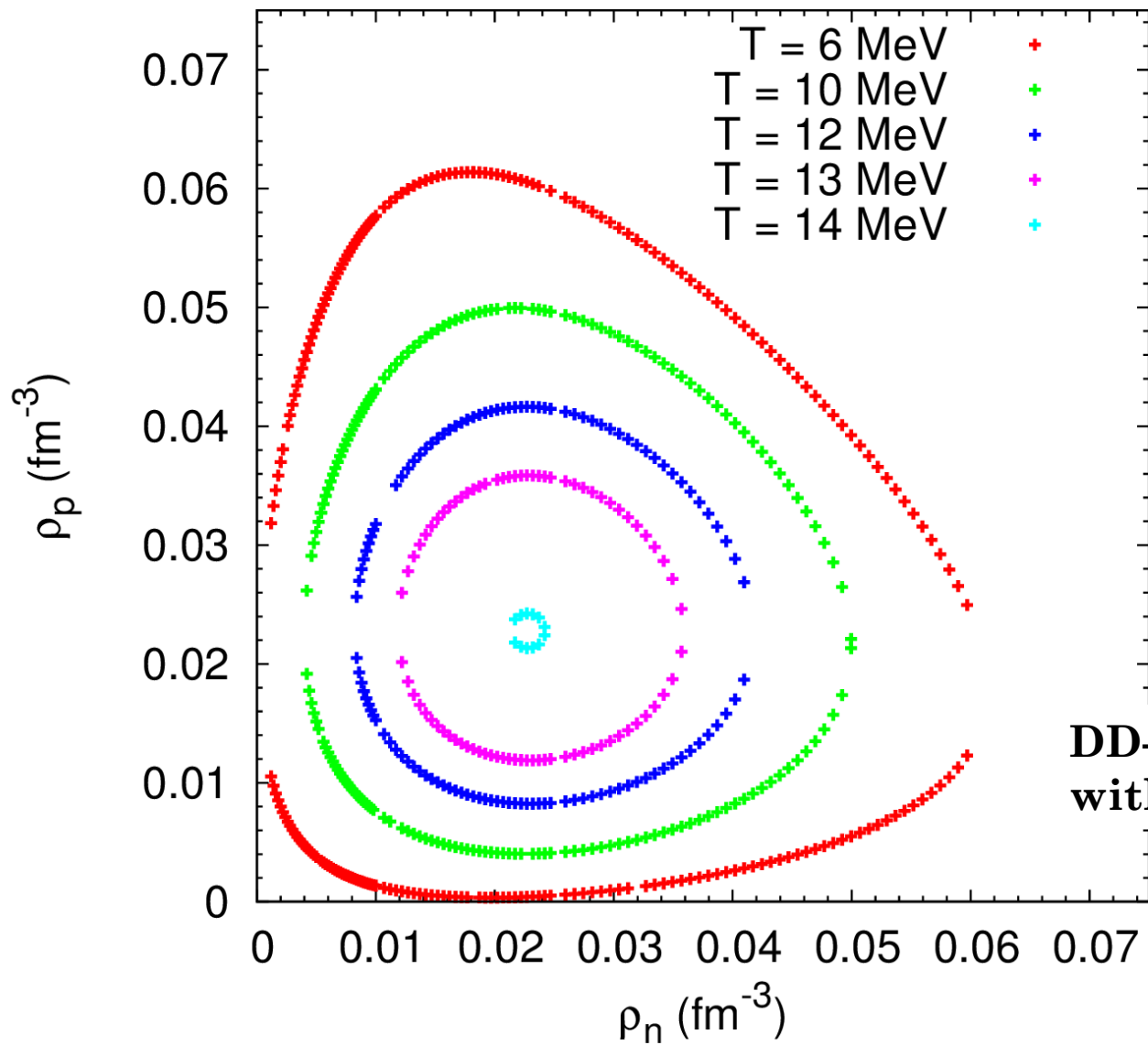
- **SPINODAL:** determined by the value of T,  $\rho$  and  $y_p$  for which  $\det(\mathcal{F}_{ij})$  goes to zero
- Stability condition: the two eigenvalues  $> 0$
- $\lambda_-$  can become  $< 0$  : system is **thermodynamically unstable**
- Looking for solution:

$$\lambda_- = 0$$

- It happens in two points in  $\rho_n$  vs  $\rho_p$  space
  - **boundaries of the spinodal (unstable region)**

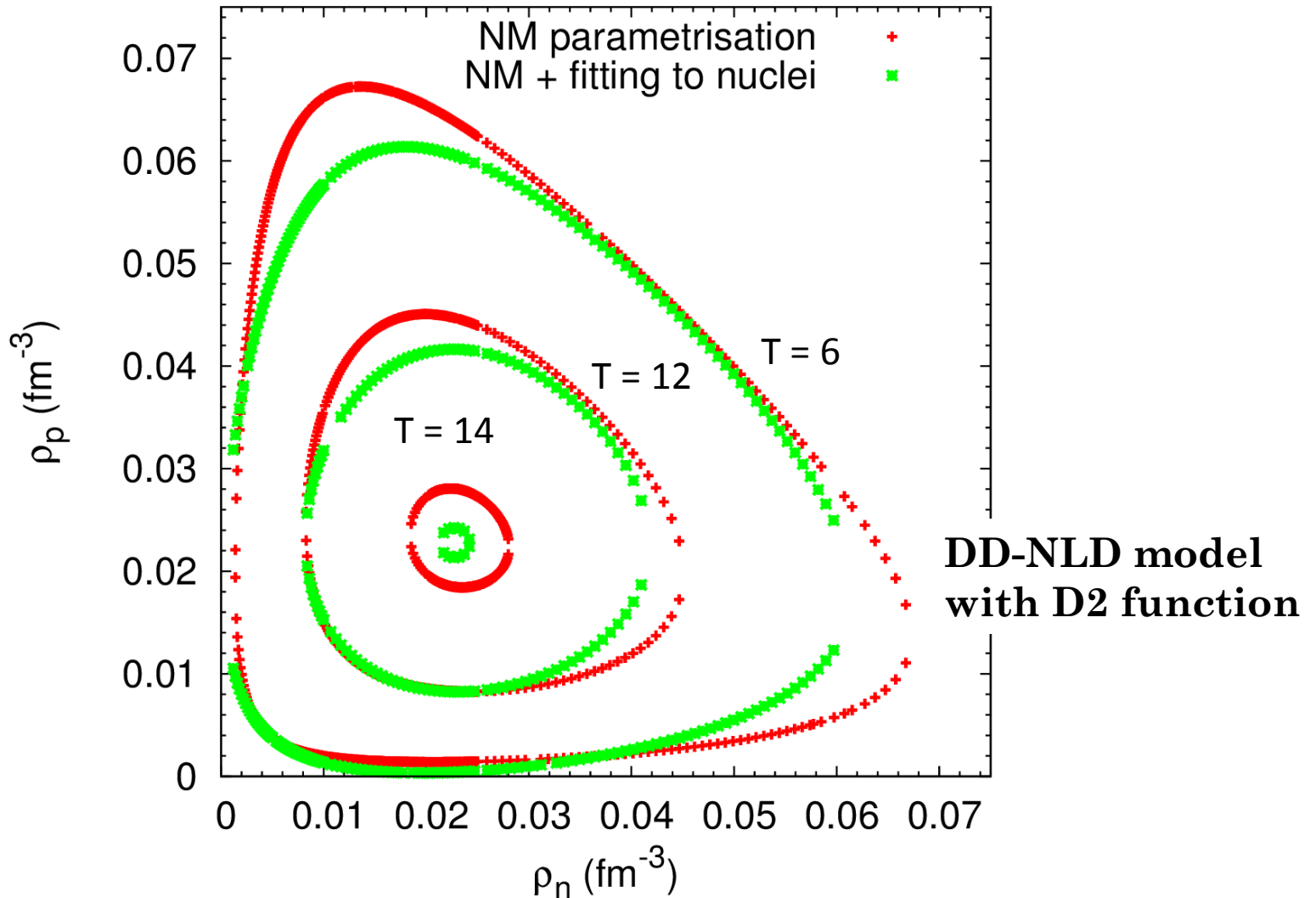




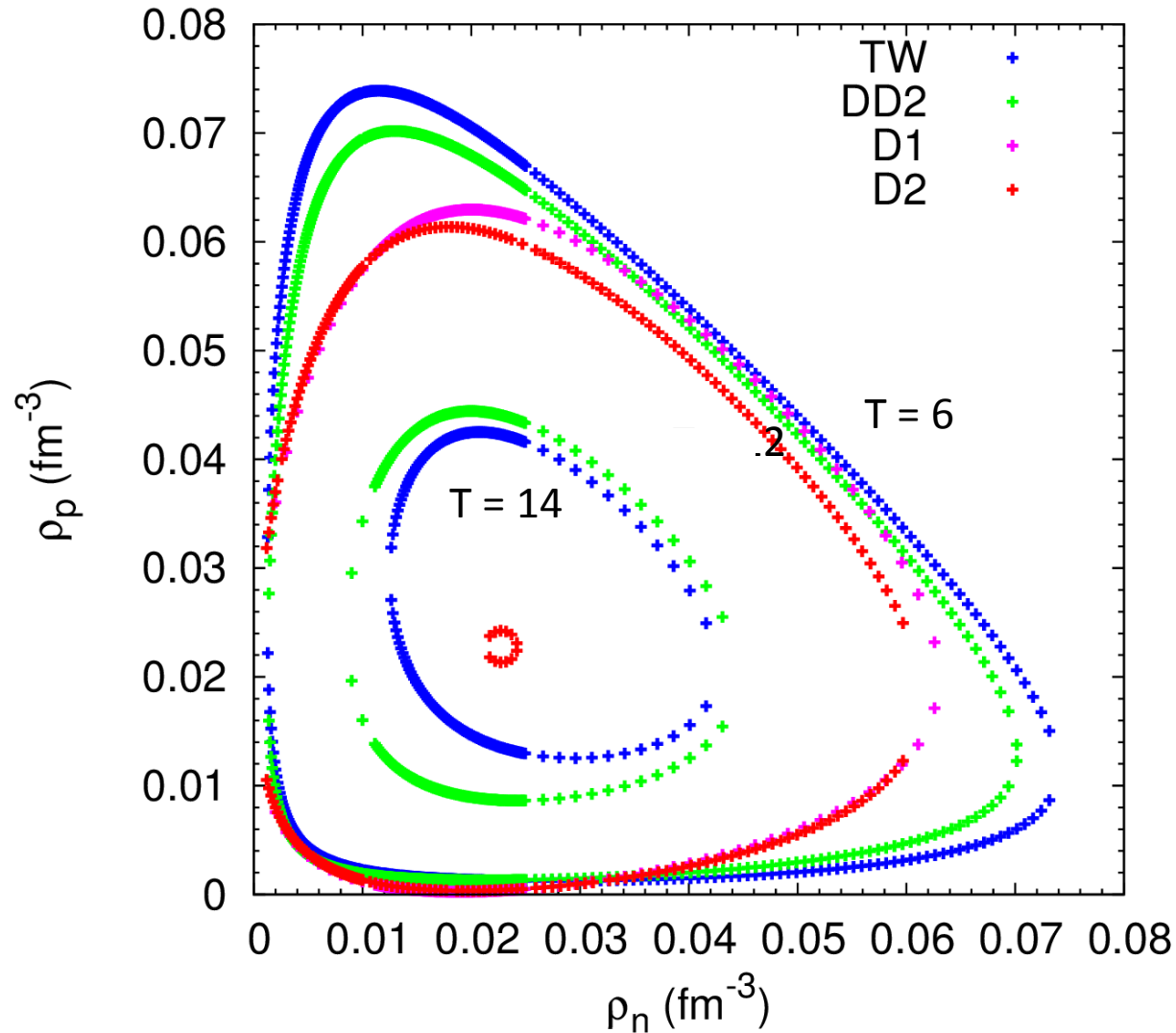


**DD-NLD model  
with D2 function**

# Different parameterizations



# Comparison to previous models



# CONCLUSIONS

- The pressure increases with increasing  $T$  and proton fraction for both (D1 and D2) models in same manner
- With increasing temperature, the envelope of the spinodals decreases for all the models considered
- For  $T=14$  the D2 model with energy dependent self-energies shows the smallest spinodal compared to the other models

# FUTURE WORK

- ...work in progress...
- Further studies of sub-saturational region and the change of spinodal
- Binodal calculation (coexistence region)
- Explore the parameter space?
- ...



# FUTURE WORK

- ...work in progress...
- Further studies of sub-saturational region and the change of spinodal
- Binodal calculation (coexistence region)
- Explore the parameter space?
- ...

Thank you 😊  
Questions?

