Demokritos - APCTP workshop

September 30 - October 4, 2024

How to define tension in Gravity

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Outline of this talk

1. Some generalities on AdS/DCFT
2. Energy/mass in gravity
3. Two independent notions of tension
4. Some examples
5. Supersymmetry
6. Summary

1. Introduction: **AdS/D(efect)CFT**

Defects are (external or dynamical) probes of a Quantum Field Theory

- The simplest, point *(p=0)* defects correspond to the insertion of local operators

- Line *(p=1)* defects have played an important role in the history of QFT:

Magnetic impurity in a metal (Kondo model): birth of Wilson's Renormalization Group

Wilson loops (heavy quarks): order parameter for confinement

Quantum dots: Key ingredients in many quantum devices

Introduction: **AdS/DCFT**

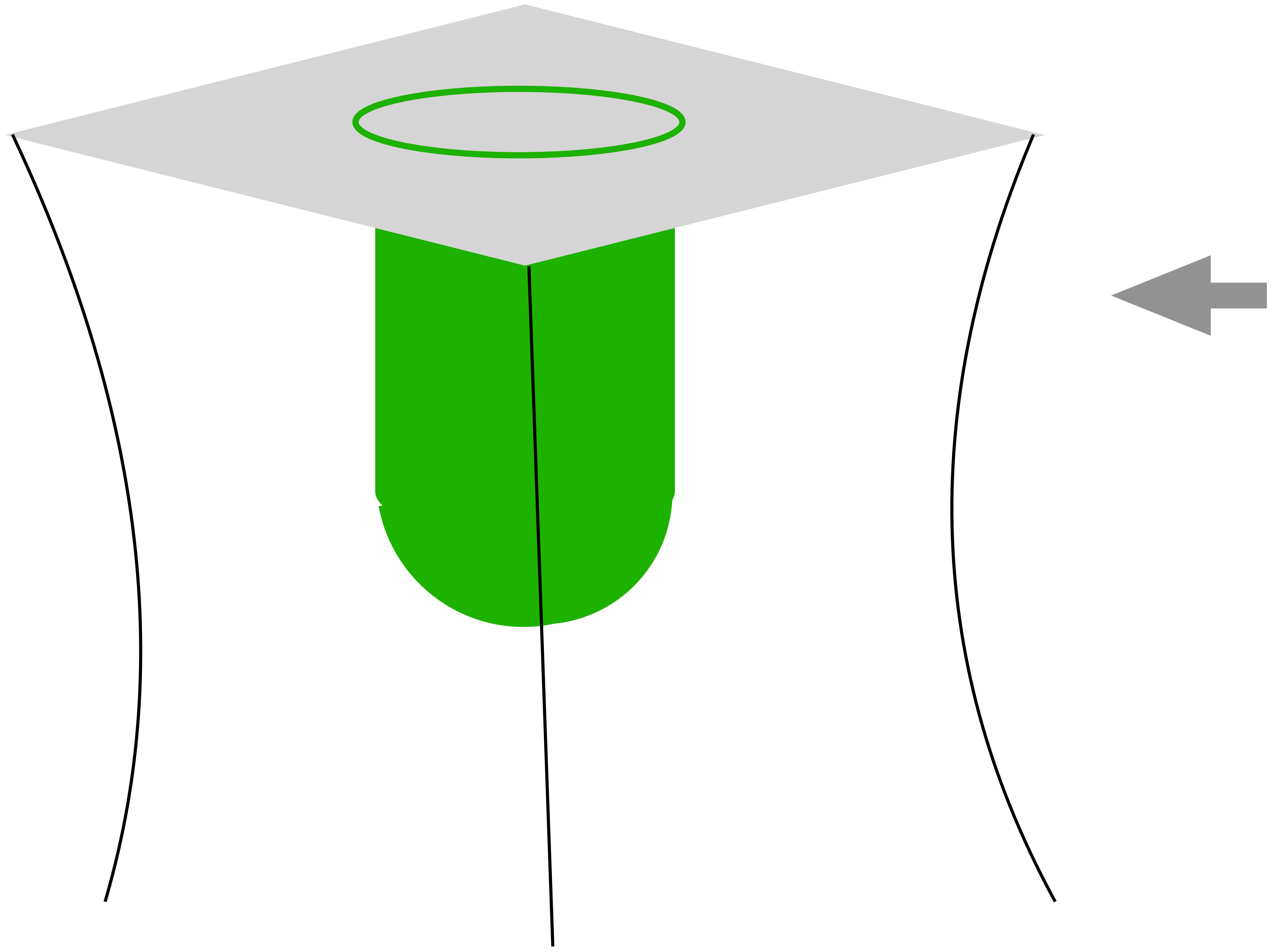
- Volume *(p=3)* defects: domain walls between coexisting phases

More generally in d spacetime dimensions can have defects stretching along $p=0,1, \ldots ,d-1$ of them. Defects extend the set of observables way beyond the correlation functions of local operators, and have become an important component of modern QFT. They are e.g. at the basis of the generalized & non- invertible symmetries that are being systematically studied nowadays

Gaiotto, Kapustin, Seiberg, Willett, arXiv:1412.5148 [hep-th]

Introduction: **AdS/DCFT**

Defects made their way very early in holography (alias AdS/CFT) as anchors of *p*-branes in the dual gravitational theory.



e.g. a string worldsheet (*p=1*) intersects the boundary of AdS (holographic screen) on a defect line

Maldacena, arXiv:9803002 [hep-th]

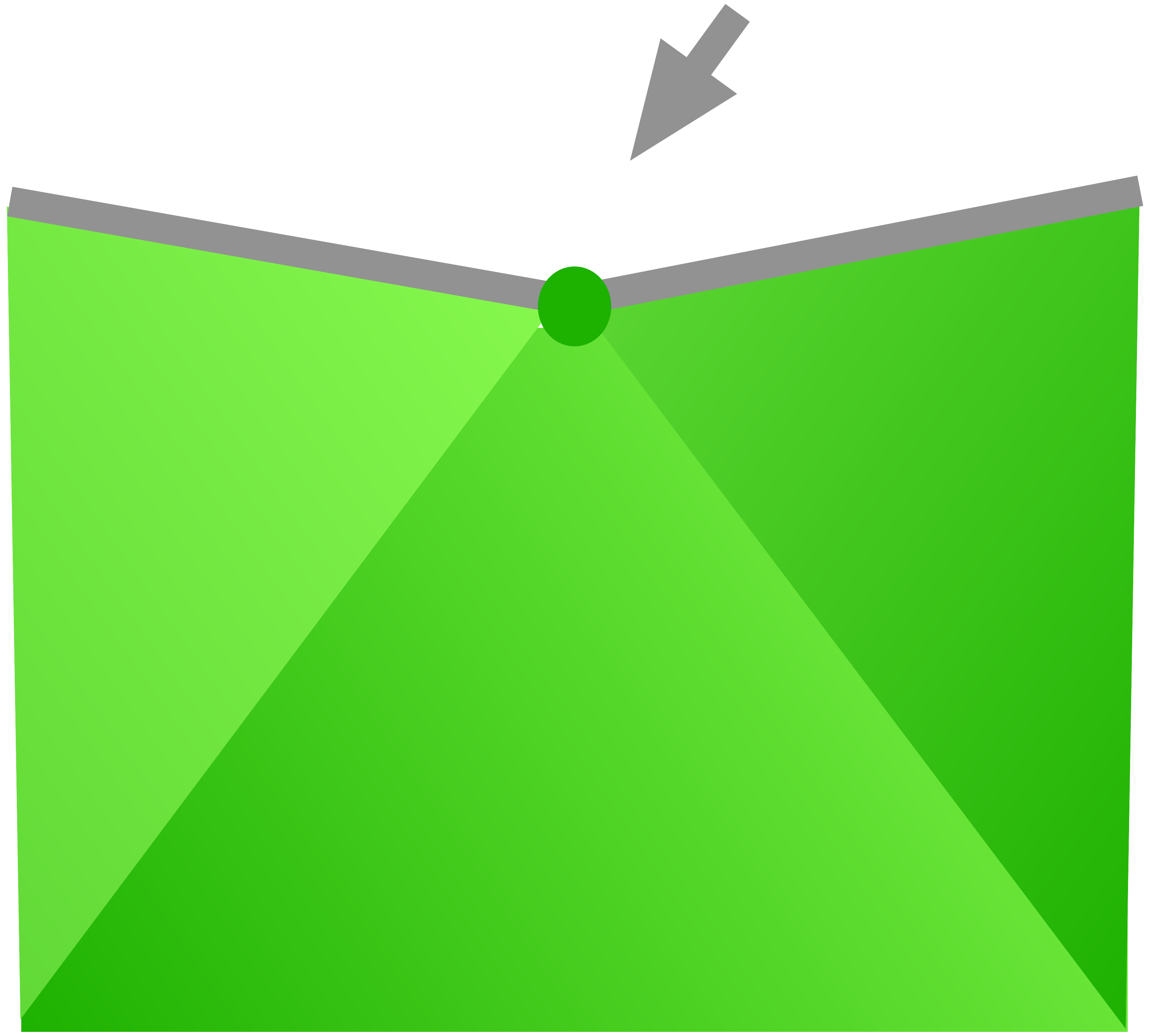
Karch,Randall, arXiv:0105132 [hep-th]

DeWolfe, Freedman, Ooguri, arXiv:0111135 [hep-th]

CB, de Boer, Dijkgraaf, Ooguri, arXiv:0111210 [hep-th]

Introduction: **AdS/DCFT**

At first the branes were considered as classical and thin, but in a full-fledged quantum theory of gravity they are thick and quantized. A cartoon of a smooth gravitational domain wall is here. Note that because of the infinite blueshift at the bnry the anchor is always thin.



∃ a large number of exact sugra solutions dual to DCFTs. The list is long, see in particular

Gutperle, D'Hoker *and collaborators*

Introduction: **AdS/DCFT**

In this talk I will focus on a specific question that arises in the context of AdS/DCFT:

Is there an invariant definition of p-brane tension in gravity ?

This is based on work with my student Zhongwu Chen

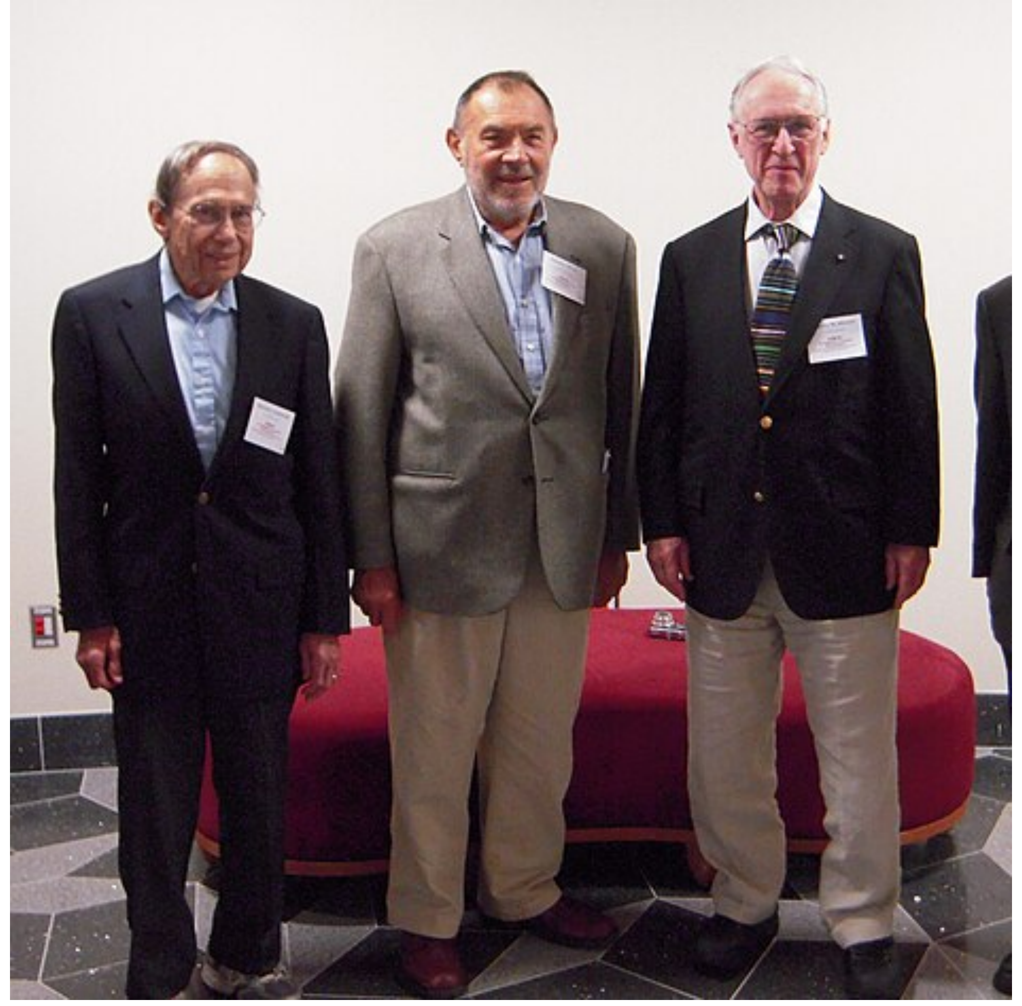
CB, Chen, arXiv:2404.14998 [hep-th]

and ongoing work also with Lorenzo Bianchi

2. Mass in AdS

Begin by recalling that there is no local definition of mass/energy in gravity

But in asymptotically-flat spacetime this can be given an invariant meaning in terms of the fall-off of the metric at infinity, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$



$$E = \frac{1}{16\pi G} \int_{S^2 \text{ at } \infty} (\partial^k h_{jk} - \partial^j h_{kk})\, d^2 S_j$$

Arnowitt, Deser, Misner 1960

Mass in AdS

The ADM definition must be revised when gravitational radiation escapes at null ∞

AdS with reflecting (*Diriclhlet*) boundary conditions is a trap, so no such problem.

An ADM-like definition is therefore possible in terms of the asymptotic metric

Abbott, Deser 1982

Hawking, Horowitz 1996

In AdS/CFT:

ADM Energy in aAdS	$\longleftrightarrow$	dilatation charge Δ of CFT operator $\mathcal{O}$ that creates the dual state

Mass in AdS

For a free scalar particle in *unit-radius* AdS_4

$$\Delta = \frac{3}{2} + \sqrt{m_0^2 + \frac{9}{4}} = m_0 \left[1 + \frac{3}{2m_0} + \frac{9}{8m_0^2} + \cdots \right.$$

quantum, negligible if

$$\lambda_{\text{Compton}} \ll 1$$

& taking into account gravitational backreaction :

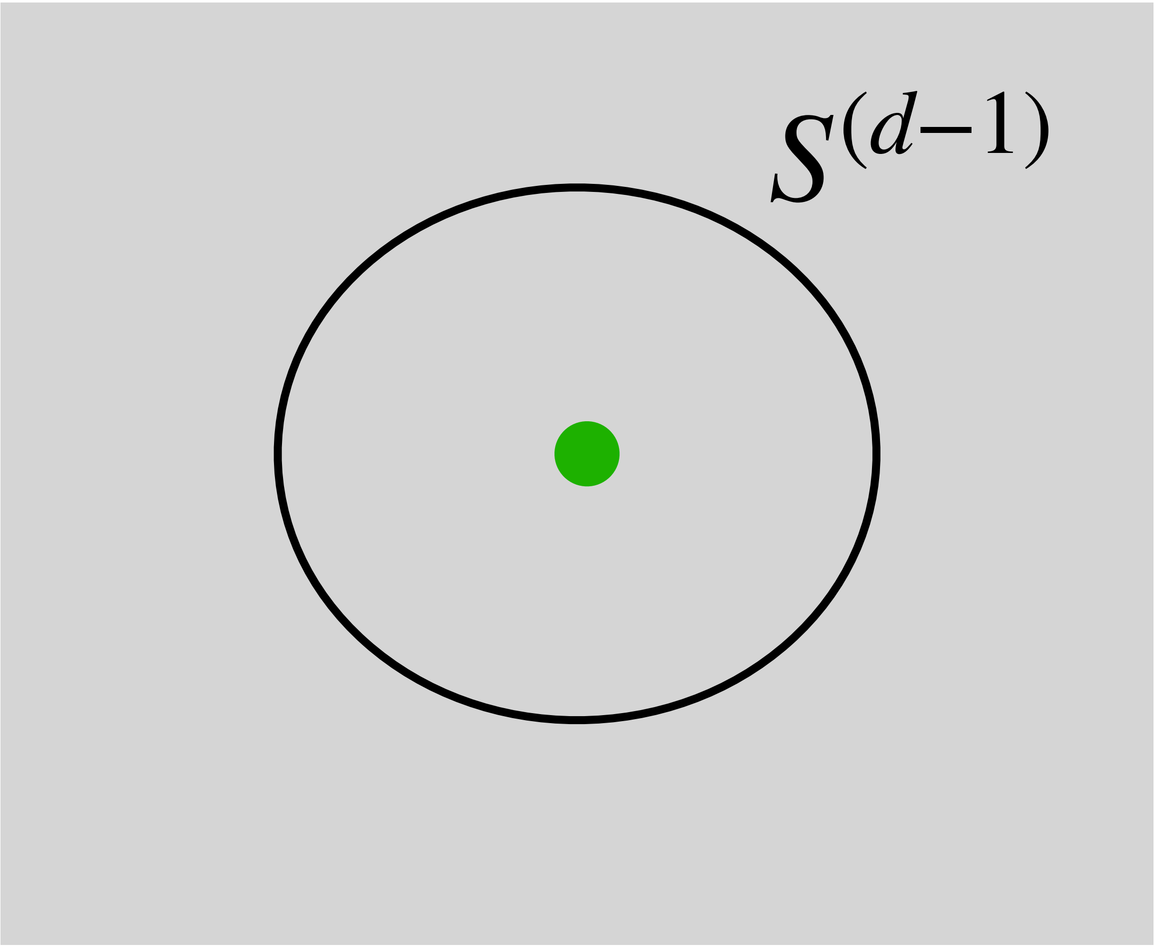
$$\left. \cdots \quad + G_N m_0 + (G_N m_0)^2 + \cdots \right]$$

negligible if

$$r_{\text{Schwarzschild}} \ll 1$$

Mass in AdS

In CFT compute Δ as a Noether charge:



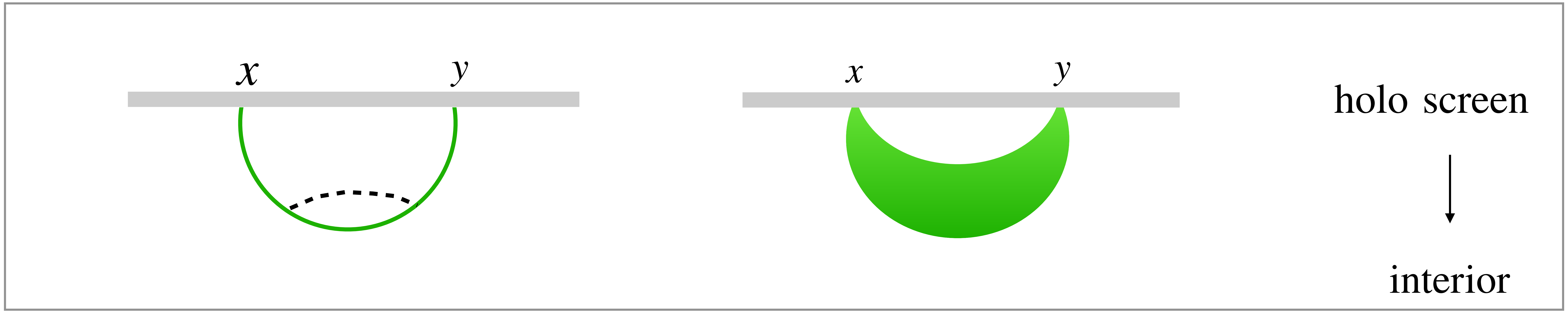
dilatation current

$$\Delta_{\mathcal{O}} := \oint ds^j\, x^k \langle T_{kj}\, \mathcal{O} \rangle$$

The $\Delta_{\mathcal{O}}$ are part of the invariant CFT data

Mass in AdS

In gravity can compute Δ from $\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = \frac{1}{|x-y|^{\Delta}}$



probe particle worldline
+ corrections

backreacting
"banana geometry"

For given Δ count the microscopic entropy S(Δ) in CFT

$\Longrightarrow$ first glimpse of the UV structure of Black Holes

Strominger, Vafa 1996

3. Two invariant tensions

Quantum gravity is (believed to be) a theory of relativistic extended objects

Does their tension, σ , admit a similar invariant definition like mass ?

The bare tension is a parameter in the effective Lagrangian of a thin brane

$$\mathscr{L}_{\text{eff}} = \sigma_0 \int d^{p+1}\zeta \sqrt{\det\left(g_{\mu\nu}\partial_a Y^\mu \partial_b Y^\nu\right)}$$

One expects $\sigma \simeq \sigma_0$ for a classical probe brane; beyond this limit there are classical gravitational and quantum corrections, suppressed by powers of $G\sigma_0$ and $1/\sigma_0$. *How to resum them in an invariant way ?*

Two invariant tensions

In contrast to the case of point-particles, $\exists$ for $0 < p < d-1$ two natural and independent definitions of invariant tension :

I. Gravitational (`ADM like') tension

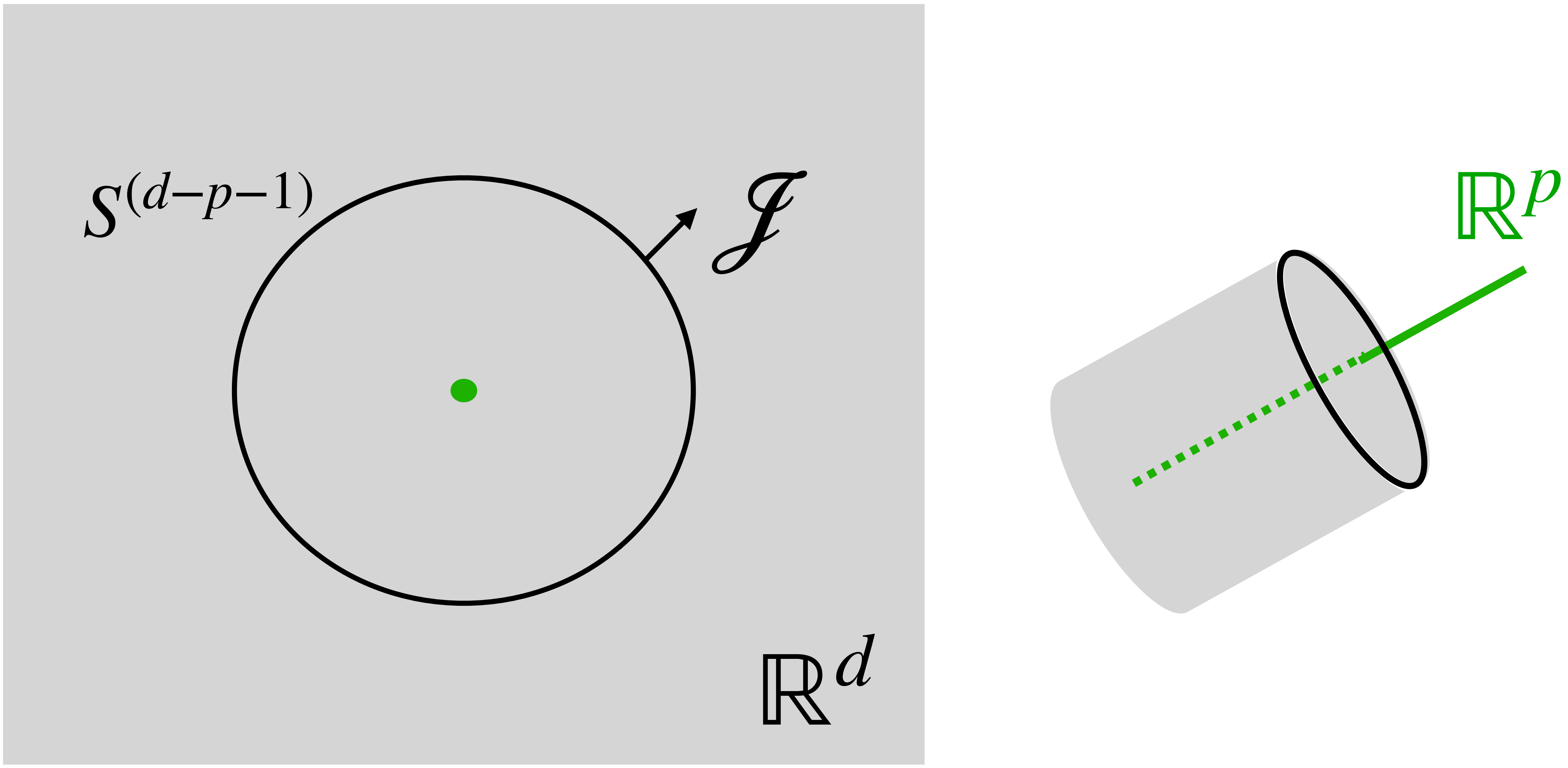
Given by the 1-point function of the dilatation current in the DCFT vacuum; related in gravity to the asymptotic behaviour of the metric far from the defect

II. Stiffness (`inertial' tension)

Given by the 2-point function of the displacement field which deforms the defect worldvolume in the CFT; related to the collective coordinates of the solution in the dual AdS gravity

Two invariant tensions

I. Gravitational (agrees with ADM mass for *p=0*)



$\partial \mathrm{AdS}_{d+1}$ = holographic screen

dilatation current

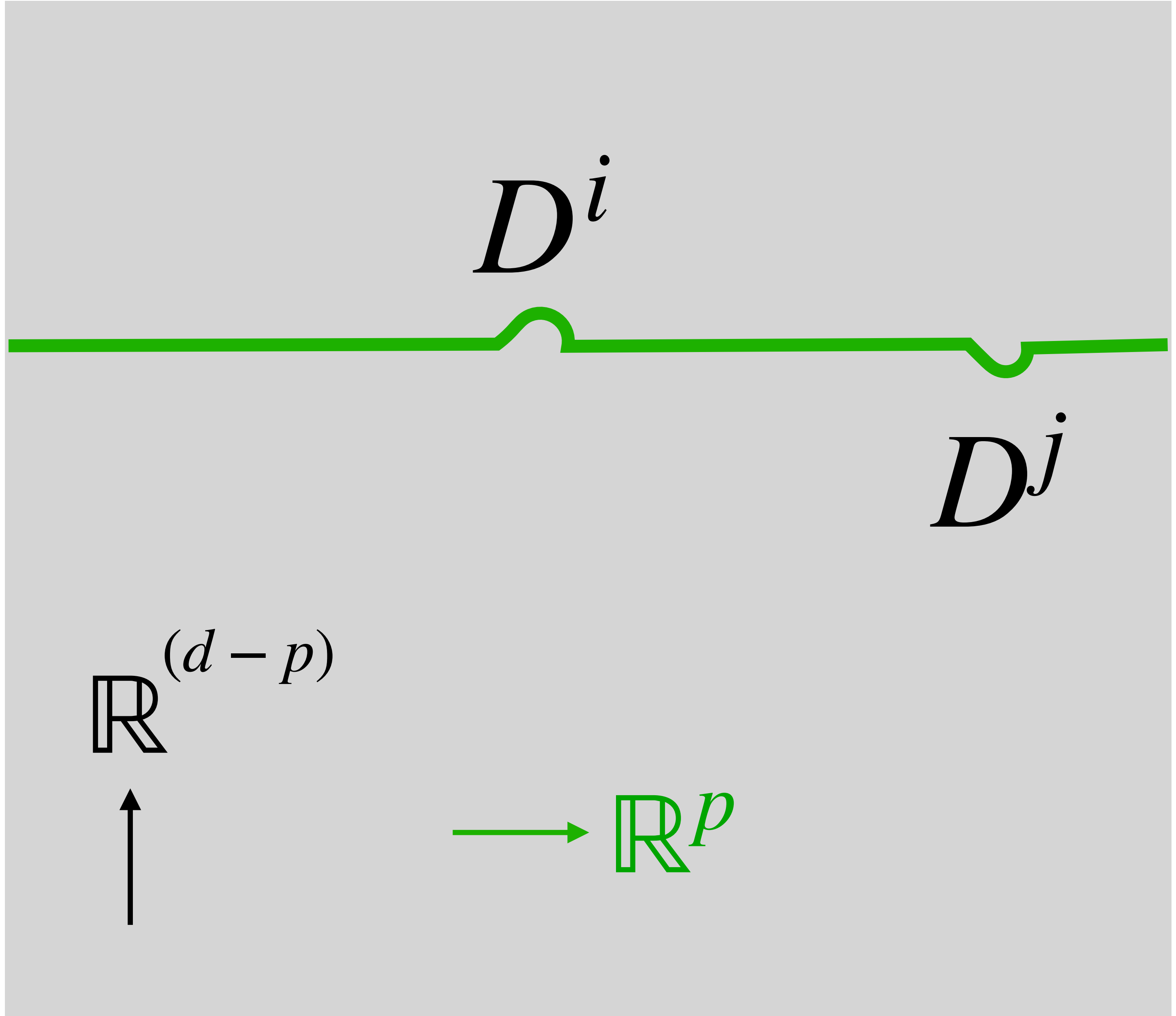
$$\sigma_{(\mathrm{gr})} := \left(\frac{d-1}{d-p-1}\right) \oint ds^j \, \langle \mathcal{J}_j \rangle_{\mathrm{D}}$$

$$x^k \, \langle T_{kj} \rangle_{\mathrm{D}} = a_T \, (\text{universal})$$

vev of em tensor
(piece of DCFT data)

Two invariant tensions

II. Stiffness (does not exist for point particles)



$$\sigma_{(\text{stiff})} := C_D \frac{\pi^{p/2}\,\Gamma(\frac{p}{2}+1)}{(p+2)\,\Gamma(p+1)}$$

displacement norm

(piece of DCFT data)

$$\langle D^j(x) D^k(y) \rangle \sim \frac{C_D\, \delta^{jk}}{(x-y)^{p+1}}$$

Two invariant tensions

Remark 1: The DCFT data is guaranteed to be invariant. Howwever the existence of a dual DCFT is not necessary; it can be only a proxy for the asymptotic behaviour of the gravitational fields.

Remark 2: The definition of $\sigma_{(\mathrm{gr})}$ reduces to (and generalizes) early efforts to define an invariant brane tension by assuming `transverse asymptotic' flat or AdS metrics.

Deser, Soldate 1989

Myers, arXiv:9903203 [hep-th]

Traschen, Fox, arXiv:0103106 [gr-cq]

Townsend, Zamaklar, arXiv:0107228 [hep-th]

Harmark, Obers, arXiv:0403103 [hep-th]

Two invariant tensions

Remark 3: The DCFT data includes the 1pt functions of all defect operators and their scaling dimensions; their norms can be normalized by convention to 1. The displacement operator is an exception because of the Ward identity

$$\partial_i T^{ij} = \delta_D(x)\, D^j$$

Remark 4: Key in the above definitions are the two prefactors marked in yellow. They were fixed from the relevant Witten diagrams of the effective gravitational theory

$$\frac{1}{16\pi G}\int \sqrt{g}R \;+\; \sigma_0 \int_D \sqrt{\hat{g}}$$

and the requirement that in the classical probe limit

$$\sigma_{(\mathrm{gr})} \simeq \sigma_{(\mathrm{stiff})} \simeq \sigma_0$$

Two invariant tensions

What makes this computation non-trivial is the absence of global Fefferman-Graham coordinates in which the standard AdS/CFT dictionary is defined. For $\langle T_{ij}\rangle_{\rm CFT} \sim \langle h_{ij}\rangle_{\rm grav}$ one uses the standard Poincaré coordinates

$$ds^2_{\rm AdS} = \frac{\delta_{\mu\nu}\, dy^\mu dy^\nu}{(y^0)^2} \qquad \text{with} \quad \mu, \nu = 0, 1, \cdots, d$$

and the brane sitting at $\mathbf{y}_\perp = (y^{p+1}, \cdots, y^d) = 0$.

But in these coordinates the residual $\mathrm{SO(2,p)} \times \mathrm{SO(d-p)}$ symmetry is not manifest, and $\mathbf{Y}_\perp(\zeta)$ is not the dual of the displacement operator.

A better choice is $\langle D^j D^k\rangle_{\rm CFT} \sim \langle \mathbf{X}_\perp{}^j\, \mathbf{X}_\perp{}^k\rangle_{\rm grav}$

Two invariant tensions

$$ds^2_{\text{AdS}} = \frac{\delta_{\alpha\beta}\, dx^\alpha dx^\beta}{(x^0)^2}\left(\frac{1+\frac{1}{4}\mathbf{x}_\perp{}^2}{1-\frac{1}{4}\mathbf{x}_\perp{}^2}\right)^2 + \frac{\delta_{ij}\, dx^i dx^j}{(1-\frac{1}{4}\mathbf{x}_\perp{}^2)}$$

$$\text{with} \quad \alpha,\beta = 0,1,\cdots,p \quad \text{and} \quad i,j = p+1,\cdots,d$$

Giombi, Roiban, Tseytlin arXiv:1706.00756 [hep-th]

Thus there is no universal AdS cutoff for both bulk and brane fields, and the correct normalization of the displacement is not clear. We sidestepped this difficulty by checking explicitly the (broken and unbroken) conformal Ward identities that equate schematically

$$\langle TD \rangle \quad \text{to} \quad \langle T \rangle + \langle DD \rangle$$

Billo, Goncalves, Lauria, Meineri arXiv:1601.02883 [hep-th]

4. Examples

Maldacena-Wilson line in $\mathcal{N} = 4$ SYM

heavy quark coupling to scalars

The two relevat pieces of DCFT data can be computed exactly, using supersymmetric localization, for all values of N_c and $\lambda = g^2 N_c$. The result is given by a modified Laguerre polynomial

$$C_D = -18 a_T = \frac{6}{\pi^2} \lambda \partial_\lambda \log \langle W_\odot \rangle \quad \text{where} \quad W_\odot = \frac{1}{N_c} e^{\lambda/8N_c} L^1_{N_c - 1}(-\frac{\lambda}{4N_c})$$

Pestun arXiv:0906.0638 [hep-th]

Erickson, Semenoff, Zarembo arXiv:0003055 [hep-th]

Drukker et al

Correa, Henn, Maldacena, Sever arXiv:1202.4455 [hep-th]

Examples

Note that $B = \dfrac{C_D}{12}$ is the Bremsstrahlung function that controls the radiation of an accelerating quark, $\mathscr{E}_{\text{rad}} = 2\pi B \int dt\ a^2$

In the limit $\lambda, N_c \to \infty$ one finds $\sigma_{(\text{gr})} = \sigma_{(\text{stiff})} \simeq \dfrac{\sqrt{\lambda}}{2\pi}$

F-string tension ✓

Expanding the Laguerre polynomial gives an infinite series of quantum and gravitational corrections, but surprisingly the equality $\sigma_{(\text{gr})} = \sigma_{(\text{stiff})}$ persists. Will come back to this in the following section.

Examples

Interfaces in 1+1 dimensions

q – wire junctions
constrictions of Hall fluids

Here the *codim=1*, so only $\sigma_{(\text{stiff})} = \frac{\pi}{6} C_D$ can be defined.

C_D is an important parameter that gives the ratio of transmitted/reflected energy at the interface; this latter is universal in 1+1 d

Quella, Runkel, Watts arXiv:0611296 [hep-th]

Meineri, Penedones, Rousset arXiv:1904.10974 [hep-th]

Together with the Affleck-Ludwig entropy $\log g$, and the Cardy-Calabrese parameter c_{eff}, it controls key long-distance properties of an interface.

Examples

One can compute C_D in holography, for a thin but fully back-reacting brane on which geometries are matched by the Israel conditions, with the result

$$C_D = \frac{6\sigma_0/\pi}{1 + 4\pi G_N \sigma_0}$$

CB, Chapman, Ge, Policastro arXiv:2006.11333 [hep-th]

CB, Chen, V. Papadopoulos arXiv:2107.00965 [hep-th]

Baig, Karch arXiv:2206.01752 [hep-th]

CB, Baiguera, Chapman, Policastro, Schwartzman arXiv:2212.14058 [hep-th]

Note that in the limit $G_N \to 0$ $\quad \sigma_{\text{(stiff)}} \simeq \sigma_0$ ✓

Examples

Graham-Witten anomalies

These are Weyl anomalies made out of the (intrinsic & extrinsic) curvatures of *p=2,4* defects. E.g. for surface defects *(p=2)*

$$T^m_m\Big|_{\text{Defect}} = \frac{1}{24\pi}\left(\mathbf{a}^{(2)}R + \mathbf{d}_1^{(2)}\bar{K}^i_{ab}\bar{K}^{ab}_i - \mathbf{d}_2^{(2)}W^{ab}_{ab}\right)$$

Graham, Witten arXiv: 9901021 [hep-th]

Schwimmer, Theisen arXiv: 0802.1077 [hep-th]

The coefficients $\mathbf{d_1}, \mathbf{d_2}$ can be related, respectively, to C_D, a_T

Furthermore, in the thin-probe limit, one can compute them with techniques of conformal geometry (Willmore energy) with the result

Examples

$$\mathbf{d}_1^{(2)} = \mathbf{d}_2^{(2)} = 6\pi\sigma_0$$

$$\mathbf{d}_1^{(4)} = -\pi^2\sigma_0 \; ; \qquad \mathbf{d}_2^{(4)} = -\frac{\pi^2\sigma_0}{d-4}$$

Graham, Reichert arXiv: 1704.03852 [hep-th]

Chalabi, Herzog, O'Bannon, Robisnon, Sisti arXiv: 2111.14713 [hep-th]

Collecting all numerical coefficients one can show that in all cases

$$\sigma_{(\mathrm{gr})},\ \sigma_{(\mathrm{stiff})} \simeq \sigma_0$$

in the classical probe limit. ✓

5. Supersymmetry

We have seen in the case of the Maldacena-Wilson line that $\sigma_{(\mathrm{gr})} = \sigma_{(\mathrm{stiff})}$

is exact, despite the fact that each tension receives an infinite # of corrections.

This follows from supersymmetry, which relates $C_D = -18a_T$

L. Bianchi, Lemos, Meineri arXiv: 1805.04111 [hep-th]

There is a related interesting physics conundrum:

C_D is proportional to the energy radiated by an accelerating quark, whereas a_T to the energy collected at infinity. Supersymmetry makes these two energies are equal !

Without it, separating the radiated from the self-energy for a constantly accelerating quark is problematic.

Lewkowycz, Maldacena arXiv: 1312.5682 [hep-th]

Fiol, Gerchkovitz, Komargodski arXiv: 1510.01332 [hep-th]

Supersymmetry

There is no known such conundrum for *p>1* defects, but the same susy argument seems to lead to a linear relation between C_D and a_T for all *p,d*.

$$C_D = - a_{\mathrm{T}} \frac{2(d-1)(p+2)\Gamma(p+1)}{d\,\pi^{p-d/2}\,\Gamma(\frac{p}{2}+1)\Gamma(\frac{d-p}{2})}$$

L. Bianchi, Lemos arXiv: 1911.05082 [hep-th]

CB, L. Bianchi, Z. Chen in progress

Roughly speaking, conformal Ward identities fix a linear relation between the corresponding quantities of the susy ancestors of T_{ij} and D_j, which are an R-symmetry current and a scalar. These then descend to $\langle T \rangle$ and $\langle DD \rangle$ thanks to supersymmetry.

Supersymmetry

Inserting this linear relation in our formulae gives

$$\sigma_{(\mathrm{gr})} = \sigma_{(\mathrm{stiff})}$$

i.e. supersymmetry implies the equality of gravitational tension and stiffness

This is a peculiar BPS protection (usually mass = charge)

It has the flavour of the principle of equivalence

Is it just a curiosity, or does it have a deeper meaning about the need for supersymmetry in the deep UV ?

6. Summary

Take away messages:

- There exist two independent definitions of the tension of extended objects in AdS gravity, related to the metric and the displacement field.

- They control important properties of the holographic dual DCFTs

  - Supersymmetry equates them, why ?

Many thanks for your attention