

# Precision phenomenology for $2 \rightarrow 3$ scattering at the LHC: Wbb and $\gamma jj$ final states

based on:

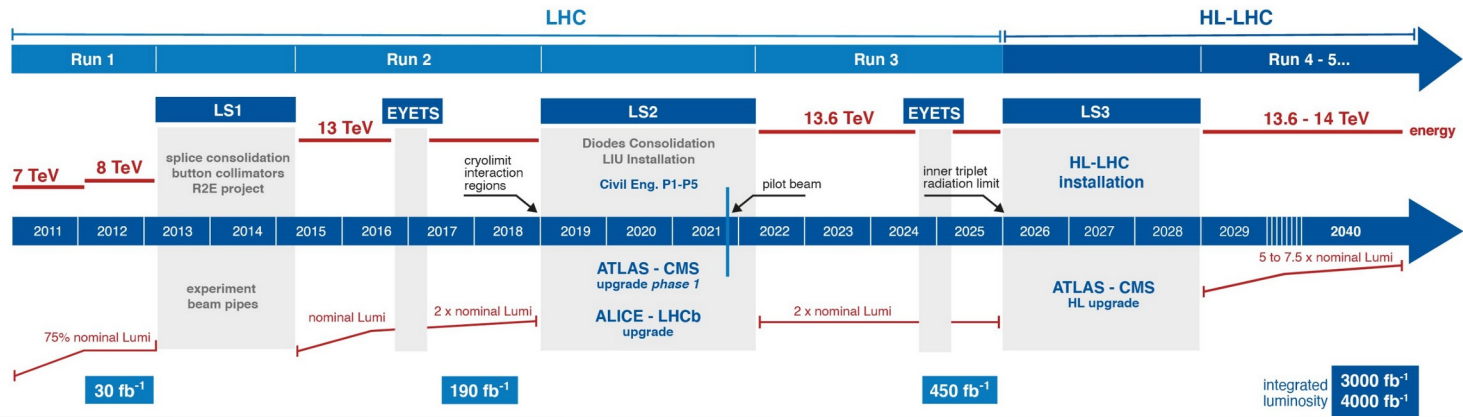
2205.01687 and 2209.03280 [hep-ph] (with R. Poncelet, A. Popescu, S. Zoia)

2304.06682 [hep-ph] (with S. Badger, M. Czakon, R. Moodie, T. Peraro, R. Poncelet, S. Zoia)

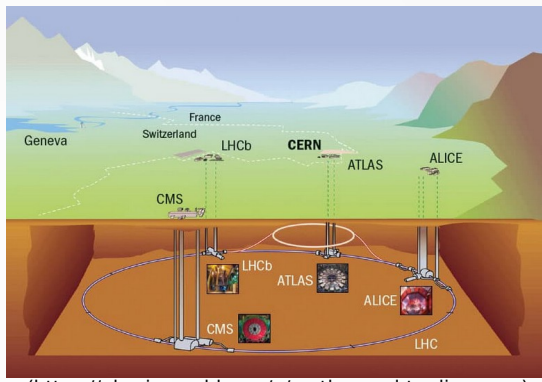
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**Demokritos-APCTP Meeting and HOCTOOLS-II mini-workshop  
October 3<sup>th</sup>, 2024**



(<https://hilumilhc.web.cern.ch/content/hl-lhc-project>)



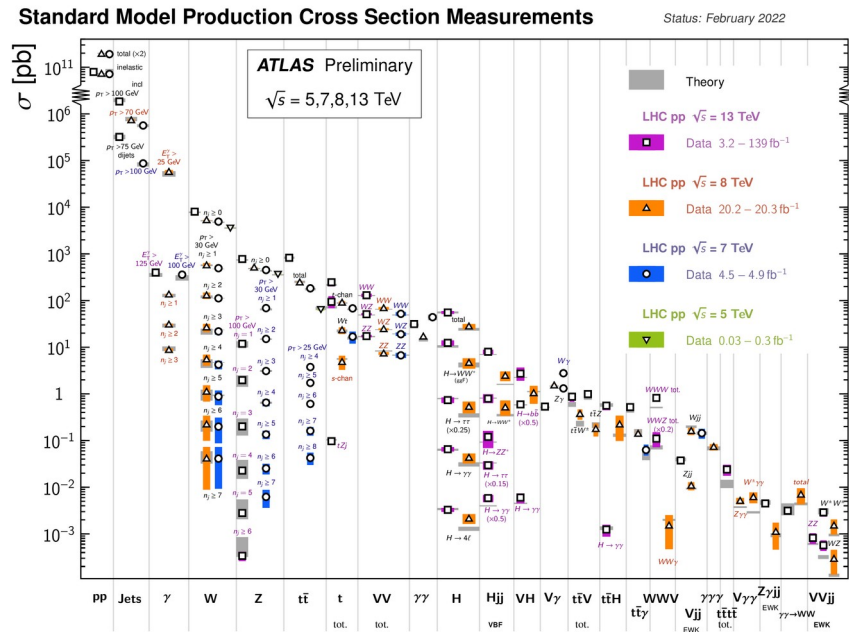
(<https://physicsworld.com/a/on-the-road-to-discovery/>)

# LHC as a precision machine

Run 1+2+3 and HL-LHC ⇒ huge amount of data

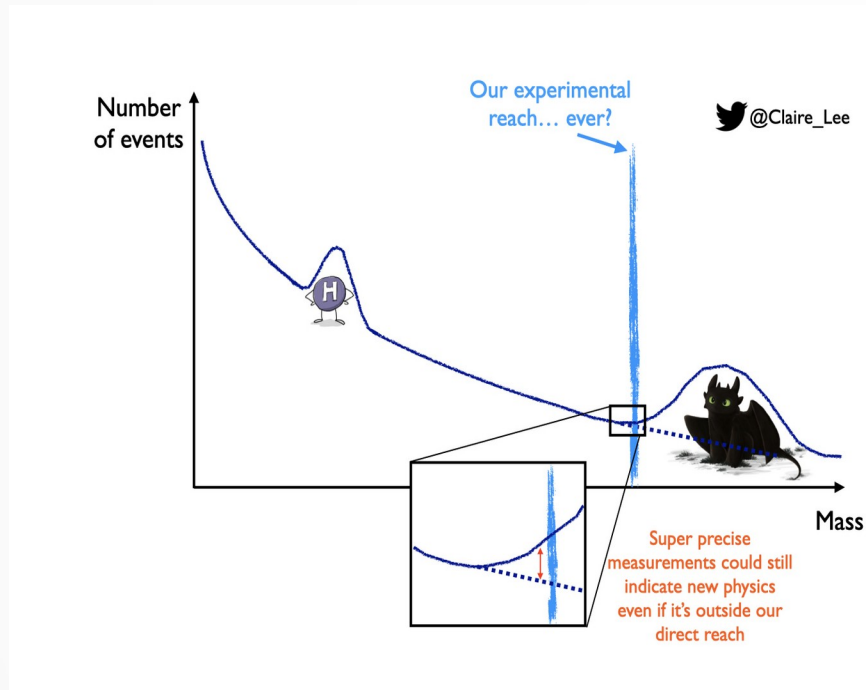
- Small uncertainties on experimental measurements (some may reach % level accuracy)

→ Observe rare processes

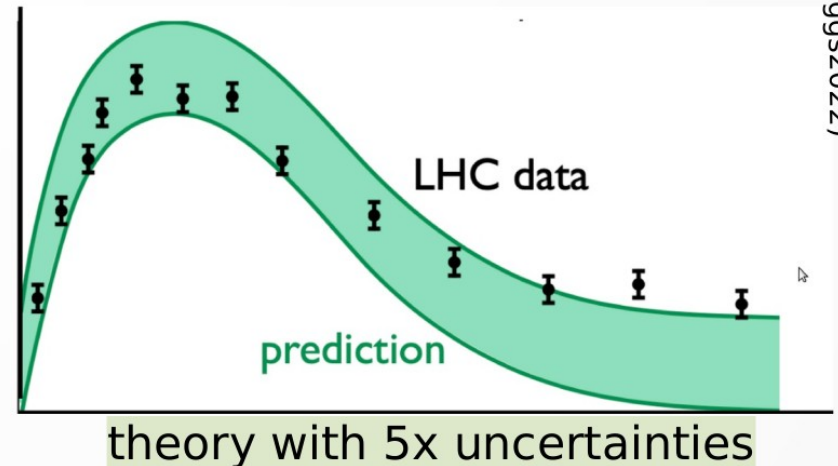
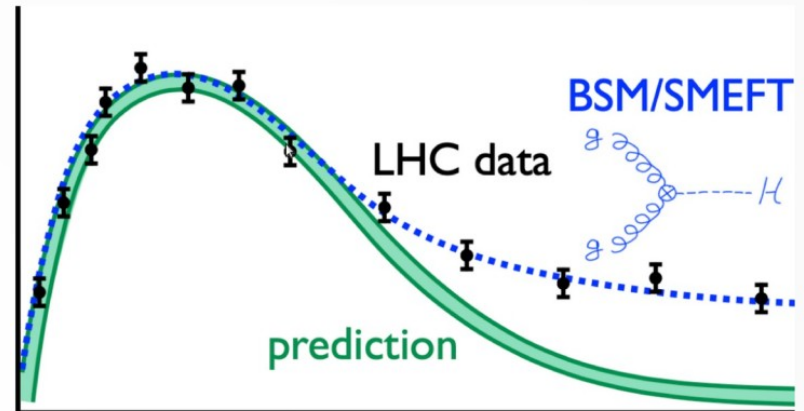


# Searching for New Physics at the LHC

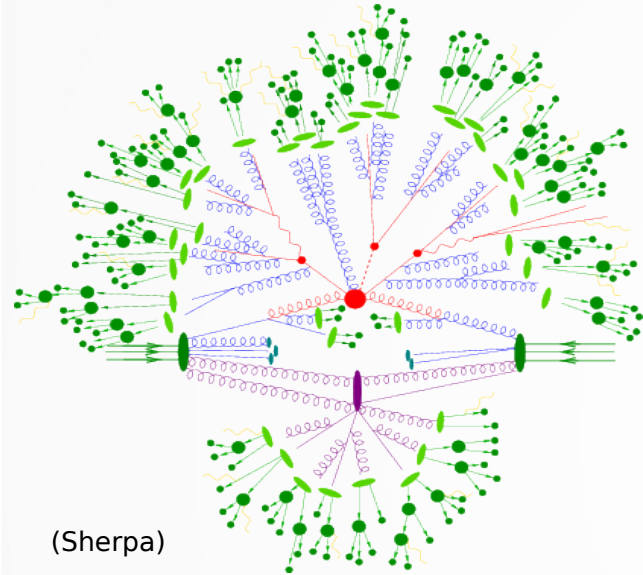
So far: no clear new physics signal at the current energy reach of LHC



looking for deviations from SM predictions through precision measurements



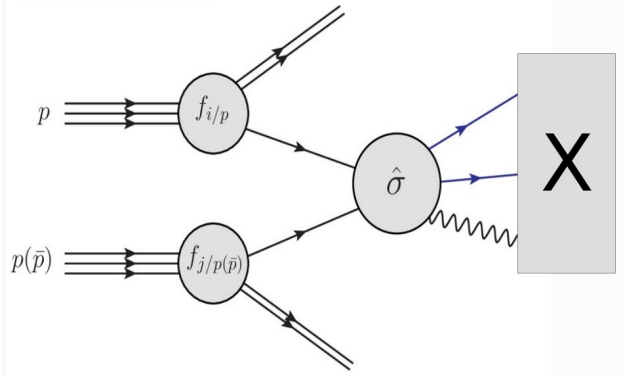
# Theoretical Predictions for the LHC



- Hard scattering
- Parton showers
- Hadronization
- Decay of Hadrons
- EM bremsstrahlung
- Underlying events

⇒ Monte Carlo event generator

Master formula for  $pp$  cross section:



$$d\sigma(pp \rightarrow X) = \sum_{ij} \int dx_1 dx_2 \underbrace{f_{i/p}(x_1) f_{j/p}(x_2)}_{\text{parton distribution functions (PDF)}} \underbrace{d\hat{\sigma}(ij \rightarrow X)}_{\text{partonic cross section}}$$

parton distribution functions (PDF)

partonic cross section

# Perturbative QFT framework

Perturbative calculation of partonic cross sections

$$d\hat{\sigma} = \underbrace{d\hat{\sigma}^{(0)}}_{\text{LO}} + \underbrace{\alpha d\hat{\sigma}^{(1)}}_{\delta\text{NLO}} + \underbrace{\alpha^2 d\hat{\sigma}^{(2)}}_{\delta\text{NNLO}} + \dots$$

LO: Leading Order

NLO: Next-to-Leading Order

NNLO: Next-to-Next-to-Leading Order

Large quantum corrections come from strong interaction (QCD)

$$\alpha_s(M_Z^2) \sim 0.1$$

**Main source of theoretical error**  $\Rightarrow$  truncation of perturbative series (scale dependence)

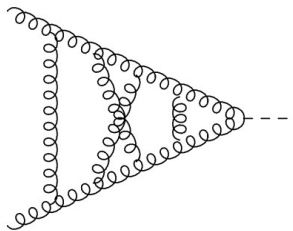
**Typical theoretical error on :**

LO > 50%

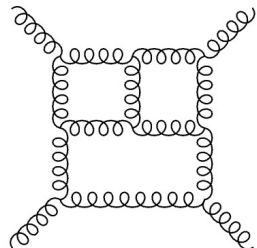
NLO QCD  $\sim$  20-30%

NNLO QCD  $\sim$  1-10%

## Loop Frontiers

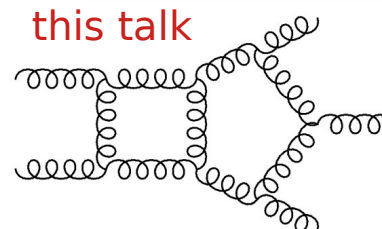


N4LO 2 $\rightarrow$ 1

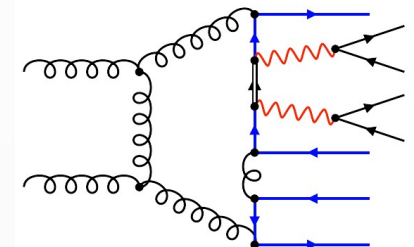


N3LO 2 $\rightarrow$ 2

## Multiplicity Frontiers

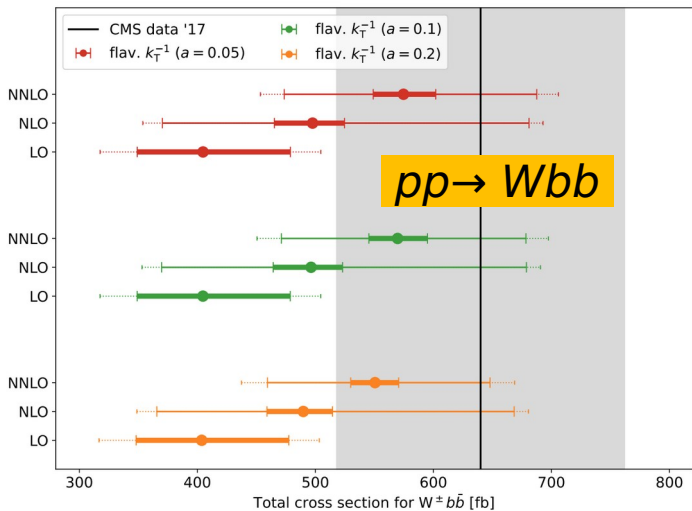
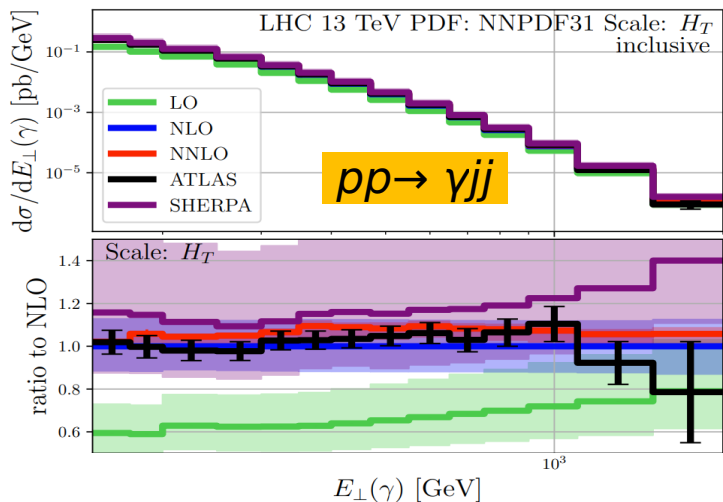


NNLO 2 $\rightarrow$ 3

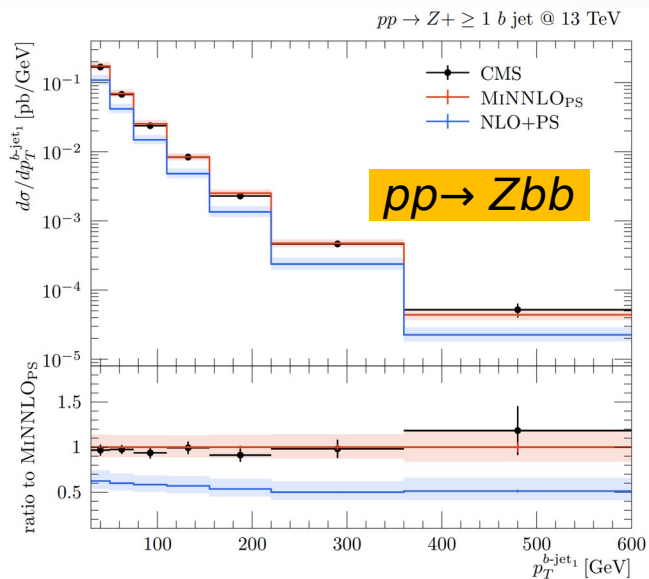


NLO 2 $\rightarrow$ 6

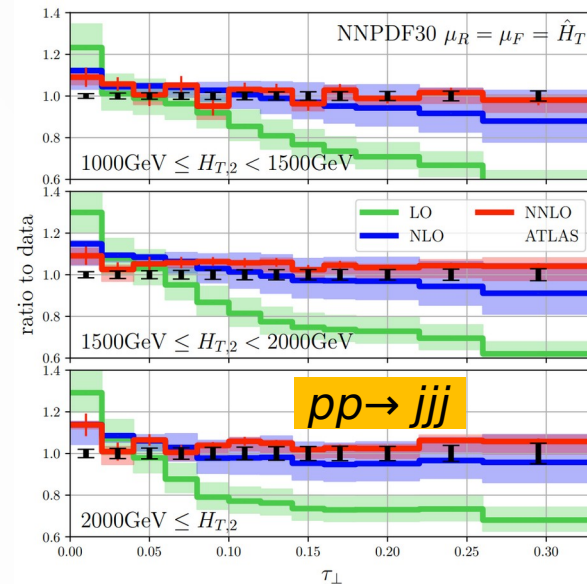
# NNLO QCD calculation for 2→3 scattering processes



[**HBH**,Poncelet,Popescu,Zoia(2022)]



[Mazzitelli,Sotnikov,Wiesemann(2024)]



[Czakon,Mitov,Poncelet(2021)][Chen,Gehrmann,Glover,Huss,Marcoli(2022)]Alvarez,Cantero,Czakon,Llorente,Mitov,Poncelet(2023)]

also:  $pp \rightarrow \gamma\gamma\gamma$ ,  $pp \rightarrow \gamma\gamma j$ ,  $pp \rightarrow Wbb$ ,  $pp \rightarrow ttW$ ,  $pp \rightarrow ttH$

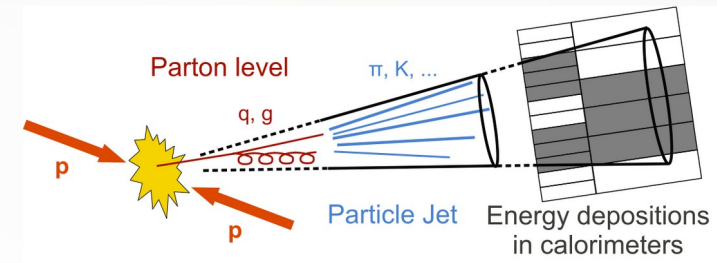
[Chawdhry,Czakon,Mitov,Poncelet(2019)][Kallweit,Sotnikov,Wiesemann(2020)]  
 [Czakon,Mitov,Poncelet(2020)][Buonocore,Devoto,Kallweit,Mazzitelli,Rottoli,Savoini(2022)]  
 [Buonocore,Devoto,Grazzini,Kallweit,Mazzitelli,Rottoli,Savoini(2022)]  
 [Catani,Devoto,Grazzini,Kallweit,Mazzitelli,Savoini(2020)]

# Wbb production at NNLO QCD

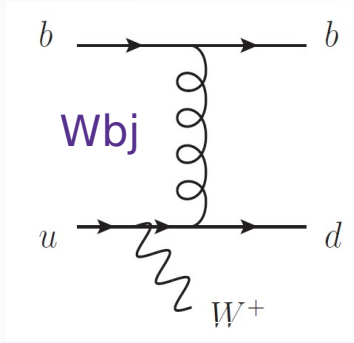
**HBH**, Poncelet, Popescu, Zoia

arXiv:2205.01687 and arXiv:2209.03280

# W/Z+b-jets production at the LHC

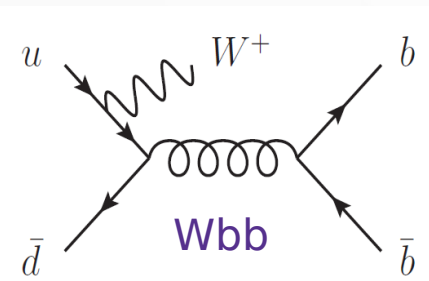
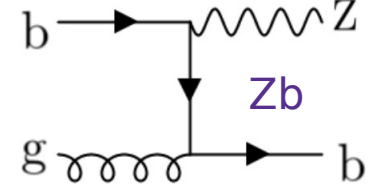


- Test perturbative QCD
- Modelling of flavoured jets (theory and experiment)
- Sensitivity to heavy flavour schemes: **4-flavour (4FS) vs 5-flavour (5FS) scheme**  
massive  $b$ 
massless  $b$

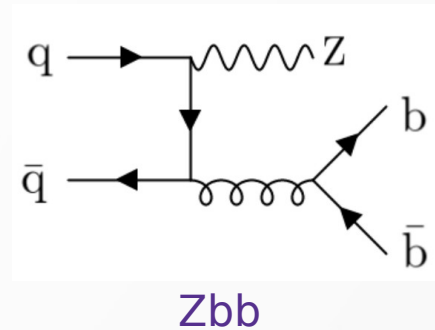
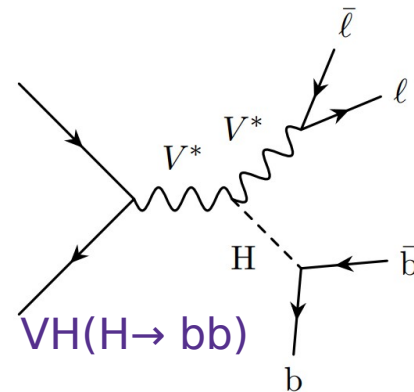


W/Z+1b jet: probe  $b$ -quark PDF

W+2b jets: background to single top production  $pp \rightarrow bt(\rightarrow bW)$



W/Z+2b jets: background to  $VH(H \rightarrow bb)$



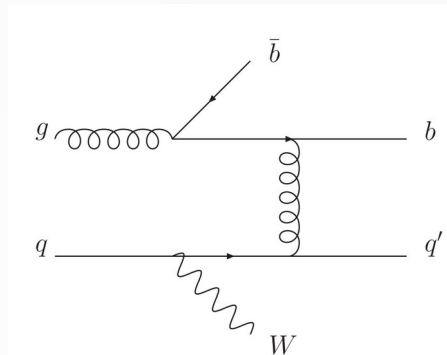


# W+2b+ ≤ 3j at NLO QCD

[Anger, Febres Cordero, Ita, Sotnikov; arXiv:1712.05721]

Inclusive Wbb production ( $pp \rightarrow Wbb + X$ )

- large NLO corrections as well as large NLO scale dependence
- due to opening of qg channel ( $qg \rightarrow Wbb + q$ )

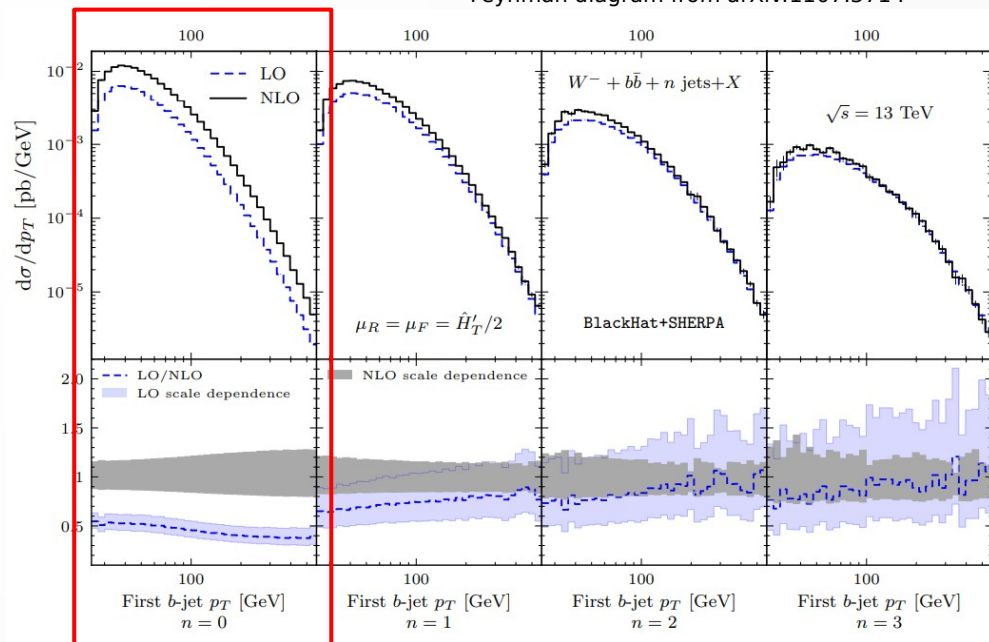


Feynman diagram from arXiv:1107.3714

| jets | $W^- b\bar{b}$ LO                 | $W^- b\bar{b}$ NLO                | $K$ -factor |
|------|-----------------------------------|-----------------------------------|-------------|
| 0    | $0.33278(12)^{+0.0619}_{-0.0490}$ | $0.67719(60)^{+0.1288}_{-0.1000}$ | 2.03        |
| 1    | $0.36153(13)^{+0.1408}_{-0.0945}$ | $0.50484(63)^{+0.0851}_{-0.0800}$ | 1.40        |
| 2    | $0.18501(44)^{+0.1053}_{-0.0626}$ | $0.22604(87)^{+0.0407}_{-0.0400}$ | 1.22        |
| 3    | $0.07204(25)^{+0.0540}_{-0.0289}$ | $0.08288(89)^{+0.0189}_{-0.0200}$ | 1.15        |

~100% NLO QCD corrections

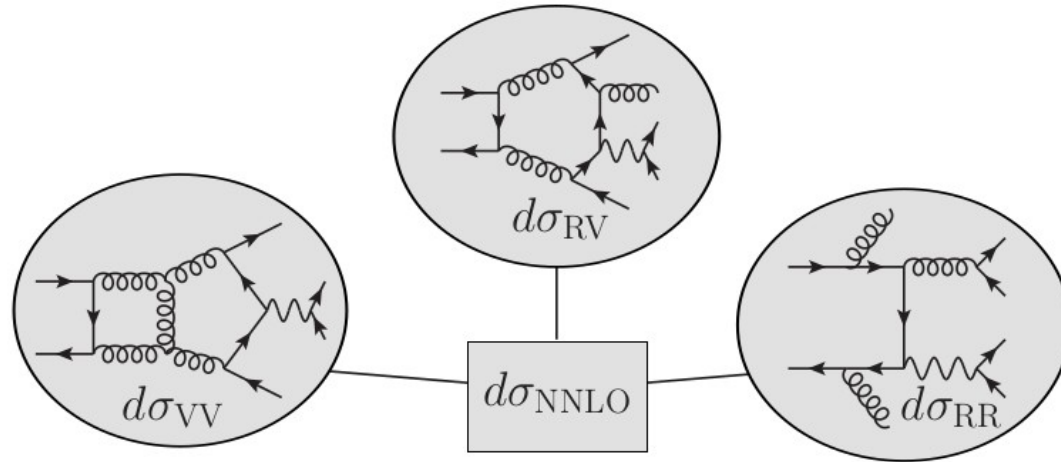
scale dependence: 19% at LO, 20% at NLO



Calls for a NNLO QCD calculation!!

# NNLO QCD corrections to $W(\rightarrow l\nu)+bb$ in 5FS

NNLO QCD calculation for  $W+2b$  jets: massless  $b$  (5FS) [HBH,Poncelet,Popescu,Zoia(2022)] ← this talk  
massive  $b$  (4FS) [Buonocore,Devoto,Kallweit,Mazzitelli,Rottoli,Savoini(2022)]

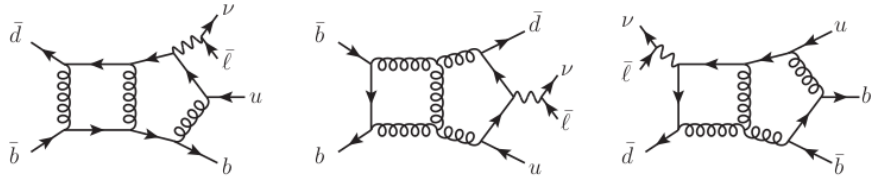


Amplitudes:

- Tree-level  $pp \rightarrow W(\rightarrow l\nu)+bbjj$ : AvH library [Bury,van Hameren(2015)]
- One-loop  $pp \rightarrow W(\rightarrow l\nu)+bbj$ : OpenLoops [Bucionni,Lang,Lindert,Maierhoefer,Pozzorini,Zhang,Zoller(2018,2019)]
- Two-loop  $pp \rightarrow W(\rightarrow l\nu)+bb$ : [Abreu,Febris Cordero,Ita,Klinkert,Page(2021)][HBH,Poncelet,Popescu,Zoia(2022)]

NNLO subtraction scheme: Sector Improved Residue Subtraction Scheme (STRIPPER) [Czakon(2010)]

# Two-loop amplitude for $u\bar{d} \rightarrow W^+ (\rightarrow \bar{\ell}\nu) b\bar{b}$



⇒ leading colour approximation and massless  $b$  quark

⇒ compute squared matrix element

$$\sum \mathcal{M}^{(0)*} \mathcal{M}^{(2)} = M_{\text{even}}^{(2)} + \text{tr}_5 M_{\text{odd}}^{(2)}$$

$$\text{tr}_5 = 4i\epsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho p_4^\sigma$$

employ CDR+Larin scheme to treat  $\gamma_5$ .

⇒ Incorporating  $W \rightarrow \ell\nu$  decay

$$M_6^{(L)} = \sum_{\text{spin}} A_6^{(0)\dagger} A_6^{(L)} = M_5^{(L)\mu\nu} \mathcal{D}_{\mu\nu} |P(s_{56})|^2$$

$$M_5^{(L)\mu\nu} = \sum_{i=1}^{16} a_i^{(L)} v_i^{\mu\nu} \quad v_i^{\mu\nu} \in \{p_1^\mu, p_2^\mu, p_3^\mu, p_W^\mu\}$$

Derive analytic expressions of the finite remainders using finite-field reconstruction method

QGRAF[Nogueira], FORM[Vermaseren], LiteRed[Lee],  
FiniteFlow[Peraro], Mathematica

$$M_k^{(2)}(\{p\}, \epsilon) = \sum_i c_{k,i}(\{p\}, \epsilon) \mathcal{I}_{k,i}(\{p\}, \epsilon)$$

↓ IBP reduction

$$M_k^{(2)}(\{p\}, \epsilon) = \sum_i d_{k,i}(\{p\}, \epsilon) \text{MI}_{k,i}(\{p\}, \epsilon)$$

↓ map to special function basis

↓ subtract UV/IR poles

↓  $\epsilon$  expansion

$$F_k^{(2)}(\{p\}) = \sum_i e_{k,i}(\{p\}) m_{k,i}(f) + \mathcal{O}(\epsilon)$$

# Fixed-order flavoured jets beyond NLO

Example NNLO: double real radiation and subtraction

$$d\sigma \ni d\Phi_{n+2} \left( \text{Double real matrix element} \right) F_{n+2} + \dots + d\tilde{\Phi}_{n+2} \mathcal{S}_2 \left( \text{Double soft subtraction term: Double soft function * tree ME} \right) F_n$$

Mistreatment of flavour pair in  $F_{n+2} \Rightarrow$  mismatch w.r.t  $F_n \Rightarrow$  double soft singularity not subtracted

Solutions:

- **Flavour- $k_T$  jet algorithm** [Banfi,Salam,Zanderighi(2006)]  $\rightarrow$  data/theory comparison require unfolding
- Practical jet flavour through NNLO [Caletti,Larkoski,Marzani,Reichelt(2022)]
- **Infrared-safe flavour anti- $k_T$  jets** [Czakon,Mitov,Poncelet(2022)]
- A dress of flavour to suit any jet [Gauld,Huss,Stagnitto(2022)]
- Flavoured jets with exact anti- $k_T$  kinematics [Caola,Grabarczyk,Hutt,Salam,Scyboz,Thaler(2023)]

# Fixed-order flavoured jets beyond NLO

Example NNLO: double real radiation and subtraction

$$d\sigma \ni d\Phi_{n+2} \left( \text{Double real matrix element} \right) F_{n+2} + \dots + d\tilde{\Phi}_{n+2} \mathcal{S}_2 \left( \text{Double soft subtraction term: Double soft function * tree ME} \right) F_n$$

**Infrared-safe flavour anti- $k_T$  jet algorithm** [Czakon,Mitov,Poncelet(2022)]

→ introduce damping function to the standard anti- $k_T$  jet algorithm

$$\mathcal{S}_{ij} = 1 - \Theta(1 - x) \cdot \cos\left(\frac{\pi}{2}x\right) \leq 1 \quad x \equiv \frac{1}{a} \frac{k_{T,i}^2 + k_{T,j}^2}{2k_{T,\max}^2}$$

if  $i, j$  have the same non-zero flavour of opposite sign.  $a$ : tunable *softness* parameter

→ minimize the effect of unfolding

# Setup (follows CMS measurement [arXiv:1608.07561])

- **5FS**, LHC 8 TeV, PDFs: NNPDF31, cuts:  $p_{T,l} > 30$  GeV,  $|\eta_l| < 2.1$ ,  $p_{T,j} > 25$  GeV,  $|\eta_j| < 2.4$
- jet algorithm: flavour- $k_T$ [Banfi,Salam,Zanderighi(2006)] and flavour anti- $k_T$ [Czakon,Mitov,Poncelet(2022)],  $R=0.5$
- central scale:  $\mu_R = \mu_F = H_T$  where  $H_T = E_T(l\nu) + p_T(b_1) + p_T(b_2)$
- final state: **inclusive (at least two b jets)** and **exclusive (exactly two b jets and no other jets)**
- scale uncertainties: **inclusive** → 7-pt variation ( $1/2 \leq \mu_R/\mu_F \leq 2$ )
- scale uncertainties: **exclusive** → 7-pt variation and uncorrelated prescription[Stewart,Tackmann(2012)]

Uncorrelated scale variations:  $\sigma_{Wb\bar{b},exc} = \sigma_{Wb\bar{b},inc} - \sigma_{Wbbj,inc}$   $\Delta\sigma_{Wb\bar{b},exc} = \sqrt{(\Delta\sigma_{Wb\bar{b},inc})^2 + (\Delta\sigma_{Wbbj,inc})^2}$

Leading colour approximation is only applied to scale independent double virtual finite remainder

$$\mathcal{V}^{(2)}(\mu_R^2) = \mathcal{V}_{LC}^{(2)}(s_{12}) + \sum_{i=1}^4 c_i \ln^i \left( \frac{\mu_R^2}{s_{12}} \right)$$

Double virtual contributions to  $\sigma$ : 5% (inc) 10% (exc), estimated SLC: 0.5% (inc) 1% (exc)

# W+2b at NNLO QCD: massless b (5FS)

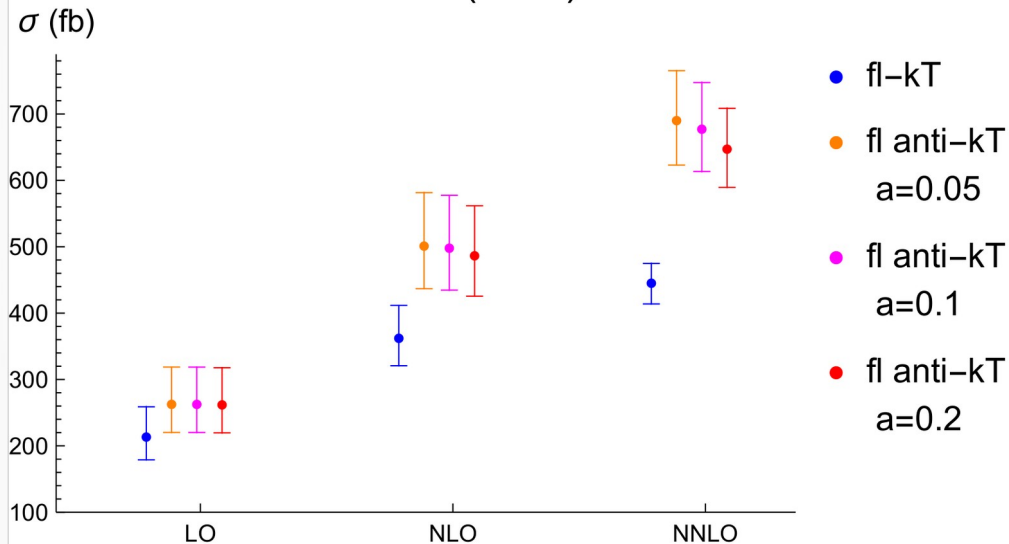
[HBH,Poncelet,Popescu,Zoia (arXiv:2205.01687,arXiv:2209.03280)]

“K-factor”

$$K_{\text{NLO}} = \sigma_{\text{NLO}} / \sigma_{\text{LO}}$$

$$K_{\text{NNLO}} = \sigma_{\text{NNLO}} / \sigma_{\text{NLO}}$$

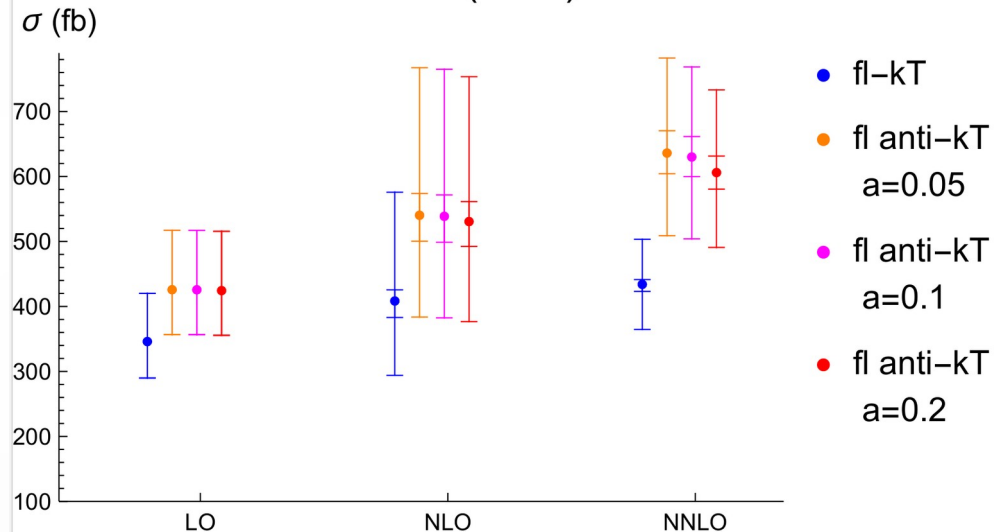
Inclusive  $W^+(\rightarrow l^+ \nu)bb$



$$K_{\text{NLO}}^{\text{fl-kT}} \sim 1.7, \quad K_{\text{NLO}}^{\text{fl-kT}^{-1}} \sim 1.9$$

$$K_{\text{NNLO}}^{\text{fl-kT}} \sim 1.2, \quad K_{\text{NNLO}}^{\text{fl-kT}^{-1}} \sim 1.3 - 1.4$$

Exclusive  $W^+(\rightarrow l^+ \nu)bb$



$$K_{\text{NLO}}^{\text{fl-kT}} \sim 1.2, \quad K_{\text{NLO}}^{\text{fl-kT}^{-1}} \sim 1.3$$

$$K_{\text{NNLO}}^{\text{fl-kT}} \sim 1.1, \quad K_{\text{NNLO}}^{\text{fl-kT}^{-1}} \sim 1.2$$

# W+2b at NNLO QCD: massless b (5FS)

[HBH,Poncelet,Popescu,Zoia (arXiv:2205.01687,arXiv:2209.03280)]

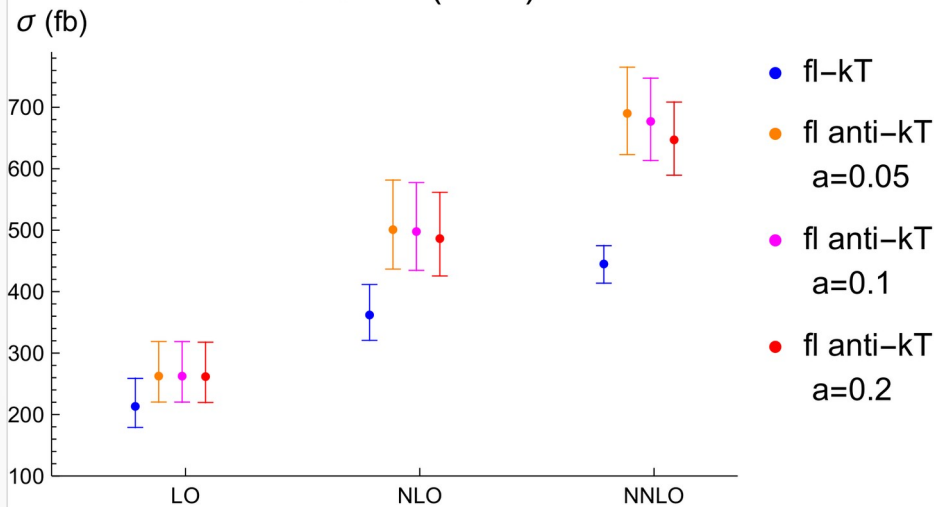
⇒ scale uncertainties: **inclusive** → 7-pt variation  $1/2 \leq \mu_R/\mu_F \leq 2$

**exclusive** → 7-pt variation and uncorrelated prescription [Stewart,Tackmann(2012)]

Uncorrelated scale variation

$$\sigma_{Wb\bar{b},exc} = \sigma_{Wb\bar{b},inc} - \sigma_{Wbbj,inc} \quad \Delta\sigma_{Wb\bar{b},exc} = \sqrt{(\Delta\sigma_{Wb\bar{b},inc})^2 + (\Delta\sigma_{Wbbj,inc})^2}$$

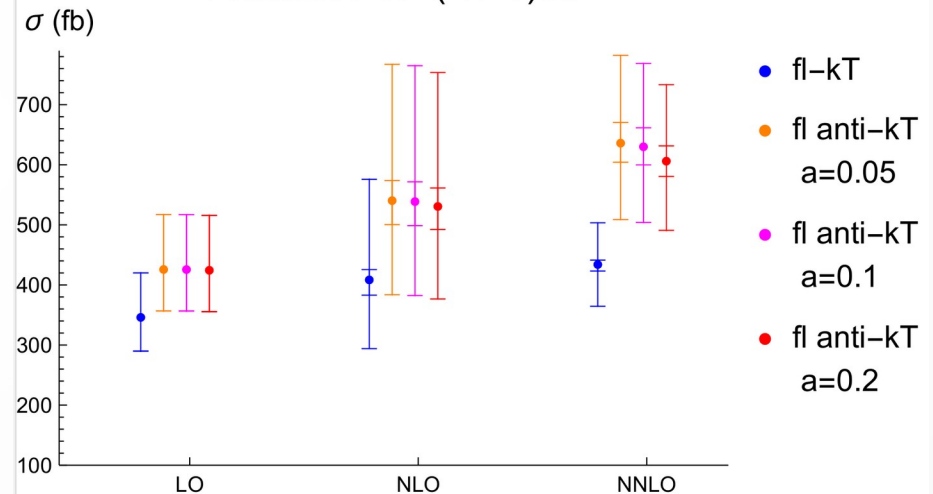
Inclusive  $W^+(\rightarrow l^+ \nu)bb$



$\delta_{scale}^{fl-k_T}$  : 20%(LO), 13%(NLO), 7%(NNLO)

$\delta_{scale}^{fl-k_T^{-1}}$  : 20%(LO), 15%(NLO), 10%(NNLO)

Exclusive  $W^+(\rightarrow l^+ \nu)bb$



$\delta_{scale}^{fl-k_T}$  : 20%(LO), 6%(NLO), 3%(NNLO)  
30%(NLO), 16%(NNLO)

$\delta_{scale}^{fl-k_T^{-1}}$  : 20%(LO), 7%(NLO), 5%(NNLO)  
35%(NLO), 22%(NNLO)

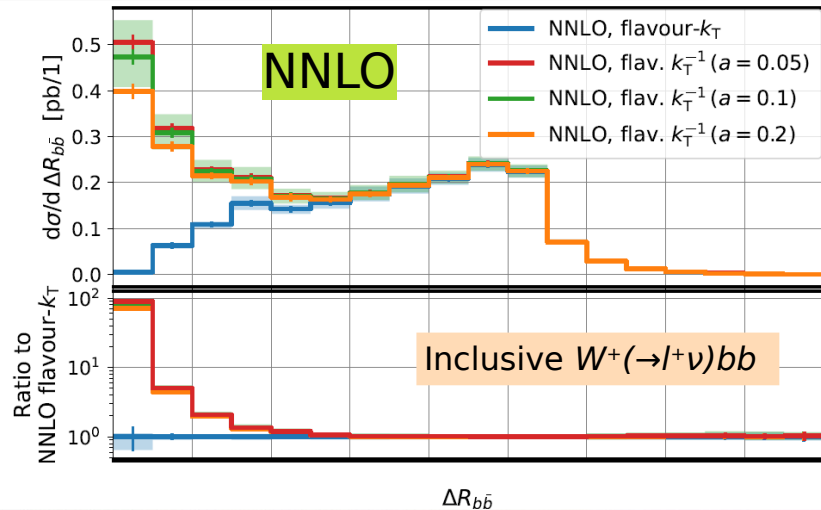
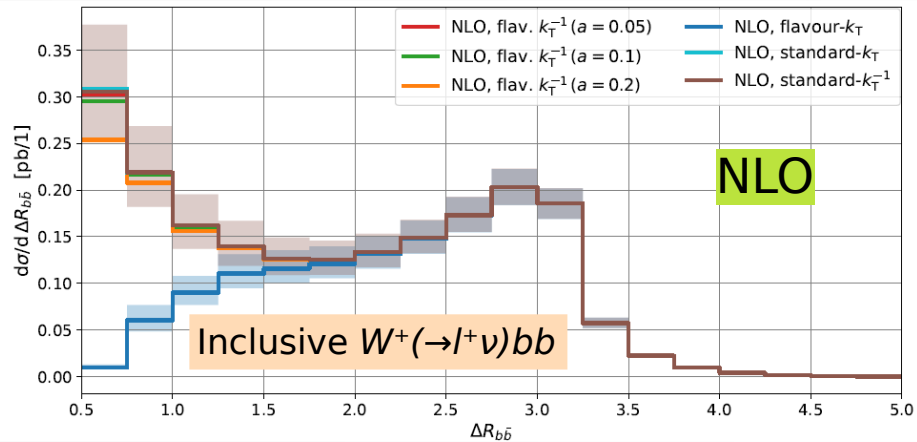
→ 7-pt  
→ correlated

→ 7-pt  
→ correlated

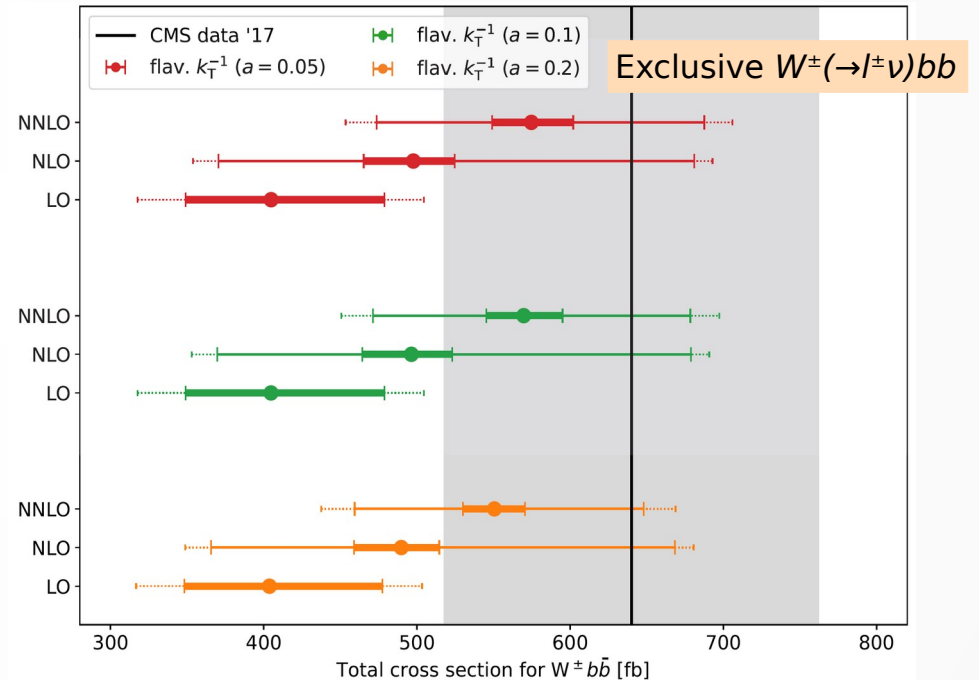


# W+2b at NNLO QCD: massless b (5FS)

[HBH,Poncelet,Popescu,Zoia (arXiv:2205.01687,arXiv:2209.03280)]



- NLO: comparison to standard  $k_T$ /anti- $k_T$  algorithm
- Supression at small  $\Delta R_{bb}$  for flavour- $k_T$  algorithm



7-pt scale variation and uncorrelated prescription,  
+ Hadronisation and MPI uncertainties

# W+2b at NNLO QCD: massive b (4FS)

[Buonocore,Devoto,Kallweit,Mazzitelli,Rottoli,Savoini (arXiv:2212.04954)]

- Two-loop amplitude with massive  $b$  is still out of reach  
→ capture leading contributions in  $m_b/Q$  using “massification” procedure [Mitov,Moch(2007)]

$$\mathcal{M}_2^m = \mathcal{M}_2^{m=0} + Z_{[q]}^1 \mathcal{M}_1^{m=0} + Z_{[q]}^2 \mathcal{M}_0^{m=0}$$

Massless two-loop  $ud \rightarrow l\nu b\bar{b}$  amplitude from [Abreu, Febres Cordero, Ita, Klinkert, Page, Sotnikov(2021)]

$Z_{[q]}$ : universal, perturbative factor, obtained from the ratio of massive to massless  $\gamma^*qq$  form factors

$$Z_{[q]}^l = f(\epsilon, \log m_b^2/Q^2) \quad \text{power corrections in } m_b \text{ and heavy loops contributions are not included}$$

- Subtraction scheme:  $q_T$  slicing [Catani,Grazzini(2007)]  $d\sigma_{\text{NNLO}} = \mathcal{H} \otimes d\sigma_{\text{LO}} + \lim_{r_{\text{cut}} \rightarrow 0} [d\sigma_{\text{R}} - d\sigma_{\text{CT}}]_{r > r_{\text{cut}}}$
- $b$ -quark mass: regulates IR divergencies in the double soft limit  
→ standard anti- $k_T$  jet algorithm can be used at NNLO
- Fiducial setup follows ATLAS  $VH(\rightarrow b\bar{b})$  boosted analysis (arXiv:2007.02873)

W+2b+X at 13.6 TeV,  $m_b = 4.92$  GeV, anti- $k_T$  (and  $k_T$ ) with  $R=0.4$

$$n_b = 2, p_T(b_1) > 45 \text{ GeV}, 0.5 < \Delta R_{bb} < 2$$

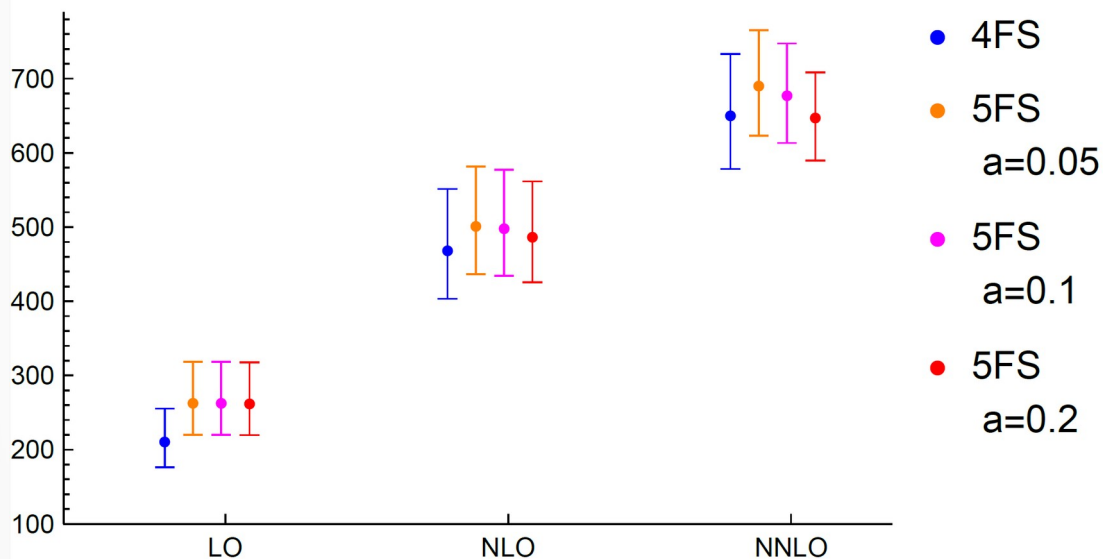
$$p_T(l) > 25 \text{ GeV}, |\eta(l)| > 2.5, p_T(W) > 150 \text{ GeV}, p_T(j) > 20 \text{ GeV if } |\eta(j)| < 2.5 \text{ or } p_T(j) > 30 \text{ GeV if } 2.5 < |\eta(j)| < 4.5$$

# W+2b at NNLO QCD: 4FS vs 5FS comparison

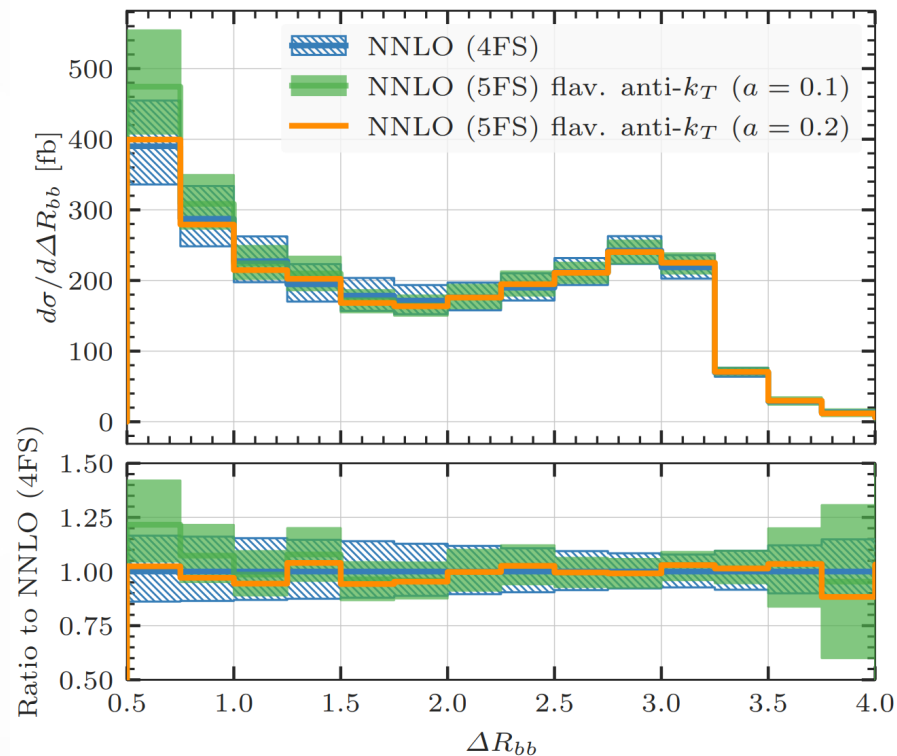
[Buonocore, Devoto, Kallweit, Mazzitelli, Rottoli, Savoini (arXiv:2212.04954)]

## Inclusive $W^+(\rightarrow l^+ \nu)bb$

$\sigma$  (fb)



→ good agreement within scale variations



# $\gamma\text{jj}$ production at NNLO QCD

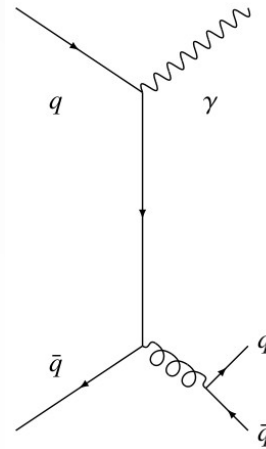
Badger, Czakon, **HBH**, Moodie, Peraro, Poncelet, Zoia

arXiv:2304.06682

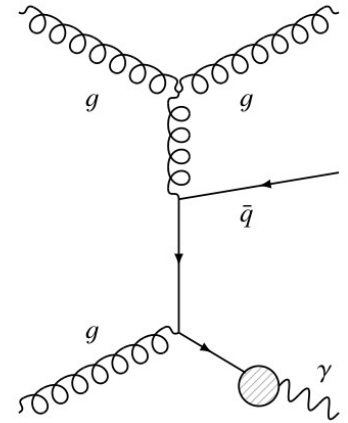
# Photon production mechanisms

## Prompt photon:

- photon **directly** produced from hard scattering  
→ test of pQCD, PDF study, BSM background
- photon from **fragmentation** of QCD partons  
→ collinear emission from final state quark  
→ non-perturbative photon FF should be modelled/extracted from data



direct

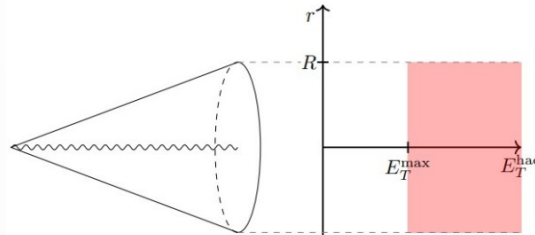


fragmentation

**Secondary photon:** hadronic activities (e.g.  $\pi^0 \rightarrow \gamma\gamma$ )  $\Rightarrow$  dominant contribution!!

$\rightarrow$  suppressed by imposing isolation cut. *Fixed-cone isolation* applied in experiment

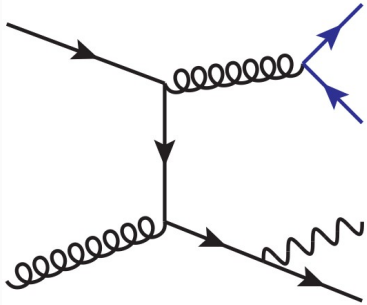
$$E_T^{\text{had}} \leq E_T^{\text{max}}(p_T^\gamma) = \varepsilon p_T^\gamma + E_T^{\text{thres}}$$



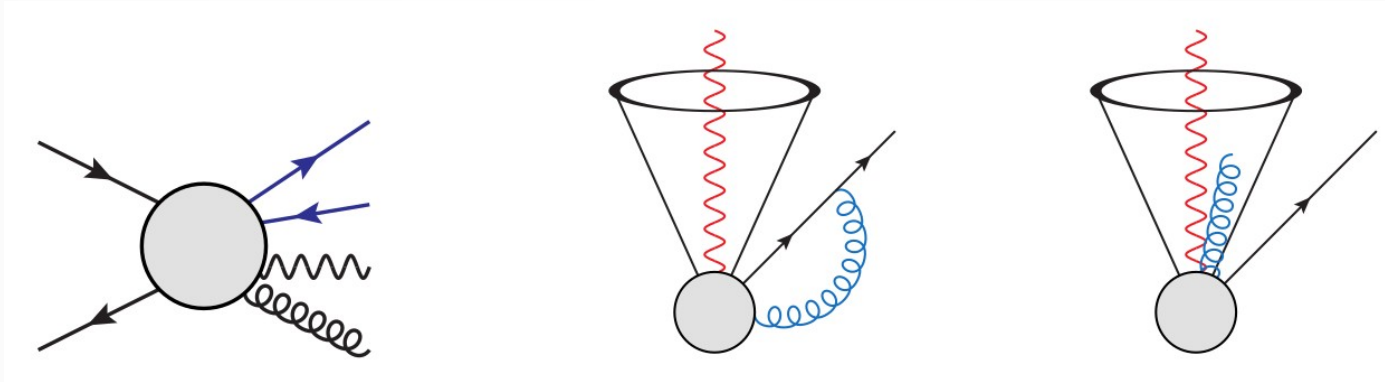
$\Rightarrow$  Non-vanishing fragmentation component

# Quark-photon final state singularities

Real correction to  $pp \rightarrow \gamma jj$  at NLO



Vetoing radiation inside photon isolation cone (sending  $E_T(\text{max}) \rightarrow 0$ ) leads to IR safety issue



Vetoing radiation inside photon isolation cone restricts gluon phase space, thus spoiling IR cancellation

# Quark-photon final state singularities

## Solution:

- Extract collinear singularities, absorb them in the photon fragmentation function

$$D_q^\gamma = -\frac{1}{\epsilon} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu^2}{M_F^2} \right)^\epsilon \frac{\alpha}{2\pi} Q_q^2 P_{\gamma q}(z)$$

The NLO cross section becomes

$$\sigma_{NLO}^\gamma = \sigma_{NLO, \text{direct}}^\gamma(M_F) + \int_0^1 dz \sum_a \sigma_f^a D_{a \rightarrow \gamma}(z, M_F)$$

- Employ smooth cone isolation ala Frixione [\[arXiv:9801442\]](https://arxiv.org/abs/9801442)

$$\sum_{\text{had}} E_T^{\text{had}} \theta(R - R_{\text{had}, \gamma}) < \epsilon_h E_T^\gamma \left( \frac{1 - \cos R}{1 - \cos R_0} \right)^n \quad \text{for all } R \leq R_0$$

- ✓ soft radiation allowed inside the cone but no collinear singularities
- ✓ remove fragmentation contribution completely (convenient for theoretical calculation)
- ✓ difficult to implement experimentally due to finite detector resolution

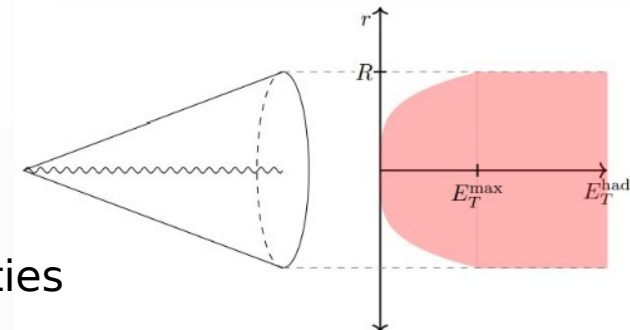
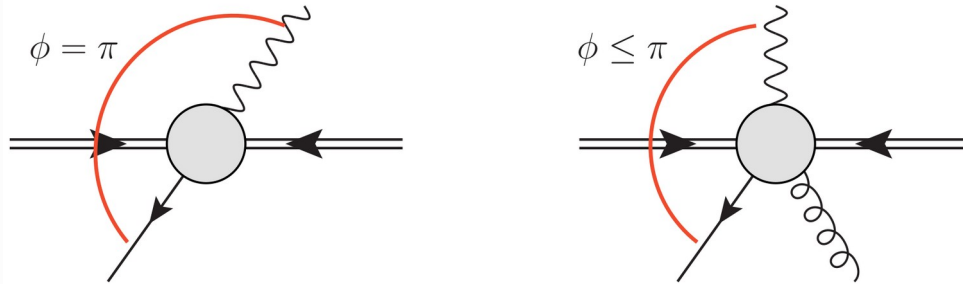


Image: Marius Hofer (SM@LHC2022)

# Photon + dijet production: motivation

- Access to angular correlations between photon and jets  
(photon and jet(s) are not back-to-back at tree level, similar to multijet processes)



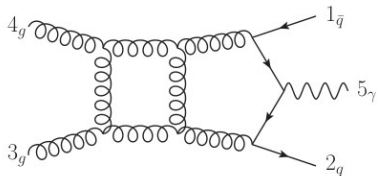
- Study different phase-space regions due photon production mechanism
  - $E_T(\gamma) > p_T(j_1)$  : photon from hard interaction
  - $p_T(j_1) > E_T(\gamma) > p_T(j_2)$  : high  $z$  fragmentation ( $z$ : photon collinear mom fraction)
  - $p_T(j_2) > E_T(\gamma)$  : low  $z$  fragmentation
- Background to Beyond Standard Model (BSM) process
$$pp \rightarrow \gamma + Y(\rightarrow jj)$$



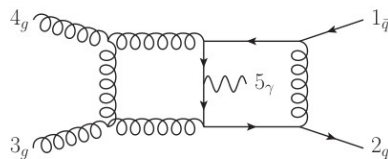
# Two-loop QCD amplitudes for $pp \rightarrow \gamma jj$ production

Two partonic channels:  $0 \rightarrow qqg\gamma$  and  $0 \rightarrow qqQQ\gamma$ , computed in full colour.

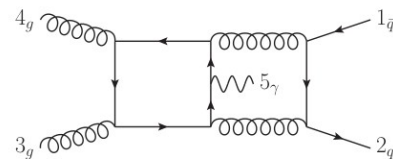
Example:  
 $0 \rightarrow qqg\gamma$



$$A_{34;q}^{(2),N_c^2}, A_{\delta;q}^{(2),N_c}$$



$$A_{34;q}^{(2),1}, A_{\delta;q}^{(2),N_c}, A_{\delta;q}^{(2),1/N_c}$$



$$A_{34;l}^{(2),N_c}, A_{34;l}^{(2),1/N_c}, A_{\delta;l}^{(2),1/N_c^2}$$

$$\mathcal{M}^{(L)}(1_{\bar{q}}, 2_q, 3_g, 4_g, 5_\gamma) = \sqrt{2} e g_s^2 n^L \left\{ (t^{a3} t^{a4})_{i_2} \bar{i}_1 \mathcal{A}_{34}^{(L)} + (t^{a4} t^{a3})_{i_2} \bar{i}_1 \mathcal{A}_{43}^{(L)} + \delta_{i_2} \bar{i}_1 \delta^{a3 a4} \mathcal{A}_\delta^{(L)} \right\}$$

Colour and  $(N_c, n_f)$   
decomposition

$$\begin{aligned} A_{34}^{(2)} = & \mathcal{Q}_q N_c^2 A_{34;q}^{(2),N_c^2} + \mathcal{Q}_q A_{34;q}^{(2),1} + \mathcal{Q}_q \frac{1}{N_c^2} A_{34;q}^{(1),1/N_c^2} + \mathcal{Q}_q N_c n_f A_{34;q}^{(2),N_c n_f} + \mathcal{Q}_q \frac{n_f}{N_c} A_{34;q}^{(2),n_f/N_c} \\ & + \mathcal{Q}_q n_f^2 A_{34;q}^{(2),n_f^2} + \left( \sum_l \mathcal{Q}_l \right) N_c A_{34;l}^{(2),N_c} + \left( \sum_l \mathcal{Q}_l \right) \frac{1}{N_c} A_{34;l}^{(2),1/N_c} + \left( \sum_l \mathcal{Q}_l \right) n_f A_{34;l}^{(2),n_f}. \end{aligned}$$

Amplitudes are written in terms of pentagon functions [Chicherin,Sotnikov(2019)]

analytic form of  
rational coefficients  
are derived using  
finite-field techniques

| amplitude              | helicity  | original | stage 1 | stage 2 | stage 3 | stage 4 |
|------------------------|-----------|----------|---------|---------|---------|---------|
| $A_{34;q}^{(2),1}$     | - + + - + | 94/91    | 74/71   | 74/0    | 22/18   | 22/0    |
| $A_{34;q}^{(2),N_c^2}$ | - + - + + | 58/55    | 54/51   | 53/0    | 20/16   | 20/0    |

# Setup (follows ATLAS measurement)

Measurement of isolated-photon plus two-jet production in pp collisions at  $\sqrt{s} = 13$  TeV with the ATLAS detector [arXiv:1912.09866]

|                               |  |                                  |                                  |
|-------------------------------|--|----------------------------------|----------------------------------|
| <b>Requirements on photon</b> | $E_T^\gamma > 150$ GeV, $ \eta^\gamma  < 2.37$ (excluding $1.37 <  \eta^\gamma  < 1.56$ )<br>$E_T^{\text{iso}} < 0.0042 \cdot E_T^\gamma + 4.8$ GeV (reconstruction level)<br>$E_T^{\text{iso}} < 0.0042 \cdot E_T^\gamma + 10$ GeV (particle level) |                                  |                                  |
| <b>Requirements on jets</b>   | at least two jets using anti- $k_t$ algorithm with $R = 0.4$<br>$p_T^{\text{jet}} > 100$ GeV, $ y^{\text{jet}}  < 2.5$ , $\Delta R^{\gamma\text{-jet}} > 0.8$  |                                  |                                  |
| <b>Phase space</b>            | <b>total</b>   | <b>fragmentation enriched</b>    | <b>direct enriched</b>           |
|                               |  | $E_T^\gamma < p_T^{\text{jet}2}$ | $E_T^\gamma > p_T^{\text{jet}1}$ |
| <b>Number of events</b>       | 755 270  | 111 666                          | 386 846                          |

Employ hybrid isolation

$$E_\perp(r) \leq E_{\perp\text{max}} = 0.0042 E_\perp(\gamma) + 10 \text{ GeV} \quad \text{for } r \leq R_{\text{max}} = 0.4$$

$$E_\perp(r) \leq E_{\perp\text{max}}(r) = 0.1 E_\perp(\gamma) \left( \frac{1 - \cos(r)}{1 - \cos(R_{\text{max}})} \right)^2 \quad \text{for } r \leq R_{\text{max}} = 0.1$$

No fragmentation component, purely pQCD through NNLO

Focus on “inclusive” (total) and “direct enriched”

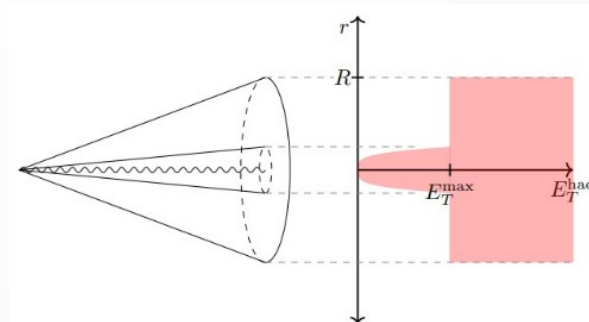


Image: Marius Hofer (SM@LHC2022)

# Data-theory comparison

$$\mu_R = \mu_F = H_T = E_{\perp}(\gamma) + p_T(j_1) + p_T(j_2)$$

$$\mu_R = \mu_F = E_{\perp}(\gamma),$$

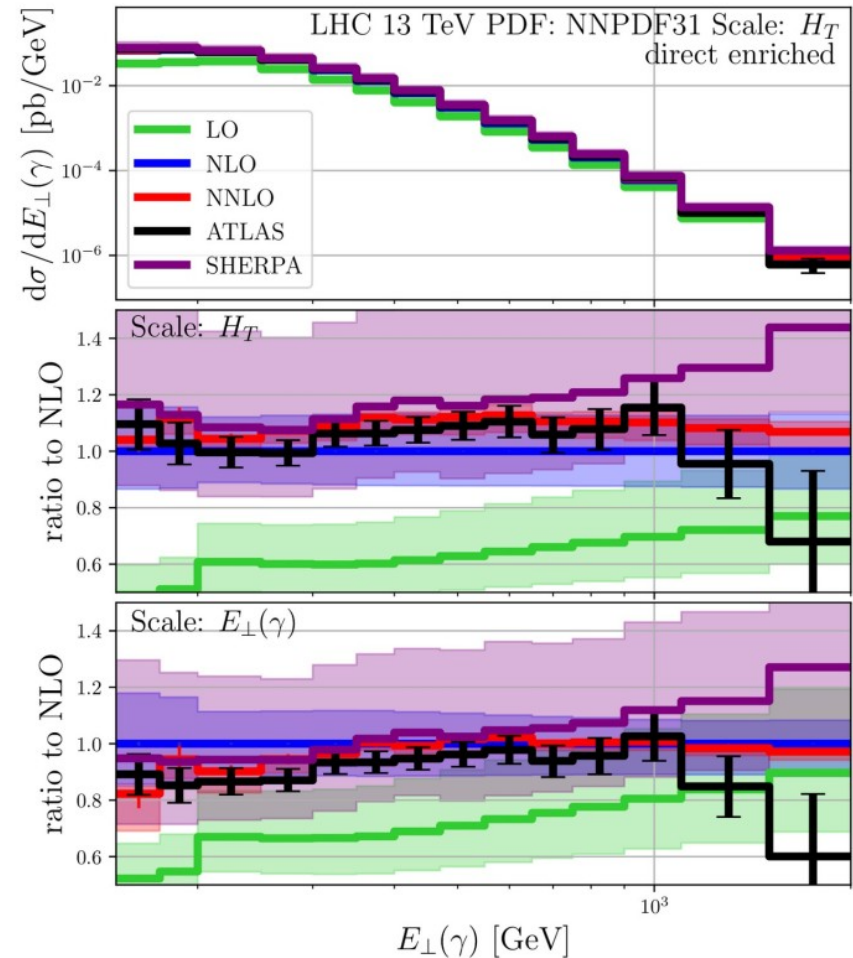
NNLO QCD prediction:

- Describe the data well
- Improved description of shape
- Small corrections
- Small scale dependence

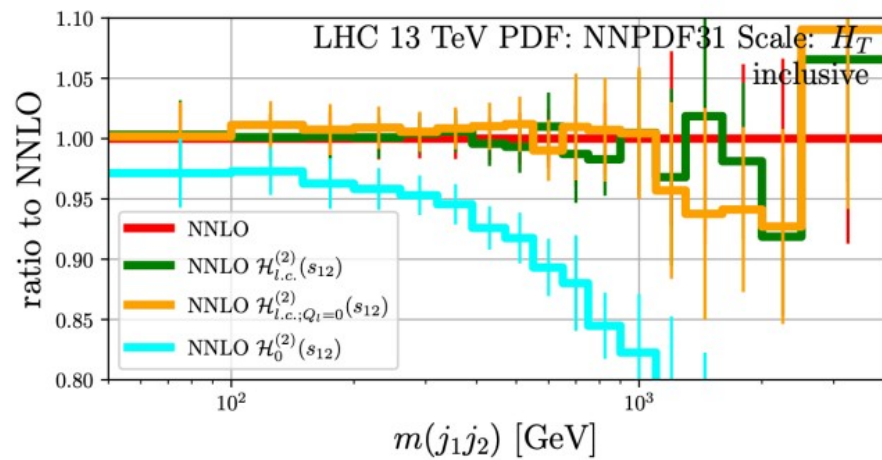
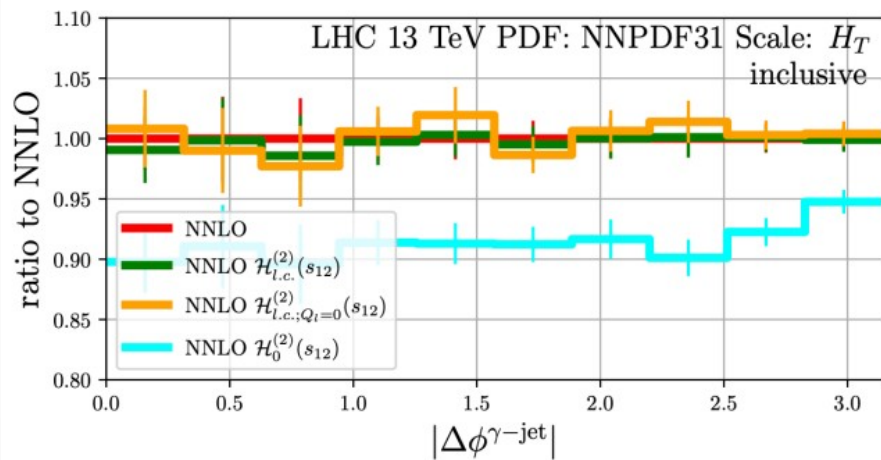
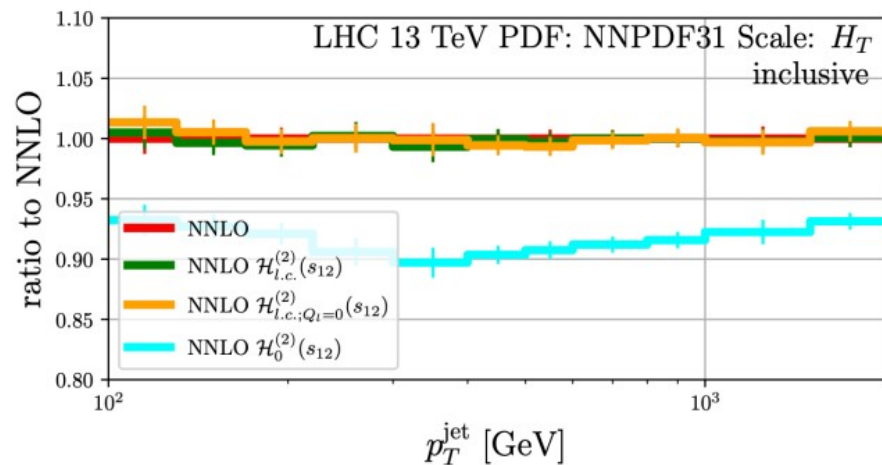
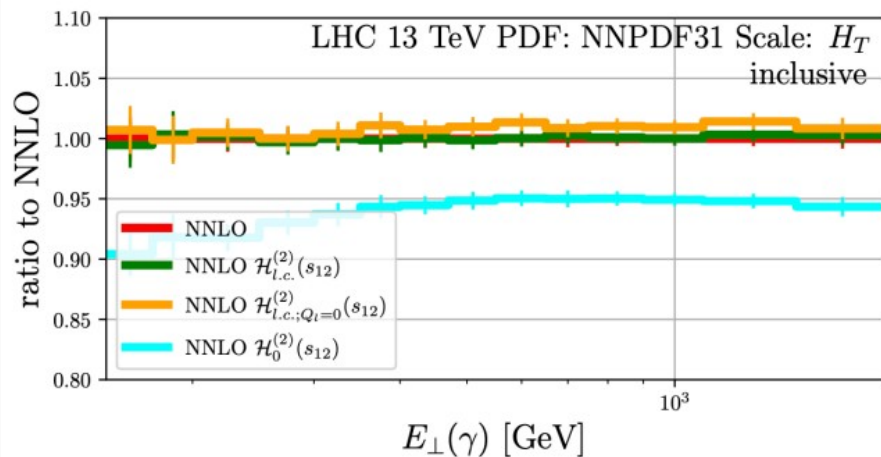
Discrepancy in the high- $E_T$  tail:

→ due to missing EW corrections

Fragmentation contribution estimated at  $< 5\%$   
⇒ from  $pp \rightarrow \gamma j$  (calculation with fragmentation vs hybrid isolation)



# Double virtual corrections: subleading colour contributions



# Summary

- ✓ NNLO QCD calculation for  $Wbb$  and  $\gamma jj$  production
  - improve theoretical uncertainties, better agreement with data
- ✓  $\gamma jj$  production: first  $2 \rightarrow 3$  NNLO QCD calculation with full colour two-loop amplitude
  - very small double virtual subleading colour contribution
- ✓ These calculations are possible thanks to:
  - (i) efficient NNLO subtraction method
  - (ii) availability of the two-loop amplitude
- ✓ Looking forward to explore other  $2 \rightarrow 3$  scattering processes