Holographic mean field theory and Kondo Lattice

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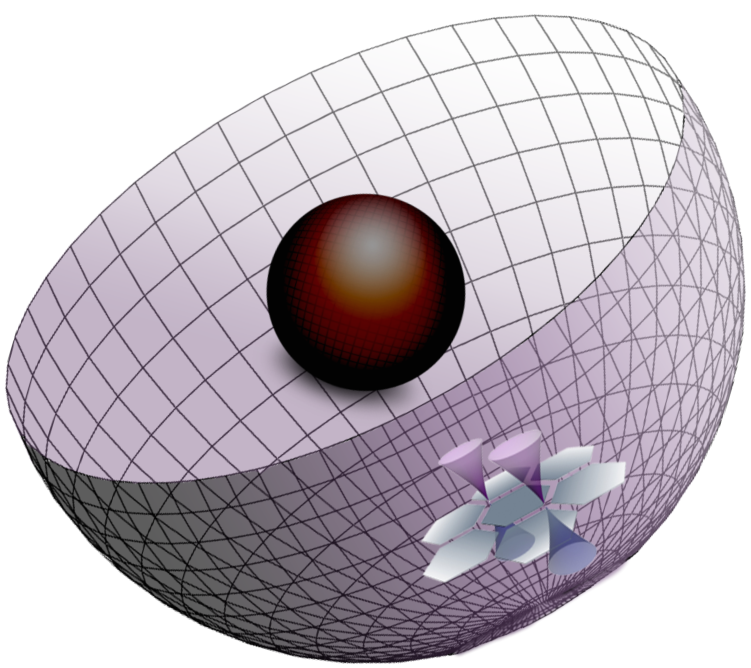
2024.09@Athen



Mean field theory and holographic Kondo lattice, 2407.01978

Mean field theory for strongly coupled systems: Holographic approach
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I. Introduction

II. Holographic mean field theory

III. Kondo Physics

IV. Topology with Holography

Frame of Physical thinking : reductionism

Δημόκριτος | Democritus

$$\text{Matter} = \sum \text{atom}$$

A frame of physical thinking:

$$\text{Complex} = \sum \text{simple}$$

Simplicity is the key to the physics.

Ways to the simplicity

- Physics=Seeking the simplicity. More than 5 parameter? Not much predictability.
- In condensed matter physics, there are 10^{25} dof.
  How CM can be a physics?
  Ans= Periodic structure + 1 electron theory based on the weakness of int.)
- Even in particle physics SM, we need something for the simplicity
  i) Group structure (symmetry)
  ii) Hierarchy + Family structure. (repetition)
  iii) Weakness of coupling. (Independence of different sectors)
- In both PP & CM, the origin of the simplicity is
  the periodicity (repetition) & independence (weakness of int.)

What happen if interaction is not weak? I

- 1. Particle nature is lost.
- 2. system is strongly entangled.

$$H_{tot} = H(x_1) + H(x_2)$$
$$\Rightarrow \psi_{tot} = \psi_i(x_1)\psi_j(x_2) \Rightarrow No\ entanglement$$

$$H_{tot} = H(x_1) + H(x_2) + H_{int}(x_1, x_2)$$
$$\Rightarrow \psi_{tot} = \sum_{ij} c_{ij}\psi_i(x_1)\psi_j(x_2) \Rightarrow entanglement$$ <-> more even c_{ij}

What if interaction is not weak? II

- Weak coupling: in $\psi_{tot} = \sum_{ij} c_{ij}\psi_i(x_1)\psi_j(x_2)$, one term dominance.

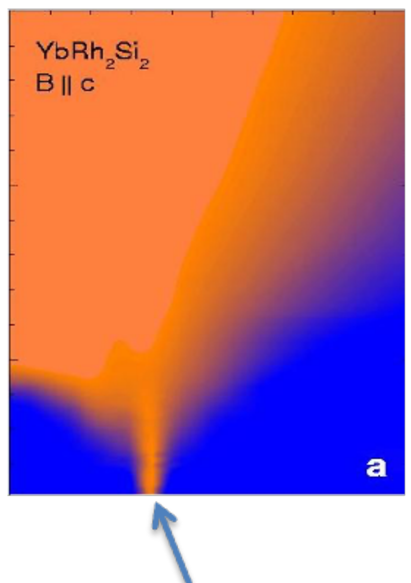
  $\Rightarrow \psi_{tot} = \psi_i(x_1)\psi_j(x_2) \quad separability$

- For strong coupling,
  all the c_{ij} in $\psi_{tot} = \sum_{ij} c_{ij}\psi_i(x_1)\psi_j(x_2)$ are evenly distributed
  => No. of the important terms increases.
  => Entire system becomes one object.

- Inseparability is the characteristic of the strongly int. Sys.

- Simplicity restored!
  What one object? The black hole.

view the whole as one body: QCP = BH



QCP

- Origin of simplification/universality in SIY = Information Loss = Democracy of scales = Emergence of physical law!

- Thermodynamic character: indeed, both have 0,1,2,3 law.

- Classification of QCP vs HSV: (z, θ) $\omega = kz$, $[s] = D - \theta$ & sym.

- Equivalence is supported by exactly solvable models: AdS/SYM

Faq in AdS/CMT

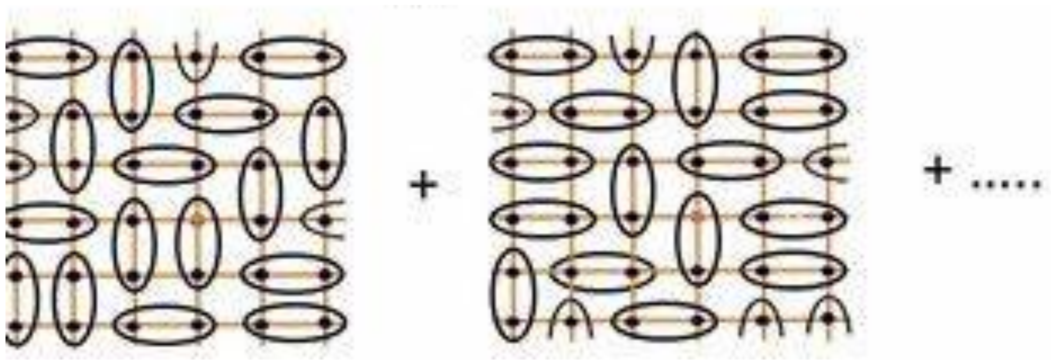
- Postulate: gravity dual exits and Dictionary works.

GKP-Witten Relation

$$Z_{\text{gauge}} = Z_{\text{AdS}}$$

$$\left\langle \exp\left(i \int \phi^{(0)} O \right) \right\rangle = e^{i \underline{S}[\phi|_{u=0}=\phi^{(0)}]}.$$

- Where is N of SU(N)?
  Large number of degen.



- Respect the bulk locality NOT the body locality.
- How to characterize a material?

II. Holographic mean field theory

- Material = lattice_structure +chem_composition

- To characterize a CM, need to introduce a lattice.
  Otherwise, we would not know what material we are dealing.

- 3 ways
  1. Explicit introduction. Brute force => PDE
  2. Explicit introduction. Tight Binding => ODE
  3. Implicit introduction by symmetry breaking

  IR Probe scale = 1 meV = 10^{-6}KeV (scale of lattice).

  => impossible to see the details of the lattice.

  Proposal: Effect of the lattice = effect of the Symmetry breaking!

Symmetry breaking and lattice

Proposal: in low E limit,
Role of lattice = R or Tr symmetry breaking
How to establish this?
Calculate the effect of the order on the Fermion spectrum
Mean field theory=Theory of symmetry breaking.

Conversely lattice can be identified as the spectrum generating symmetry breaking.

That is, material = spectrum (~ band structure)

Universal structure of MFT : Condensation and Order

$$\Delta \sim c_k c_{-k},\ BCS$$

$$\Delta \sim f_k^\dagger c_{-k},\ Kondo\ \ Condensation$$

$$M \sim c_k^\dagger \Gamma c_k,\ Charge\ density\ or\ magnetic\ ordering$$

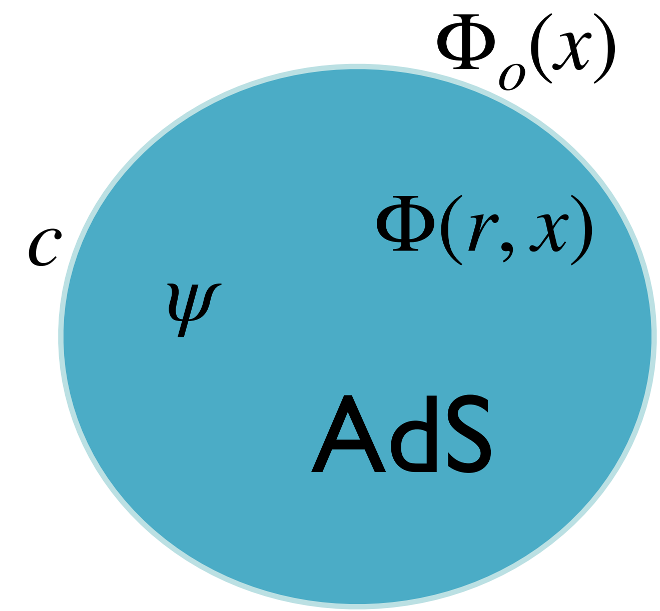
Holographic MFT= Effect of order in fermion spectrum

Order : $\langle \bar{c}\Gamma^A c\rangle \neq 0,$

Holographic dictionary:

Consider ψ *dual to* c, and

add $\Phi_A \cdot \bar{\psi}\Gamma^A\psi$ to $\mathscr{L}_0 = \bar{\psi}(\gamma^\mu i\partial_\mu - m)\psi$.



Find the configuration of Φ first, in the fixed BH gravity.

—> Study $\psi(z,x)$ in the fixed ($g_{\mu\nu}$, Φ)

to get spectrum of χ.

Structure of holographic MFT

T.Yuk

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S. Sukrakarn

$$S_{total} = S_\psi + S_{bdy} + S_{g,\Phi} + S_{int},$$

$$S_\psi = i\int d^dx \sum_{j=1}^{2} \sqrt{-g}\,\bar{\psi}^{(j)}\left(\not{D} - m^{(j)}\right)\psi^{(j)},$$

$$S_{bdy} = \frac{i}{2}\int_{bdy} d^{d-1}x\sqrt{-h}\left(\bar{\psi}^{(1)}\psi^{(1)} \pm \bar{\psi}^{(2)}\psi^{(2)}\right),$$

$$S_{g,\Phi} = \int d^dx\sqrt{-g}\left(R - 2\Lambda + |D_M\Phi_I|^2 - m_\Phi^2|\Phi|^2\right),$$

$$S_{int} = \int d^dx\sqrt{-g}\left(\bar{\psi}^{(1)}\Phi\cdot\Gamma\psi^{(2)} + h.c\right)$$

where Φ_I is order parameter field, $\bar{\psi}^{(1)}\Phi\cdot\Gamma\psi^{(2)}$ is constructed by considering all possible Lorentz symmetry.

$$\Phi\cdot\Gamma = \Gamma^{\underline{\mu_1\mu_2}\cdots\underline{\mu_I}}\Phi_{\underline{\mu_1\mu_2}\cdots\underline{\mu_I}}.$$

Classifying the MFT by the symmetry of the order

8 (half) of them have both simple pole and branch-cut types.

- $\Phi, B_i, B_{jk}, B_{tu}\ (AdS_5)$
- $\Phi, \Phi_5, B_i, B_{5i}, B_{jk}, B_{tu}\ (AdS_4)$

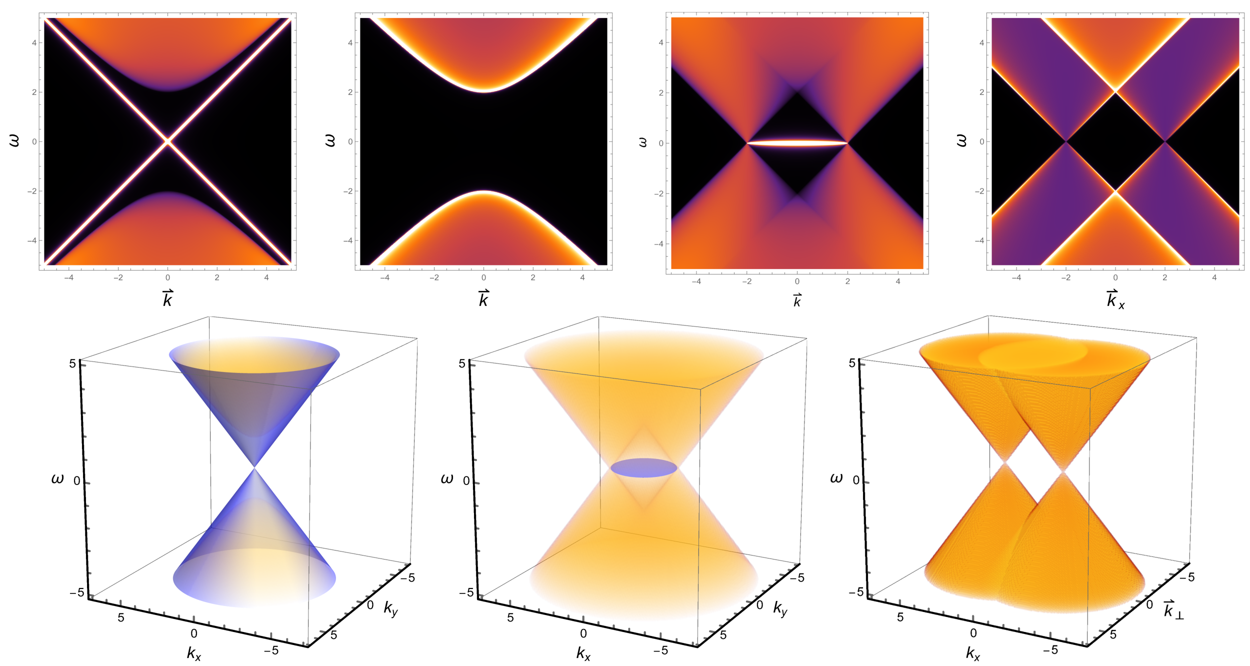
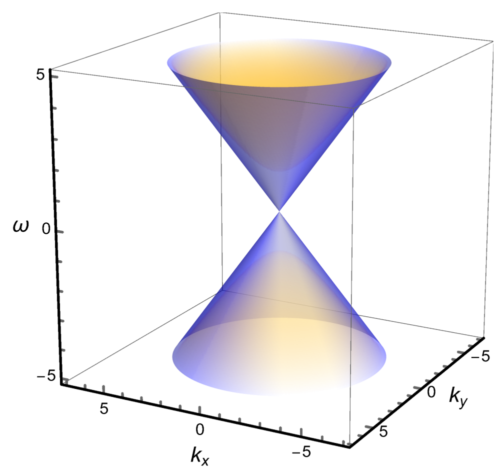
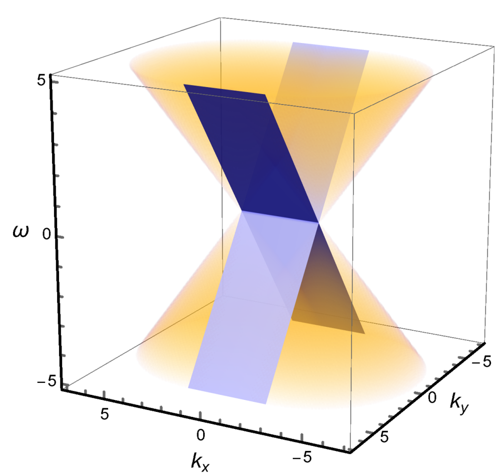
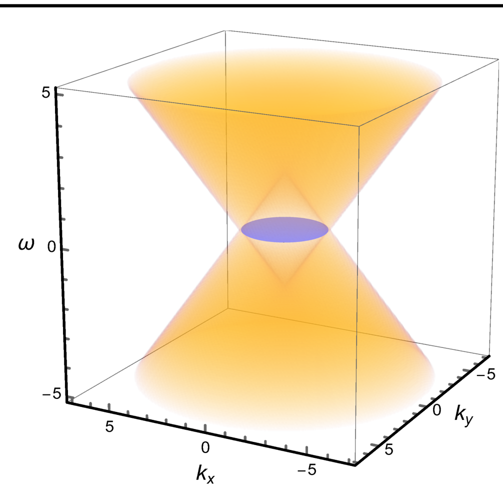


Figure: Simple pole and Branch-Cut types spectra

- 2-dimensional slice of the spectral density
- 3-dimensional spectral density

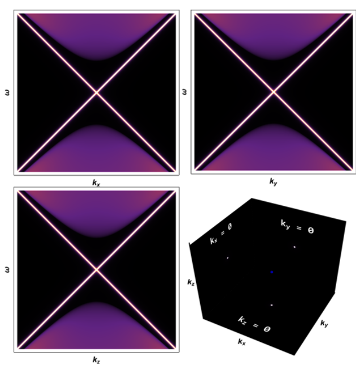
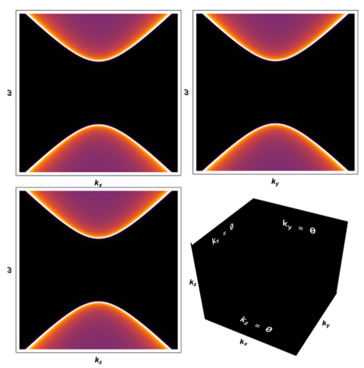
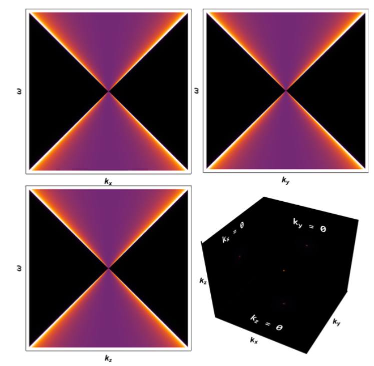
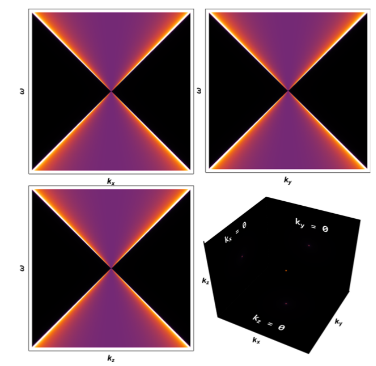
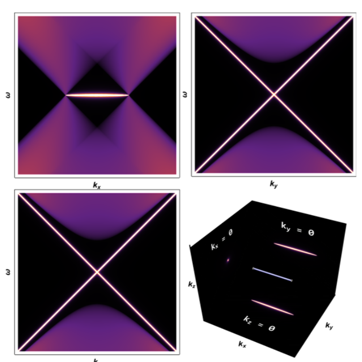
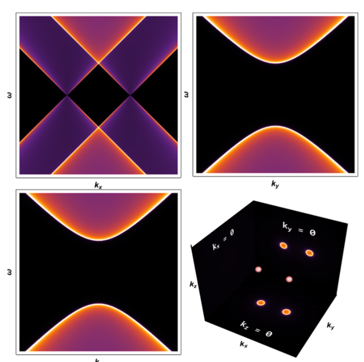
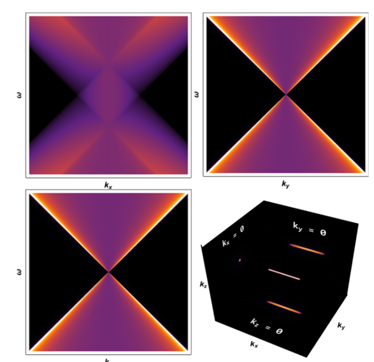
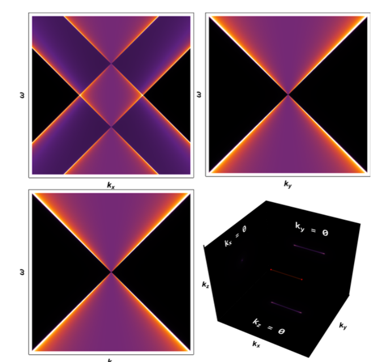
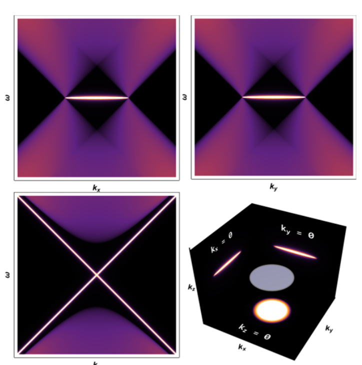
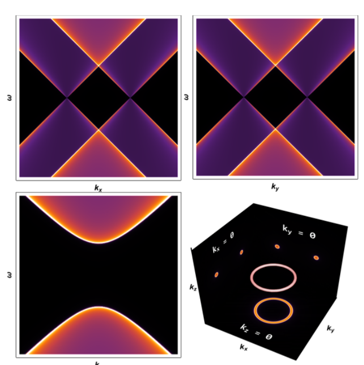
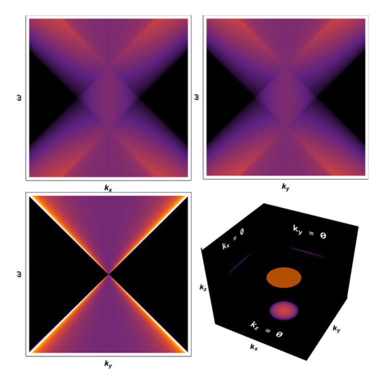
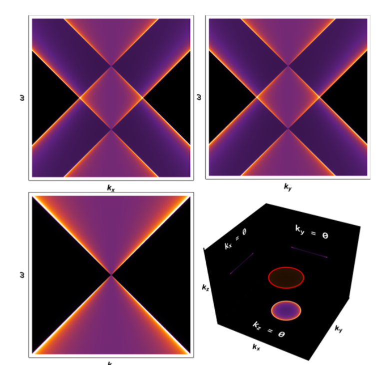
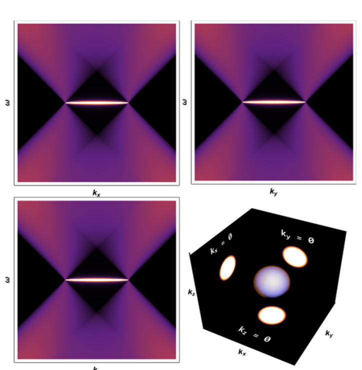
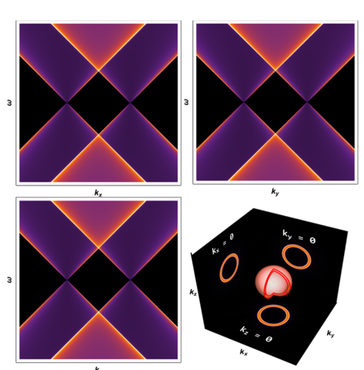
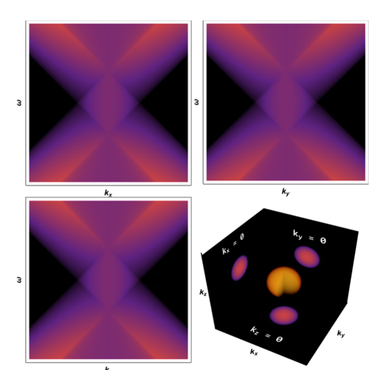
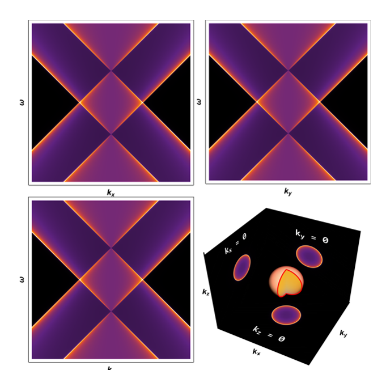
Appearing features: Gaps of s-,p-wave sym.
Flat bands of dim 1,2,3.
Nodal rings of dim 1,2

Analytic Green functions and their spectral functions (pole types 1/4)

$\Phi_{(SS)}, \Phi_{5(SA)}$		$TrG_R(k_\mu) = \dfrac{4\omega\sqrt{\vec{k}^2 - \omega^2 + M^2}}{\vec{k}^2 - \omega^2}$
$B_{x(SA)}, B_{x5(SS)}$		$TrG_R(k_\mu) = \dfrac{2\omega}{b(k_y^2 - \omega^2)}\left[(b + k_x)\epsilon_- + (b - k_x)\epsilon_+\right]$ $; \quad \epsilon_\pm = \sqrt{(b \pm k_x)^2 + k_y^2 - \omega^2}$
$B_{xy(SS)}, B_{tu(SA)}$		$TrG_R(k_\mu) = -\dfrac{2}{b\omega}\left[(b + \lvert\vec{k}\rvert)\epsilon_- + (b - \lvert\vec{k}\rvert)\epsilon +\right]$ $; \quad \epsilon_\pm = \sqrt{(b \pm \lvert\vec{k}\rvert)^2 - \omega^2}$

Interactions	Trace of analytic Green's functions (AdS_5)		Features/Classifications	Singularity types
M_0	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{M_0} = \frac{4\omega}{\sqrt{\boldsymbol{k}^2-\omega^2+M_0^2}}$	(4.3)	Gapful/s-wave gap	Branch-cut
	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{M_0} = 4\omega\frac{\sqrt{\boldsymbol{k}^2-\omega^2+M_0^2}}{\boldsymbol{k}^2-\omega^2-i\epsilon}$	(4.2)	Topological liquid	Pole
B_x	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_x^{(0)}} = \frac{2\omega}{\sqrt{(b-k_x)^2+\boldsymbol{k}_\perp^2-\omega^2}} + \frac{2\omega}{\sqrt{(b+k_x)^2+\boldsymbol{k}_\perp^2-\omega^2}}$	(4.10)	Shifting cones/p-wave gap	Branch-cut
	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_x^{(0)}} = \frac{2\omega}{b}\Big[\frac{(b+k_x)\sqrt{(b-k_x)^2+\boldsymbol{k}_\perp^2-\omega^2}+(b-k_x)\sqrt{(b+k_x)^2+\boldsymbol{k}_\perp^2-\omega^2}}{\boldsymbol{k}_\perp^2-\omega^2-i\epsilon}\Big]$	(4.11)	1D flat band	Pole
B_{xy}	$\mathrm{Tr}\,G^{(SA)}_{B_{xy}^{(-1)}} = \frac{2\omega}{\sqrt{(b-\lvert\boldsymbol{k}_\perp\rvert)^2+k_z^2-\omega^2}} + \frac{2\omega}{\sqrt{(b+\lvert\boldsymbol{k}_\perp\rvert)^2+k_z^2-\omega^2}}$	(4.15)	Nodal ring	Branch-cut
	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_{xy}^{(-1)}} = \frac{2\omega}{b}\Big[\frac{(b+\lvert\boldsymbol{k}_\perp\rvert)\sqrt{(b-\lvert\boldsymbol{k}_\perp\rvert)^2+k_z^2-\omega^2}+(b-\lvert\boldsymbol{k}_\perp\rvert)\sqrt{(b+\lvert\boldsymbol{k}_\perp\rvert)^2+k_z^2-\omega^2}}{k_z^2-\omega^2-i\epsilon}\Big]$	(4.14)	2D flat band	Pole
B_{tu}	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_{tu}^{(-1)}} = \frac{2\omega}{\sqrt{(b-\lvert\boldsymbol{k}\rvert)^2-\omega^2}} + \frac{2\omega}{\sqrt{(b+\lvert\boldsymbol{k}\rvert)^2-\omega^2}}$	(4.8)	Nodal shell	Branch-cut
	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_{tu}^{(-1)}} = -\frac{2}{b}\Big[\frac{(b+\lvert\boldsymbol{k}\rvert)\sqrt{(b-\lvert\boldsymbol{k}\rvert)^2-\omega^2}+(b-\lvert\boldsymbol{k}\rvert)\sqrt{(b+\lvert\boldsymbol{k}\rvert)^2-\omega^2}}{\omega+i\epsilon}\Big]$	(4.9)	3D flat band	Pole
B_u	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_u^{(0)}} \equiv \mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_u^{(0)}} = \frac{4\omega}{\sqrt{\boldsymbol{k}^2-\omega^2}}$	(4.5)	QCP	Branch-cut
B_{ux}	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_{ux}^{(-1)}} = 4\omega\frac{b^2+\boldsymbol{k}^2-\omega^2+f_+f_-}{f_+f_-(f_++f_-)}$; $f_\pm = \sqrt{k_x^2-\Big(b\pm\sqrt{\omega^2-\boldsymbol{k}_\perp^2}\Big)^2}$	(4.12)	Filled nodal segment	Branch-cut
	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_{ux}^{(-1)}} = 4\omega\frac{(f_++f_-)\sqrt{\omega^2-\boldsymbol{k}_\perp^2}-b(f_+-f_-)}{\sqrt{\omega^2-\boldsymbol{k}_\perp^2}(b^2+\boldsymbol{k}^2-\omega^2+f_+f_-)}$; $f_\pm = \sqrt{k_x^2-\Big(b\pm\sqrt{\omega^2-\boldsymbol{k}_\perp^2}\Big)^2}$	(4.13)	Non-singular segment	Branch-cut & nonsingular
B_{tz}	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_{tz}^{(-1)}} = 4\omega\frac{b^2+\boldsymbol{k}^2-\omega^2+h_+h_-}{h_+h_-(h_++h_-)}$; $h_\pm = \sqrt{\boldsymbol{k}_\perp^2-\Big(b\pm\sqrt{\omega^2-k_z^2}\Big)^2}$	(4.16)	Filled nodal ring	Branch-cut
	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_{tz}^{(-1)}} = 4\omega\frac{(h_++h_-)\sqrt{\omega^2-\boldsymbol{k}_\perp^2}-b(h_+-h_-)}{\sqrt{\omega^2-\boldsymbol{k}_\perp^2}(b^2+\boldsymbol{k}^2-\omega^2+h_+h_-)}$; $h_\pm = \sqrt{\boldsymbol{k}_\perp^2-\Big(b\pm\sqrt{\omega^2-k_z^2}\Big)^2}$	(4.17)	Non-singular disk	Branch-cut & nonsingular
B_t	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_t^{(0)}} = 2\Big(\frac{b+\omega}{\sqrt{\boldsymbol{k}^2-(b+\omega)^2}} - \frac{b-\omega}{\sqrt{\boldsymbol{k}^2-(b-\omega)^2}}\Big)$	(4.6)	Filled nodal shell	Branch-cut
	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_t^{(0)}} = \frac{2}{b}\Big[\sqrt{\boldsymbol{k}^2-(b-\omega)^2} - \sqrt{\boldsymbol{k}^2-(b+\omega)^2}\Big]$	(4.7)	Non-singular bowl	Branch-cut & nonsingular

Interactions	**Trace of analytic Green's functions ($\mathbf{AdS_4}$)**	**Features/Classification**
M_0/M_{05}	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{M_0} \equiv \mathrm{Tr}\,\mathbb{G}^{(SS)}_{M_{50}} = \frac{4\omega}{\sqrt{\boldsymbol{k}^2-\omega^2+M_0^2}}$	Gapful/s-wave gap
	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{M_0} \equiv \mathrm{Tr}\,\mathbb{G}^{(SA)}_{M_{50}} = 4\omega\frac{\sqrt{\boldsymbol{k}^2-\omega^2+M_0^2}}{\boldsymbol{k}^2-\omega^2-i\epsilon}$	Topological liquid
B_x/B_{5x}	$\mathrm{Tr}\,G^{(SS)}_{B_x^{(0)}} \equiv \mathrm{Tr}\,G^{(SA)}_{B_{5x}^{(0)}} = \frac{2\omega}{\sqrt{(b-k_x)^2+k_y^2-\omega^2}} + \frac{2\omega}{\sqrt{(b+k_x)^2+k_y^2-\omega^2}}$	Shifting cones/p-wave gap
	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_x^{(0)}} \equiv \mathrm{Tr}\,G^{(SS)}_{B_{5x}^{(0)}} = \frac{2\omega}{b}\Big[\frac{(b+k_x)\sqrt{(b-k_x)^2+k_y^2-\omega^2}+(b-k_x)\sqrt{(b+k_x)^2+k_y^2-\omega^2}}{k_y^2-\omega^2-i\epsilon}\Big]$	1D flat band
B_{xy}/B_{tu}	$\mathrm{Tr}\,G^{(SA)}_{B_{xy}^{(-1)}} \equiv \mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_{tu}^{(-1)}} = \frac{2\omega}{\sqrt{(b-\boldsymbol{k})^2-\omega^2}} + \frac{2\omega}{\sqrt{(b+\boldsymbol{k})^2-\omega^2}}$	Nodal ring
(anti-symmetric)	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_{xy}^{(-1)}} \equiv \mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_{tu}^{(-1)}} = -\frac{2}{b}\Big[\frac{(b+\lvert\boldsymbol{k}\rvert)\sqrt{(b-\boldsymbol{k})^2-\omega^2}+(b-\lvert\boldsymbol{k}\rvert)\sqrt{(b+\boldsymbol{k})^2-\omega^2}}{\omega+i\epsilon}\Big]$	2D flat band
B_u	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_u^{(0)}} \equiv \mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_u^{(0)}} = \frac{4\omega}{\sqrt{\boldsymbol{k}^2-\omega^2}}$	QCP
B_{ux}/B_{5u}	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_{ux}^{(-1)}} \equiv \mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_{5u}^{(-1)}} = 4\omega\frac{b^2+\boldsymbol{k}^2-\omega^2+f_+f_-}{f_+f_-(f_++f_-)}\ ;\ f_\pm = \sqrt{k_x^2-\Big(b\pm\sqrt{\omega^2-k_y^2}\Big)^2}$	Filled nodal line
	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_{ux}^{(-1)}} \equiv \mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_{5u}^{(-1)}} = 4\omega\frac{(f_++f_-)\sqrt{\omega^2-k_y^2}-b(f_+-f_-)}{\sqrt{\omega^2-k_y^2}(b^2+\boldsymbol{k}^2-\omega^2+f_+f_-)}\ ;\ f_\pm = \sqrt{k_x^2-\Big(b\pm\sqrt{\omega^2-k_y^2}\Big)^2}$	Non-singular segment
B_t/B_{5t}	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_t^{(0)}} \equiv \mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_{5t}^{(0)}} = 2\Big(\frac{b+\omega}{\sqrt{\boldsymbol{k}^2-(b+\omega)^2}} - \frac{b-\omega}{\sqrt{\boldsymbol{k}^2-(b-\omega)^2}}\Big)$	Filled nodal ring
	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_t^{(0)}} \equiv \mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_{5t}^{(0)}} = \frac{2}{b}\Big[\sqrt{\boldsymbol{k}^2-(b-\omega)^2}-\sqrt{\boldsymbol{k}^2-(b+\omega)^2}\Big]$	Non-singular disk

Order p. & Dims	Flat bands	Gaps	Order p. & Dims	Nonsingular/Gapless	ω-shiftings/Gapless
Φ $d_{eff}=0$	SS, (figure 2) 	SA, (figure 2) 	B_u $d_{eff}=0$	SS,SA 	SS,SA 
B_x $d_{eff}=1$	SA, (figure 5) 	SS, (figure 5) 	B_{ux} $d_{eff}=1$	SA, (figure 6) 	SS, (figure 6) 
B_{xy} $d_{eff}=2$	SS, (figure 7) 	SA, (figure 7) 	B_{tz} $d_{eff}=2$	SS, (figure 8) 	SA, (figure 8) 
B_{tu} $d_{eff}=3$	SA, (figure 4) 	SS, (figure 4) 	B_t $d_{eff}=3$	SA, (figure 3) 	SS, (figure 3) 

Summary of II

Features in spectrum from Sym. Breaking

Gaps of s-,p-wave sym.

Flat bands of dim 1,2,3.

Nodal rings of dim 1,2

Lattice <=> symmetry breaking

All many body theory assume: $G \sim \dfrac{Z}{\omega - \epsilon - \Sigma}.$

Some of Green fct has poles, indeed.

but some of them are not. Branch cut singularity!

=> New class of Non-Fermi liquid

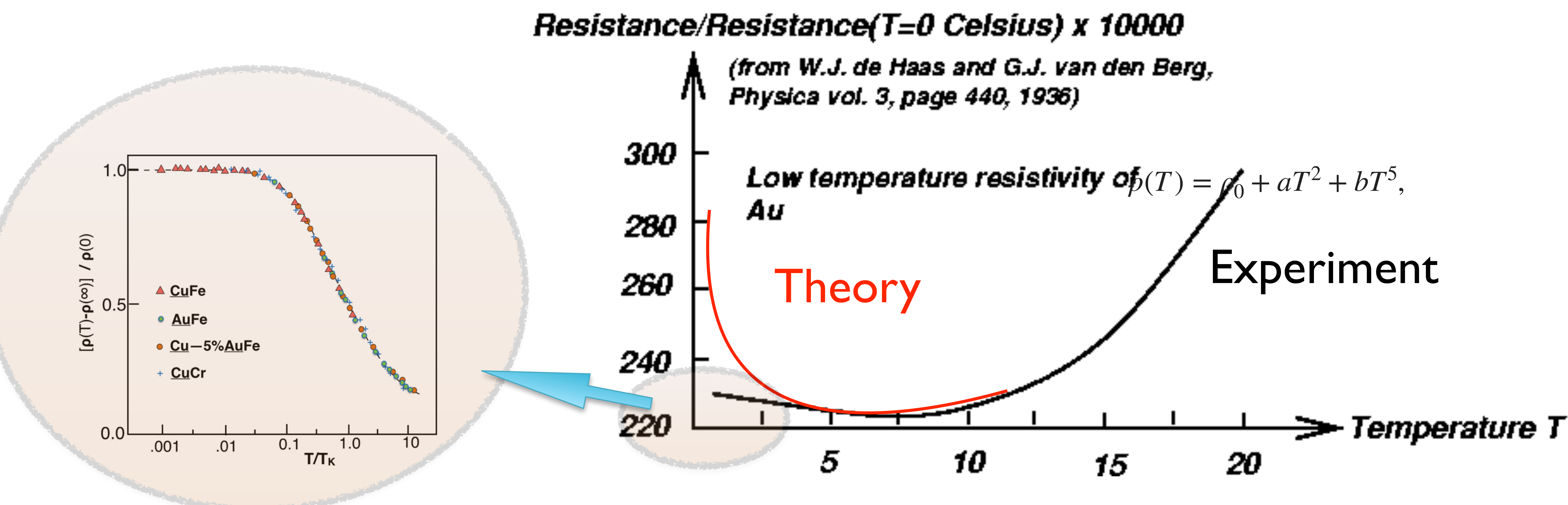
III. Kondo Physics

1. Single Kondo
2. Multi-Kondo : Random impurities
3. Muti-Kondo : Kondo lattice

What is Kondo physics

Scattering of conduction electrons in a metal
by the magnetic impurities.

1. Exp. Fact : Resistivity increase as T decrease after certain temp.



2. Theory: **Kondo**: $\rho(T) = \rho_0 + aT^2 + bT^5 + c_m \ln \frac{\mu}{T},$ Divergence as $T \to 0$.

Saturation of ρ in $T \to 0$

RG: imp-itinerant e coupling goes strong in IR: complete screening

The Hamiltonian of the Anderson model can be described by

$$H_0 = \sum_{k,\sigma} \varepsilon_k c_{k\sigma}{}^{+} c_{k\sigma} + \varepsilon_d \sum_{\sigma} n_{d\sigma} + U n_{d\uparrow} n_{d\downarrow}$$

$$H' = \frac{1}{\sqrt{N}} \sum_{k,\sigma} (V_{kd} c_{k\sigma}{}^{+} d_\sigma + V_{dk} d_\sigma{}^{+} c_{k\sigma}) \quad \text{(perturbation)}$$

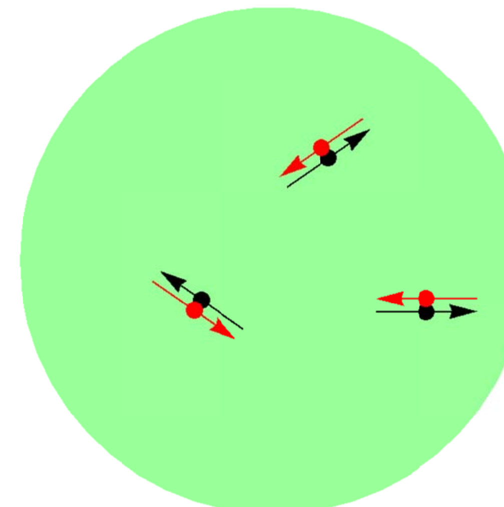
where

$$n_{d\sigma} = d_\sigma{}^{+} d_\sigma \qquad (\sigma = \uparrow, \downarrow).$$

$$\{d_\sigma, d_\sigma{}^{+}\} = 1, \qquad \{c_{k\sigma}, c_{k'\sigma}{}^{+}\} = \delta_{k,k'}$$

Schrieffer-Wolff transformation —> Kondo Hamiltonian

$$H = \sum_{k\sigma} \epsilon_k c^{\dagger}{}_{k\sigma} c_{k\sigma} + \overbrace{g\, \psi^{\dagger}(0) \vec{\sigma} \psi(0) \cdot \vec{S}_f}^{\Delta H}$$



$$\frac{dg}{d \ln \mathcal{D}} = \beta(g) = -2g^2$$

$$g(D') = \frac{g_0}{1 + 2g_0 \ln(D_0/D')}$$

$$T_K = D_0 e^{-1/2g_0} = D_0 e^{-1/\rho_0 J}$$

Anderson's poor man's scaling(1970) —> K. Wilson : numerical RG (1975)

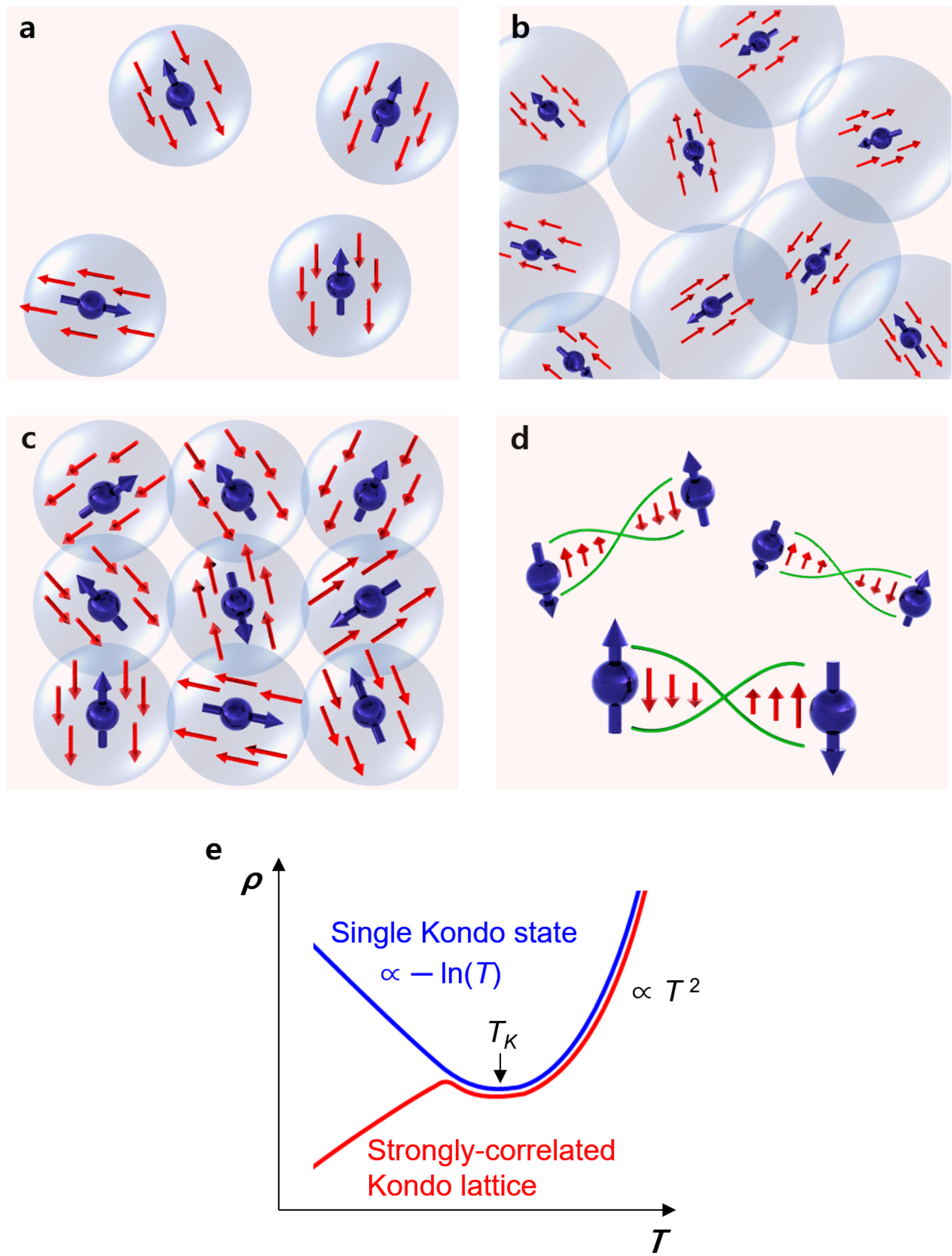
Classification of Multi Kondo : Random vs regular impurities

single Kondo

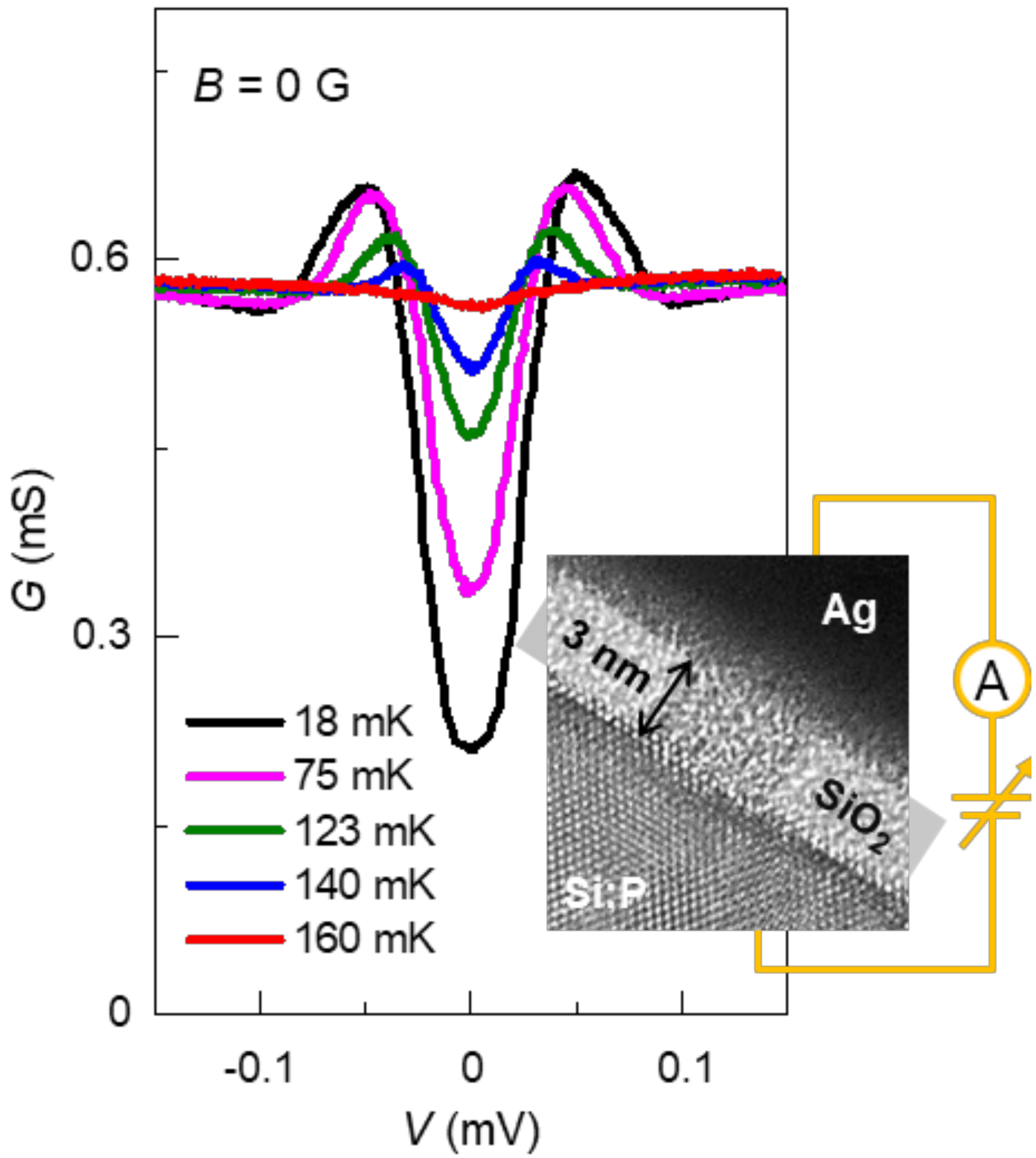
Random imp.
Kondo-Conden:
gap

Kondo Lattice
Both
heavy fermion/
Kondo insulator

RKKY
weak coupling



III.2. Discovery of a tiny gap in a dirty semiconductor



Difficulty of our system as Kondo lattice

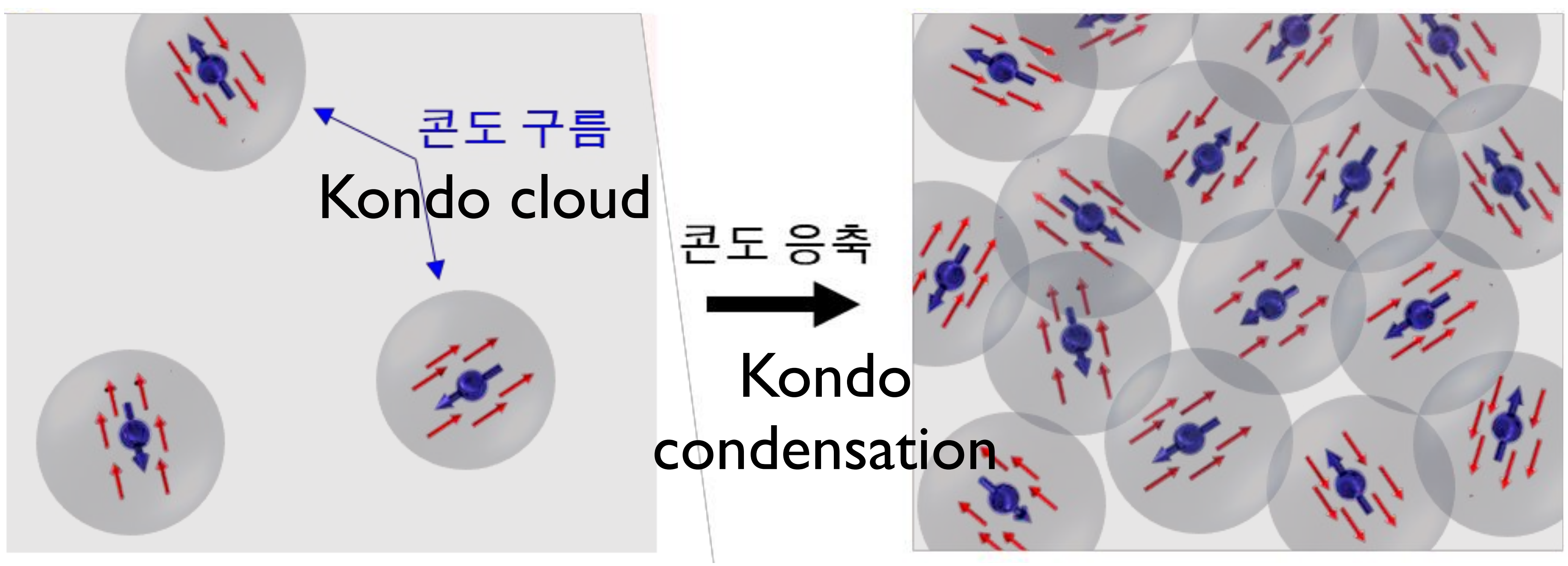
- If no periodicity—> No momentum !
  No band.
  The whole picture of Kondo-lattice break down.
  No calculational scheme.
  In fact, random singlet picture
  —> No gap!

- However, ……
  A gap is found in random impurity
  similar to
  Indirect gap of Kondo lattice

Our proposal: dense Random multi-Kondo
Overlapping Kondo cloud => Kondo condensation :

$$H=\sum_{i\sigma}\varepsilon_i^f f_{i\sigma}^\dagger f_{i\sigma}+Un_{i\uparrow}n_{i\downarrow}+\sum_{\vec{k}\sigma}\varepsilon_{\vec{k}\sigma}c_{\vec{k}\sigma}^\dagger c_{\vec{k}\sigma}+\sum_{i,\vec{k},\sigma}V_k\left(e^{i\vec{k}\vec{R}_i}f_{i\sigma}^\dagger c_{\vec{k}\sigma}+e^{-i\vec{k}\vec{R}_i}c_{\vec{k}\sigma}^\dagger f_{i\sigma}\right)$$

- Cooper pair=cc : $< cc > \neq 0 \rightarrow$ superconductivity
- Kondo pair = $f^\dagger c$: $< f^\dagger c > \neq 0 \rightarrow$ Kondo condensation



- Yamamoto et. al.
"Observation of the Kondo screening cloud"
Nature 2020

Kondo condensation model and its result

$$S_D = \int d^{d+1}x\sqrt{-g}\bar{\psi}(\Gamma^M D_M - m - \Phi)\psi + \int d^{d+1}x\sqrt{-g}\left(|\partial_\mu \Phi|^2 - m^2\Phi^2\right)$$

$$D_M = \partial_M + \frac{1}{4}\omega_{abM}\Gamma^{ab} - iqA_M,$$

$$\Phi \sim f^\dagger c$$

$$\Phi = \frac{\Phi^{(0)}}{r} + \frac{\Phi^{(1)}}{r^2} + \cdots$$

$$\Phi^{(0)} = 0, \quad \Phi^{(1)} = M_0\sqrt{1 - T/T^*}$$

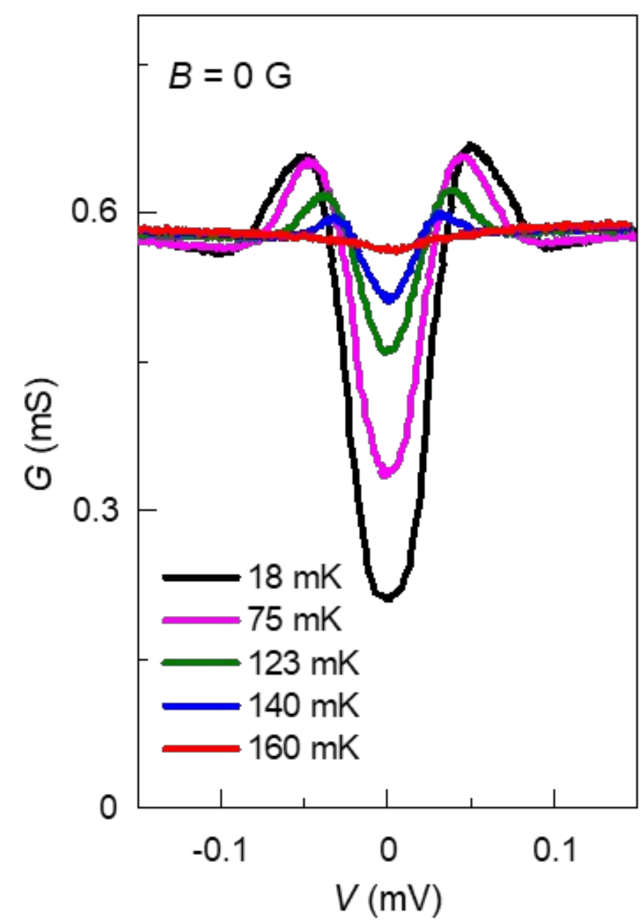
$$ds^2 = -r^2 f(r)dt^2 + \frac{1}{r^2 f(r)}dr^2 + r^2 d\vec{x}^2$$

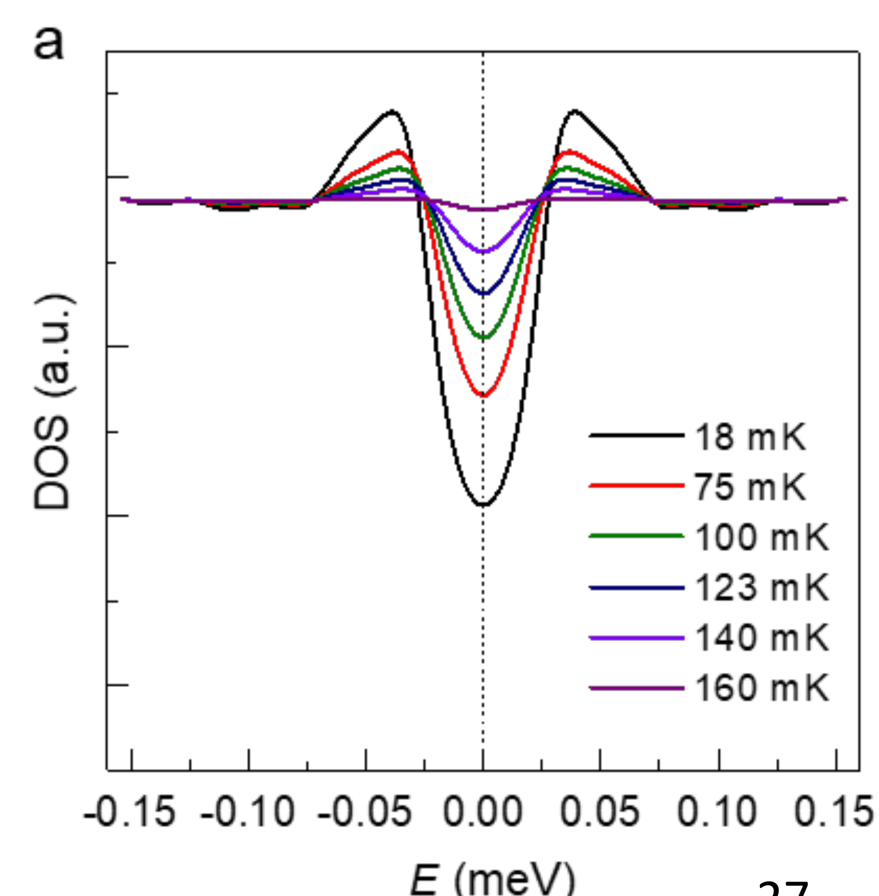
$$f(r) = 1 - \frac{r_0{}^3}{r^3} - \frac{r_0\mu^2}{r^3} + \frac{r_0{}^2\mu^2}{r^4} \ .$$

$$T \to \frac{k_B T}{\hbar v_F}L = \frac{\hat{T}}{Kelvin}\frac{L}{2.3\times10^6 nm}$$

$$B \to \frac{e}{\hbar}BL^2 = \frac{\hat{B}}{Tesla}\frac{L^2}{(25.7nm)^2}$$

$$M_0 \to \frac{M_0}{(\hbar v_F)^2}L^2 = \frac{9v_0^2}{4v_F^2}\frac{L^2}{(nm)^2}\frac{M}{(eV)^2}$$





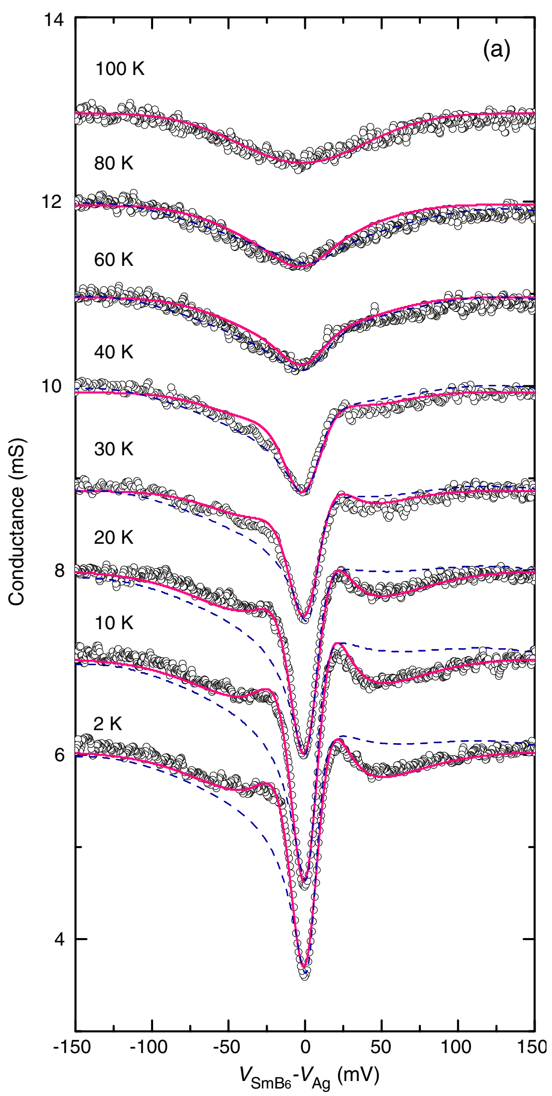
nature physics

Article https://doi.org/10.1038/s41567-022-01930-3

Observation of Kondo condensation in a degenerately doped silicon metal

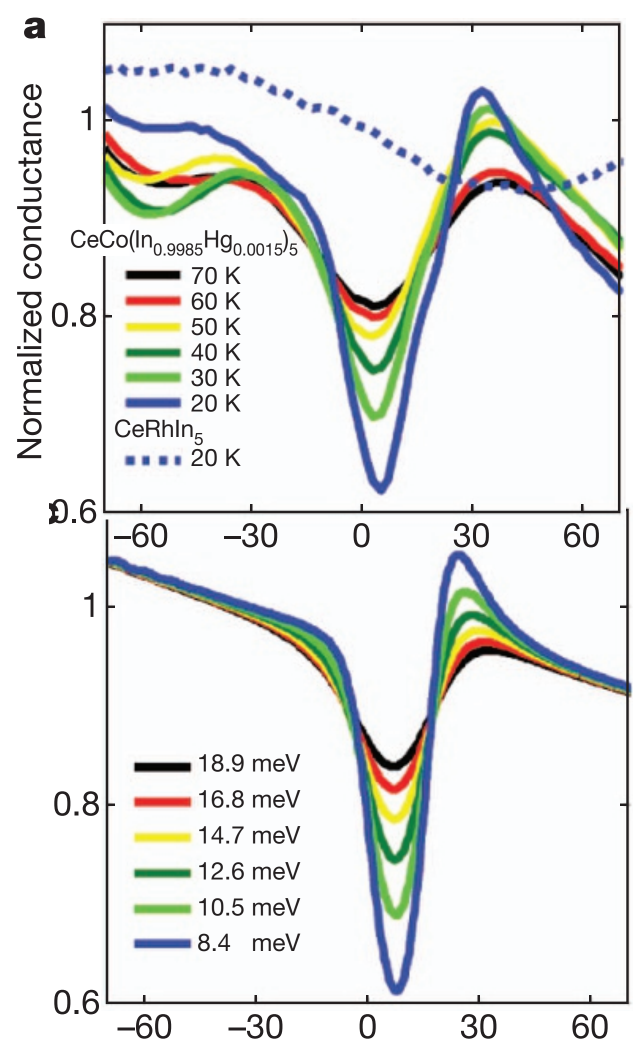
Hyunsik Im[1,2], Dong Uk Lee[3], Yongcheol Jo[1], Jongmin Kim[1], Yonuk Chong[4], Woon Song[5], Hyungsang Kim[1], Eun Kyu Kim[3], Taewon Yuk[3], Sang-Jin Sin[3], Soonjae Moon[3], Jonathan R. Prance[6], Yuri A. Pashkin[6] & Jaw-Shen Tsai[2,7]

Remark: random vs lattice
K-condensation vs K-insulator



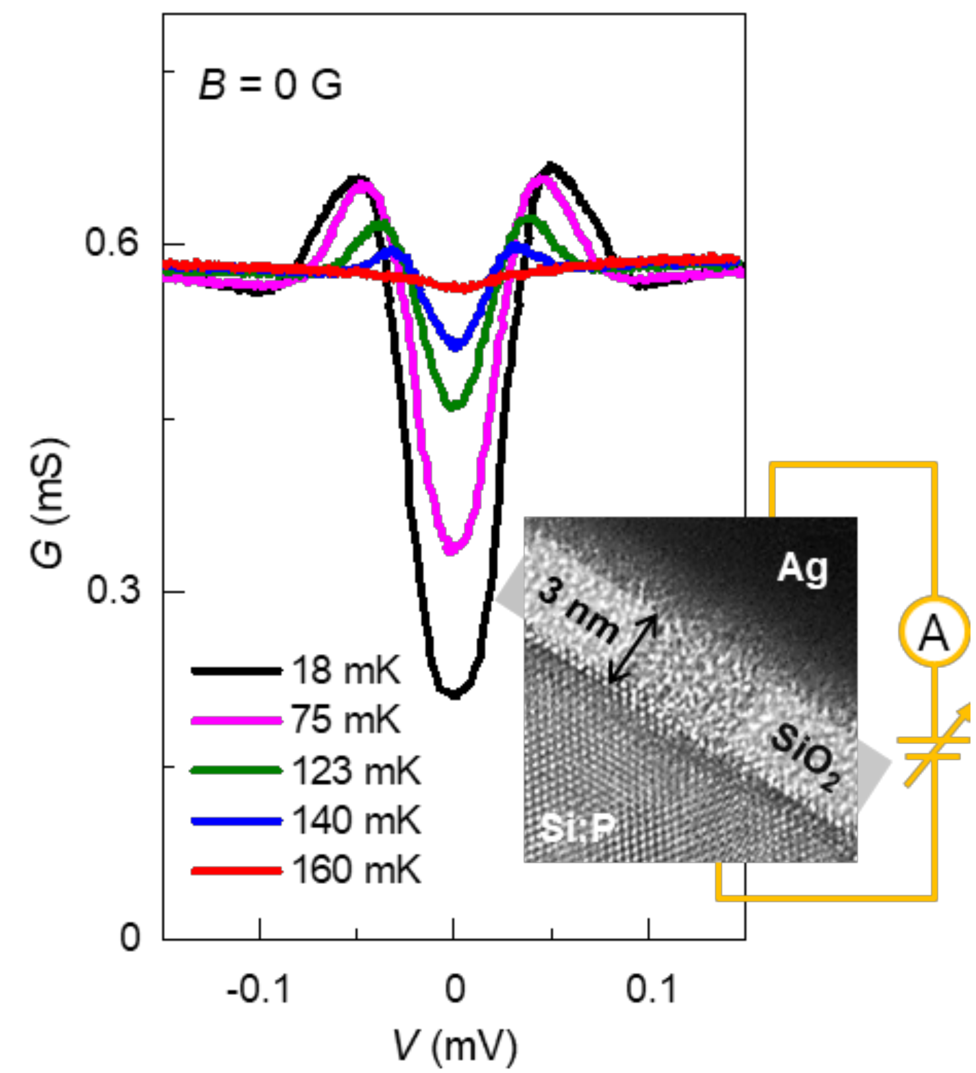
SmB6

PHYSICAL REVIEW X **3**, 011011 (2013)



$CeCoIn_{1-x}Hg_x; x = 0.0015$

nature11204

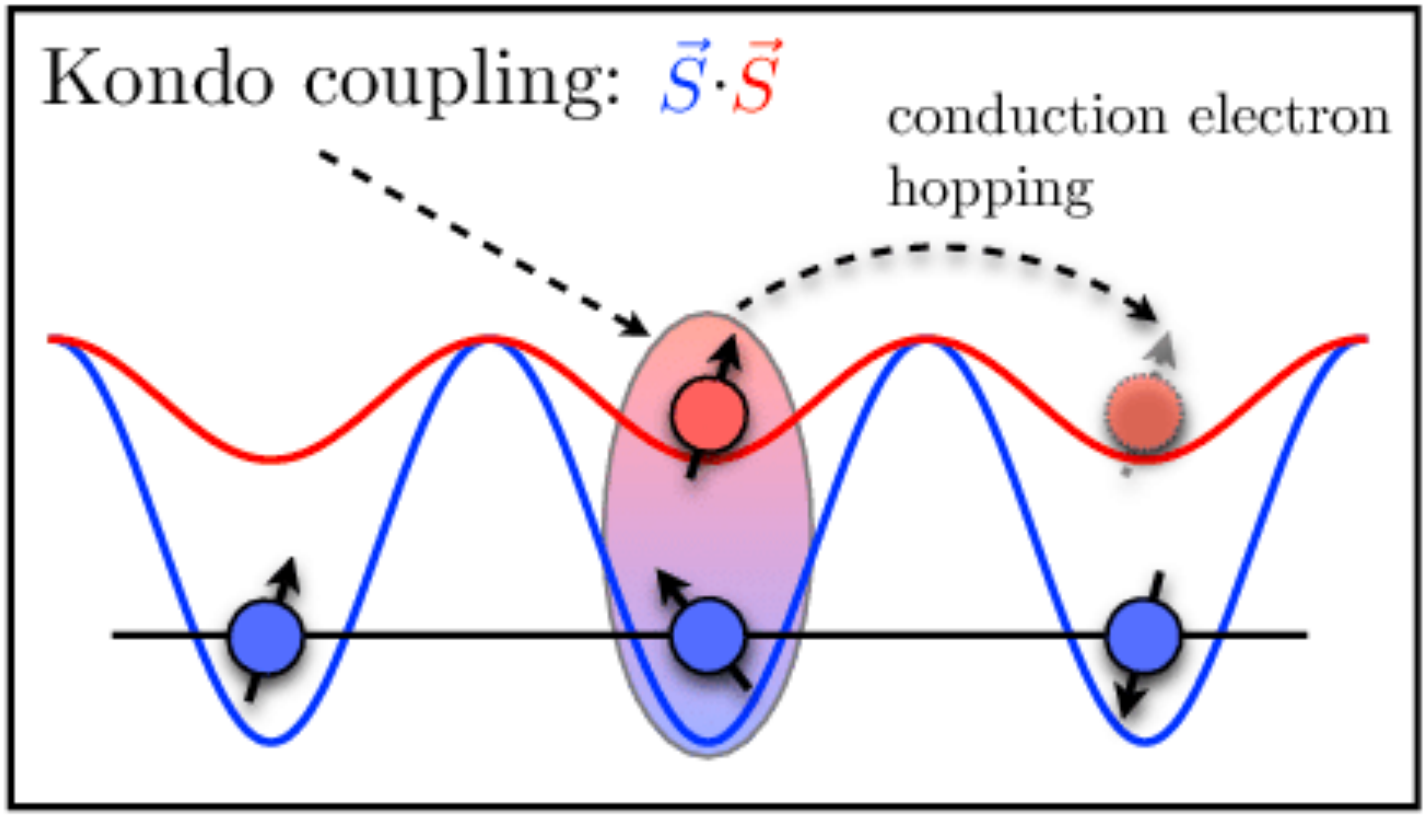


Si:P

K-lattice : asymmetric gap

K-cond: symmetric gap

III.3 Physics of Kondo lattice



Essence of the Kondo Lattice physics:

Electron trapped and propagate rarely from site to site.

On a larger length scale, a very slow coherent motion

= a quasi-particle with a large effective mass.

MFT for the Kondo lattice

$$\mathcal{L} = \psi^\dagger \left(i\frac{\partial}{\partial t} + \frac{\nabla^2}{2m} + \mu \right) \psi + \chi^\dagger \left(i\frac{\partial}{\partial t} - \lambda \right) \chi$$
$$+ \frac{g_l}{2}(\psi^\dagger\psi)^2 - g_s(\psi^\dagger\psi)(\chi^\dagger\chi) - g_v(\psi^\dagger\vec{\sigma}\psi)\cdot(\chi^\dagger\vec{\sigma}\chi).$$

Using the Fierz identity,

$$\mathcal{L} = \psi^\dagger \left(i\frac{\partial}{\partial t} + \frac{\nabla^2}{2m} + \mu \right) \psi + \chi^\dagger \left(i\frac{\partial}{\partial t} - \lambda \right) \chi$$
$$+ \frac{g_l}{2}(\psi^\dagger\psi)^2 + g_s'(\psi^\dagger\chi)(\chi^\dagger\psi) + g_v'(\psi^\dagger\vec{\sigma}\chi)\cdot(\chi^\dagger\vec{\sigma}\psi), \qquad g_s' := \frac{g_s + 3g_v}{2}, \quad g_v' := \frac{g_s - g_v}{2}.$$

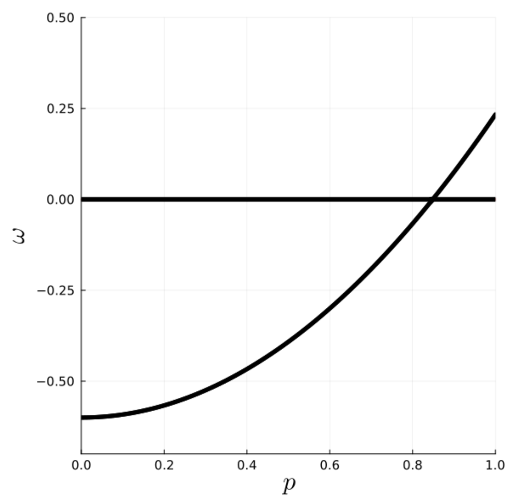
$$\mathcal{L}_{\mathrm{MF}} = \Psi^\dagger D\Psi - U,$$

$$\Psi^\dagger := \begin{pmatrix} \psi^\dagger & \chi^\dagger \end{pmatrix}, \quad \Psi := \begin{pmatrix} \psi \\ \chi \end{pmatrix}, \qquad \langle\psi^\dagger\psi\rangle \equiv -\frac{M}{g_l}, \quad \langle\psi^\dagger\chi\rangle \equiv \frac{\Delta_s}{g_s'}, \quad \langle\psi^\dagger\vec{\sigma}\chi\rangle \equiv \frac{\vec{\Delta}_v}{g_v'},$$

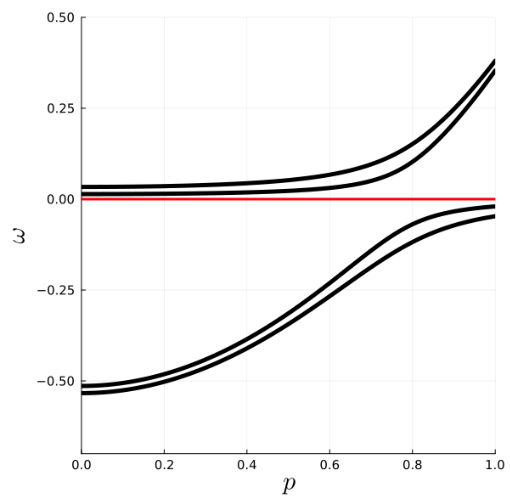
$$D := \begin{pmatrix} i\frac{\partial}{\partial t} + \frac{\nabla^2}{2m} + \mu - M & \Delta_s^* + \vec{\sigma}\cdot\vec{\Delta}_v^* \\ \Delta_s + \vec{\sigma}\cdot\vec{\Delta}_v & i\frac{\partial}{\partial t} - \lambda \end{pmatrix},$$

$$U := \frac{M^2}{2g_l} + \frac{|\Delta_s|^2}{g_s'} + \frac{|\vec{\Delta}_v|^2}{g_v'}.$$

MFT for the Kondo lattice (continued)



(a) $\omega(p)$ without condensation.



(b) $\omega(p)$ with condensation.

$T_K \sim V^2/D$: $1-Kondo\ Temp$.

FS in gap-> K insulator, otherwise Heavy Fermion w/ larger FS

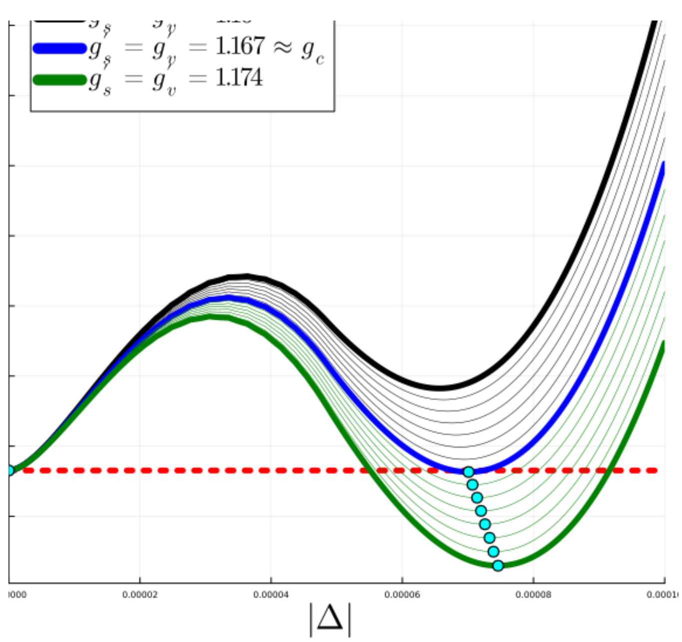
MFT for the Kondo lattice 2

$$\Omega = U + \frac{1}{V}\sum_{|\vec{p}|<\Lambda}\sum_{i=1}^{4}\left\{-\frac{1}{2}|\omega_i(\vec{p})| - \frac{1}{\beta}\ln\left[1+e^{-\beta|\omega_i(\vec{p})|}\right]\right\}$$

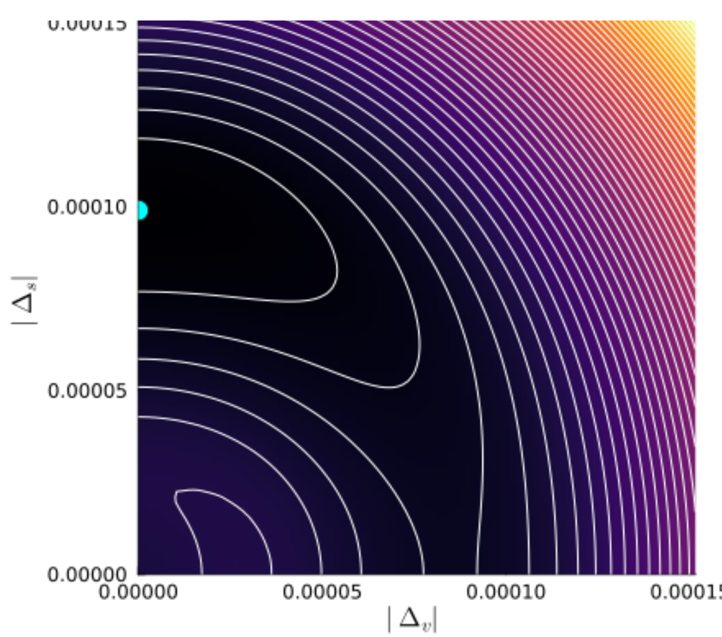
$$= U - \frac{1}{4\pi^2}\int_0^\Lambda \mathrm{d}p p^2 \sum_{i=1}^{4}|\omega_i(p)| - \frac{1}{2\pi^2\beta}\int_0^\Lambda \mathrm{d}p p^2 \sum_{i=1}^{4}\ln\left[1+e^{-\beta|\omega_i(p)|}\right],$$

$$\omega_{i=1,\cdots,4} = \mathcal{E}_+ \pm \sqrt{\mathcal{E}_-^2 + |\Delta_s|^2 + |\vec{\Delta}_v|^2 \pm \sqrt{(|\Delta_s|^2+|\vec{\Delta}_v|^2)^2 - |\Delta_s^2 - \vec{\Delta}_v\cdot\vec{\Delta}_v|^2}},$$

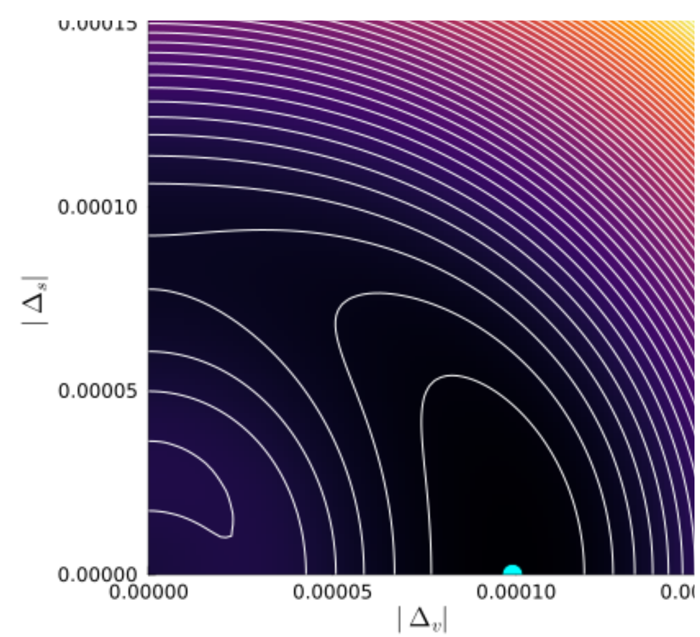
$$\mathcal{E}_\pm := \frac{1}{2}\left[\left(\frac{p^2}{2m} - \mu + M\right) \pm \lambda\right].$$



(a) Ω versus $|\Delta|$.



(b) Ω with strong $g_s' > g_v' > g_c$.



(c) Ω with strong $g_v' > g_s' >$

Holographic Kondo Lattice

YoungKwon Han

$$S_{\mathrm{tot}} = S_{\mathrm{bg}} + S_{\mathrm{spin}},$$

$$S_{\mathrm{bg}} = S_{\mathrm{bg,bdy}} + \int \mathrm{d}^4x\sqrt{-g}\left(R + \frac{6}{L^2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

$$+ \int \mathrm{d}^4x\sqrt{-g}[-(\partial_\mu\Phi_{\mathrm{s}})(\partial^\mu\Phi_{\mathrm{s}}) - m_{\mathrm{s}}^2\Phi_{\mathrm{s}}^2 - (\partial_\mu\Phi_{\mathrm{ps}})(\partial^\mu\Phi_{\mathrm{ps}}) - m_{\mathrm{ps}}^2\Phi_{\mathrm{ps}}^2],$$

$$S_{\mathrm{spin}} = S_{\mathrm{spin,bdy}} + \sum_{j=1}^{2}\int \mathrm{d}^4x\sqrt{-g}i\bar{\psi}^{(j)}\left[\frac{1}{2}\left(\overrightarrow{\not{D}}^{(j)} - \overleftarrow{\not{D}}^{(j)}\right) - m_j\right]\psi^{(j)}$$

$$+ \int \mathrm{d}^4x\sqrt{-g}\begin{pmatrix}\bar{\psi}^{(1)}\\ \bar{\psi}^{(2)}\end{pmatrix}^T\begin{pmatrix}g_1\Phi_{\mathrm{ps}}\cdot\Gamma^5 & V\Phi_{\mathrm{s}}\cdot i\mathbb{I}_4\\ V\Phi_{\mathrm{s}}\cdot i\mathbb{I}_4 & g_2\Phi_{\mathrm{s}}\cdot i\mathbb{I}_4\end{pmatrix}\begin{pmatrix}\psi^{(1)}\\ \psi^{(2)}\end{pmatrix},$$

$$S_{\mathrm{spin,bdy}} = \frac{1}{2}\int \mathrm{d}^3x\sqrt{-h}[\bar{\psi}^{(1)}(i\mathbb{I}_4)\psi^{(1)} + \bar{\psi}^{(2)}\Gamma^{\underline{\mathrm{xy}}}\psi^{(2)}],$$

$$\not{D}^{(j)} = \Gamma^a e_a{}^B\left(\partial_B + \frac{1}{4}\omega_{Bcd}\Gamma^{cd} - iq_jA_B\right), \qquad h = gg^{uu},$$

$$L = 1, \qquad \bar{\psi}^{(j)} = \psi^{(j)\dagger}\Gamma^{\underline{\mathrm{t}}},$$

$$\Gamma^{\underline{\mathrm{t}}} = \sigma_1\otimes i\sigma_2 = \begin{pmatrix}0 & i\sigma_2\\ i\sigma_2 & 0\end{pmatrix}, \qquad \Gamma^{\underline{\mathrm{x}}} = \sigma_1\otimes\sigma_1 = \begin{pmatrix}0 & \sigma_1\\ \sigma_1 & 0\end{pmatrix},$$

$$\Gamma^{\underline{\mathrm{y}}} = \sigma_1\otimes\sigma_3 = \begin{pmatrix}0 & \sigma_3\\ \sigma_3 & 0\end{pmatrix}, \qquad \Gamma^{\underline{\mathrm{u}}} = \sigma_3\otimes\sigma_0 = \begin{pmatrix}\sigma_0 & 0\\ 0 & -\sigma_0\end{pmatrix}$$

$$\Gamma^5 = i\Gamma^{\underline{\mathrm{t}}}\Gamma^{\underline{\mathrm{x}}}\Gamma^{\underline{\mathrm{y}}}\Gamma^{\underline{\mathrm{u}}}, \qquad \Gamma^{ab} = \frac{1}{2}[\Gamma^a, \Gamma^b].$$

Holographic Kondo Lattice 2

$$+\int \mathrm{d}^4x\sqrt{-g}\begin{pmatrix}\bar{\psi}^{(1)}\\ \bar{\psi}^{(2)}\end{pmatrix}^{T}\begin{pmatrix}g_1\Phi_{\mathrm{ps}}\cdot\Gamma^5 & V\Phi_{\mathrm{s}}\cdot i\mathbb{I}_4\\ V\Phi_{\mathrm{s}}\cdot i\mathbb{I}_4 & g_2\Phi_{\mathrm{s}}\cdot i\mathbb{I}_4\end{pmatrix}\begin{pmatrix}\psi^{(1)}\\ \psi^{(2)}\end{pmatrix},$$

- g_1 is the coupling strength of $\psi^{(1)}(\Phi_{\mathrm{ps}}\cdot\Gamma^5)\psi^{(1)}$ that makes a hyperbolic spectrum of the light fermion dual to $\psi^{(1)}$ (to see why we have not chosen the scalar-type interaction, see appendix D).

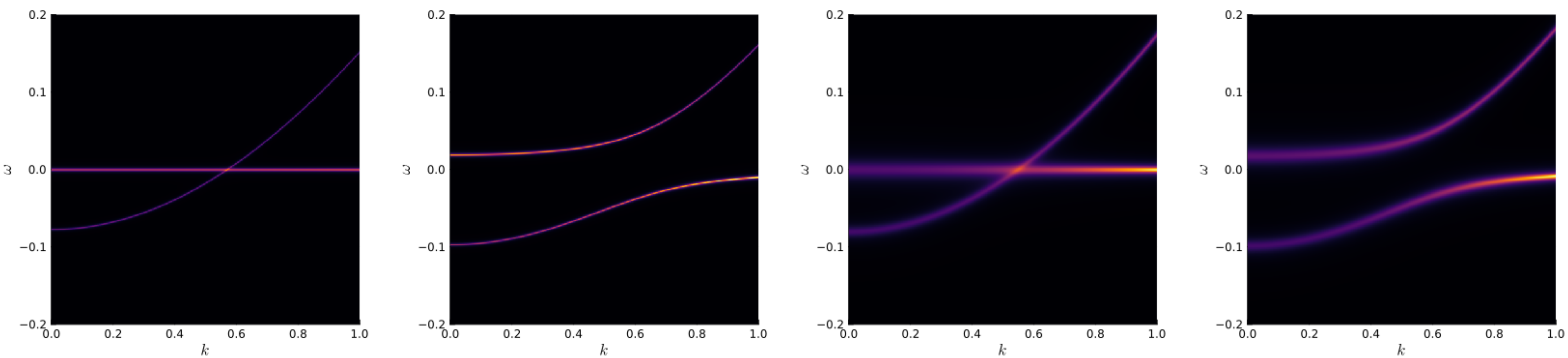
- We consider the standard-mixed quantization to flatten the spectrum of the heavy fermion dual to $\psi^{(2)}$ (see eq. (3.4) and refs. [70, 71, 83]). The flat spectrum comes from the cancellation of the spinor components making the compact localized states (CLS) [71, 84].

- g_2 is the coupling strength of $\bar{\psi}^{(2)}(\Phi_{\mathrm{s}}\cdot i\mathbb{I}_4)\psi^{(2)}$ that isolates the flat spectrum from others (see appendix D and ref. [71]).

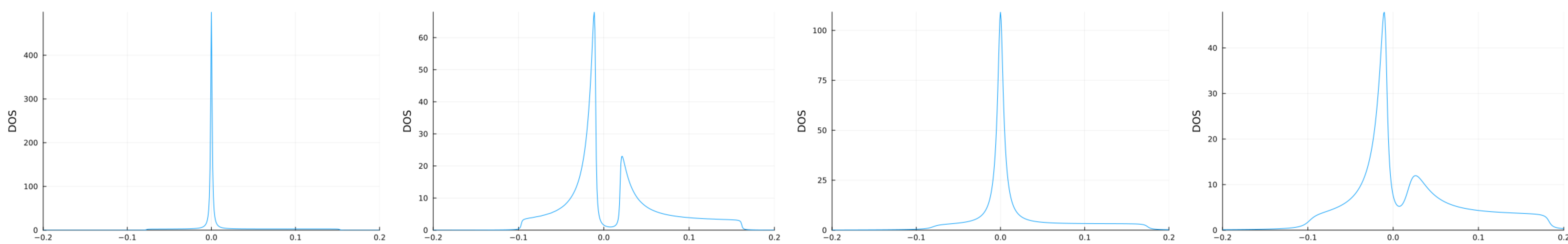
- V is the coupling constant of the inter-flavor interaction $\bar{\psi}^{(1)}(\Phi_{\mathrm{s}}\cdot i\mathbb{I}_4)\psi^{(2)}$ hybridizing the light and heavy fermions.

Holographic Kondo Lattice 3

$$\left[\begin{pmatrix} \overrightarrow{\not{D}} - m_1 & 0 \\ 0 & \overrightarrow{\not{D}} - m_2 \end{pmatrix} - i \begin{pmatrix} g_1 \Phi_{\mathrm{ps}} \cdot \Gamma^5 & V \Phi_{\mathrm{s}} \cdot i\mathbb{I}_4 \\ V \Phi_{\mathrm{s}} \cdot i\mathbb{I}_4 & g_2 \Phi_{\mathrm{s}} \cdot i\mathbb{I}_4 \end{pmatrix}\right] \begin{pmatrix} \psi^{(1)} \\ \psi^{(2)} \end{pmatrix} = 0.$$



(a) $A(\omega, k), V = 0.$ (b) $A(\omega, k), V = 0.5.$ (c) $A(\omega, k), V = 0.$ (d) $A(\omega, k), V = 0.5.$



(e) $g(\omega), V = 0.$ (f) $g(\omega), V = 0.5.$ (g) $g(\omega), V = 0.$ (h) $g(\omega), V = 0.5.$

IV. Topology in interacting system

- Topological Hamiltonian Method and Eigenvectors $(\omega = 0)$

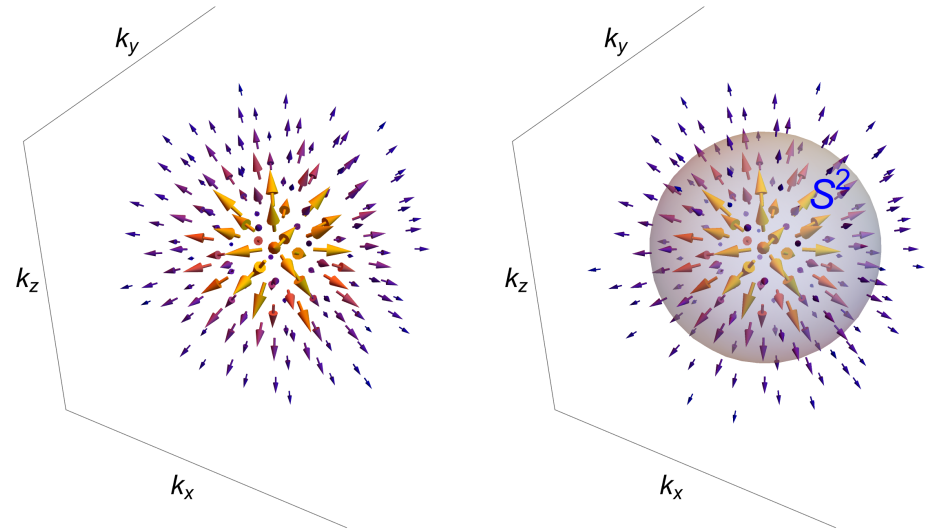
$$\mathcal{H}_t(\boldsymbol{k}) = -\mathbb{G}^{-1}(0, \boldsymbol{k})$$

where eigenvector of H_t and H share the same eigenvector, $|n\rangle$.

$$\mathcal{F}_c = \nabla \times \langle n| \partial_{\boldsymbol{k}} |n\rangle \tag{2}$$

- Alternative method: "Cubic of Green's function"

$$\mathcal{F}_c = \frac{1}{3!} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \epsilon_{\mu\nu\rho c} \mathrm{Tr}\left[\mathbb{G}(\partial_\mu \mathbb{G}^{-1})\mathbb{G}(\partial_\nu \mathbb{G}^{-1})\mathbb{G}(\partial_\rho \mathbb{G}^{-1})\right] \tag{3}$$



Monopole Number:

$$C_n = \oint \mathcal{F}_c \cdot dS = i \oint \nabla \times \langle n| \partial_{\boldsymbol{k}} |n\rangle \cdot dS$$

Critical case ($\Phi = 0$)

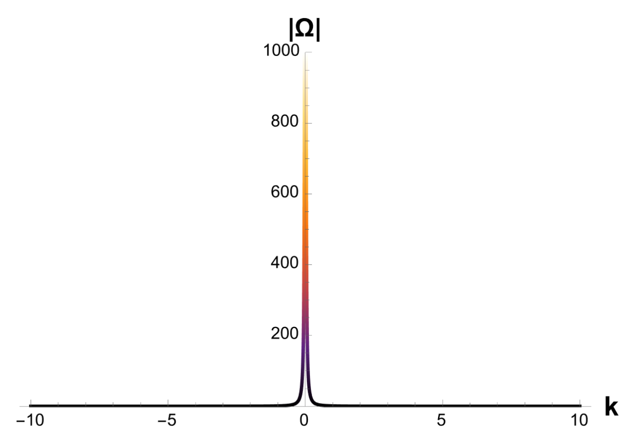
$$\mathcal{A}^{11} = \mathcal{A}^{22} = \frac{|\boldsymbol{k}| - k_x}{2|\boldsymbol{k}|(k_y^2 + k_z^2)}(0, -k_z, k_y)^{\mathrm{T}} \tag{5.1}$$

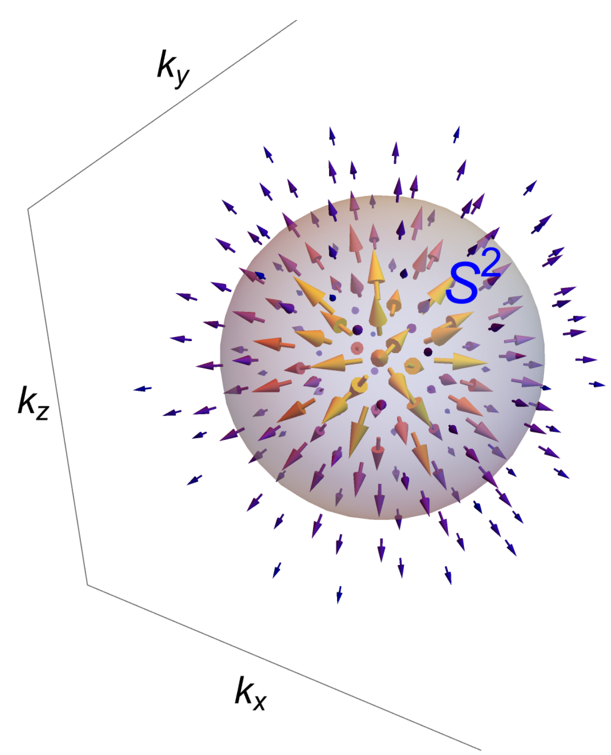
$$\mathcal{A}^{33} = \mathcal{A}^{44} = \frac{|\boldsymbol{k}| + k_x}{2|\boldsymbol{k}|(k_y^2 + k_z^2)}(0, -k_z, k_y)^{\mathrm{T}} \tag{5.2}$$

$$\mathcal{A}^{13} = \mathcal{A}^{24} = \mathcal{A}^{31*} = \mathcal{A}^{42*} = \frac{\sqrt{\boldsymbol{k}^2 - k_x^2}}{2\boldsymbol{k}^2(k_y^2 + k_z^2)}(-i(k_y^2 + k_z^2), ik_xk_y + |\boldsymbol{k}|k_z, ik_xk_z - |\boldsymbol{k}|k_y)^{\mathrm{T}} \tag{5.3}$$

F=dA+A^A =>for Abelian case, denote F= Ω

$$\boldsymbol{\Omega} = \frac{1}{\boldsymbol{k}^{3/2}}(k_x, k_y, k_z)^{\mathrm{T}} \qquad \text{flux} = \int_{\mathcal{S}} \boldsymbol{\Omega} \cdot d\boldsymbol{S} = 2\pi$$





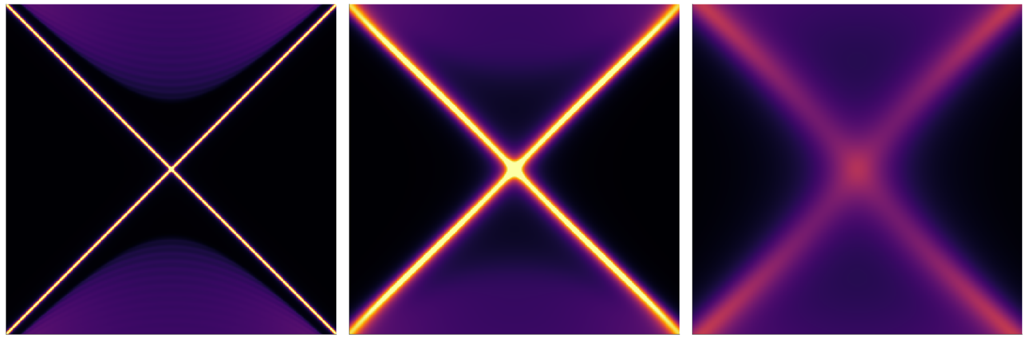
Topological Liquid : scalar order without gap

$$S_\psi = \int d^5x \sum_{j=1}^{2} \sqrt{-g}\; \bar{\psi}^{(j)} \Big(\frac{\overrightarrow{\not{D}} - \overleftarrow{\not{D}}}{2} - m^{(j)} \Big) \psi^{(j)}, \tag{5}$$

$$S_{g,\Phi} = \int d^5x \sqrt{-g} \Big(R - 2\Lambda - \nabla_M \Phi^2 - m_\Phi^2 |\Phi|^2 \Big) \tag{6}$$

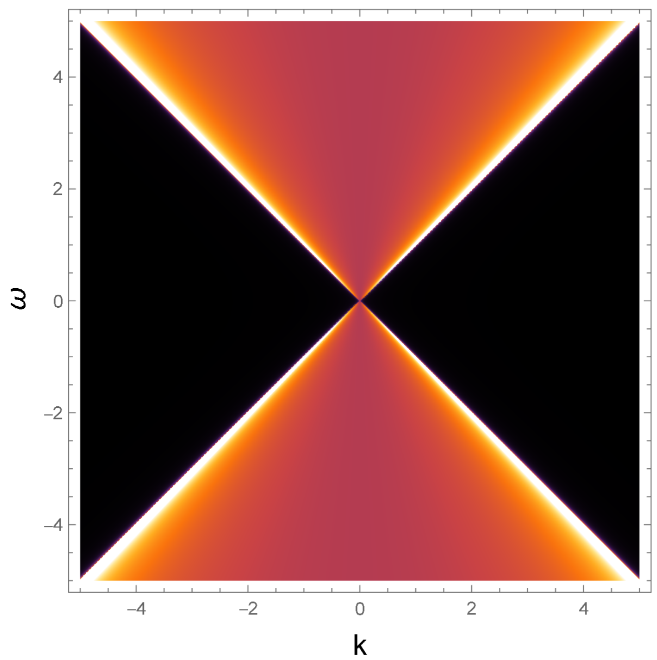
$$S_{int} = \int d^5x \sqrt{-g} \Big(i\Phi \bar{\psi}^{(1)} \psi^{(2)} + h.c \Big). \tag{7}$$

where $\not{D} = \Gamma^M D_M$, $D_M = (\partial_M - iqA_M + \frac{1}{4}\omega_{M\alpha\beta}\Gamma^{\alpha\beta})$

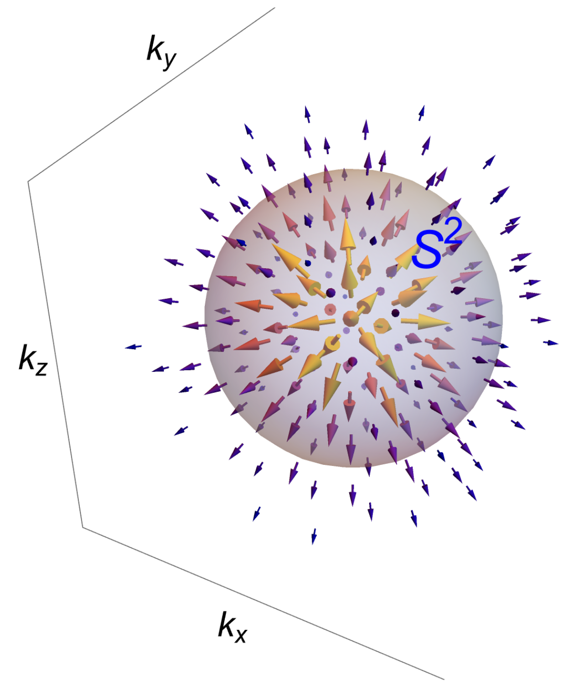


Scalar Interaction case(SS quantization)

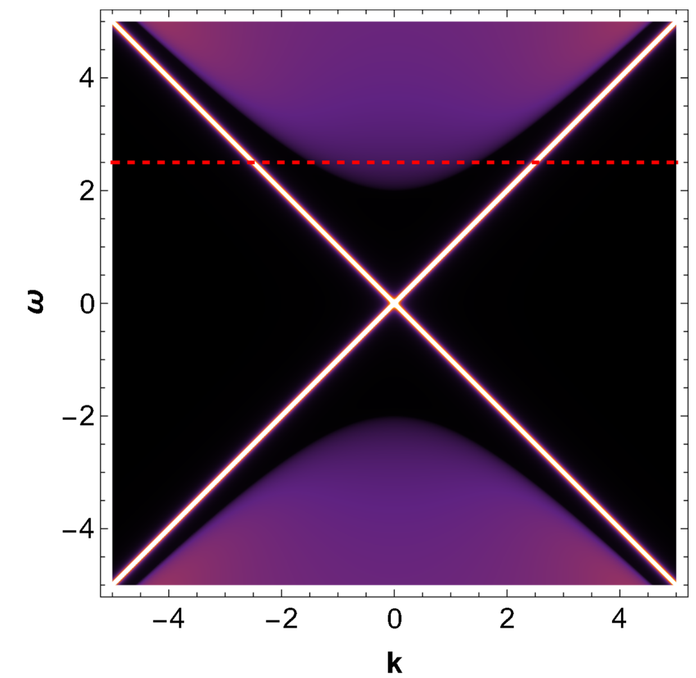
Spectrum is pole type, differ from critical case.



Spectrum of free Fermion



Berry curvature for both cases



Spectrum of scalar coupled Fermion

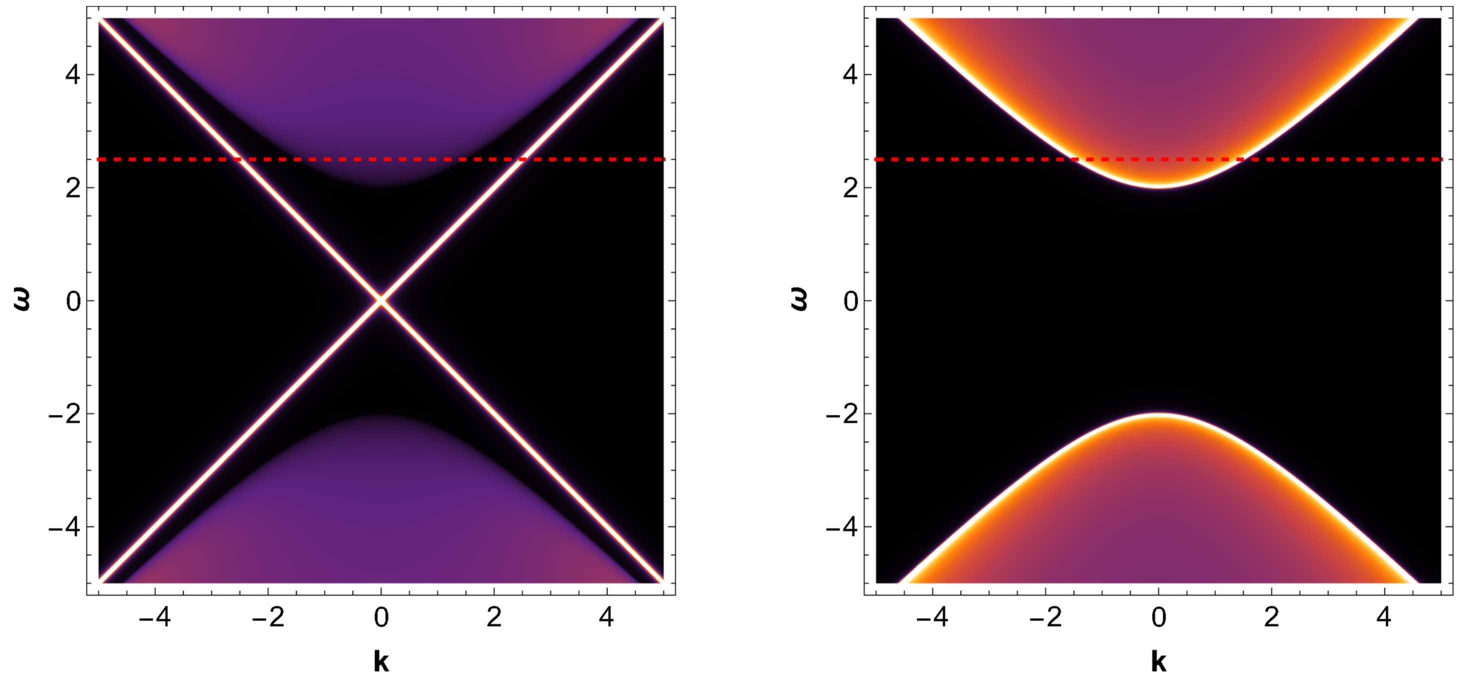
However, Berry Curvature is Identical to critical case.

The same Dirac monopole

Scalar Interaction case (SA quantization)

- Gapped spectrum
- Trivial topology

$$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{M_0} = \frac{4\omega}{\sqrt{\boldsymbol{k}^2 - \omega^2 + M_0^2}},$$

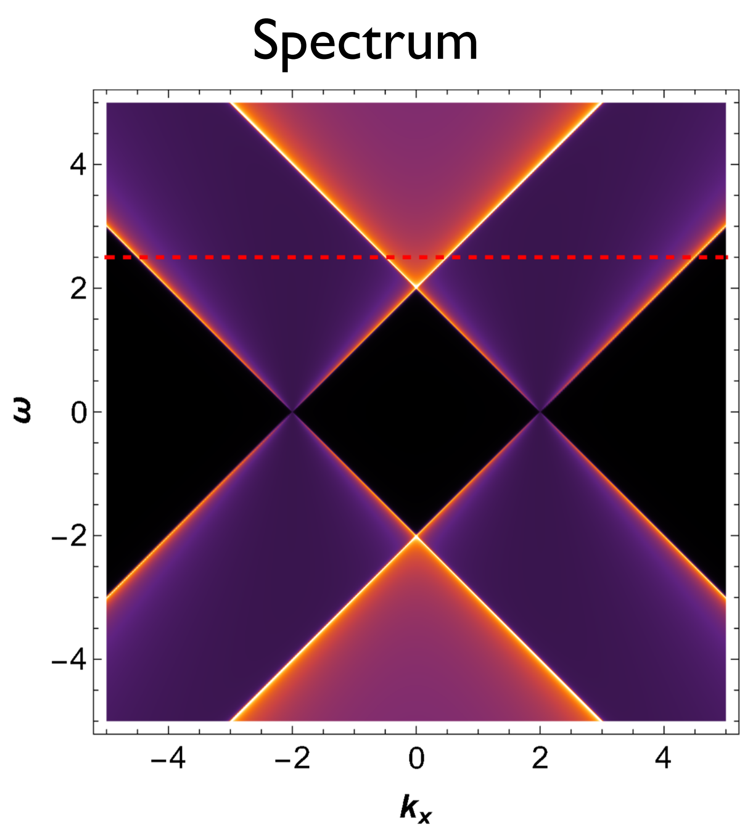


(a) $M_0^{(SS)}, \omega\text{-}\boldsymbol{k}$ (b) $M_0^{(SA)}, \omega\text{-}\boldsymbol{k}$

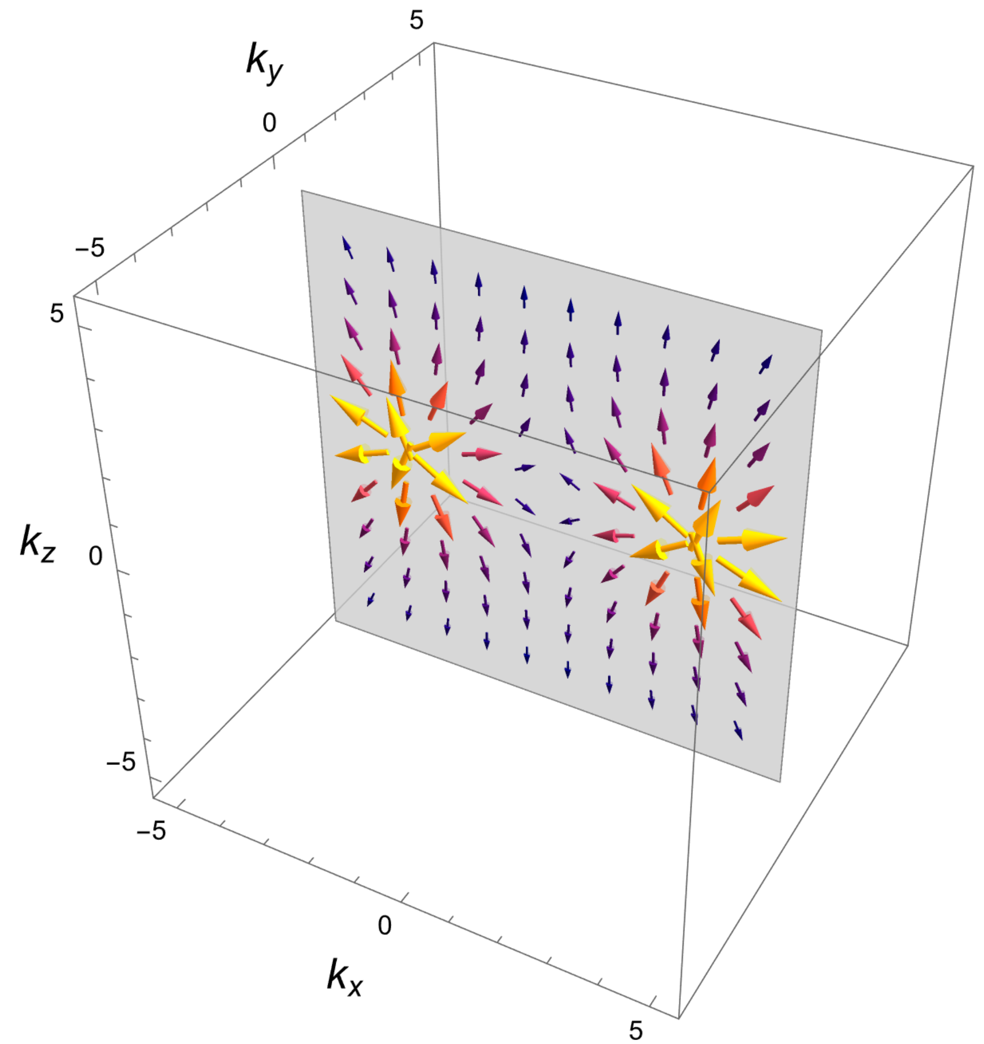
Vector Interaction : Separated Dirac monopole

Berry curvature

$$\mathbf{\Omega} = \frac{1}{2\left((b_x+k_x)^2+k_y^2+k_z^2\right)^{3/2}}(k_x+b_x,k_y,k_z)^{\mathrm{T}} + \frac{1}{2\left((b_x-k_x)^2+k_y^2+k_z^2\right)^{3/2}}(k_x-b_x,k_y,k_z)^{\mathrm{T}}$$



(a) $B_x^{(0)(SS)}$, ω-k_x



(b) Berry curvature on k_z-k_x plane

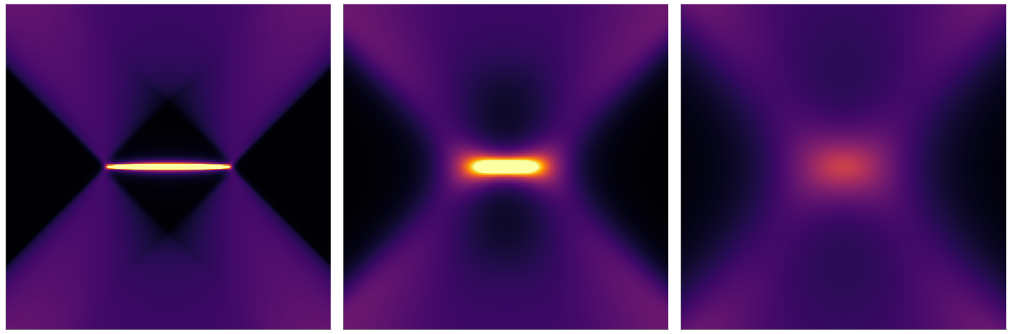
B_{xy} case

$$S_\psi = \int d^5x \sum_{j=1}^{2} \sqrt{-g}\; \bar{\psi}^{(j)} \left(\frac{\overrightarrow{\not{D}} - \overleftarrow{\not{D}}}{2} - m^{(j)} \right) \psi^{(j)}, \tag{8}$$

$$S_{g,B_{\mu\nu}} = \int d^5x \sqrt{-g} \left(R - 2\Lambda - |D_M \Phi_I|^2 - m_\Phi^2 |\Phi|^2 \right), \tag{9}$$

$$S_{int} = \int d^5x \sqrt{-g} \left(B_{\mu\nu} \bar{\psi}^{(1)} \Gamma^{\mu\nu} \psi^{(2)} + h.c \right). \tag{10}$$

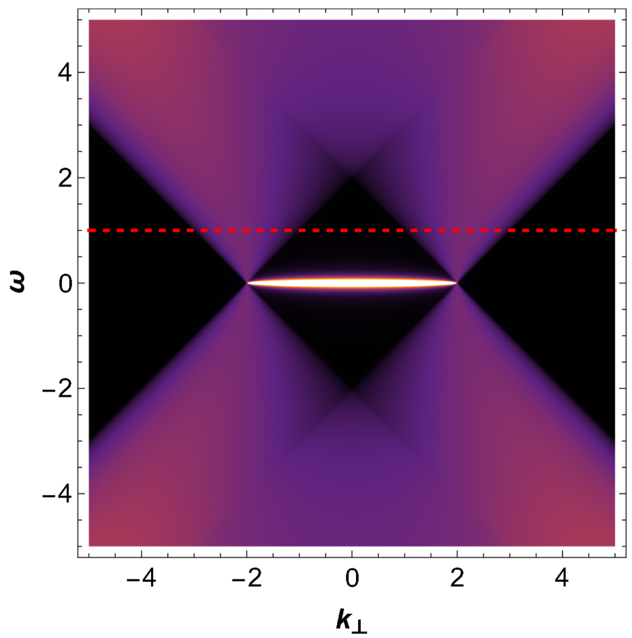
where $\not{D} = \Gamma^M D_M$, $D_M = (\partial_M + \frac{1}{4}\omega_{M\alpha\beta}\Gamma^{\alpha\beta})$, and $B = B_{xy}(u)\; dx \wedge dy$



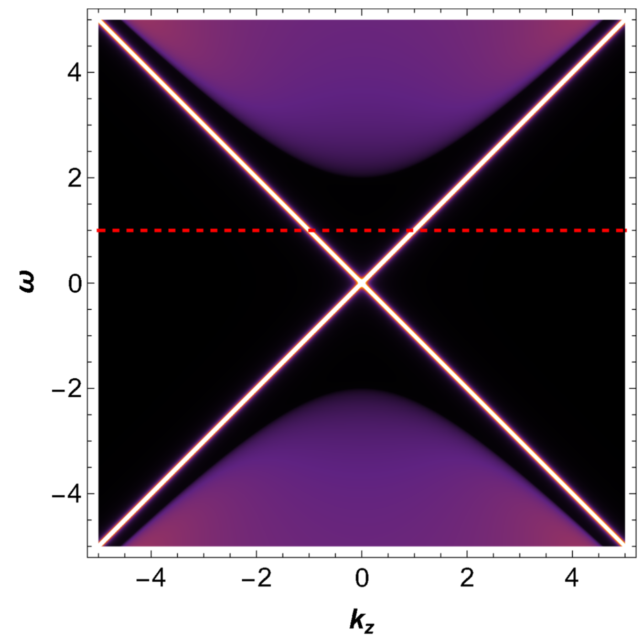
Topology of Flat band

Spectrum =2d Disk

$$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B^{(-1)}_{xy}} = \frac{2\omega}{b}\Big[\frac{(b+|\boldsymbol{k}_\perp|)\sqrt{(b-|\boldsymbol{k}_\perp|)^2+k_z^2-\omega^2}+(b-|\boldsymbol{k}_\perp|)\sqrt{(b+|\boldsymbol{k}_\perp|)^2+k_z^2-\omega^2}}{k_z^2-\omega^2-i\epsilon}\Big].$$

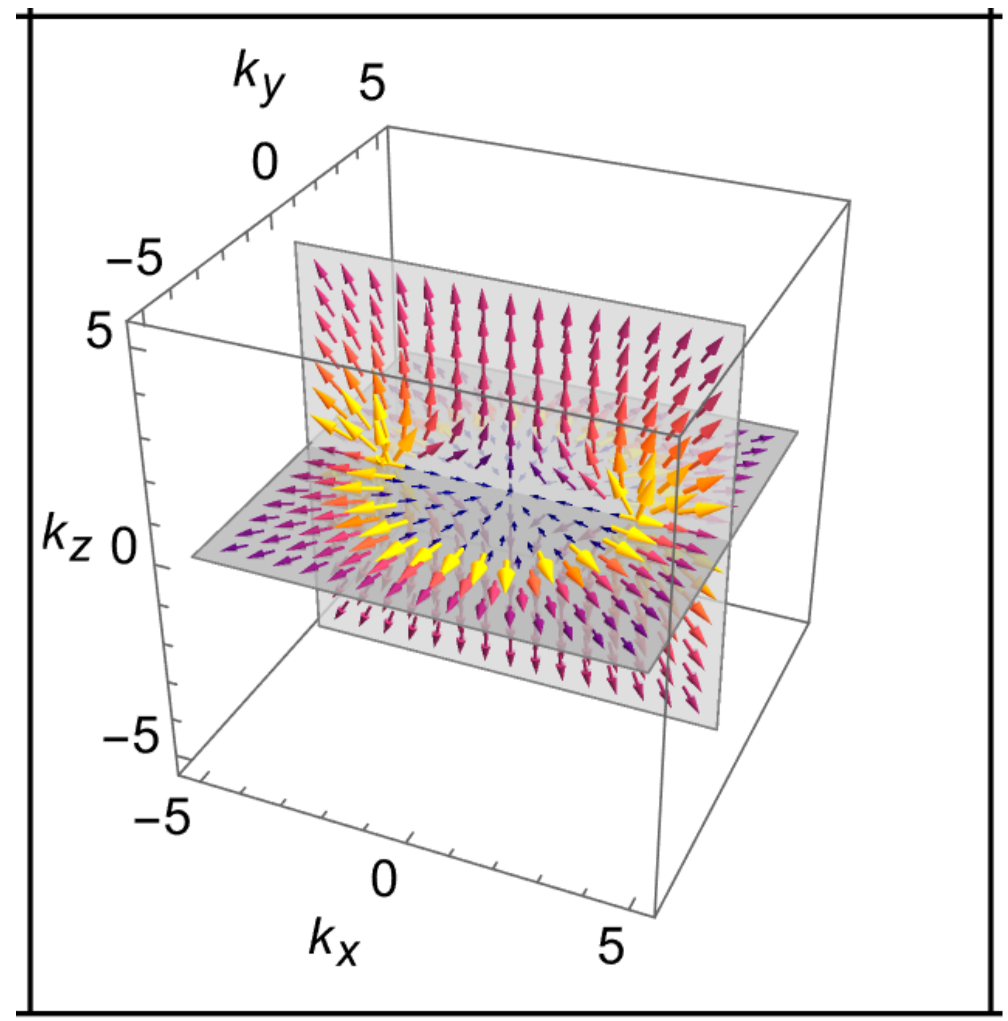


(e) $B^{(-1)(SS)}_{xy}, \omega\text{-}\boldsymbol{k}_\perp$



(f) $B^{(-1)(SS)}_{xy}, \omega\text{-}k_z$

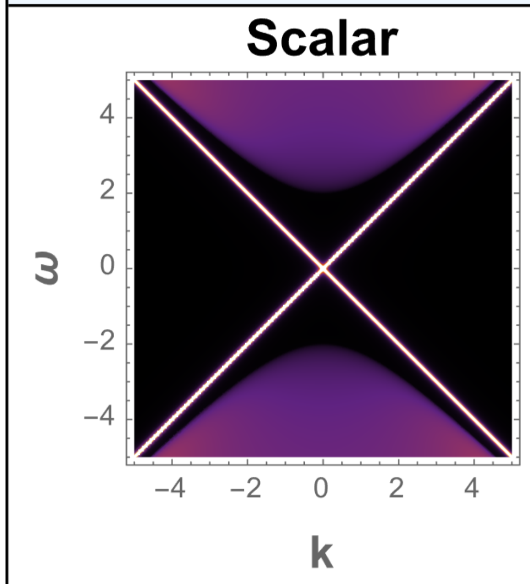
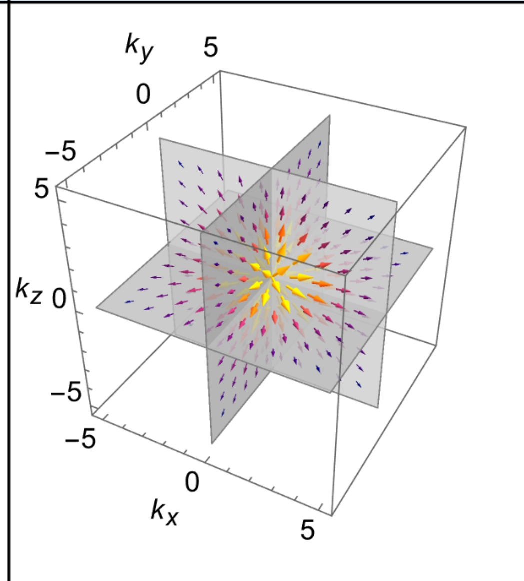
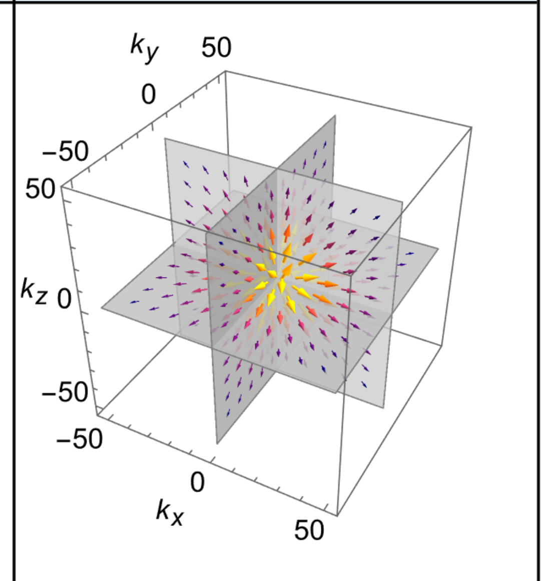
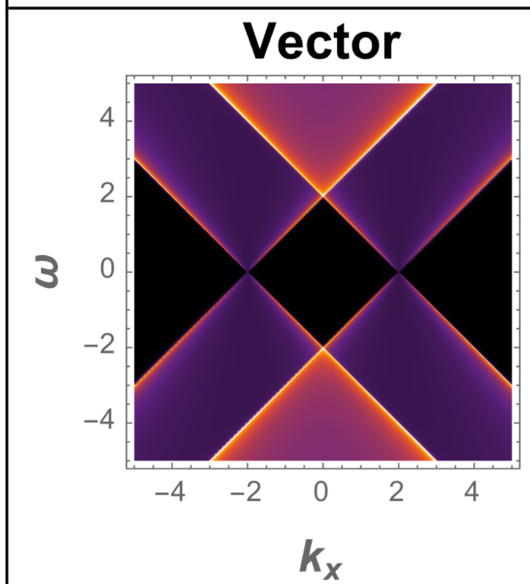
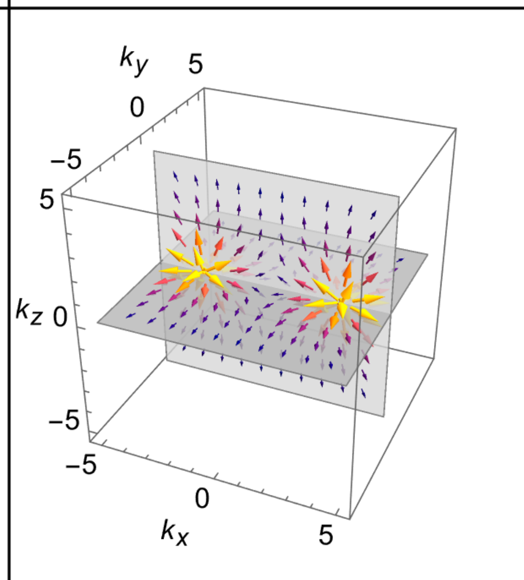
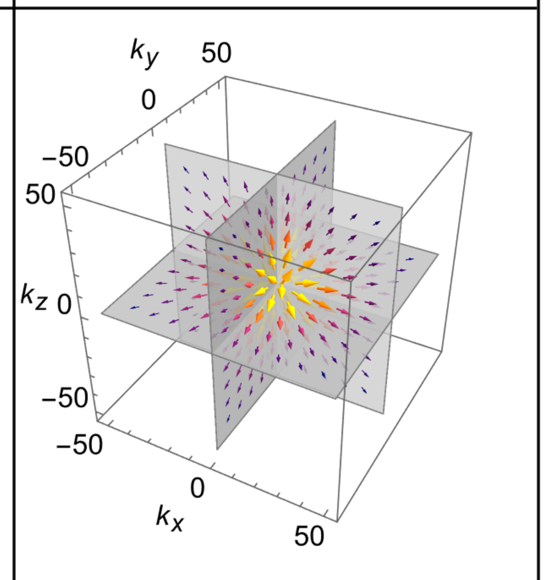
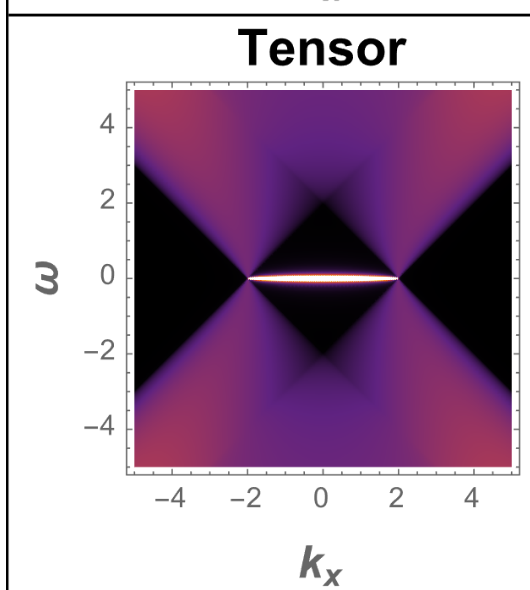
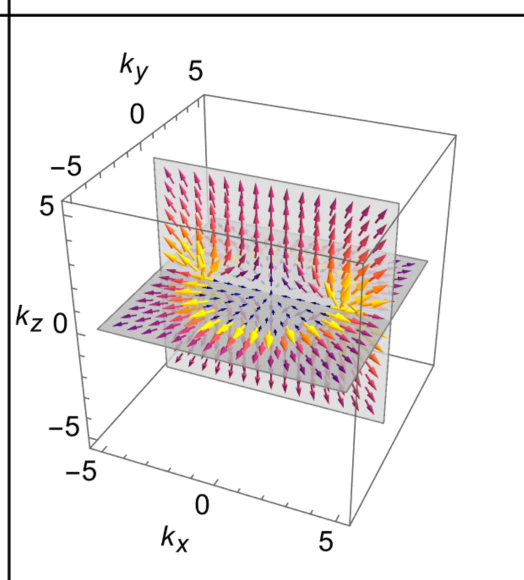
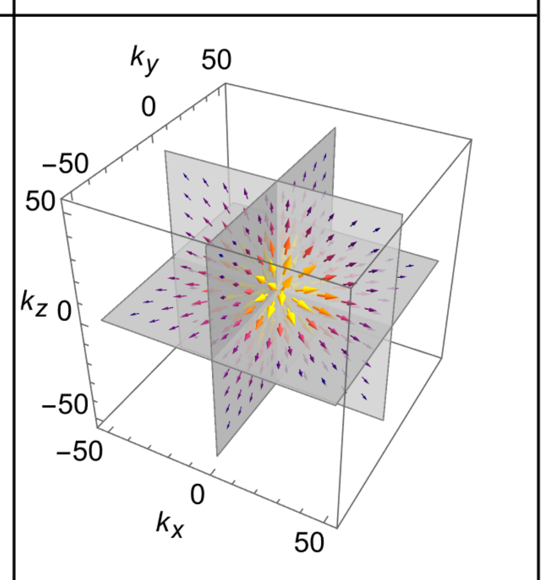
Berry curvature=monopole Ring



$$\boldsymbol{\Omega} = \frac{k_z(f_- - f_+)^2(\boldsymbol{k}^2-b^2)}{4\sqrt{2}|\boldsymbol{k}_\perp|f_-f_+\big((\boldsymbol{k}^2-b^2)(\boldsymbol{k}^2-b_{xy}^2-f_-f_+)\big)^{3/2}}\begin{pmatrix}\big(|\boldsymbol{k}_\perp|(f_- - f+) + b_{xy}(f_- + f_+)\big)\sin\theta \\ \big(|\boldsymbol{k}_\perp|(f_- - f+) + b_{xy}(f_- + f_+)\big)\cos\theta \\ \frac{4k_z(b_{xy}^2+\boldsymbol{k}_\perp^2+k_z^2)\Big((\boldsymbol{k}_\perp^2+k_z^2)(f_- - f_+)+b_{xy}|\boldsymbol{k}_\perp|(f_-+f_+)\Big)}{(\boldsymbol{k}_\perp^2+k_z^2-b_{xy}^2)(f_- - f_+)^2}\end{pmatrix}$$

$$f_\pm = \sqrt{(b_{xy}\pm\boldsymbol{k}_\perp)^2+k_z^2} \text{ and } \boldsymbol{k}_\perp = \sqrt{k_x^2+k_y^2}.$$

Summary (AdS_5 or 3d topology)

Interaction types/SF	Near the origin	Far from the origin
		
		
		

Single monopole

Separated monopole

Monopole ring

AdS_4 : scalar vs pseudo-scalar

the scalar $\Gamma \cdot \Phi = iM_0$ Green's function is given by

$$\mathbb{G} = \begin{pmatrix} \frac{k_x+\omega}{-M_0+\sqrt{k_x^2+k_y^2+M_0^2-\omega^2}} & \frac{k_y}{M_0-\sqrt{k_x^2+k_y^2+M_0^2-\omega^2}} \\ \frac{k_y}{M_0-\sqrt{k_x^2+k_y^2+M_0^2-\omega^2}} & \frac{k_x+\omega}{M_0-\sqrt{k_x^2+k_y^2+M_0^2-\omega^2}} \end{pmatrix}$$

$$\mathrm{Tr}\,\mathbb{G} = \frac{2\omega}{-M_0 + \sqrt{k_x^2 + k_y^2 + M_0^2 - \omega^2}}$$

Spectrum-> gap (g>0)
Topological Liquid (g<0)

But in both case
$\Omega_{xy} = 0$

the 1-flavor with psudo scalar $\Gamma \cdot \Phi = \Gamma^5 M_5$ can give a gap

$$\mathbb{G} = \frac{1}{\sqrt{k_x^2 + k_y^2 + M_5^2 - \omega^2}} \begin{pmatrix} k_x + \omega & -k_y + iM_5 \\ -k_y - iM_5 & -k_x + \omega \end{pmatrix},$$

$$\mathrm{Tr}\,\mathbb{G} = \frac{2\omega}{\sqrt{k_x^2 + k_y^2 + M_5^2 - \omega^2}}$$

Spectrum-> gap

$$\boldsymbol{\Omega} = \frac{M_5}{2(k_x^2 + k_y^2 + M_5^2)^{3/2}}$$

$$c_1 = \frac{1}{2\pi}\int F = 1$$

Topology in finite temperature

1. Non-interacting (single particle) theory:
   Finite temperature is ensemble average.
   Each band has its own topological number c_n .
   Therefore the topological number = average of c_n :

$$c(T) = \sum p_n(T) c_n$$

   Actually Uhlmann defined a T-dependent c.

Q: But does it make sense for a topology to be dependent on T, a continuous deformation?

Q: What holography says about it?

Monopole number at Finite T in holography

Method 1: A & F are T-independent, though G depends on T.
Method 2: GdG^{-1} depends on T.

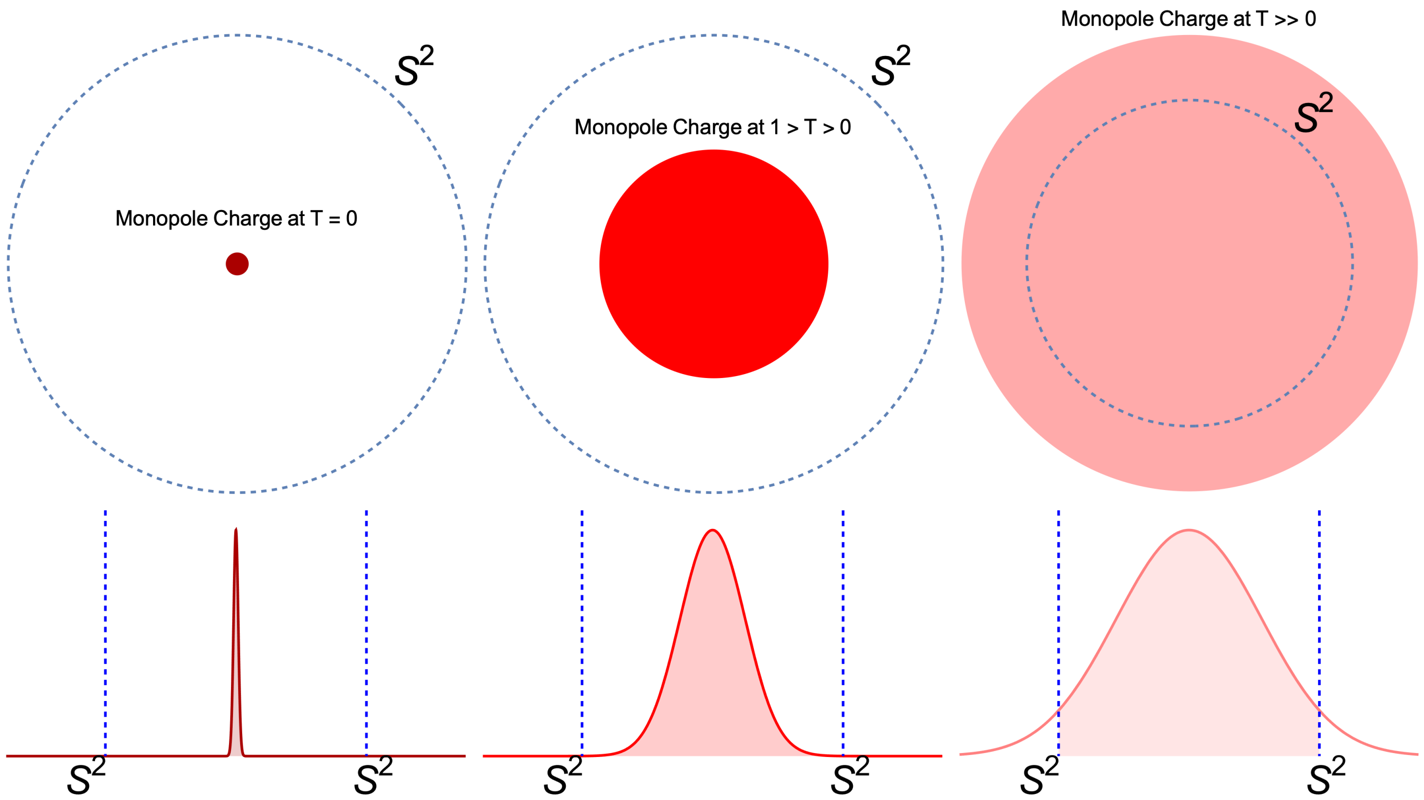


Figure: Monopole charge with increasing of temperature, with a fixed sphere surface

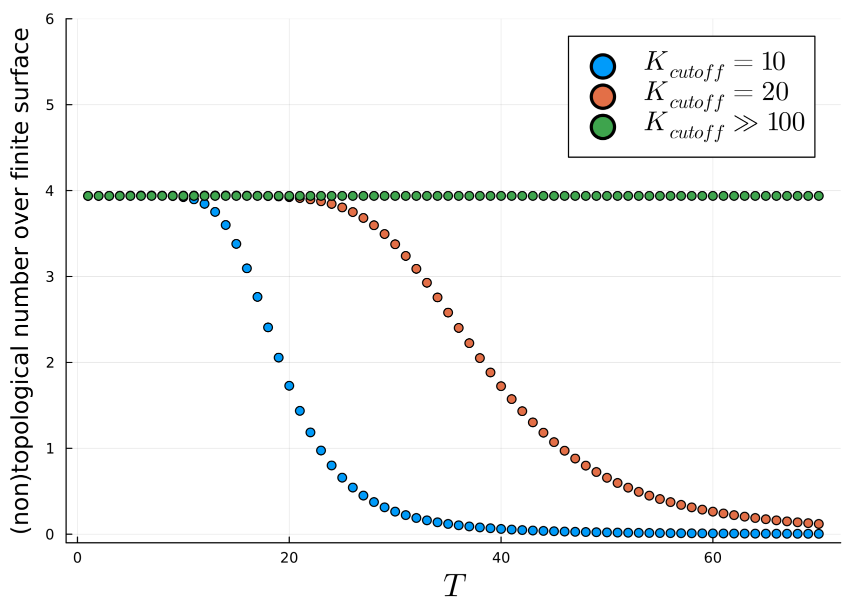


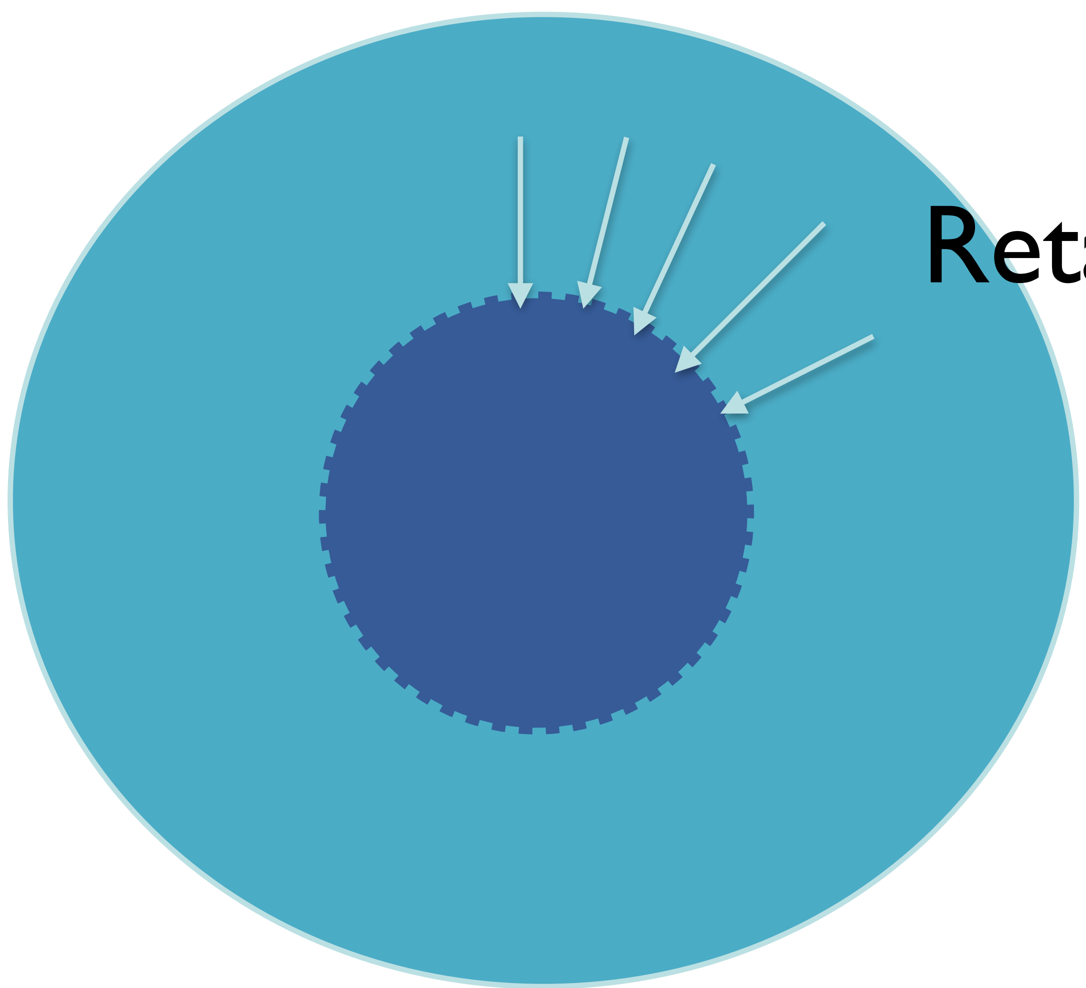
Figure: monopole numbers over the evolution of temperature by various integration sphere radius.

Flux over Large enough Surface => temperature independent result.

Observation

1. In holography, $c_1(T) = c_1(0)$.

2. Why this happen?
In AdS/CFT dictionary,
finite temperature ~ black hole ~ (a pure) state!



Retarded = inflating BC

Conclusion

1. Lattice = symmetry breaking mechanism =>spectrum generation
   CLS=Atom, essence of both=localization of electron
   identify f orbital = flat band by CLS.
2. Topology of strongly interaction can be handled and holography gives a T-independent Topology.
3. Kondo lattice = flat band hybridized with s-band.

Thank you