INPP Demokritos-APCTP meeting and HOCTOOLS-II mini-workshop
30 September ~ 4 October, 2024

National Centre for
Scientific Research (NCSR)

Dilaton-Einstein-Gauss-Bonnet Gravity and its Cosmological Implicationin

Bum-Hoon Lee
(이 범 훈 李範勳)
Sogang University

apctp asia pacific center for theoretical physics

ASIA PACIFIC CENTER FOR

THEORETICAL PHYSICS

Brief Facts about Korea

People & Language: Korean (~4,500 yrs in the area)
Area (South): ~100,000 km^2
cf) Area of Uzbekistan ≈ 447,000 km^2
Population (South)**:** 51 million

Recent History:

1945: Divided into North and South
1950~1953: Korean Conflict
1960~1970: Modernization (Migration to cities)
1970~1980: Industrialization (Heavy Industries)
1990~2019: High-tech oriented

Leading Industries:

Electronics, Automobile, Ship-building, Steel, Chemicals, Construction, Textiles

Economy: GNI: 31.3 k$/capita in 2018
Religion: Christian (~30%), Buddhism (~30%)
Education: > 80% high-school seniors go to college
Theoretical Physics Institute: KIAS, IBS, APCTP(Asia Pacific Center for Theoretical Physic) etc.

Our Vision and Mission

International Organization for Science Research and Collaboration, established in 1996 with 10 member countries, endorsed by APEC.

Vision

- Asia-Pacific Physics Community should play a global leadership in Theoretical Physics.

Mission

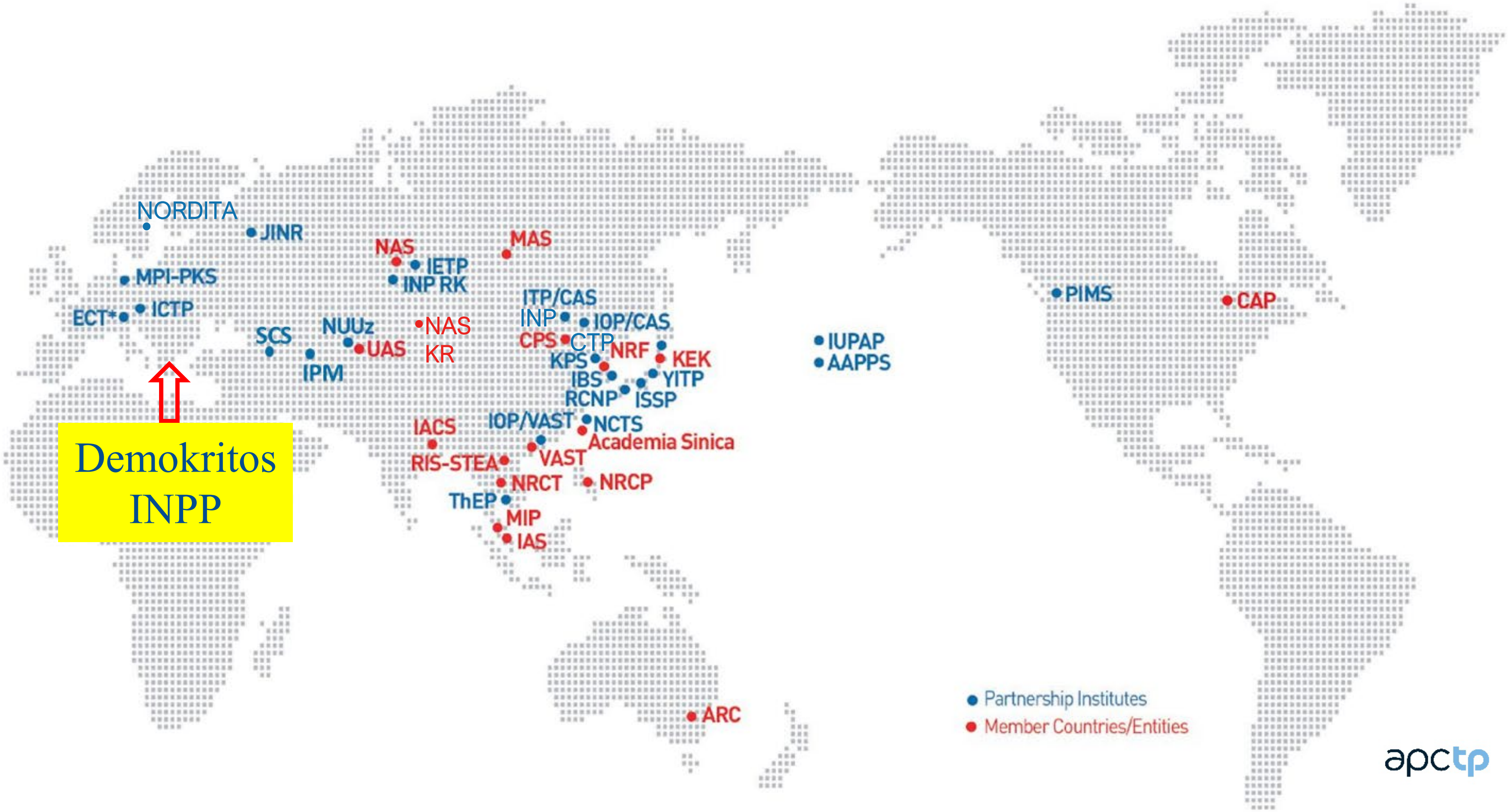
- Function as Hub Center to create a network of exchange and collaboration for Physicists in the AP-region.
- Train young Physicists in the AP-region.
- Contribute to increase the global Common Wealth.

History

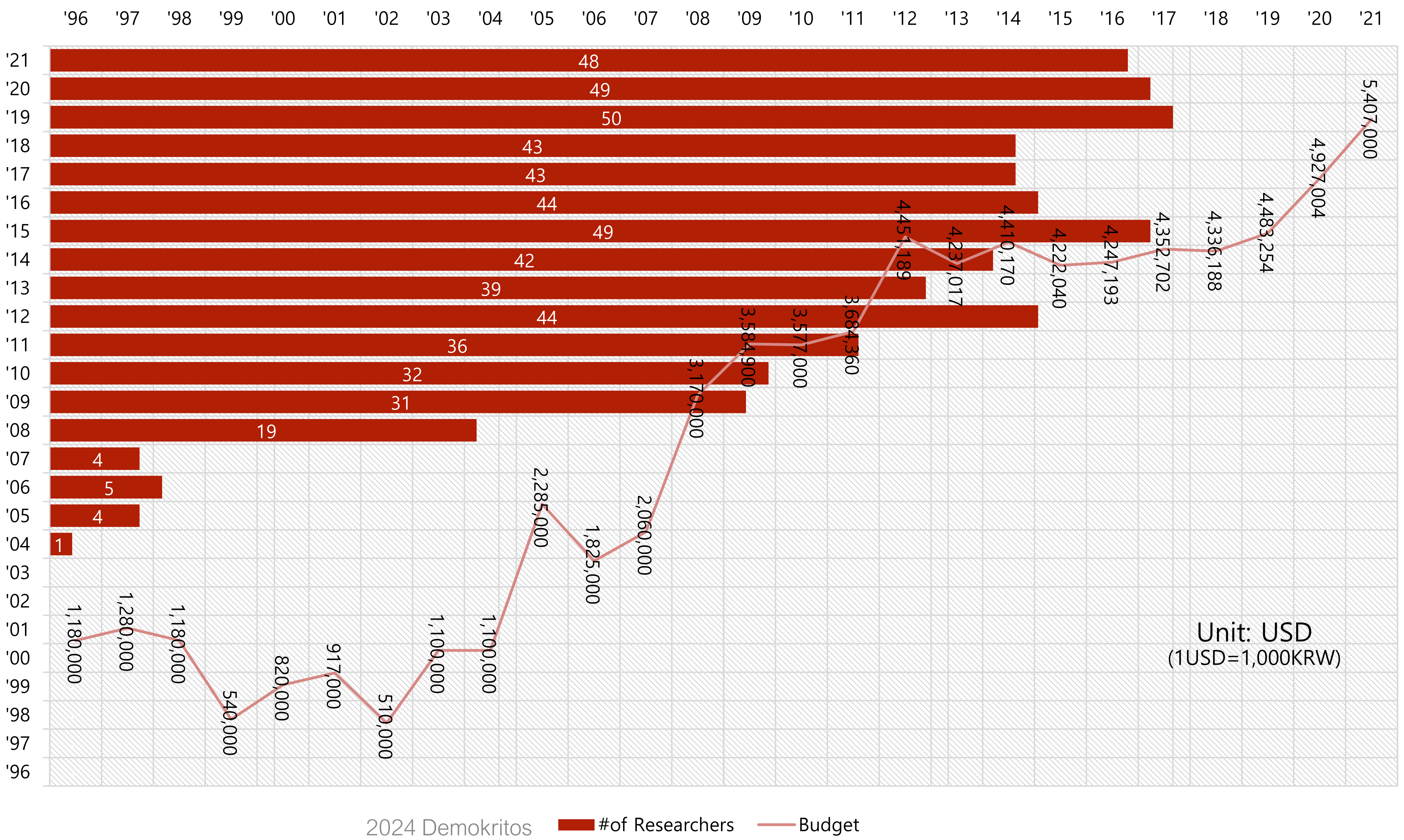
Nobel Laureate in Physics, 1957

1989	Proposal to establish **an international center for theoretical physics** in the **Asia-Pacific region**
1994	IUPAP, AAPPS supports the establishment of APCTP in Korea. (AAPPS: Association of Asia Pacific Physical Societies)
1996	Inauguration of APCTP (APEC S&T Ministers Meeting endorsed) 10 member countries (Australia, China, Japan, Malaysia, Philippines, Korea, Singapore, Taipei, Thailand, Vietnam) Prof. **C.N. Yang** (1st President and Chairperson)
2001	Relocated in POSTECH Prof. A. Arima (2nd Chairperson of BOT)
2004	Prof. R. B. Laughlin (2nd President) Lao PDR (2006), Mongolia (2006)
2007-2013	Prof. P. Fulde (3rd, 4th President) India (2008), Uzbekistan (2011)
2013	Prof. Seunghwan Kim (5th President) Kazakhstan (2013)
2014	Join the APEC PPSTI working group (PPSTI: Policy Partnership on Science, Technology and Innovation)
2015	Prof. Bum-Hoon Lee (6th President)
2016	Opening of **AAPPS headquarter.** Canada (2016)
2018~	Prof. Yunkyu Bang (7th President), Prof. Noboru Kawamoto (7th Chairperson)

18 Member Entities & 35 Partnership Institutions



Progress of APCTP (Quantitative)



Structure of In-House Research (2023)

10 Junior Research Groups (JRG) ~ 39 PhDs

1. Observational Cosmology
2. Duality in String/M-Theory and Quantum Gravity
3. String Theory and Quantum Chromodynamics
4. Black Holes, Quantum Gravity and String Theory
5. Holography and Black holes
6. Interfaces and Defects in Strongly Coupled Matter
7. Magnetized Plasma Physics and Astrophysics
8. Thermodynamics of Microscopic Non-equilibrium Systems
9. Scattering Amplitude and Precision Collider Phenomenology
10. To be filled.

+ Young Scientist Training (YST) Program ~ 16 PhDs

No. of APCTP Researchers by Nationality

Australia	1
China	5
Chinese Taipei	1
Cuba	1
Finland	1
India	12
Indonesia	1
Iran	2
Italy	1
Japan	5
Korea, Republic of	20
Mexico	1
Sweden	1
Turkey	1
United Kingdom	2
Total	**55**

Structure of In-House Research (2023)

4 Senior Advisory Groups (**SRG**) ~ **40** Professors

1. High Energy and Particle Physics:
(Bumhoon Lee, Kimyung Lee, Robert de Mello Koch,Jacob Sonneschein, etc)

1. Condensed Matter Physics and Quantum Material :
(Naoto Nagaosa, A.V. Balatsky, Isaac Kim, KS Kim, HW Lee, etc)
2. Astrophysics and Cosmology :

(Misao Sasaki, Y M Cho, JE Kim, KM Lee, L P Zayas, Frank Ferrari, Antal Jevicki, etc)

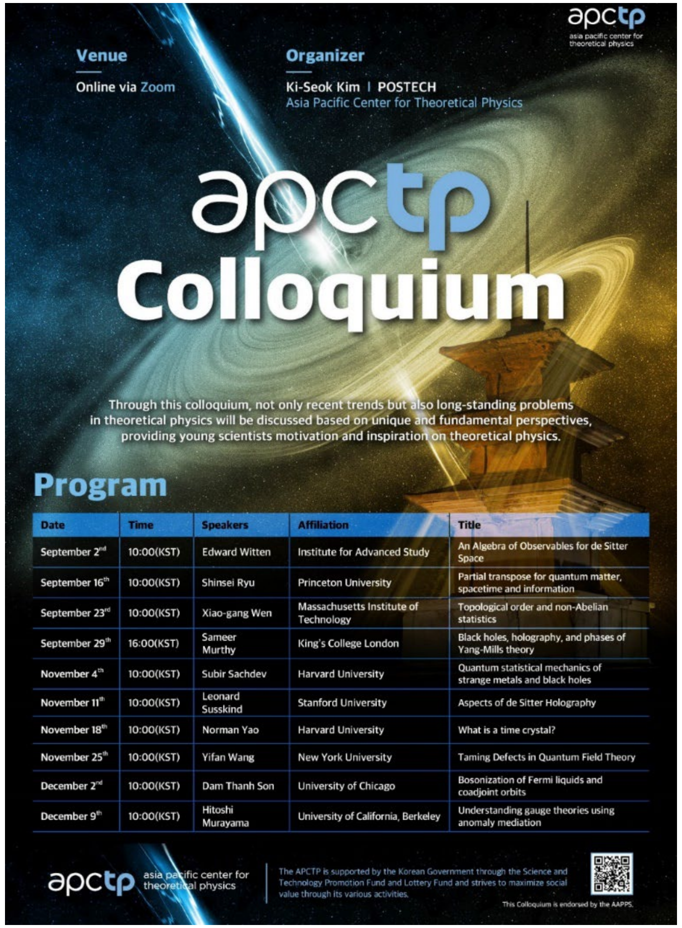
3. Non-Equilibrium Physics and Statistical Physics :
(Fuchun Zhang, Ralf Jevicki, Eli Barkai, HK Kee, etc.)

Aim: short or long-term visiting position
Providing collaborations with and mentoring to the Center's Young Researchers

2024 Demokritos

Scientific Programs of APCTP (2022) (45)

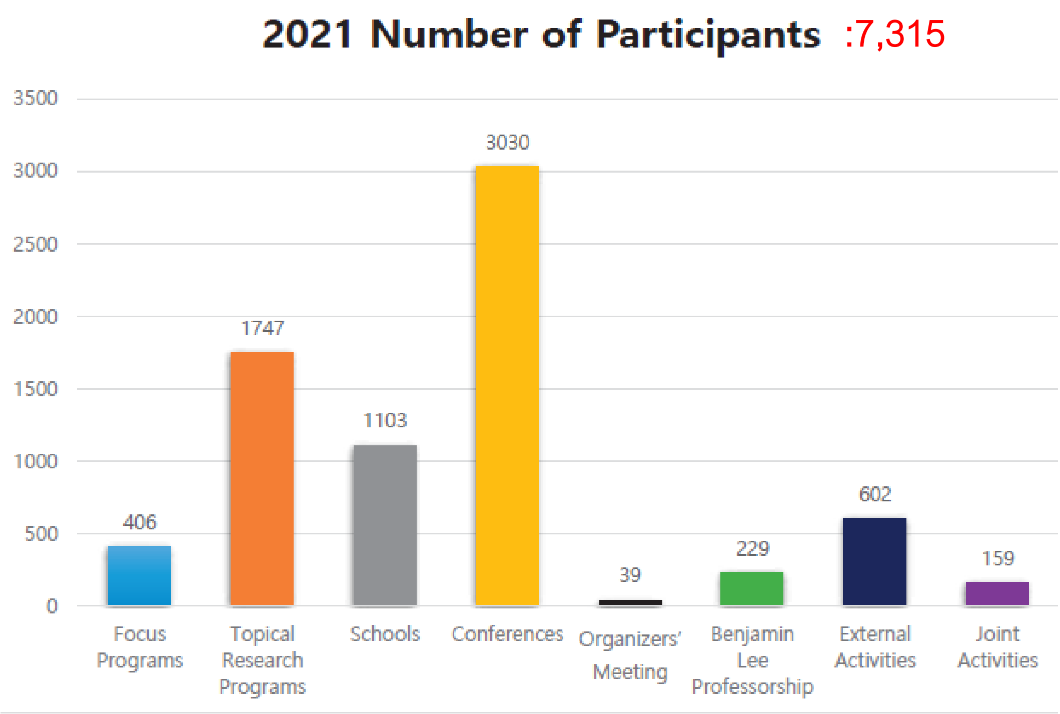
World Class APCTP Colloquium (2022)

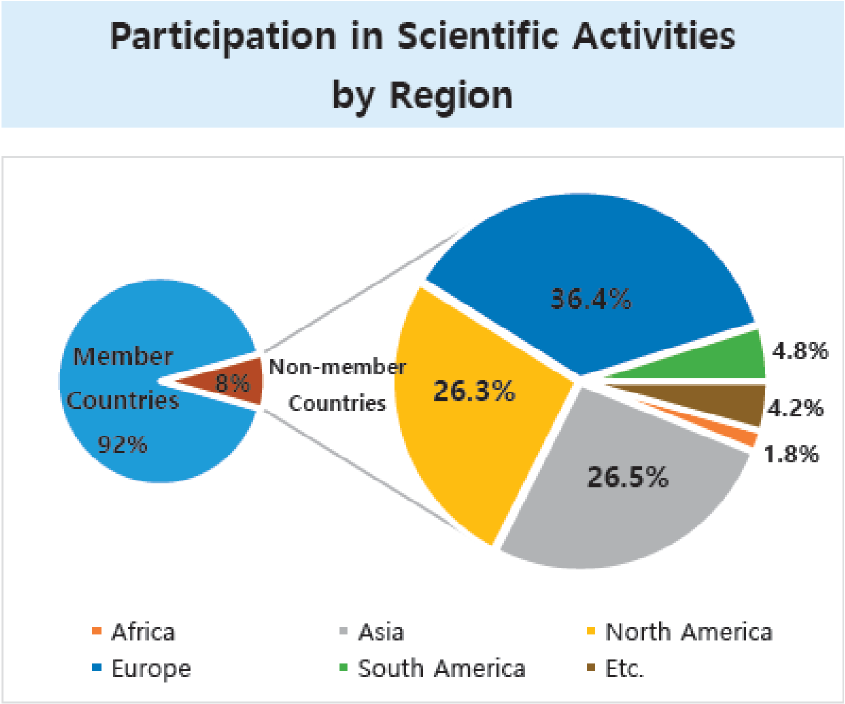


APCTP: Scientific Activities Statistics (2021)

- **Academic Activity Hub (~50 programs)**

Conferences, Workshops, Focus programs, Schools, Topical research programs, etc.



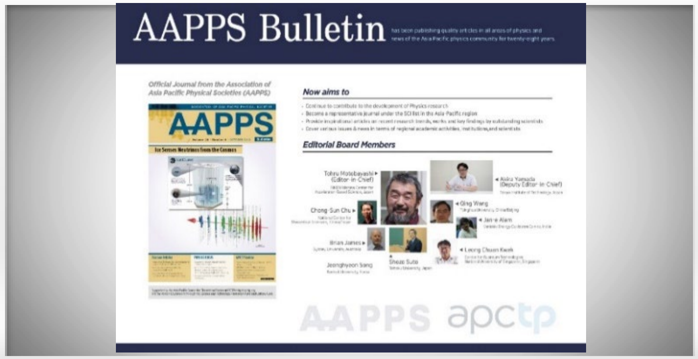


Science Diplomacy and Cooperation

Mission Contribute to increase the Global Common Wealth through Science.

We are working with AAPPS, APEC-PPSTI, IUPAP, AAAS, ASEAN

- ✓ Publication of *AAPPS Bulletin Journal*
- ✓ Develop Strategic Agenda in Science Diplomacy with APEC-PPSTI



Expected outcomes

- ✓ Build a science diplomatic bridge which connects the Asia Pacific region and Other regional Blocs.
- ✓ Form an active platform for global cooperation on science related social issues.

Collaborations with ICTP (International Center for Theoretical Physics)

Established in 1964 during the peak of Cold War
by *Abdus Salam*, endorsed by UNESCO and IAEA.
financially supported by the Italian government.

The first example of International cooperation based on Basic science

Why Theoretical Physics ?
Most non-political, Common asset of all Human Civilization
→ can be shared and spread with the least conflicts.

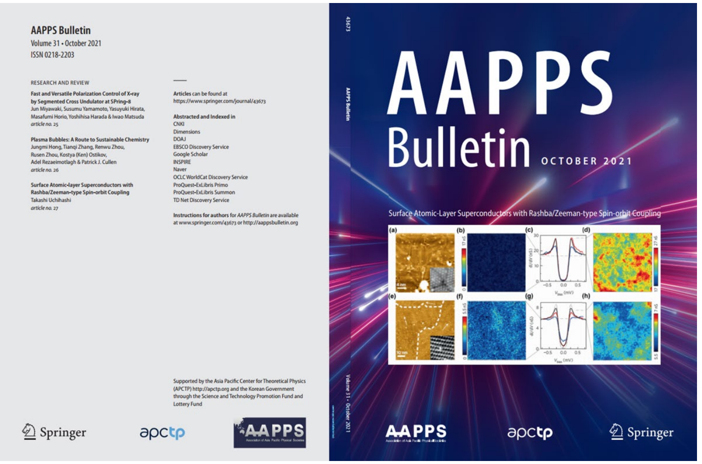
ICTP's mission is to:

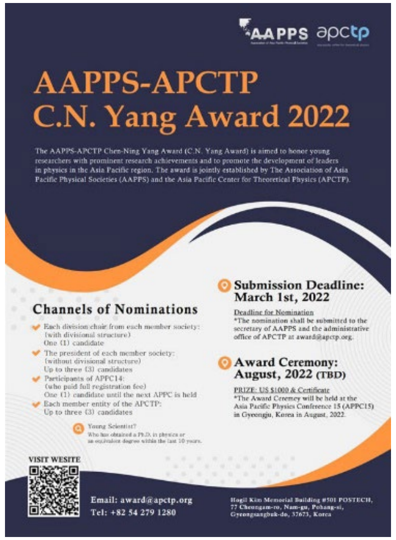
Foster the growth of advanced studies and research in physical and mathematical sciences, especially in support of excellence in developing countries.
Develop high-level scientific programs keeping in mind the needs of developing countries, and provide an international forum of scientific contact for scientists from all countries.
Conduct research at the highest international standards.

ICTP has played an important role for the communication between E-W and N-S.

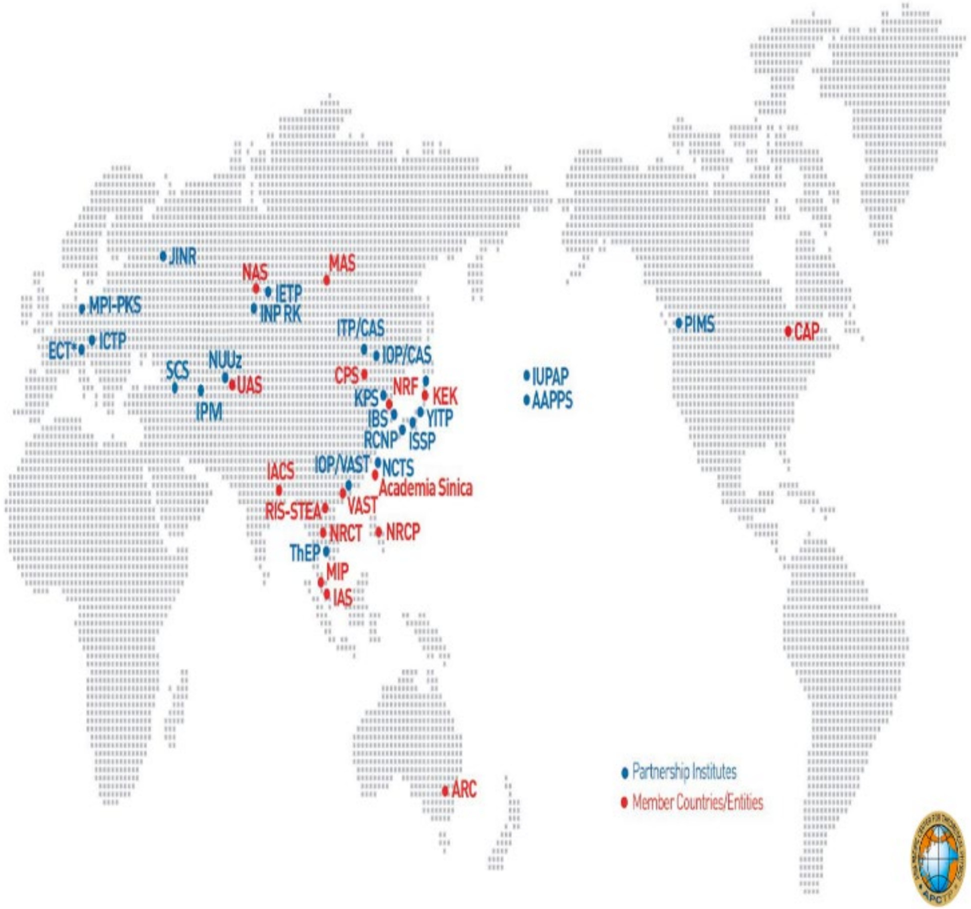
Collaborations with the AAPPS

1. Operating HQ Office of AAPPS
2. Jointly publish/promote the AAPPS bulletin as an International Research Journal
3. Support Divisional activities (**DPP, DACG, DNP, DCMP)**
4. Jointly awarding C N Yang Award
5. Support APPC conference every 3 years
6. Etc.





4. Various forms of Science Diplomacy



1. Support Less active countries in the AP-regions:
   (YST, APCTP-schools, South-Asia network program w/ ICTP)

2. Collaborations w/ Int'l science organizations:
   AAPPS, IUPAP, EPS, APS, etc

3. Targeted Int'l Collaborations :
-- JINR (Russia), NORDITA (Sweden)
-- Large Facility related programs :
   CERN (Europe), EIC (US), PAL (Pohang)

4. More Future programs ??

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Global Common Wealth and Cooperation through Theoretical Physics

Ministry of Science and ICT

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Contents

I. Motivation: Gravity beyond Einstein - Is it needed? :- Alternatives to Λ CDM ?

1. Gravity : Theoretical Aspect

- GR is an **effective theory** valid below UV cut-off, $M_{Pl} \sim 10^{19} GeV$

Ex) String theory $\xrightarrow[\text{Low Energy}]{}$ Einstein Gravity + higher curvature terms (α'-expansion)

- **Extreme fine-tuning ($\Lambda = 2,9\mathrm{x}10^{-122} \ell_P^{-2}$)** for Present accelerating Expansion (c.c. or DE)

– Holographic QCD & CMT

Maldacena, Witten 98; Gubser,Klebanov,Polyakov 98 etc.

Goal : Using the 5 dim. dual classical gravity, study 4 dim. strongly interacting QCD & CMT

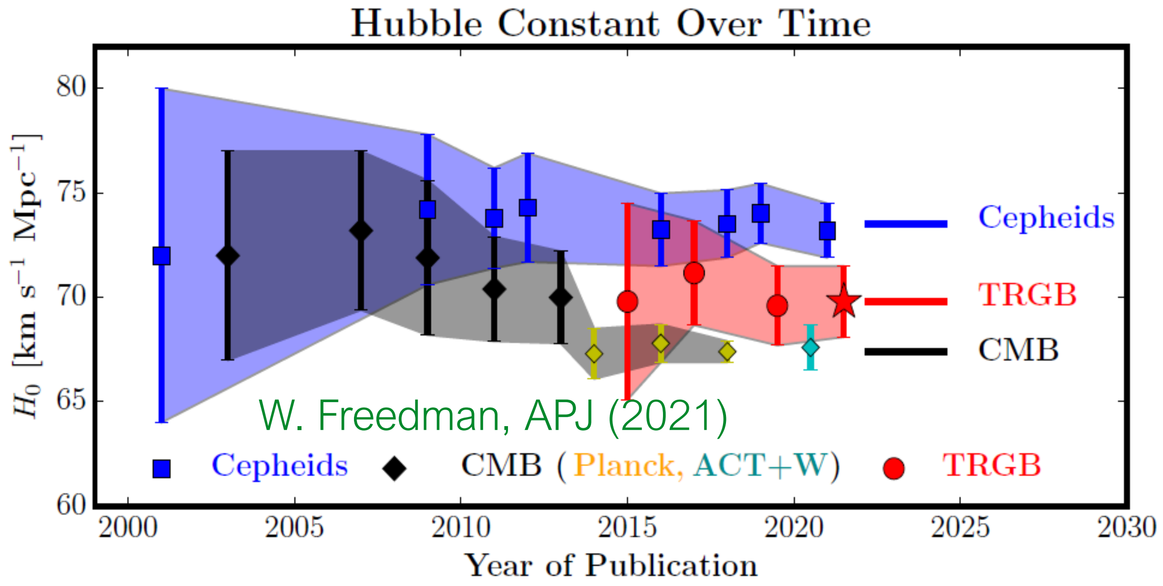
Needs the dual geometry (beyond Einstein) !

* BHs in high dimensions are quite diverse !

2. Cosmology (Λ CDM) - Observational Aspect

Observational H_0 tension

H_0 = 67.4 ± 0.5 km/s/Mpc (CMB),
= 73.5 ± 1.4 km/s/Mpc (SN & Cepheids)



3. Modified Gravity beyond Einstein

Q : Is it working better ? ⇒ We investigate

1) the Black Hole properties &

2) the implication to the cosmology.

II. Gravity with Gauss-Bonnet (G-B) term

II-1) Lovelock theory (dim. $D = 2t + 1$ or $2t$)

Lagrangian with only **1) metric 2)2nd order e.o.m** (for no ghosts and instabilities) will be in the following form

$$\mathcal{L}_D = \sqrt{-g} \sum_{n=0}^{t} \alpha_n L^n$$

L^n : Lovelock term, topological in 2n D

Ex) $L^1 = R$ Einstein-Hilbert term topol in 2 D

$L^2 = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}$

$= R_{GB}^2$ Gauss-Bonnet term. topol in 4 D

$$L^m = \frac{1}{2^m} \delta^{\mu_1\nu_1\mu_2\nu_2\cdots\mu_m\nu_m}_{a_1b_1a_2b_2\cdots a_mb_m} R_{a_1b_1}{}^{\mu_1\nu_1} R_{a_2b_2}{}^{\mu_2\nu_2} \cdots R_{a_mb_m}{}^{\mu_m\nu_m}$$

Euler characteristic of dim 2m topol in $2m$ D

$$\delta^{\mu_1\nu_1\mu_2\nu_2\cdots\mu_m\nu_m}_{a_1b_1a_2b_2\cdots a_mb_m} = (2m)!\, \delta^{\mu_1}_{[a_1} \delta^{\nu_1}_{b_1} \cdots \delta^{\mu_m}_{a_m} \delta^{\nu_m}_{b_m}$$

Ex) D-dim

$\mathcal{L}_2 = L^1 = \sqrt{-g}\ R$ topological

$\mathcal{L}_3 = L^1 = \sqrt{-g}\ R$

$\mathcal{L}_4 = L^1 + L^2 = \sqrt{-g}(R + R_{GB}^2) \approx \sqrt{-g}\ R$

$\mathcal{L}_5 = L^1 + L^2 = \sqrt{-g}(R + R_{GB}^2)$

$\mathcal{L}_6 = L^1 + L^2 + L^3 = \sqrt{-g}(R + R_{GB}^2 + R_{E.C}^3) \approx \sqrt{-g}(R + R_{GB}^2)$

$\mathcal{L}_7 = L^1 + L^2 + L^3 = \sqrt{-g}(R + R_{GB}^2 + R_{E.C}^3)$

Lovelock's theorem (in dim =4 (& 3)

The Einstein eqns (w/ c.c.) are the only possible 2nd-order eqns derived in 4 dim. solely from the metric.

Modification of GR needs to relax the assumptions of Lovelock's theorem.
→ Adding a **new degree of freedom (such as scalars)** other than the metric

II-2) Horndeski Theory - the most general scalar-tensor theory w/ 2nd-order field eqn in 4D

$$\mathcal{L} = G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + G_{4X}[(\Box\phi)^2 - \phi_{\mu\nu}\phi^{\nu\mu}]$$
$$+ G_5(\phi, X)G^{\mu\nu}\phi_{\mu\nu} - \frac{G_{5X}}{6}[(\Box\phi)^3 - 3\Box\phi\phi_{\mu\nu}\phi^{\nu\mu} + 2\phi_{\mu\nu}\phi^{\nu\lambda}\phi_{\lambda}{}^{\mu}]$$

higher derivative theories may have ghosts and Ostrogradsky instability :

Note : Horndeski theory is classified by 4 arbitrary functions $\{G_i(\phi, X),\ i = 2,3,4,5\}$.

Examples:

(i) Einstein Gravity is obtained by taking $G_4 = \frac{M_P{}^2}{2}$ (other $G_i = 0$)

$$S = \int d^4x \sqrt{-g}\frac{M_P{}^2}{2}R$$ Linear in curvature scalar

(ii) Brans-Dicke $f(R)$, k-inflation/k-essence, Quintessence gravity, etc

(iii) Gauss-Bonnet Term $S = -\frac{1}{2}\int d^4x \sqrt{-g}\,\xi(\phi)R_{GB}^2$ where $R_{GB}^2 = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$

can be shown to be realized (at the level of the e.o.m)

Horndeski, *Int. J. Theor. Phys.* **10** 363–84 (1974)
Charmousis, Copeland, Padilla & Saffin *Phys. Rev. Lett.* **108** 051101 (2012)

the Dilaton-Einstein-Gauss-Bonnet (DEGB) Gravity $f(\phi) = \alpha e^{\gamma\phi}$ polynomial etc.

$$S_{dEGB} = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa}R - \frac{\Lambda e^{\lambda\phi(r)}}{2\kappa} + f(\phi)R_{GB}^2 - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_m^{matt}\right]$$

Goal : To understand the physics due to the main parameters

II-3) Einstein Gauss-Bonnet Gravity

1) The general theory with quadratic curvature terms Λ in $d > 4$

$$S_{quad} = \int d^d x \sqrt{-g}\left[\frac{1}{2\kappa}\left(R - 2\Lambda + aR^2 + bR_{\mu\nu}R^{\mu\nu} + cR_{\mu\nu\rho\sigma}R^{\mu\nu\varrho\sigma}\right) + \mathcal{L}_m^{matt}\right]$$

The e.o.m. doen't include the derivatives of the curvatures only if $b = -4a$ & $c = a$

Gauss-Bonnet term
$R_{GB}^2 = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$

2) the Einstein-Gauss-Bonnet (EGB)- Λ Gravity (GB-AdS) in $d > 4$

$$S_{EGB-\Lambda} = \int d^d x \sqrt{-g}\left[\frac{1}{2\kappa}\left(R - 2\Lambda + \alpha R_{GB}^2\right) + \mathcal{L}_m^{matt}\right]$$

Note : $\Lambda = -\frac{(d-1)(d-2)}{2\ell^2}$

$\kappa = 8\pi G, \quad g = \det g_{\mu\nu}$

$[\alpha]$= [length]^2

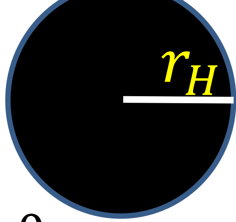
3) the Dilaton-Einstein-Gauss-Bonnet (DEGB) Gravity in $d = 4$ $\qquad f(\phi) = \alpha e^{\gamma\phi}$ polynomial etc.

$$S_{dEGB} = \int d^4 x \sqrt{-g}\left[\frac{1}{2\kappa}\left(R - 2\Lambda e^{\lambda\phi(r)} + f(\phi)R_{GB}^2\right) + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_m^{matt}\right]$$

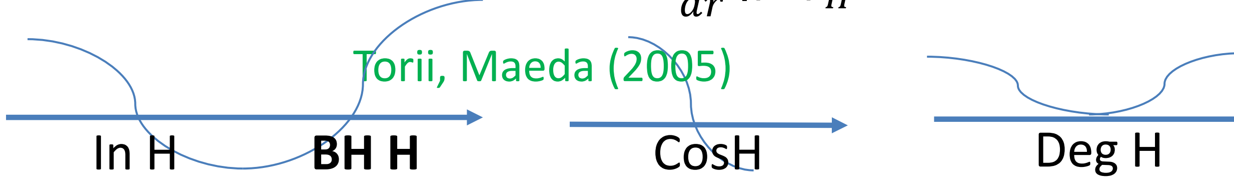
Goal : To understand the physics due to the main parameters

III. Black Holes (in d-dim)

$$ds^2 = -f(r)\,dt^2 + f^{-1}(r)\,dr^2 + r^2 d\Sigma_k^{d-2}$$



- Black hole horizon : if $\frac{df}{dr}|_{r=r_H} 0$
- Inner horizon : within a BH hor w/ $\frac{df}{dr}|_{r=r_H} < 0$
- Cosmological H : the outer most hor w/ $\frac{df}{dr}|_{r=r_H} < 0$
- Degenerate H : a horizon with $\frac{df}{dr}|_{r=r_H} = 0$



Horizon :a null hypersurface defined by $f(r_H) = 0$ w/ finite curvatures)

III-1) **Einstein theory** – Schwarzschild BH

Action

$$S = \int d^d x \sqrt{-g}\left[\frac{1}{2\kappa} R\right]$$

Eqns of motion

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$

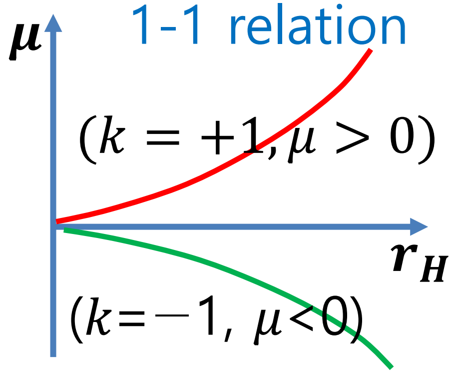
$$\kappa = 8\pi G,\ g = \det g_{\mu\nu}$$

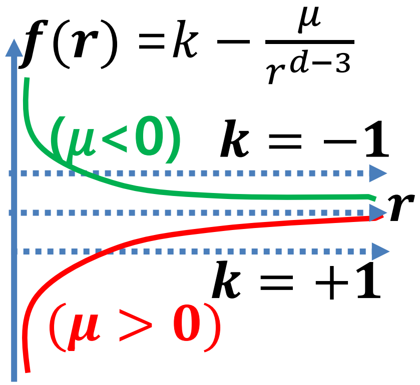
Black Hole solution

$$f(r) = k - \frac{\mu}{r^{d-3}} \xrightarrow{d=4;k=1} 1 - \frac{\mu}{r}\ (\mu > 0),$$

Horizon $(f(r_H) = 0)$& $(\boldsymbol{\mu}$–$\boldsymbol{r_H})$ **relation**

$$\mu = k r_H^{d-3} \xrightarrow{d=4;k=1} r_H \quad (\mu > 0)$$





Schwarzschild BH only for $k = +1, \mu > 0$

Note: ADM mass M

$$\mu = \frac{16\pi G}{(d-2)\Sigma_k^{d-2}} M$$

Ex) $\mu = 2GM\ (d = 4)$

$$\mu = \frac{8}{3\pi} GM\ (d = 5)$$

$(\boldsymbol{\mu = 0})$

$$ds^2 = -k\,dt^2 + k^{-1}\,dr^2 + r^2 d\Sigma_k^{d-2}$$

$k = +1$, Minkowski

$k = -1$, Hyperbolic

Note: $[S] = ML$;

$$[G] = \frac{L^{d-3}}{M};\ [\mu] = L^{d-3};$$

Σ_k^{d-2}: Einstein mfld $(R_{ij} \propto h_{ij})$, codim.2, curvature = k

Ex) $\Sigma_1^2 = S^2;\ \Sigma_0^2 = T^2; \Sigma_{-1}^2 = H^2$

$$d\Sigma_k^{d-2} = h_{ij}(x)\,dx^i dx^j$$

$$d\Sigma_k^2 = \begin{cases} d\Omega_{d-2}^2 & \text{for } k = +1 \\ \Sigma\, dx_i^2 & \text{for } k = 0 \\ dH_{d-2}^2 & \text{for } k = -1 \end{cases}$$

$$\Sigma_k^{d-2} = \int d^{d-2}x \sqrt{|h_{ij}|}$$

Singularity (spacelike) at $r = 0$

The Kretschmann invariant

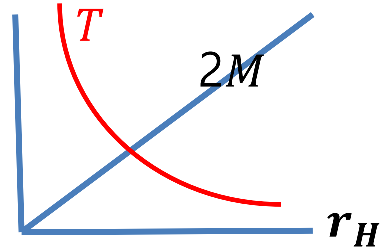
$$I \equiv R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \mathcal{O}\left(\frac{\mu^2}{r^{2(d-1)}}\right)$$

Black Hole	Thermodynamics
Mass	Energy
Area	Entropy
Surf grav	Temperature

Horizon

$$k r_H^{d-3} = \mu \xrightarrow{d=4;k=1} 2GM$$

or $M = \frac{1}{2G} r_H = \frac{1}{8\pi G}\frac{1}{T}$



Unstable (thermal)

$T;\ r_H;\ M$

Thermodynamics

Hawking Temperature

$$kT_H = \frac{\hbar \kappa_{SG}}{2\pi} = \frac{\hbar}{4\pi} f'(r_H) = \frac{\hbar(d-3)k}{4\pi r_H}$$

$$\xrightarrow{d=4;k=1} \frac{\hbar}{4\pi r_H} = \frac{\hbar c^3}{8\pi G M}$$

Surface Gravity $\kappa_{SG} = \frac{f'(r_H)}{2}$

$$f(r) = k - \frac{\mu}{r^{d-3}} \qquad \mu = k r_H^{d-3}$$

$$\Rightarrow f'(r_H) = (d-3)\frac{\mu}{r_H^{d-2}}$$

$$= (d-3)\frac{k}{r_H}$$

Note: For a BH, with the metric

$$ds^2 = -f(r)\,dt^2 + f^{-1}(r)\,dr^2 + r^2 d\Sigma_k^{d-2}$$

Near enough to the horizon,

$$f(r) = f'(r_H)(r - r_H) = 2\kappa_{SG}(r - r_H)$$

The Euclidean BH metric after "Wick rotation"

$$ds^2 = 2\kappa_{SG}(r - r_H)d\tau^2 + \frac{1}{2\kappa_{SG}(r-r_H)}dr^2 + r^2 d\Sigma_k^{d-2}$$

$$= d\rho^2 + \kappa_{SG}^2\rho^2 d\tau^2 + r^2 d\Sigma_k^{d-2};$$

$$\rho = \frac{1}{\kappa_{SG}}\sqrt{2\kappa_{SG}(r - r_H)}$$

For no conical singularity at the origin,

$$\tau\text{-period} = \frac{2\pi}{\kappa_{SG}} = \frac{4\pi}{f'(r_H)} \equiv \beta = \frac{1}{T_H}$$

Entropy

$$S = \frac{\text{Area}}{4\hbar G}$$

$$= \frac{\pi r_H^2}{\hbar G} = \frac{1}{16\pi\hbar G}\frac{1}{T^2}$$

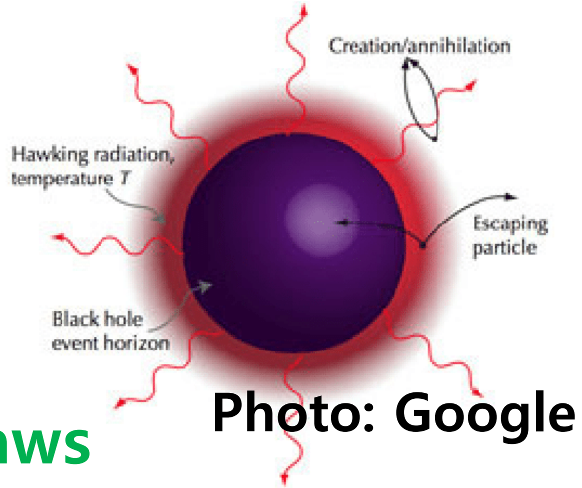


Photo: Google

Thermodynamic Laws

$$F(T) = M - TS = \frac{r_H}{4G} = \frac{1}{16\pi G}\frac{1}{T};\ dF = -SdT,$$

$$C_V = \frac{dM}{dT} = -\frac{1}{8\pi G}\frac{1}{T^2} < 0$$: **Unstable**

(Hawking radiation)

A **BH** in **asymptotically flat space** is **thermodynamically unstable (**Hawking Radiation).

Question: How to make the BH thermodynamically stable?

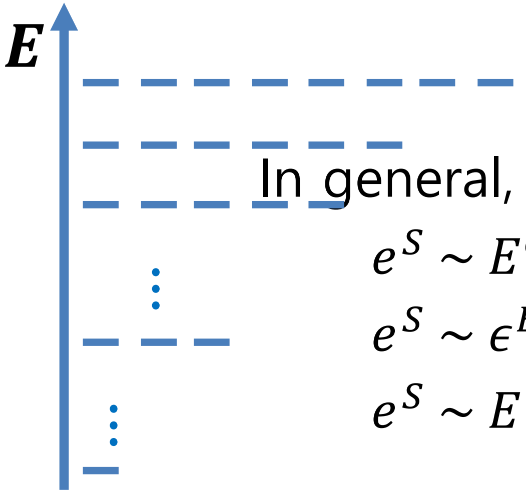
Method 1) Place the BH inside a finite spherical cavity. T is fixed at the surface of the cavity,
Method 2) Put the BH in AdS space ($\Lambda < 0$), which stabilizes BH by acting as a reflecting box.

Question: Information loss? No unitary evolution for the Hawking radiation? cf) Page curve
Does the Hawking radiation change the pure quantum state into a mixed state?

Thermodynamics

Entropy

e^S = # of configurations (states)



Temperature

$$\frac{1}{T} = \frac{dS}{dE}$$

In general, the entropy S

$e^S \sim E^{\epsilon}$, then, $S \sim \ln E \ll E^2$

$e^S \sim \epsilon^{E}$, then, $S \sim E \ll E^2$

$e^S \sim E! \sim E^E$, then, $S \sim E \ln E \ll E^2$

Black Holes

Entropy ($r_H = 2GM$) $S = Area/4 = \pi r_H^2 = 4\pi G^2 M^2$

$e^S \sim \epsilon^{E^2}$

Stability

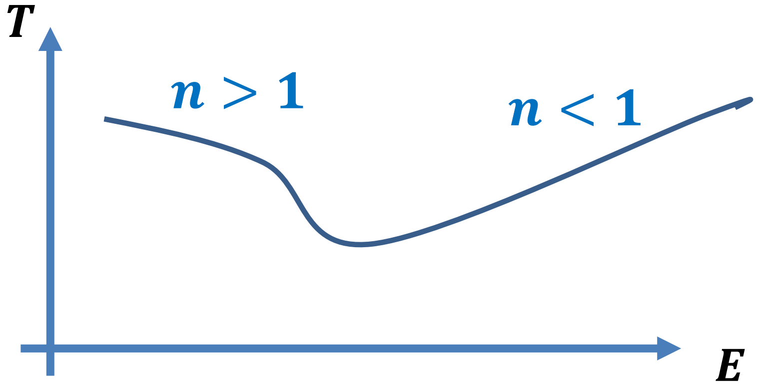
$$\frac{\partial^2 \mathcal{F}}{\partial x \partial y} \geq 0$$

Ex) Heat Capacity

$$C = \frac{dE}{dT}$$

$S \sim E^n \rightarrow T \sim E^{1-n}$,

$$C = \frac{dE}{dT} \sim E^n$$



III-2) Schwarz $\mathbf{AdS}_d$ Black Holes

Note:
Dimension (c=1) $[S] = ML\,;[G] = \frac{L^{d-3}}{M}; [\mu] = L^{d-3}; [\ell^2] = L^2$

$$\Lambda = -\frac{(d-1)(d-2)}{2\ell^2} < 0$$ Cosmol Const for AdS

Action

Birmingham (1999); Emparan (1999)

$$S = \frac{1}{2\kappa}\int_{\mathcal{M}} d^d x\,\sqrt{-g}\,[\,(R - 2\Lambda)\,]$$
$$+\frac{1}{\kappa}\int_{\partial\mathcal{M}} d^{d-1}x\,\sqrt{-h}\,K + S_{ct}$$

K = Trace of the extrinsic curvature (Tr of the 2nd fundamental form)

h the induced metric on the boundary

Eqns of motion (vacuum)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda\,g_{\mu\nu} = 0$$

$\kappa = 8\pi G,$
$g = \det g_{\mu\nu};$

or

Einstein manifold solution:

$$R = \frac{d}{d-2}2\Lambda; \quad R_{\mu\nu} = \frac{2\Lambda}{d-2}g_{\mu\nu}$$
$$R = -\frac{d(d-1)}{\ell^2}; \quad R_{\mu\nu} = -\frac{(d-1)}{\ell^2}g_{\mu\nu}$$

Note :

1) The BH is an Einstein spacetime, if the horizon is an Einstein space of +, 0, − curvature.
 i.e., $R_{\mu\nu} = -\frac{(d-1)}{\ell^2}g_{\mu\nu}$ (Einstein space w/ $\Lambda < 0$)
 if hor is Einstein mfld $R_{ij}(h) = (d-3)kh_{ij};\ k = +1,0,-1$

2) $\mu = 0 \Longrightarrow$ (locally) AdS $R_{\mu\nu\rho\sigma} = -\frac{1}{\ell^2}\left(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}\right)$
if hor : const curvature $R_{ijkl}(h) = k\left(g_{ik}g_{jl} - g_{il}g_{jk}\right)$

3) Solutions classified by k and μ.

III-2) Schwarz AdS_d Black Holes

Action

Birmingham (1999); Emparan (1999)

$$S = \frac{1}{2\kappa}\int d^d x\,\sqrt{-g}\,[\,(R - 2\Lambda)\,]$$

$$+\frac{1}{\kappa}\int d^{d-1}x\,\sqrt{-h}\,K + S_{ct}$$

Eqns of motion (vacuum)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda\, g_{\mu\nu} = 0$$

$\kappa = 8\pi G$, $g = \det g_{\mu\nu}$;

or

Einstein manifold solution:

$$R = \frac{d}{d-2}2\Lambda; \quad R_{\mu\nu} = \frac{2\Lambda}{d-2}g_{\mu\nu}$$

$$R = -\frac{d(d-1)}{\ell^2}; \quad R_{\mu\nu} = -\frac{(d-1)}{\ell^2}g_{\mu\nu}$$

Black Hole solution

$$ds^2 = -f(r)\,dt^2 + f^{-1}(r)\,dr^2 + r^2 d\Sigma_k^{d-2}$$

$$f(r) = k - \frac{\mu}{r^{d-3}} + \frac{r^2}{l^2}$$

$$\mu = \frac{16\pi G}{(d-2)\Sigma_k^{d-2}}M;\ M\text{:ADM mass}$$

Note:

Dimension (c=1) $[S] = ML$; $[G] = \frac{L^{d-3}}{M}$; $[\mu] = L^{d-3}$; $[\ell^2] = L^2$

$$\Lambda = -\frac{(d-1)(d-2)}{2\ell^2} < 0$$ Cosmol Const for AdS

K = Trace of the extrinsic curvature

Note :

1) The BH is an Einstein spacetime, if the horizon is an Einstein space of +, 0, − curvature.

i.e., $R_{\mu\nu} = -\frac{(d-1)}{\ell^2}g_{\mu\nu}$ (Einstein space w/ $\Lambda < 0$)

if hor is Einstein mfld $R_{ij}(h) = (d-3)kh_{ij}$; $k = +1, 0, -1$

2) $\mu = 0 \Longrightarrow$ (locally) AdS $R_{\mu\nu\rho\sigma} = -\frac{1}{\ell^2}\left(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}\right)$

if hor : const curvature $R_{ijkl}(h) = k\left(g_{ik}g_{jl} - g_{il}g_{jk}\right)$

3) Solutions classified by k and μ.

Σ_k^{d-2}: Einstein mfld (codim.2) ($R_{ij} = (d-3)kh_{ij}$)

Metic $d\Sigma_k^{d-2} = h_{ij}\,(x)\,dx^i dx^j$

$$= \begin{cases} d\Omega_{d-2}^2 & k = +1 \text{ sphere} \\ \sum_{i=1}^{d-2} d\,x_i^2 & k = 0 \text{ plane} \\ dH_{d-2}^2 & k = -1 \text{ hyperbolic space} \end{cases}$$

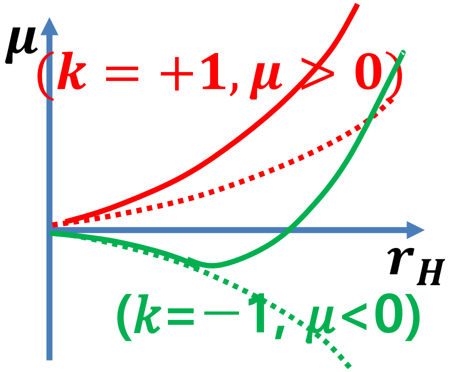
Volume $\Sigma_k^{d-2} = \int d^{d-2}x\sqrt{|h_{ij}|}$

Horizon $f(r_H) = 0$ & **(μ–r_H) relation** (Schwarz **AdS BH**)

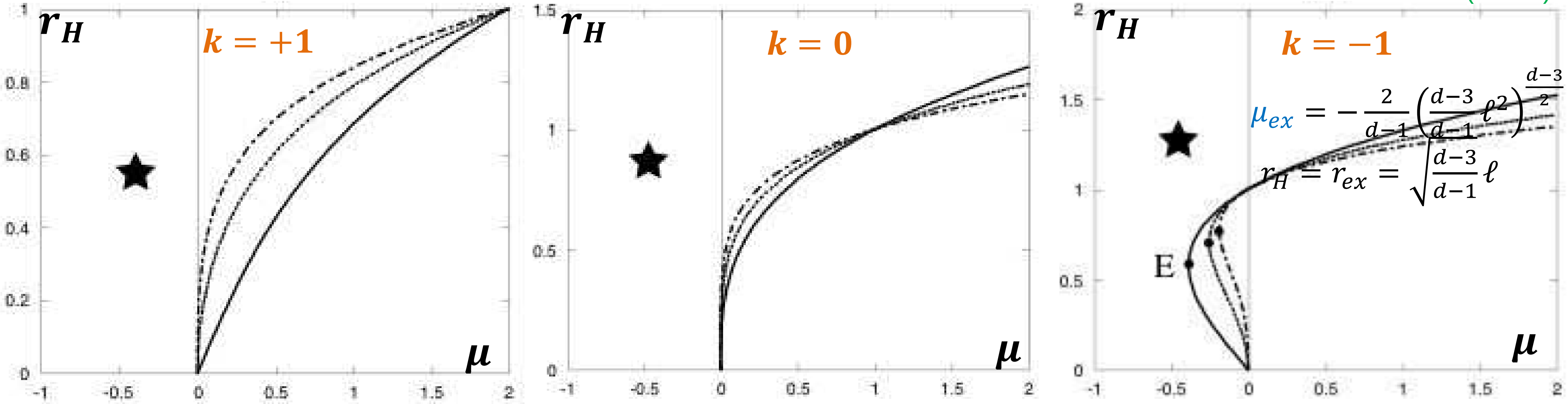
$$f(r) = k - \frac{\mu}{r^{d-3}} + \frac{r^2}{l^2}$$

$$\mu = r_H^{d-3}\left(k + \frac{r_H^2}{\ell^2}\right)$$

Solutions classified by k and μ.



Torii, Maeda
PRD(2005)



Thermodynamics - Schwarz **AdS Black Holes:Phases** (1-parameter)

Hawking Temperature $\mu = r_H^{d-3}\left(k + \frac{r_H^2}{\ell^2}\right)$ $f(r) = 1 - \frac{\mu}{r^{d-3}} + \frac{r^2}{l^2}$

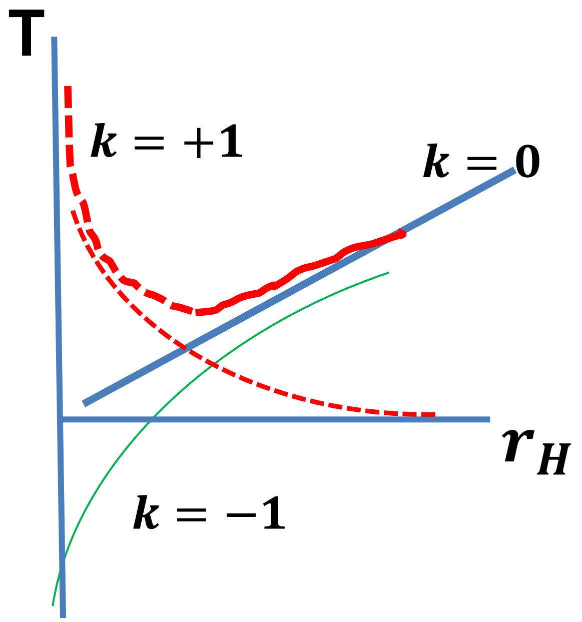
$$T_H = \frac{1}{4\pi} f'(r_H) = \frac{1}{4\pi}\left((d-3)\frac{\mu}{r_H^{d-2}} + 2\frac{r_H}{\ell^2}\right) = \frac{1}{4\pi}\left(\frac{(d-3)k}{r_H} + (d-1)\frac{r_H}{\ell^2}\right)$$

or $\beta = \frac{4\pi\ell^2 r_H}{(d-1)r_H^2 + k(d-3)\ell^2}$

Or $r_H = \frac{2\pi\ell^2 T_H}{d-1}\left[1 + \sqrt{1 - k\frac{(d-1)(d-3)}{4\pi^2\ell^2 {T_H}^2}}\right]$

Surface Gravity

$\kappa_{SG} = \frac{f'(r_H)}{2}$



Note :

For $k = +1$, (Schw. AdS BH)

(1) $T \geq T_{min} = \frac{\sqrt{2}}{\pi\ell}$,

(2) Two branches:

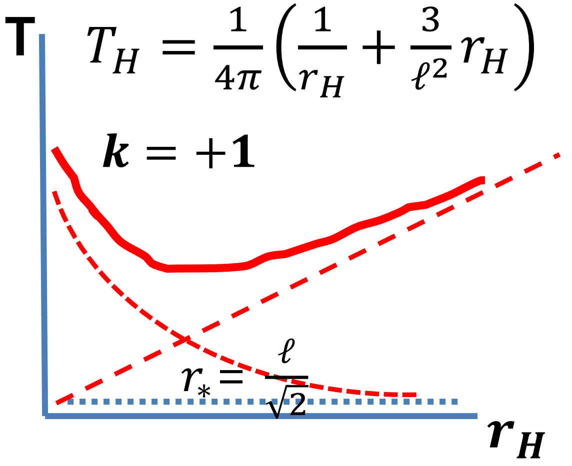
Small BH ($r_H \ll \ell$) is unstable (like SSBH), while

Large BH ($r_H \gg \ell$) is stable.

(3) Hawking-Page Tr.

Ex) AdS4 $(k = +1)$

$T_H = \frac{1}{4\pi}\left(\frac{1}{r_H} + \frac{3}{\ell^2} r_H\right)$



Small BH:C<0, unstable

Large BH : Stable,

Hawking-page Transition

Gravitational **Partition function**) (the Euclidean path integral) : **Canonical Ensemble**

$$Z[\beta] = \int [dg][d\Phi_{matter}]\, e^{-I_{Euc}} = e^{-\beta F} \qquad -\ln Z = I_{Euc} = \beta F$$

(for X_2 = **AdS SS BH** wrt X_1 = **AdS$_d$**/Z)

$$I_{Euc} = -\frac{1}{16\pi G}\int d^d x \sqrt{-g}\,[R - 2\Lambda] = \frac{(d-1)}{8\pi G \ell^2}\big(V_2(R) - V_1(R)\big)$$
$$= \frac{\Sigma_1^{d-2}}{4G}\,\frac{\ell^2 r_H^{d-2} - r_H^d}{(d-1)r_H^2 + (d-3)\ell^2}$$

$$2\Lambda = -\frac{(d-1)(d-2)}{\ell^2}$$

For $T < T_0$, only thermal AdS4 (no BH)

$$F = -T\ln Z = -\frac{\pi^4}{30} g T^4 \ell^3 + \mathcal{O}(\ell T^2)$$

For $T > T_0$, two AdS4 BHs

large BH : stable

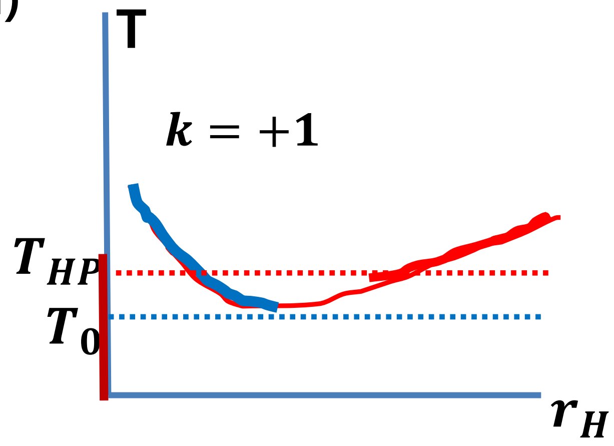
small BH : unstable->Thermal AdS

For $T_0 < T < T_{HP} = \frac{1}{\pi\ell}$, $F > 0$:

BH evaporate into thermal AdS

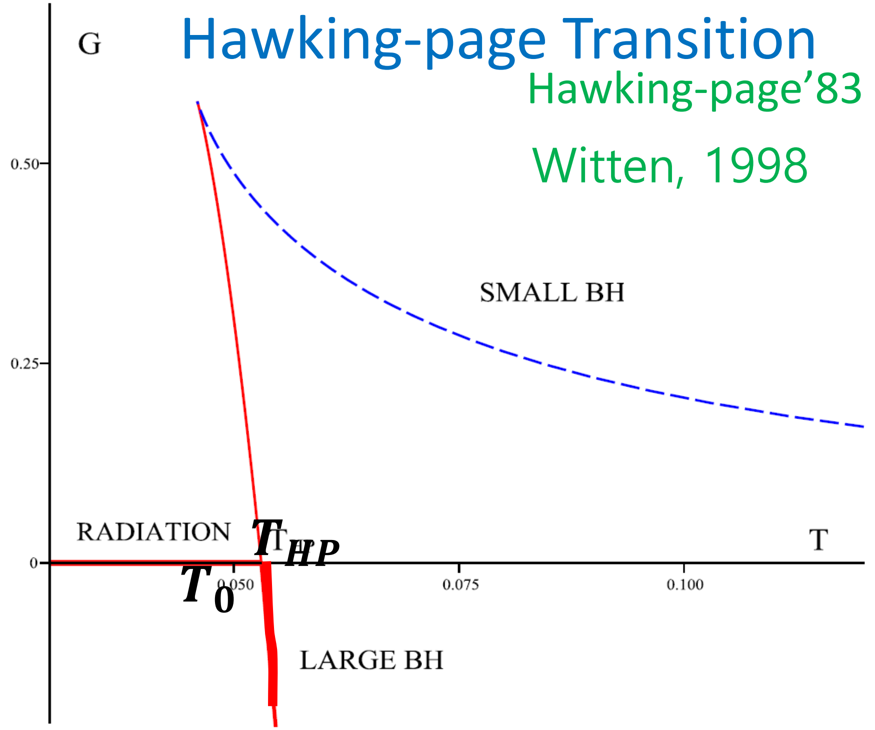
For $T > T_{HP}$, $F < 0$ (large BH):

thermal AdS tunnels into BH



Small BH:C<0, unstable

Large BH Stable,



Phase Transitions are described in terms of the geometry change.

RN**AdS BH**

Black Hole solution

$$ds^2 = -f(r)\ dt^2\ + f^{-1}(r)\ dr^2\ + r^2 d\Sigma_k^2$$

$$f(r) = k - \frac{\mu}{r^{d-3}} + \frac{q^2}{r^{2(d-3)}} + \frac{r^2}{l^2};$$

$$A = \left(-\frac{1}{c}\frac{q}{r^{d-3}} + \Phi\right)dt$$

$$c = \sqrt{\frac{2(d-3)}{d-2}} \quad \text{and } \Phi = \frac{1}{c}\frac{q}{r_H^{d-3}}$$

Horizon-Mass

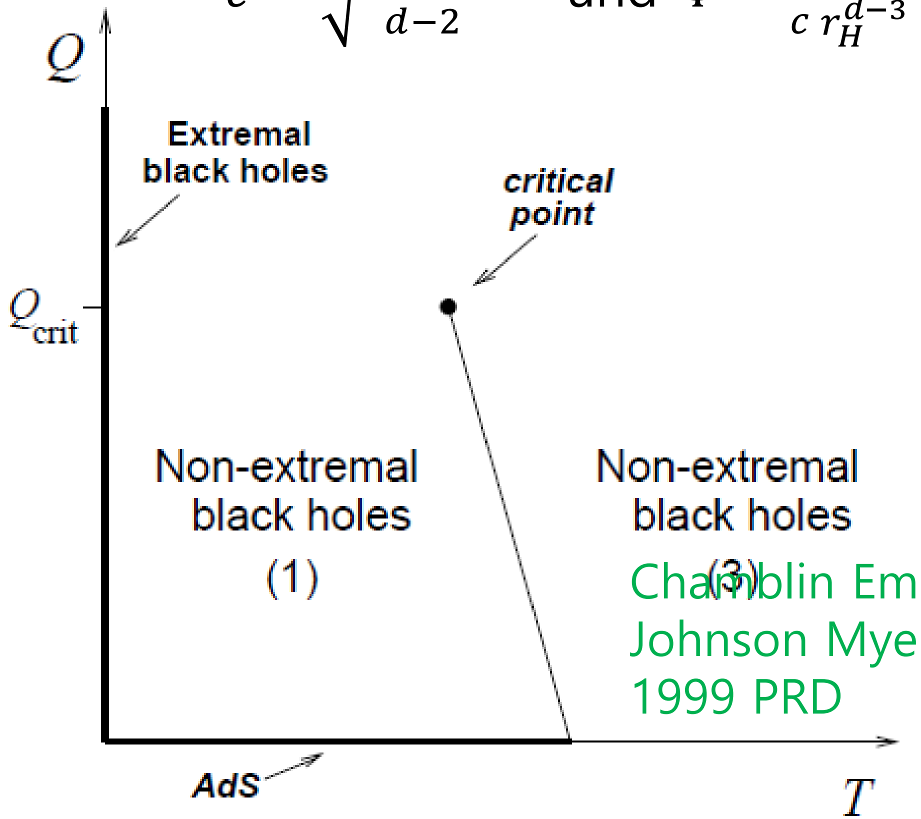
$$\mu = r_H^{d-3}\left(k + \frac{q^2}{r_H^{2(d-3)}} + \frac{r_H^2}{\ell^2}\right) = k r_H^{d-3} + \frac{q^2}{r_H^{(d-3)}} + \frac{r_H^{d-1}}{\ell^2}$$

Hawking Temperature

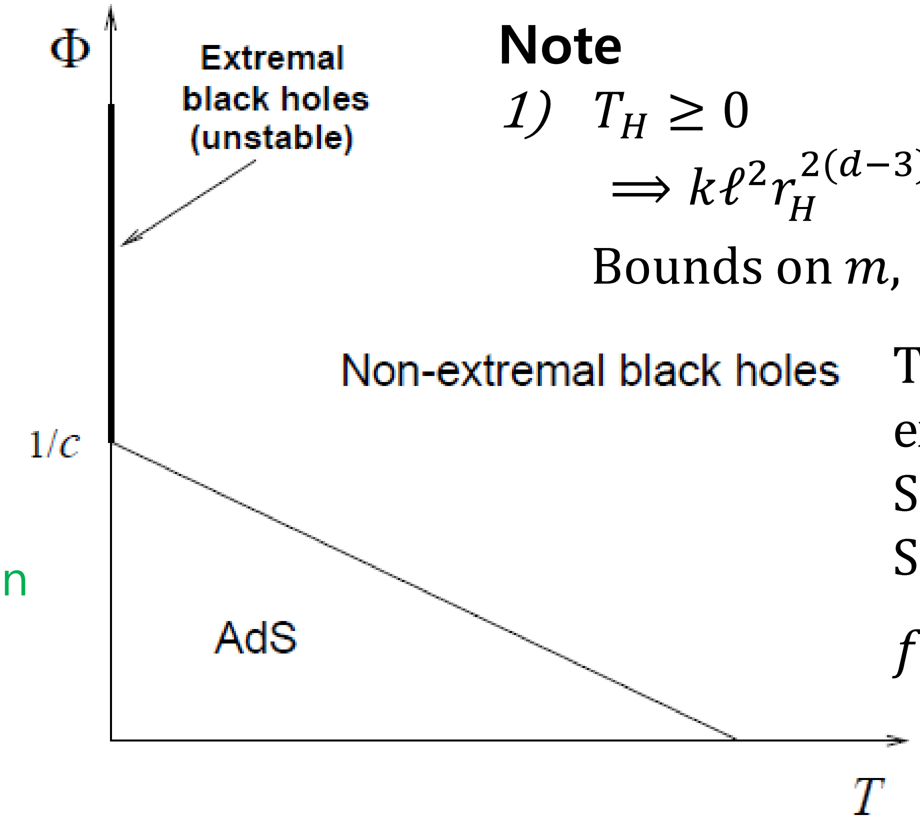
$$T_H = \frac{1}{4\pi} f'(r_H) = \frac{1}{4\pi}\left(k(d-3)\frac{1}{r_H} - \frac{(d-3)q^2}{r_H{}^{2(d-3)+1}} + \frac{d-1}{\ell^2} r_H\right)$$

Entropy

$$S = \frac{Area}{4G} = \frac{\Sigma_{d-2}^1}{4G} r_H^{d-2} = \frac{\pi}{2(d-2)\tilde{\Gamma} G} r_H^{d-2}$$



Chamblin Emparan
Johnson Myers
1999 PRD



Note

1) $T_H \geq 0$

$$\Longrightarrow k\ell^2 r_H^{2(d-3)} + \frac{d-1}{d-3} r_H^{2(d-2)} \geq \ell^2 q^2$$

Bounds on m, $m \geq m_e(q, \ell) > 2q$

The inequality saturated is extremal BH, nonSUSY.
SUSY : bounds $m \geq 2q$,
SUSY solution:

$$f(r) = \left(1 - \frac{q}{r^{d-3}}\right)^2 + \frac{r^2}{l^2} > 0$$

: naked singularity

III-3) RN**AdS in Einstein-Gauss-Bonnet (d>4)**

R. -G. Cai, Phys. Rev. D (2002).

I. Jeon, B-HL, W. Lee, M. Mishra, 2407.20016

Action

$$S_{EGB-\Lambda} = \int d^d x \sqrt{-g}\left[\frac{1}{2\kappa}\left(R + \frac{(d-1)(d-2)}{l^2} + \alpha_{GB} R_{GB}^2\right) + \mathcal{L}_m^{matt}\right]$$

Black Hole solution

$$ds^2 = -f(r)\, dt^2 + f^{-1}(r)\, dr^2 + r^2 d\Sigma_k^2$$

$$f(r) = k + \frac{r^2}{2\tilde{\alpha}}\left(1 \mp \sqrt{1 - \frac{4\alpha}{\ell^2}}\sqrt{1 + \frac{\mu}{r^{d-1}} - \frac{q^2}{r^{2(d-2)}}}\right)$$

$$A(r) = \left(-\frac{1}{c}\frac{q}{r^{d-3}} + \Phi\right) dt \qquad c = \sqrt{\frac{2(d-3)}{d-2}} \qquad \text{and } \Phi = \frac{1}{c}\frac{q}{r_H^{d-3}}$$

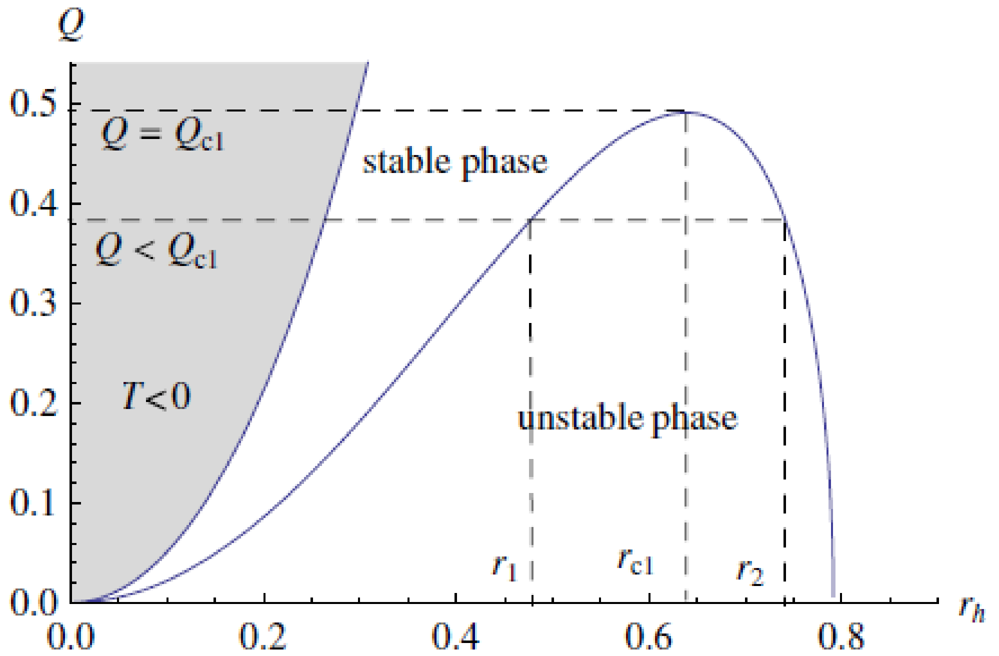
Horizon

$$M = \frac{(d-1)Q^2 r_H^8 + 2\pi r_H^{2d}(d-3)\left((d^2-3d+2)(k r_H^2 + k^2\alpha) - 2\Lambda r_H^4\right)}{8\pi^2(d^2-4d+3) r_H^{d+5}} \Sigma_{\mathrm{d-2}}^k$$

Hawking Temperature

$$T_H = \frac{1}{4\pi} f'(r_H) = \frac{-Q^2 r_H^8 + 2\pi r_H^{2d}\left((d-2)k\left((d-3)r_H^2 + (d-5)k\alpha\right) - 2\Lambda r_H^4\right)}{32\pi^2(d-2) r_H^{2d+1}(2k\alpha + r_H^2)}$$

Near Extremal behavior etc.
I. Jeon, BHL, W. Lee, M. Mishra, in preparation



III-4) dEGB theory - Black Holes $(d = 4)$

$$S_{dEGB} = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa}\left(R - 2\Lambda e^{\lambda\phi(r)} + f(\phi)R_{GB}^2\right) + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_m^{matt}\right]$$

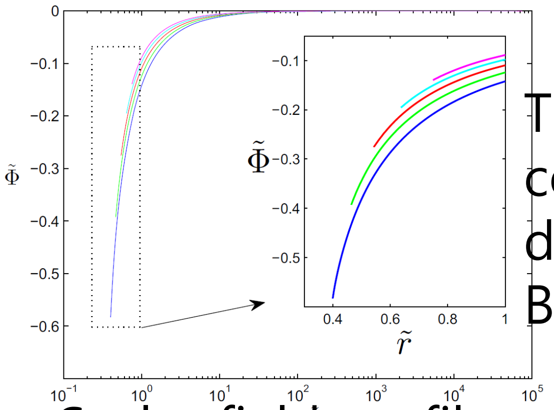
Guo, Ohta & Torii, Prog.Theor.Phys. (2008); (2009); (2010);
Maeda, Ohta Sasagawa, PRD(2009);(2011) Ohta Torii, PRD (2013).

Note :

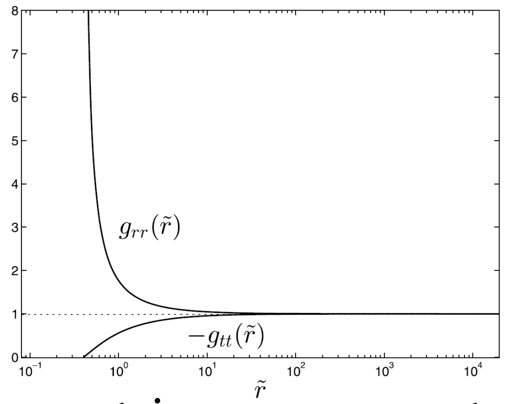
1) For $\gamma \longrightarrow 0$, DEGB $\longrightarrow$ EGB (the GB becomes the bdry term)
2) The symmetry under $\gamma \rightarrow -\gamma$, $\phi \rightarrow -\phi$ allows choosing γ positive.
3) α scaling r → r/$\sqrt{|\alpha|}$ absorbs α dependency.
   Sign of α is important (can't be absorbed)
4) DEGB BH
- Hair Charge Q ≠ 0, and is
- not independent charge
- : secondary hair.

DEGB BH solutions (γ = 1/6, α = 1/16)

(α > 0)



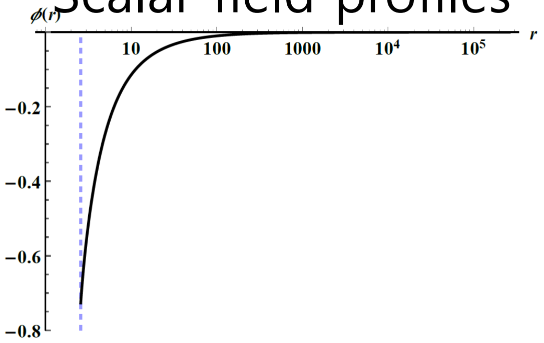
The five colors correspond to different DEGB BH solutions.



Scalar field profiles

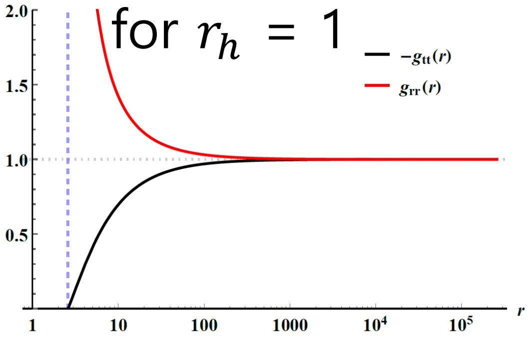
the metric components for r_h = 1

(α < 0)



(b) $\phi(r)$ vs. r

BHL, W. Lee, D. Rho, PRD (2019)

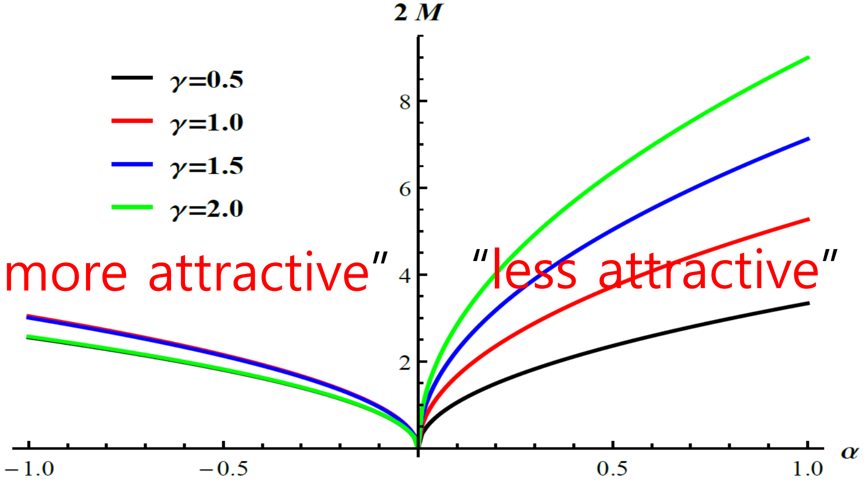


(a) $-g_{tt}(r)$ and $g_{rr}(r)$ vs. r

New Properties of the Black Holes

∃ Scalar Hair,

∃ M_{min}. such that BH mass $M \geq M_{min}$

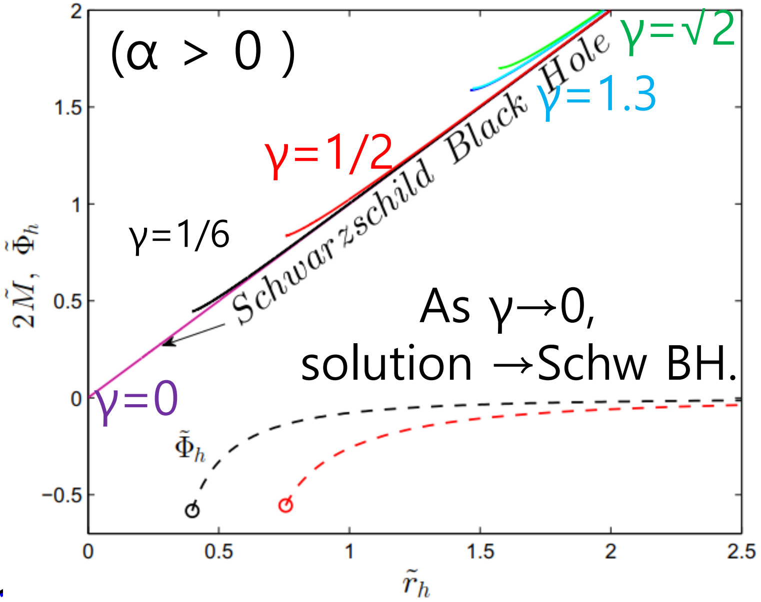


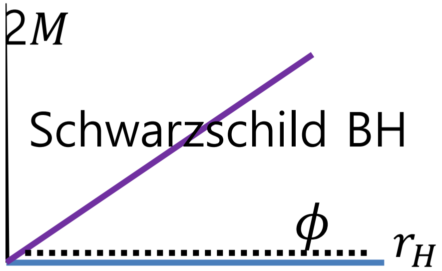
"relatively more attractive"   "less attractive"

(α < 0)
M_{min} ↗ then ↘ as γ ↗
(max. of M_{min} at γ=γ_{max})

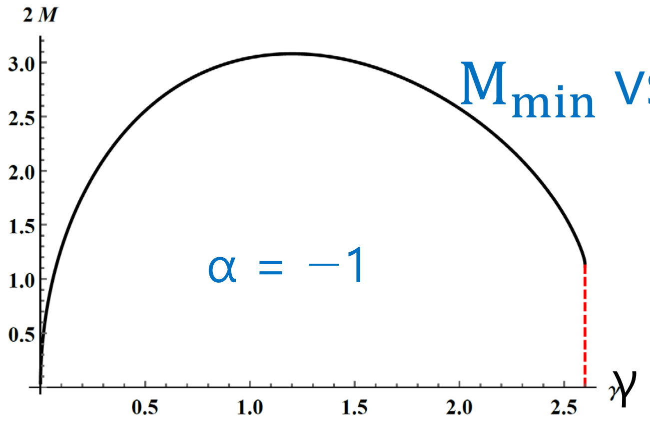
(α > 0)
M_{min} ↗ as γ ↗

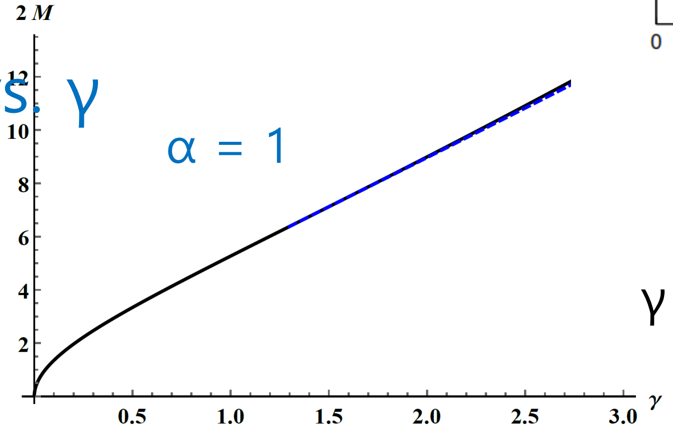
minimum mass → New Phase?





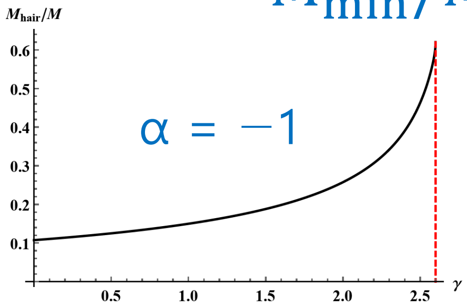
M_{min} vs. γ



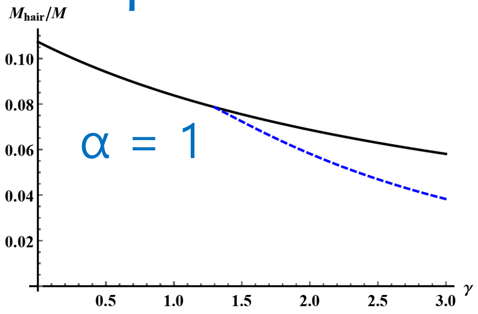


γ

M_{min}/M vs. γ



(a) The hairy mass ratio vs. γ with α = −1.



(b) The hairy mass ratio vs. γ with α = 1.

GB term → makes gravity "less attractive" (for α >0) (making the black hole "smaller") !!!

Einsten Gauss-Bonnet (EGB) theory

W.Ahn, B. Gwak, BHL, W.Lee, Eur.Phys.J.C (2015)

Action

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa} R + \alpha R^2_{GB} - \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right]$$

$R^2_{\rm GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ The Gauss-Bonnet term

$\kappa \equiv 8\pi G$

$g = \det g_{\mu\nu}$

Eqns of motion

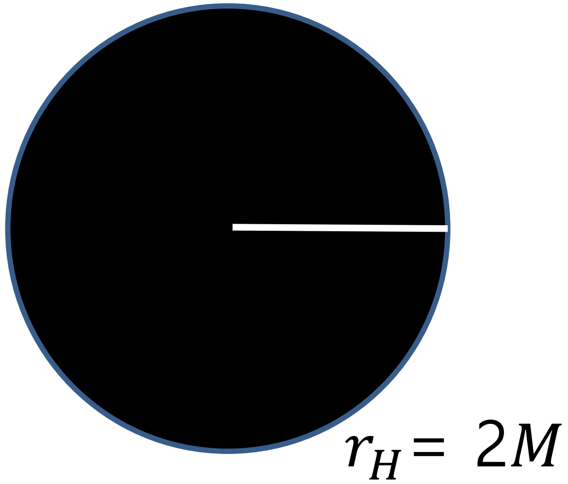
$$G_{\mu\nu} = \kappa T_{\mu\nu} = \kappa\left(\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2} g_{\mu\nu}\partial_\rho\Phi\partial^\rho\Phi\right)$$

$$\frac{1}{\sqrt{-g}}\partial_\mu[\sqrt{-g}\, g^{\mu\nu}\partial_\nu\Phi] = 0$$

Black Hole solution

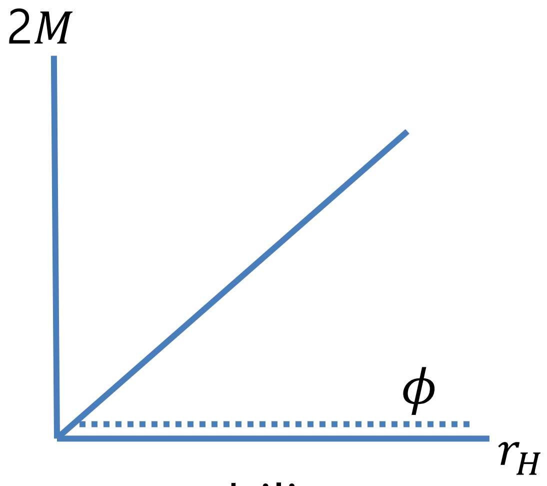
$$ds^2 = -(1-\frac{2M}{r})\, dt^2 + \frac{dr^2}{(1-\frac{2M}{r})} + r^2 d\Omega^2$$

$\phi = 0$ No hair



Horizon

$r_H = 2M$



Note :

1) For the coupling α = 0, the theory becomes the Einstein gravity.

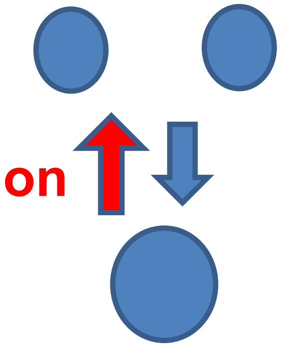
2) GB term is a surface term, not affecting the e.o.m. Hence,
The black hole solution is the same as that of the Schwarzschild one.

3) However, the GB term contributes to the black hole entropy and influence stability.

(In)stability of the DEGB Blackholes under fragmentation

B. Gwak & BHL, PRD (2015).

B.Gwak, BHL, D. Rho, PL.B (2016)



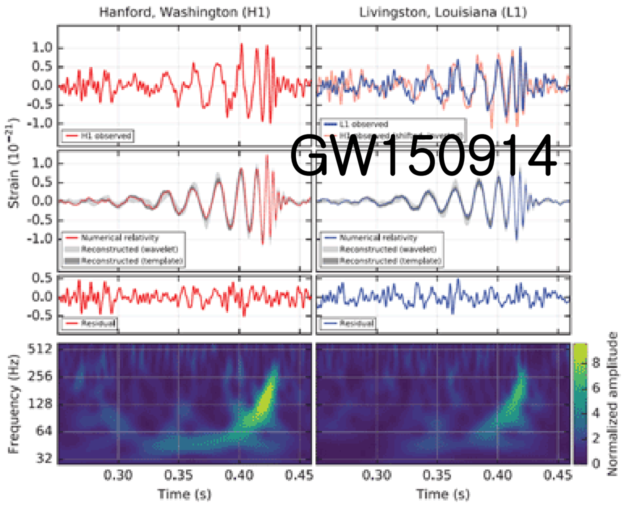
B) fragmentation

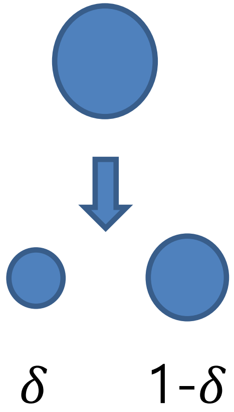
?

A) Merging

+ Gravitational Wave

Observed!





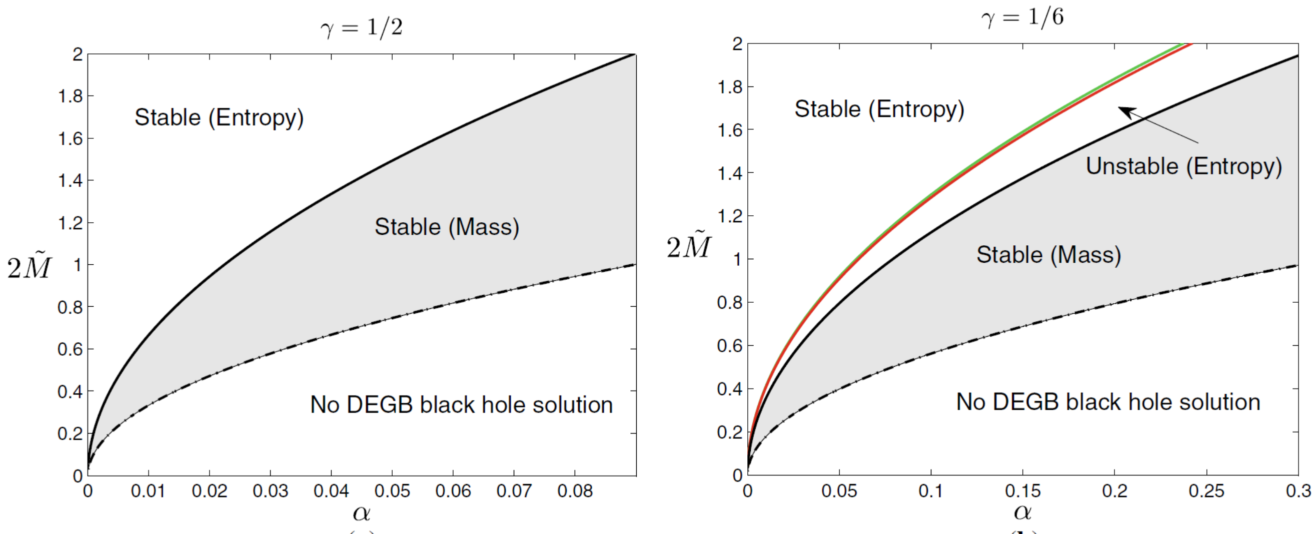
B) Fragmentation Process : one BHs → two BH ?

There exists parameter range where the BHs are unstable under the fragmentation.

Schwarzschild BHs is marginally stable under shooting off the infinitesimal mass BH .

$$\frac{S_f}{S_i} = \frac{M_1^2+M_2^2}{(M_1+M_2)^2} = \frac{(\delta r_h)^2+((1-\delta)r_h)^2}{r_h^2} = \delta^2 + (1-\delta)^2 \leq 1$$

(equality only when $M_2 \cdot M_2 = 0$)



(a) (b)

Note :

1) It cannot decay into black holes with mass smaller than the minimum mass M_{min}. Hence, $\delta_m \leq \delta \leq 1/2$, $\delta_m = {M_{min}}/{M}$.

2) The BHs with $M < 2M_{min}$ are absolutely stable. The black hole can be fragmented only when its mass exceeds twice of minimum mass.

IV. Dilaton-Einstein-Gauss-Bonnet (dEGB) Cosmology

A. Biswas, A. Kar, **BHL**, H. Lee, W. Lee, **S. Scopel**, L. Yin **JCAP08 (2023) 023**

A. Biswas, A. Kar, **BHL**, H. Lee, W. Lee, **S. Scopel**, L. Yin **arXiv 2405.15998**

Action

$$S_{dEGB} = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa}R + f(\phi)R^2_{GB} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_m\right]$$

$$R^2_{GB} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

Gauss-Bonnet term

$$\kappa \equiv 8\pi G,\ [\kappa] = \sqrt{\frac{[L]}{[M]}}$$

$$\mathcal{L}_m = \mathcal{L}_{SM} + \mathcal{L}_{CDM} - \frac{1}{\kappa}\Lambda \to \mathcal{L}_{rad}$$

Note:

1) If $f(\phi)$ = const, the theory is reduced to a **quintessence model.**

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_m^{rad} + \mathcal{L}_m\right]$$

(dEGB $\xrightarrow{\text{GB term dropped}}$ **Quintessence**)

2) If $f(\phi)$ = const and ϕ = const, the theory is reduced to **Standard ΛCDM**.

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa}R + \mathcal{L}_m - \frac{1}{\kappa}\Lambda\right] \quad \mathcal{L}_m = \mathcal{L}_{rad} + \mathcal{L}_{matt} + \mathcal{L}_{CDM}$$

(dEGB $\xrightarrow{\text{GB, } \phi \text{ dropped}}$ **ΛCDM**)

3) WIMPs

WIMPs decouple in the rad dom era, hence will take $\mathcal{L}_m = \mathcal{L}_{rad} + \mathcal{L}_{DM}^{WIMP}$.

$$S_{dEGB} = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa}R + f(\phi)R^2_{GB} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_m^{rad} + \mathcal{L}_{DM}^{WIMP}\right]$$

4) The spatially flat Friedmann-Lemaˆıtre-Robertson-Walker (FLRW) metric,

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

(*) Geometric units κ = 8πG = 1, c = 1 Then [α]= (length)^2, [ϕ]= [γ]=dimensionless.

The Einstein and scalar Eqs.

$w_I = \frac{p_I}{\rho_I}$ $\quad (w_{rad} = \frac{1}{3})$

$$H^2 = \frac{\kappa}{3}\left(\rho_{\{\phi+GB\}} + \rho_m\right)$$
$$= \frac{\kappa}{3}\left(\frac{1}{2}\dot{\phi}^2 - 24\dot{f}H^3 + \rho_m\right) = \frac{\kappa}{3}\rho_{tot}$$
$$\dot{H} = -\frac{\kappa}{2}\left[\left(\rho_{\{\phi+GB\}} + p_{\{\phi+GB\}}\right) + (\rho_m + p_m)\right]$$
$$= -\frac{\kappa}{2}\left[\dot{\phi}^2 + 8\frac{d(\dot{f}H^2)}{dt} - 8\dot{f}H^3 + (\rho_m + p_m)\right]$$
$$\equiv -\frac{\kappa}{2}(\rho_{tot} + p_{tot}) = -\frac{\kappa}{2}\rho_{tot}(1 + w_{tot})$$
$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) + V'_{GB} = 0$$

where:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 = p_\phi \; (V(\phi) = 0) \quad \rho_{rad} = 3\, p_{rad} = \frac{\pi^2}{30}\, g_* T^4$$
$$\rho_{GB} = -24\dot{f}H^3 = -24f'H^3 = -24\alpha\gamma e^{\gamma\phi}\dot{\phi}H^3$$
$$p_{GB} = 8\left(f''\dot{\phi}^2 + f'\ddot{\phi}\right)H^2 + 16f'\dot{\phi}H\left(\dot{H} + H^2\right)$$
$$= 8\frac{d(\dot{f}H^2)}{dt} + 16\dot{f}H^3 = 8\frac{d(\dot{f}H^2)}{dt} - \frac{2}{3}\rho_{GB}$$
$$V'_{GB} \equiv -f'R^2_{GB} = -24f'H^2(\dot{H} + H^2) = 24\alpha\gamma e^{\gamma\phi}qH^4$$

the continuity equation

$$\dot{\rho}_I + 3H(\rho_I + p_I) = \dot{\rho}_I + 3H(1 + w_I)\rho_I = 0$$

Deceleration parameter

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1}{2}(1 + 3w_{tot})$$

acceleration⟶ ⟵ deceleration

w_I: -1 $\quad -1/3$ $\quad 0$ $\quad +\frac{1}{3}$ $\quad +1$

Note

ρ_{GB} p_{GB} w_ϕ $\rho_{\{\phi+GB\}}$ & $p_{\{\phi+GB\}}$: NOT necessarily +tive.

Bdry Conditions at BBN

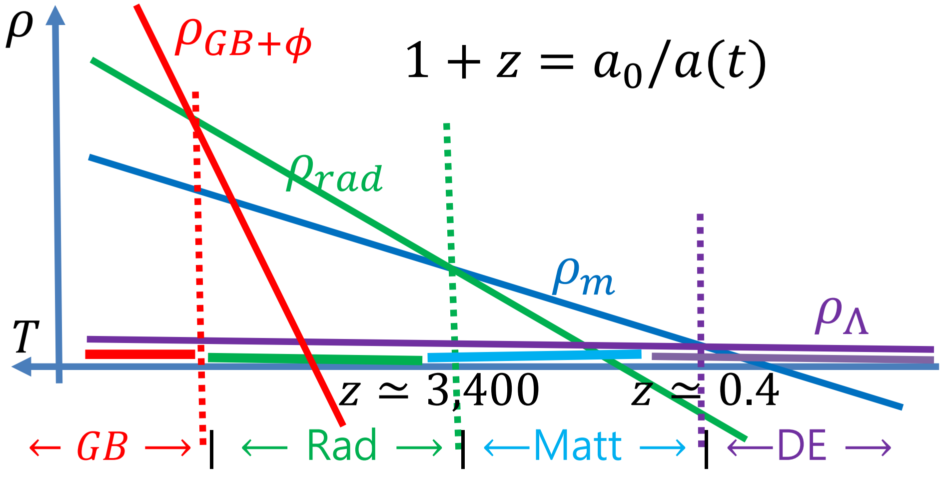
$\phi_{BBN} = 0$

$\dot{\phi}_{BBN} \geq 0$: For magnitude, Use $\eta = \frac{\rho_\phi(T_{BBN})}{\rho_{tot}(T_{BBN})}$

($\eta \leq 3 \times 10^{-2}$ from $N_{eff} \leq 2.99 \pm 0.17$)

H_{BBN}: from $8\sqrt{6\kappa\eta}f'(0)H^4_{BBN} + (1-\eta)H^2_{BBN} + \frac{\kappa}{3}\rho_{rad}(T_{BBN}) = 0$

New Phases



Goal : Constrain the **Modified Gravity (dEGB)**

Investigate the cosmological effects of the **Modified Gravity (dEGB)** during the various phases of the cosmological evolution

1) With $V(\phi)$: Inflation in DEGB theory ($\mathcal{L}_m^{matt}$=0)

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa^2} R - \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \frac{1}{2}\xi(\phi)R_{GB}^2\right]$$

The duration of inflation gets shorter as ξ_0 increases.
(making the effective potential steeper)

S. Koh, BHL, Tumurtushaa
PRD98 (2018) 10, 103511
S. Koh, BHL, W. Lee, Tumurtushaa
PRD90 (2014) no.6, 063527

Blue shifted spectrum

2) Reconstruction of $V(\phi)$ in Inflationary Models with a GB term

How to get the inflationary potential from the cosmol data?
"Inverse Scattering" Problem

V $\rightleftarrows$ n_s, r

S. Koh, BHL, Tumurtushaa
PRD 95 (2017)

3) Primordial Grav Waves & Reheating parameters in G-B inflation

PRIMORDIAL GRAVITATIONAL WAVES INDUCED BY THE BLUE-TILTED AND RED-TILTED TENSOR SPECTRA

4) w/o $V(\phi)$ WIMPs in DEGB cosmology

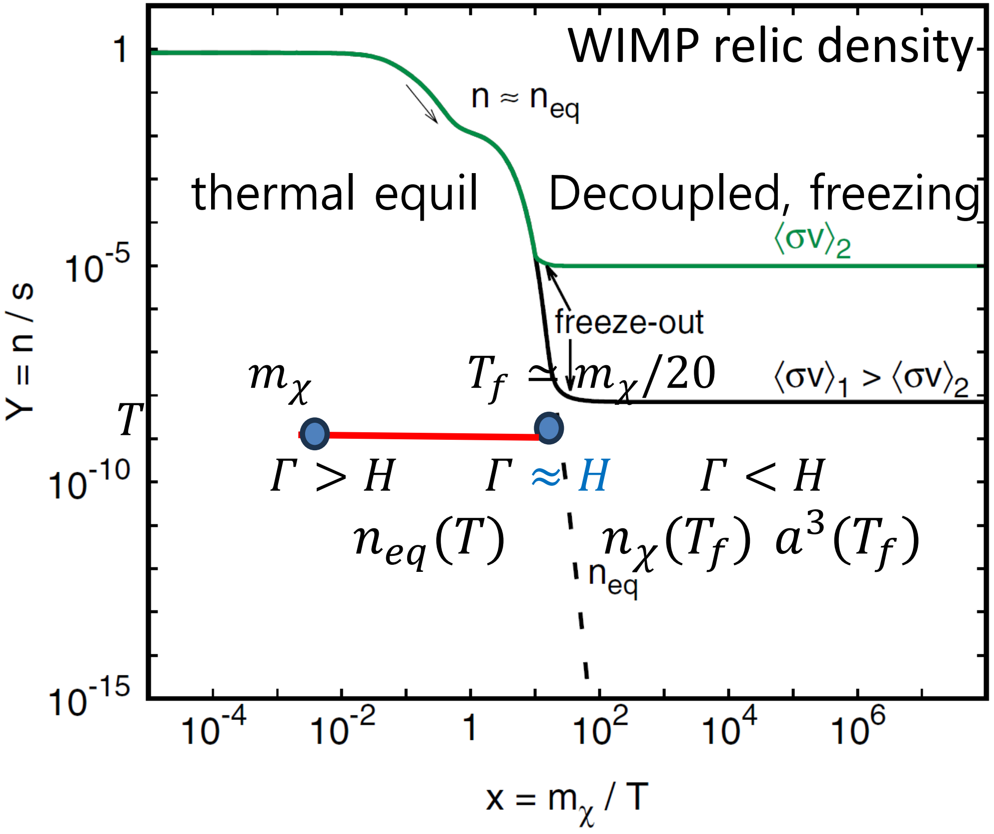
Big Bang Nucleosynthesis (BBN) : initial condition

BBN ($T_{BBN} \simeq 1\ MeV$) strongly constrains any departure from Standard Cosmology.
All events that take place at $T > T_{BBN}$ can be used to shed light on physics beyond GR and the SM.

WIMPs χ in DEGB cosmology

Biswas, Kar, **BHL**, Lee, Lee, **Scopel**, Velasco-Sevilla, Yin (2023)

$$S_{dEGB} = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa}(R - 2\Lambda(\phi) + f(\phi)R_{GB}^2) + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_m^{matt}\right] \quad f(\phi) = \alpha e^{\gamma\phi(r)}$$



$\Gamma = n_\chi < \sigma v >$: **The WIMP annihilation rate** to SM particles

The larger is $< \sigma v >_f$ **, the lower relic abundance** $\Omega_\chi h^2$

- find $< \sigma v >_{relic}$ of $< \sigma v >_f$ as a fn of m_χ which yields WIMP relic density $\Omega_\chi h^2 \simeq 0.12$ (the observational CDM density).

  Ex) $\Omega_\chi h^2 \simeq 0.12$ (CDM density in ΛCDM) gives $< \sigma v >_f \simeq 3 \times 10^{-26}\ cm^3 s^{-1} \simeq pb \cdot c$. (if all DM are WIMPs)

- Nonobservation of the WIMP annihilation in our Galaxy to γ, $e^\pm, p, \bar{p}$ & $\nu, \bar{\nu}$, ⇝ an upper bound $< \sigma v >_{ID}$ on $< \sigma v >_{gal}$ as a fn of m_χ.

- **The favoured param** of the dEGB cosmol are those satisfying

$$< \boldsymbol{\sigma v} >_{gal} / < \boldsymbol{\sigma v} >_{ID} \lesssim \mathbf{1}$$

GWs from BH-BH & BH-NS merger events

	LMXB	GW (BBH)		GW (NSBH)	
		O1–O2	O1–O3	GW200115	combined
$\alpha_{GB}^{1/2}$ [km]	1.9	5.6	1.7,	1.33	1.18

Yagi, (2012) Nair, Perkins, Silva, Yunes, (2019)
Perkins,Nair,Silva,Yunes(2021), Lyu, Jiang, Yagi, (2022)

the constraints from the GW signals from BH-BH and BH-NS merger events

- ϕ freezes at $T_L \ll T_{BBN}$ to a background value $\phi(T_L)$, while near a BH or a NS, ϕ is distorted compared to $\phi(T_L)$, that can modify the GW signal in a merger event.
- the data from the LIGO-Virgo for constraints $\alpha_{GB}^{1/2} \leq \mathcal{O}(2\text{ km})$ or $\alpha_{GB}^{1/2} \leq 1.18\text{ km}$

Lyu Jiang Yagi PRD (2022)

the constraints from compact binary mergers

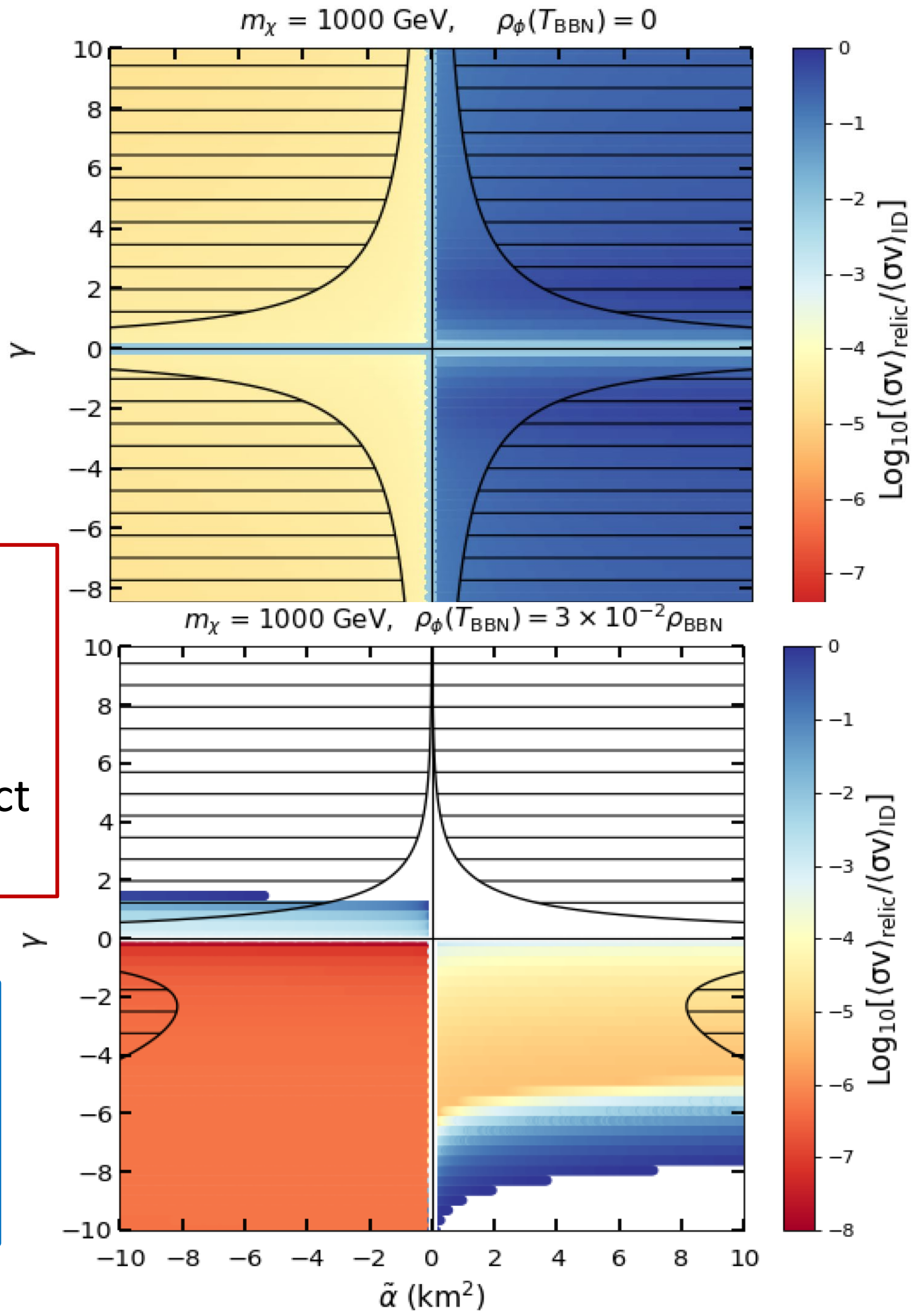
$|f'(\phi(T_L))| \leq \sqrt{8\pi}\alpha_{GB}^{max}$ w/ $\alpha_{GB}^{max} = (1.18)^2\text{km}^2$

- If $\dot{\phi}\,(T_{BBN}) = 0,$ then $|\tilde{\alpha}\gamma| \leq \sqrt{8\pi}\alpha_{GB}^{max}$
- If $\dot{\phi}\,(T_{BBN}) \neq 0,$ then $|\tilde{\alpha}\gamma e^{\gamma\frac{\dot{\phi}_{BBN}}{H_{BBN}}}| \leq \sqrt{8\pi}\alpha_{GB}^{max}$

- The bounds from WIMP indirect detection are complementary to late-time BBH merger constraints.
- As m_χ increases for fixed ϵ, $\frac{<\sigma v>_f}{<\sigma v>_{ID}}$ decreases (more favored).
- As ϵ increases for fixed m_χ, $<\sigma v>_f/<\sigma v>_{ID}$ usually increase,

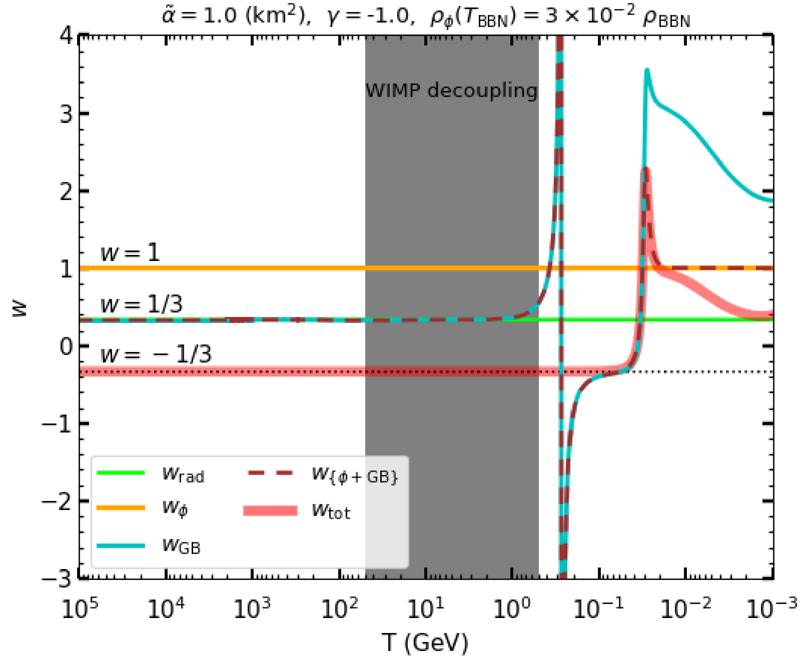
White regions ($\frac{<\sigma v>_{relic}}{<\sigma v>_{ID}} > 1$) are disfavoured by WIMP indirect detection.

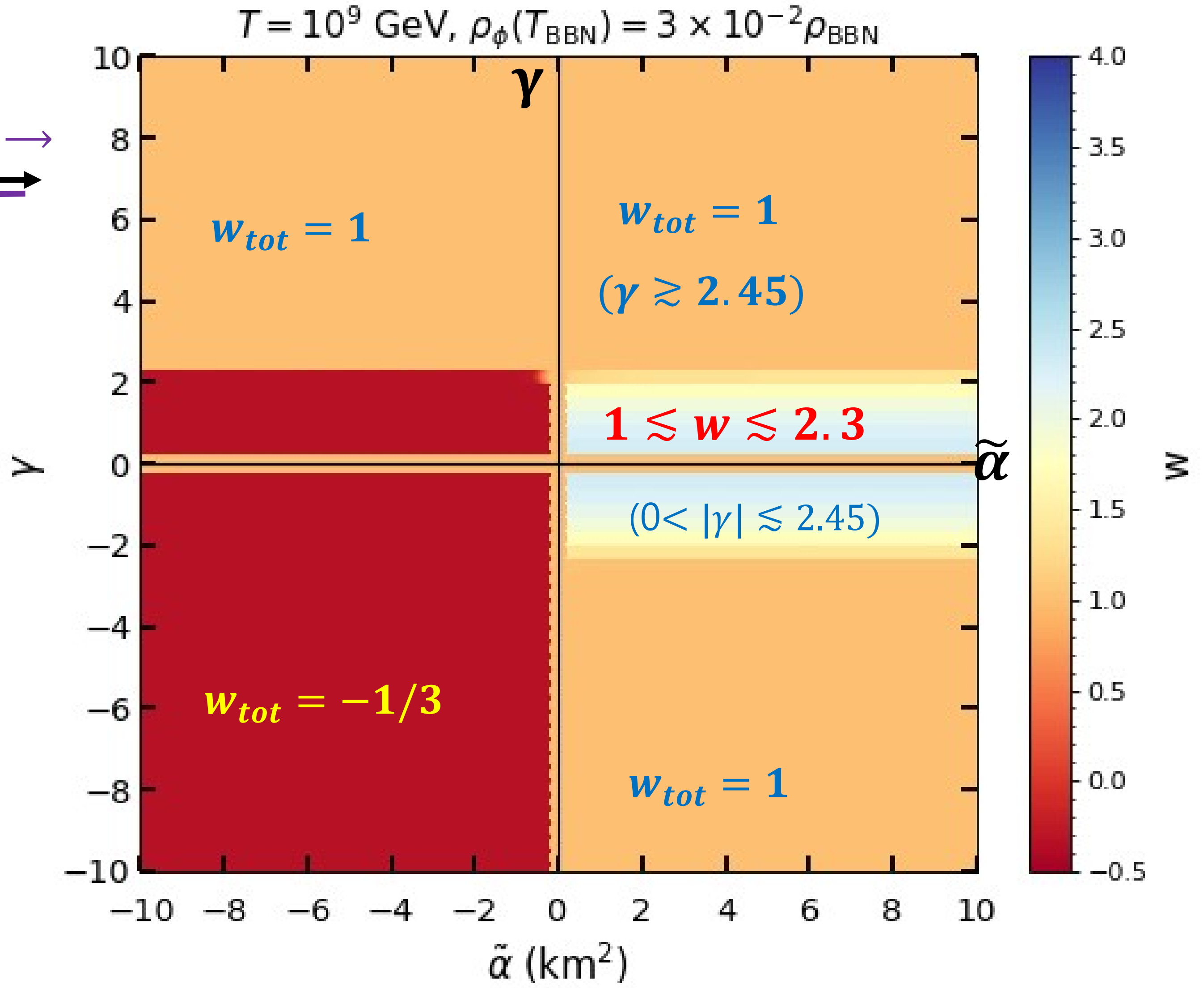
Hatched areas of the $\tilde{\alpha}$-γ parameter space are disallowed by the constraint



High T behavior of dEGB cosmology

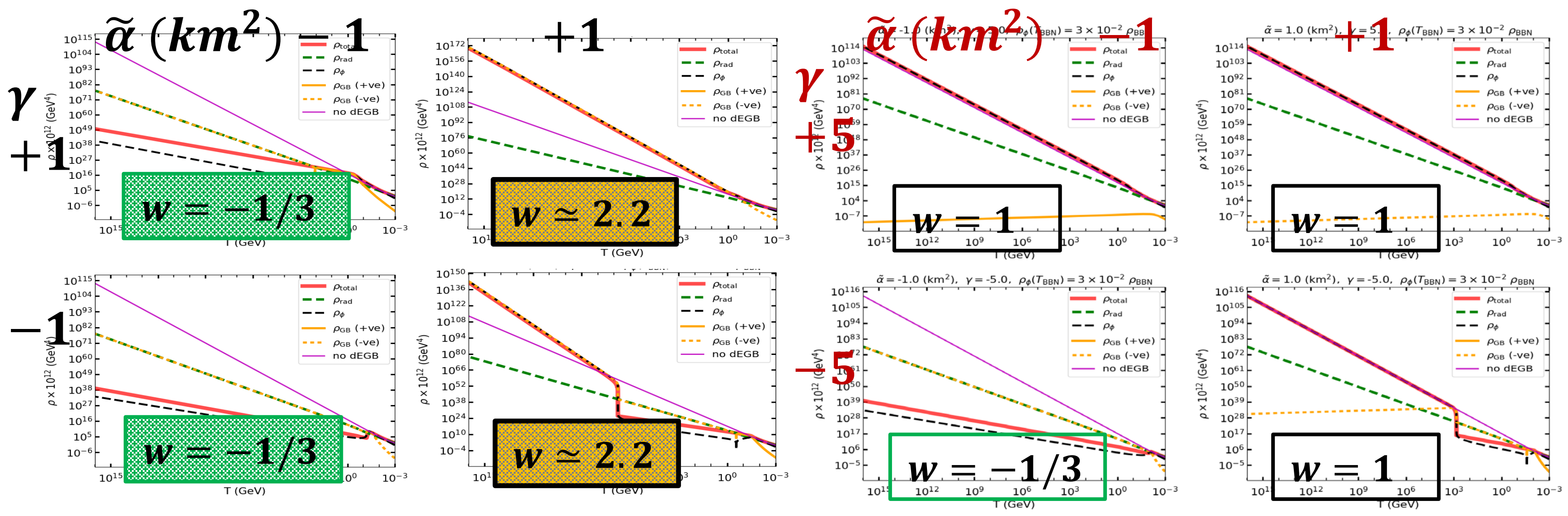
1) New Phases appear
   Ex) Super Kination phase ($w > 1$)
   Kination Phase ($w = 1$)
   Slow rolling phase ($w \approx -1/3$)
2) These are attractor/fixed point solutions)
3) May affect observation -New Physics
   Ex) GWs

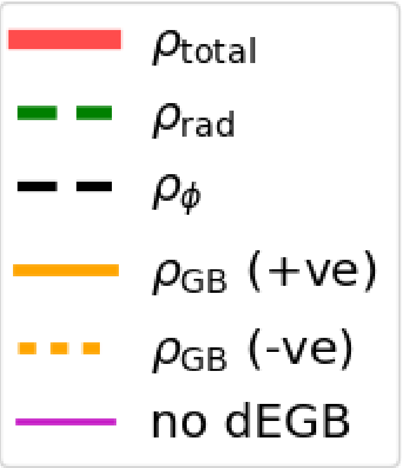




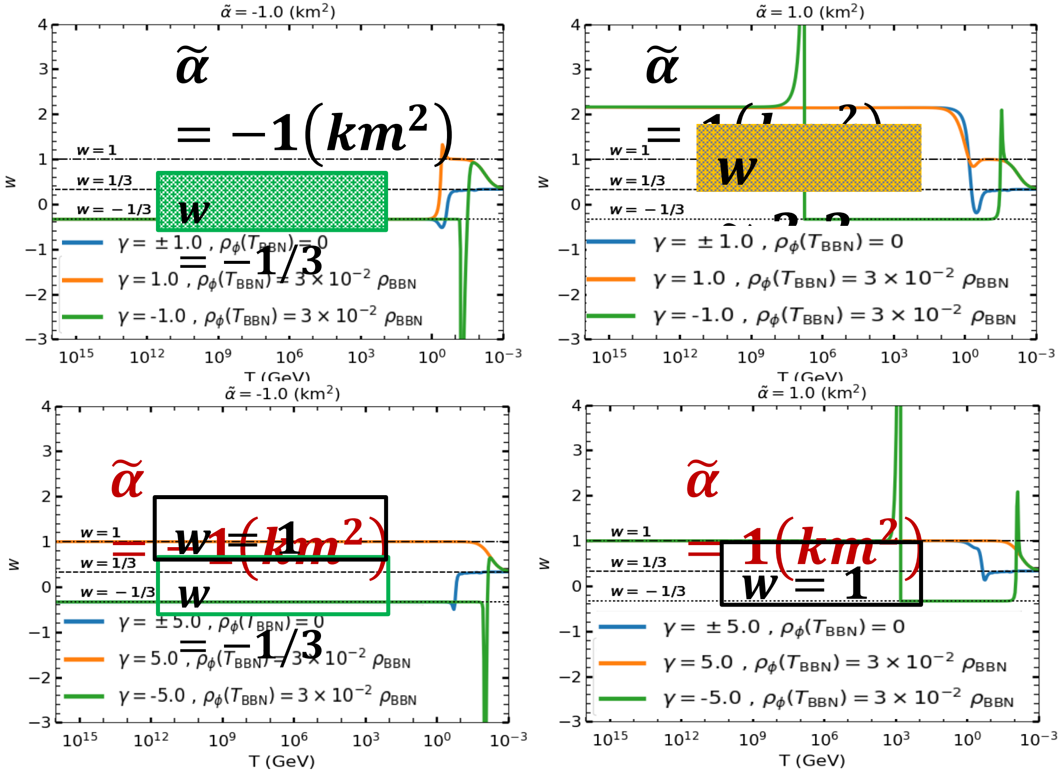
A. Biswas, A. Kar, **BHL**, H. Lee, W. Lee, **S. Scopel**, L. Yin **arXiv 2405.15998**

$\rho_\phi(T_{BBN}) = 3 \times 10^{-2} \rho_{BBN}$



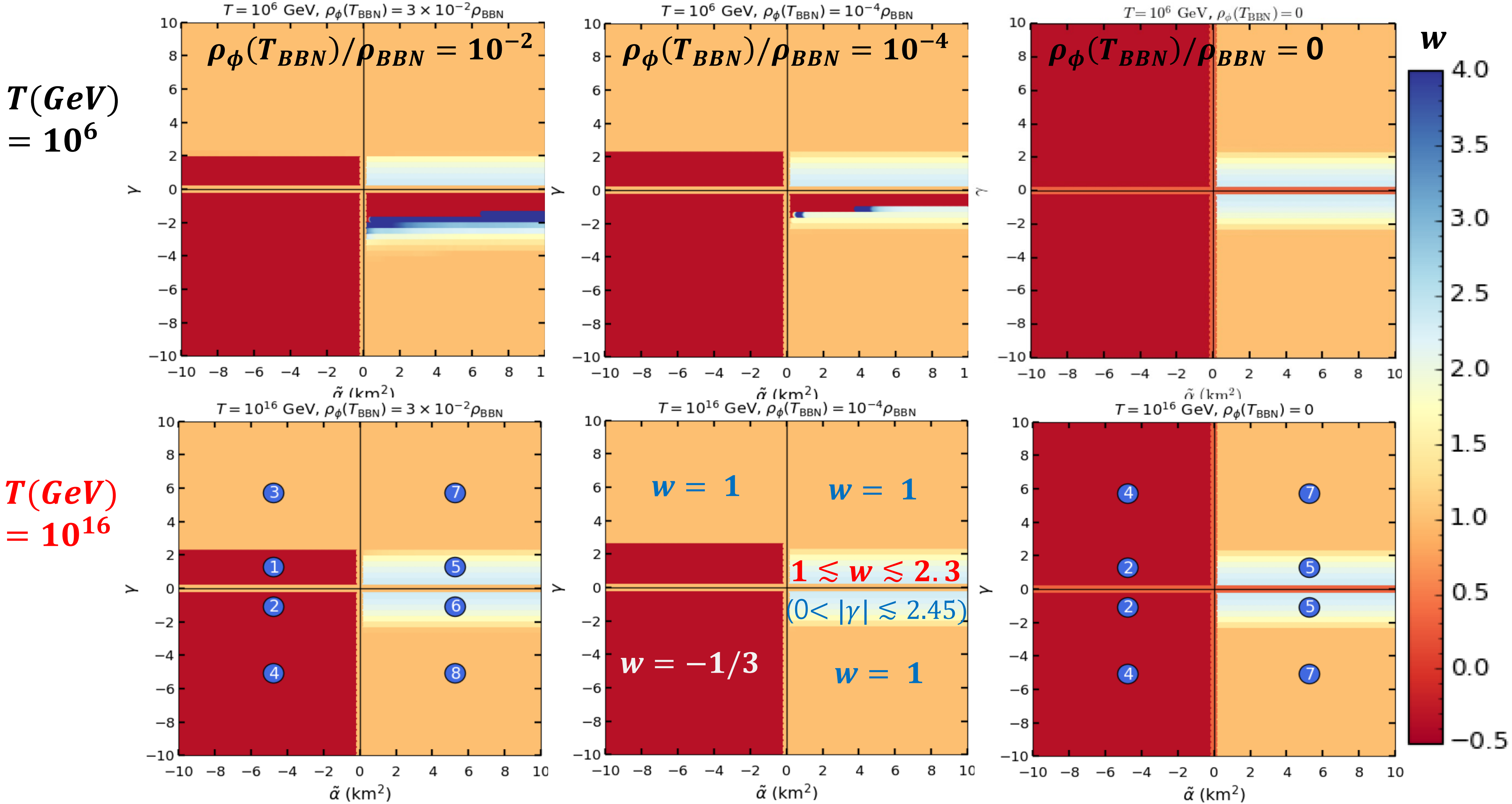


- At high enough T, ρ_{tot} reaches an asymptotics (w =const).
- At high T, $|\rho_{GB}|$ either tracks the dominant component(*) or negligible
- (*) = ρ_{rad} for $w = -\frac{1}{3}$ (for $\tilde{\alpha} < 0$ and $|\gamma| < \sqrt{6}$ or $\gamma < -\sqrt{6}$), $\rho \ll \rho_{rad}, |\rho_{GB}|$
  = ρ_ϕ for $w \gtrsim 1$ (for $\tilde{\alpha} > 0$ (any γ) or for $\tilde{\alpha} < 0$ & $\gamma > \sqrt{6}$)



1) For $(\tilde{\alpha}, \gamma, \rho_\phi(T_{BBN}))$, asymptotically, $w = -\frac{1}{3}$ or $1 \le w \lesssim 2.3$

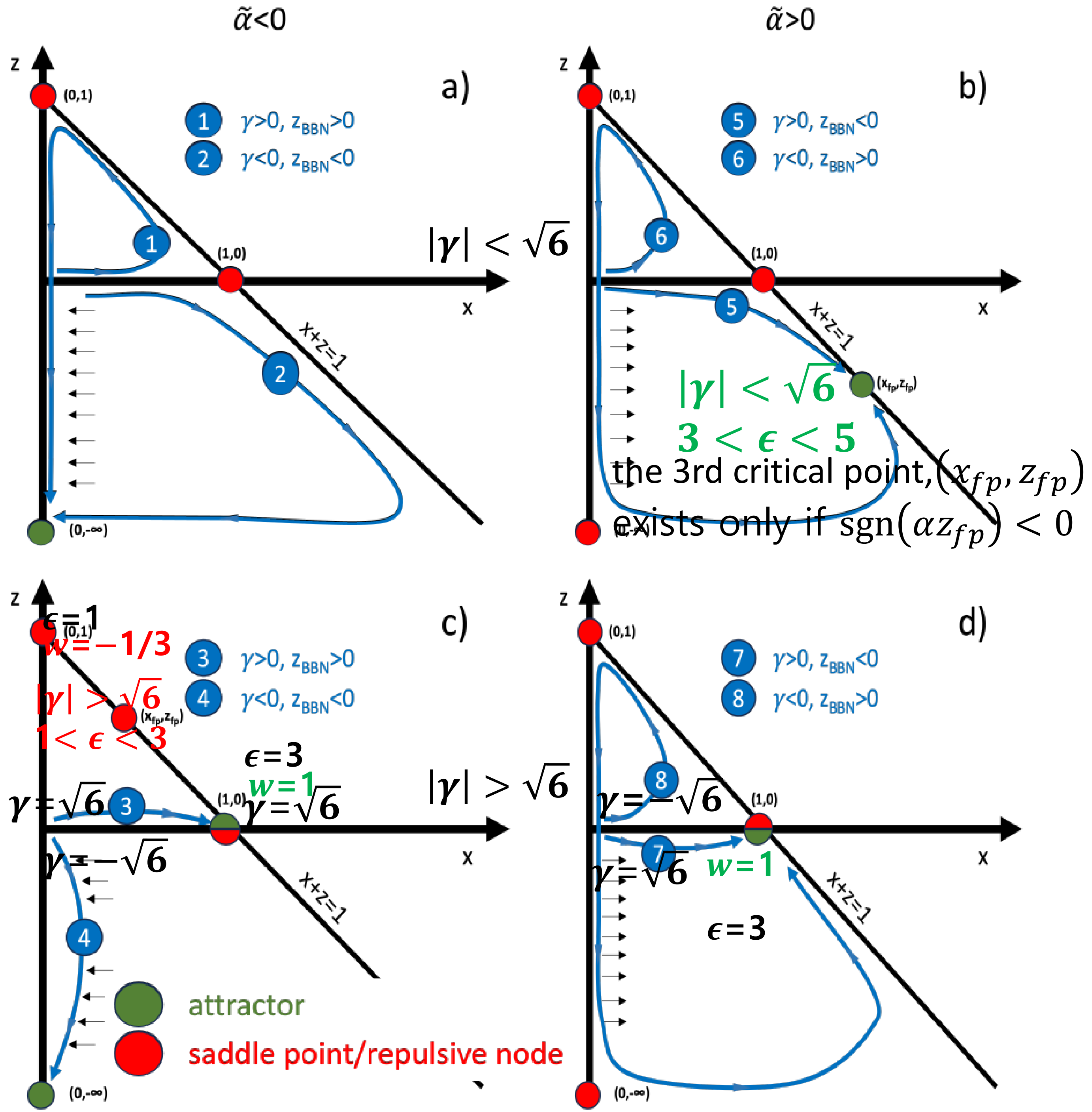
2) The asymptotic value of w depends only on the sign of $\tilde{\alpha}$, but not on its actual value.



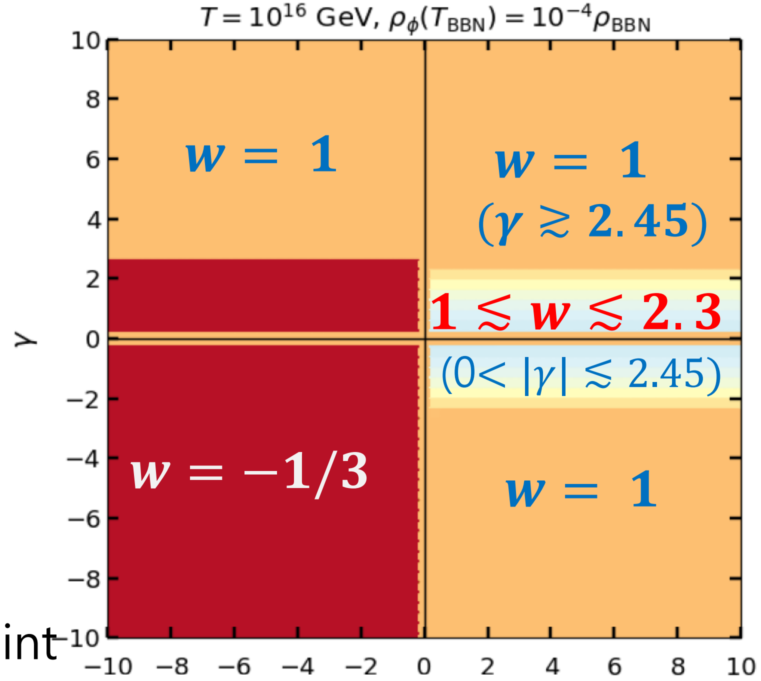
Consider the case of $\alpha > 0$:

1) for $|\gamma| < \sqrt{6}$ the only stable critical point of the system is (x_{fp}, z_{fp}),

2) while for $|\gamma| > \sqrt{6}$ it is (1, 0) (for $z \to 0-$).



$|\gamma| < \sqrt{6}$



(x_c, z_c)	stability
(0, 1)	saddle point
(1, 0)	$|\gamma| < \sqrt{6\kappa} \to$ saddle point,
	$|\gamma| > \sqrt{6\kappa} \to \begin{cases} \alpha > 0 & \begin{cases} z \to 0^+ & \text{saddle point} \\ z \to 0^- & \text{attractor} \end{cases} \\ \alpha < 0 & \begin{cases} z \to 0^+ & \text{attractor} \\ z \to 0^- & \text{saddle point.} \end{cases} \end{cases}$

Constraints from Gravitational Waves

J. Ghiglieri and M. Laine, JCAP (2015), [1504.02569].
Ghiglieri, Jackson, Laine, Zhu, JHEP (2020), [2004.11392]

- Any plasma of relativistic particles in thermal equilibrium emits a stochastic GW background (SGWB)
- SGWB : potential probe of Cosmological models before BBN. Ex) the Standard Model : peak around 80 GHz
  (Present detectors are only sensitive to few Hertz, some proposals exist to extend to the GHz range.

The magnitude and spectral shape of the SGWB produced at a given time
(during the thermal–radiation dominated epoch until the electroweak crossover, at T_{EWCO} = 160 GeV)

$$\frac{1}{a^4}\frac{d}{dt}\left(a^4\rho_{\rm GW}(t)\right) = \left(\frac{\partial}{\partial t}+4H\right)\rho_{\rm GW}(t) = 4\frac{T^4}{M_{PL}^2}\int\frac{d^3k}{(2\pi)^3}\eta(T,k),$$

$\eta(k,T)$:the shear viscosity of the plasma.

$$\eta(\hat{k},T) = \begin{cases} \frac{1}{8\pi}\frac{16}{g_1^4\ln(5T/m_{D_1})}, & k\lesssim \alpha_1^2 T, \\ \eta_{\rm HTL}(\hat{k},T)+\eta^T(\hat{k},T), & k\gtrsim 3T, \end{cases},$$

$\hat{k} = k/T$

$\eta_{HTL}(\hat{k},T)$:Hard Thermal Logarithmic (HTL),
$\eta_T(\hat{k},T)$: the thermal corrections.

The fraction of energy liberated into GW radiation per frequency octave,

$h = H_0/(100km/s/Mpc)$
$T_0 = 2.7K$ f: freq. measured today

$$\Omega_{GW}(f,T_0)h^2 \equiv \frac{1}{\rho_{crit}(T_0)}\frac{d\rho_{GW}(T_0)}{d\ln f}h^2$$

$\Omega_{\gamma,0} = 2.4729(21)\times 10^{-5}$
the present photon density

$$= \Omega_{\gamma 0}h^2\frac{\lambda}{M_{PL}}\int_{T_{\rm EWCO}}^{T_{\rm Max}} dT\left(\frac{g_{*0}}{g*(T)}\right)^{4/3} T^2\,\hat{k}(f,T)^3\frac{\eta(\hat{k},T)}{\sqrt{\rho(T)}}\beta(T),$$

$\lambda = 30\sqrt{3}/\pi^4$, $g_{*0} = 2$,
$g_{*S}(T_0) = 3.91$, $g_{*S}(T_{EWCO}) = 106.75$

$$\beta = \left(1+\frac{1}{3}\frac{d\ln g_{*s}}{d\ln T}\right)$$

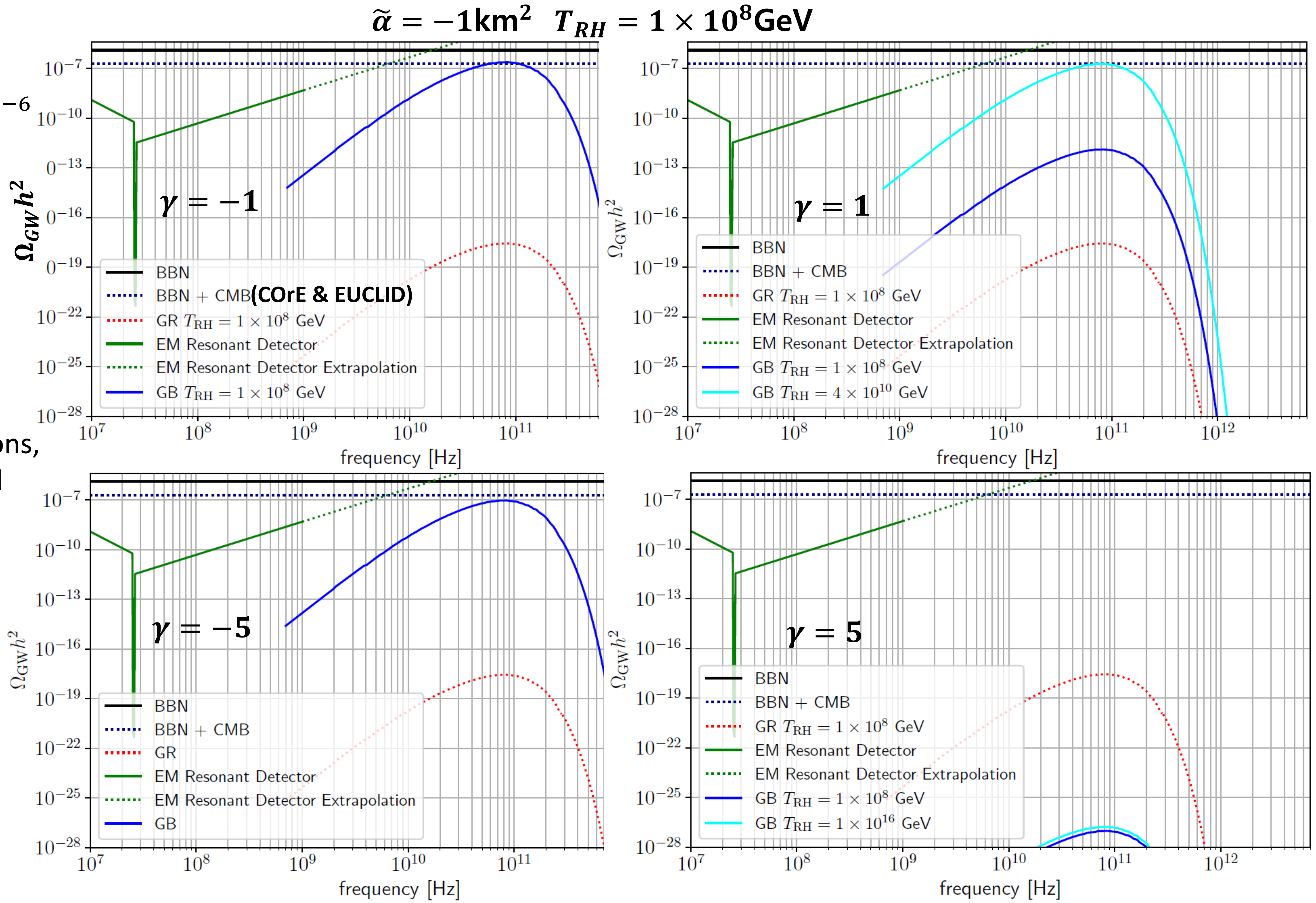
$$\hat{k}(f,T) = \left[\frac{g_{*s}(T)}{g_{*s}(T_0)}\right]^{\frac{1}{3}}\frac{2\pi f}{T_0}. \qquad f = \frac{1}{2\pi}\left[\frac{g_{*s}(T_0)}{g_{*s}(T_{\rm EWCO})}\right]^{\frac{1}{3}}\left(\frac{T_0}{T_{\rm EWCO}}\right)k_{\rm EWCO},$$

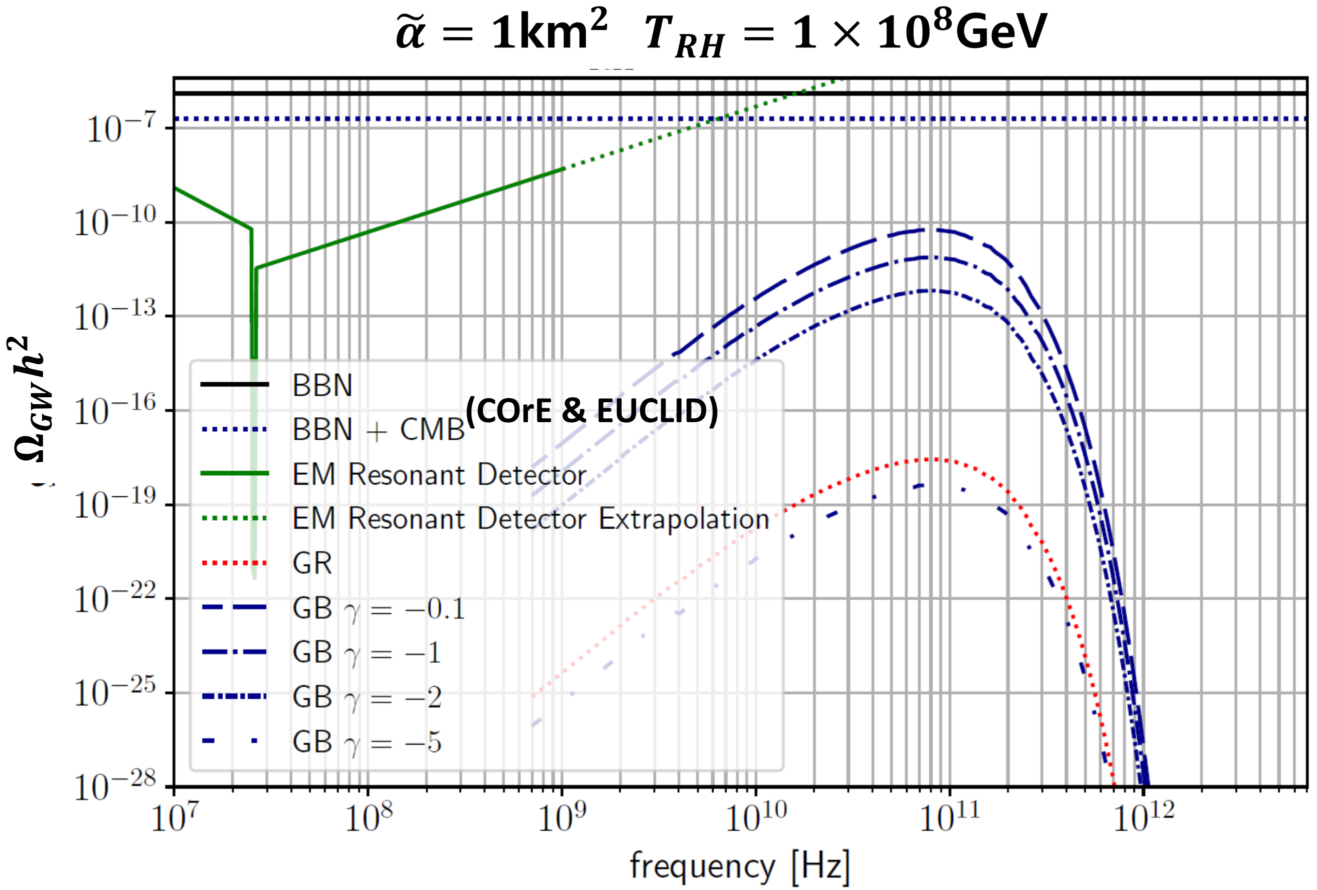
$\eta(\hat{k},T)$ has a peak at $\hat{k}_{peak}\simeq 3.92$ (at production) independent of T or $f_{peak}\simeq$ 74GHz (today)

The BBN bound: $\Omega(f,T_0)h^2 < 1.3\times 10^{-6}$

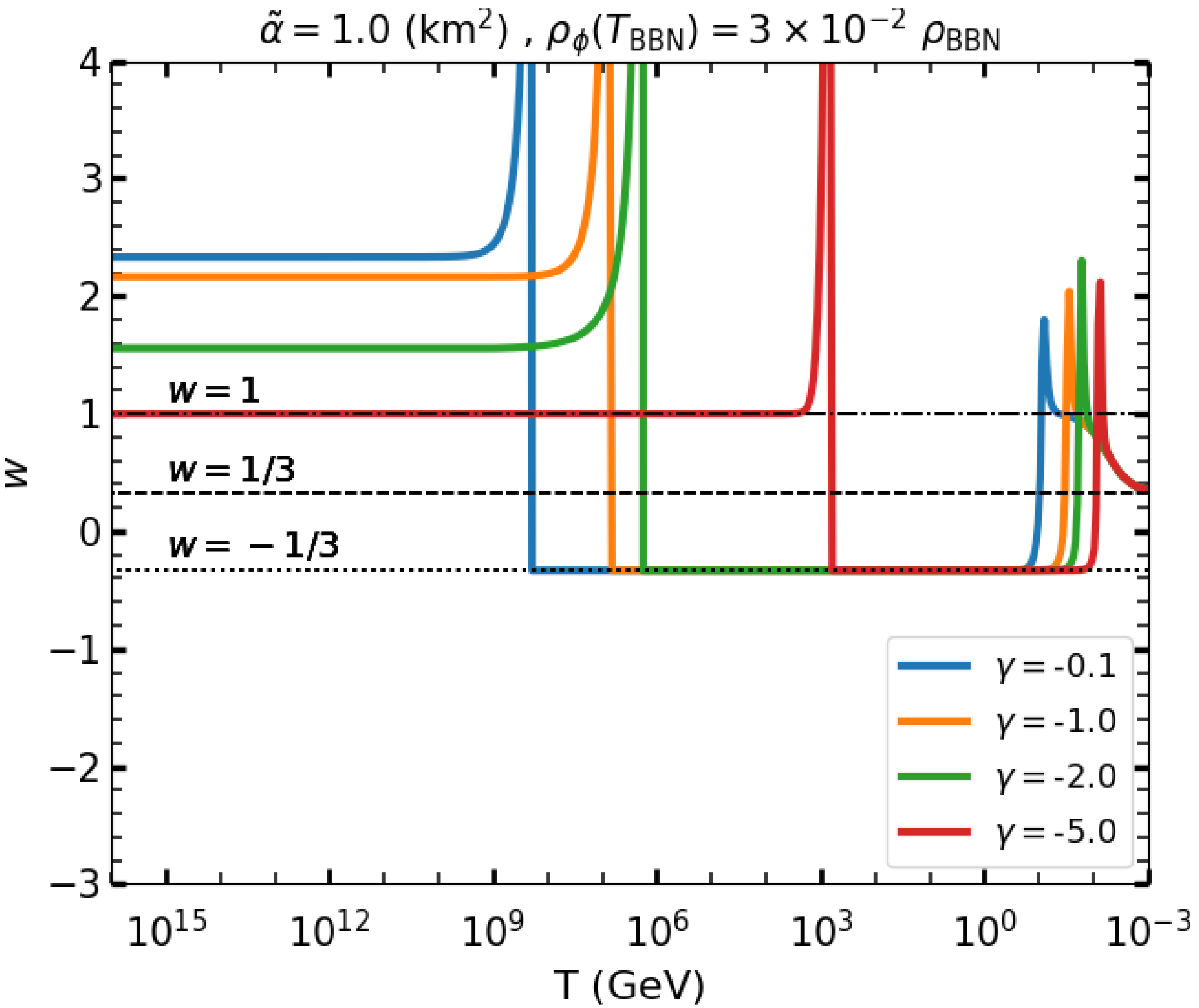
- $T_{max} = 10^9$ GeV.
- the BBN bound $\Omega(f, T_0)h^2 < 1.3 \times 10^{-6}$
- future (COrE, EUCLID) $\Omega(f, T_0)h^2 \lesssim \mathcal{O}(10^{-7})$

The GW peak value largely exceeds unity whenever asymptotic e.o.s. corresponds to slow roll (w = −1/3). In such parameter regions, can set an upper bound on $T_{max} \ll 10^{16}$ GeV.





The signal is enhanced as |γ| → 0 because the metastable slow–roll solution with w=-1/3 lasts longer.



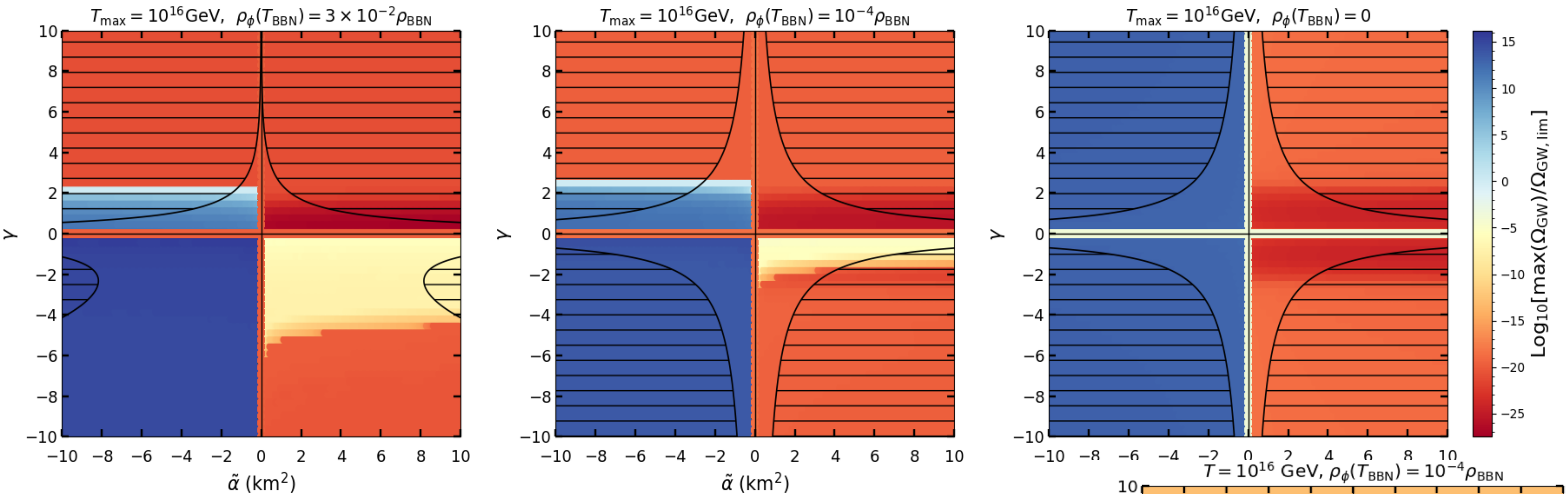
As |γ| →0 the system follows the metastable solution w = −1/3 for a larger interval of T before jumping to a different regime, implying an increasing GW stochastic background.

Summary of GW bounds

- A sizeable GWs are produced when radiation is the dominant component, $\rho \propto T^4$.
- As a consequence, $d\Omega_{GW}/d\ln a \propto T$, UV dominated, i.e. by the GWs emitted at high T.
- In the standard Cosmol, ρ_{rad} dominates at all $T > T_{EWCO}$, Ω_{GW} is a monotonically growing fn of T_{max}. potentially put bounds on T_{RH}.
- For standard Cosmology the ensuing stochastic background turns out to be below the BBN bound even for values as high as $T_{RH} \simeq 10^{16}$GeV.
- For a non–standard cosmology where radiation dominance stops above some temperature $T_{rad,max}$ the stochastic background is dominated by the GWs produced at $T_{rad,max}$, and increasing T_{max} beyond $T_{rad,max}$ does not change the final result, so that the detection are even worse.

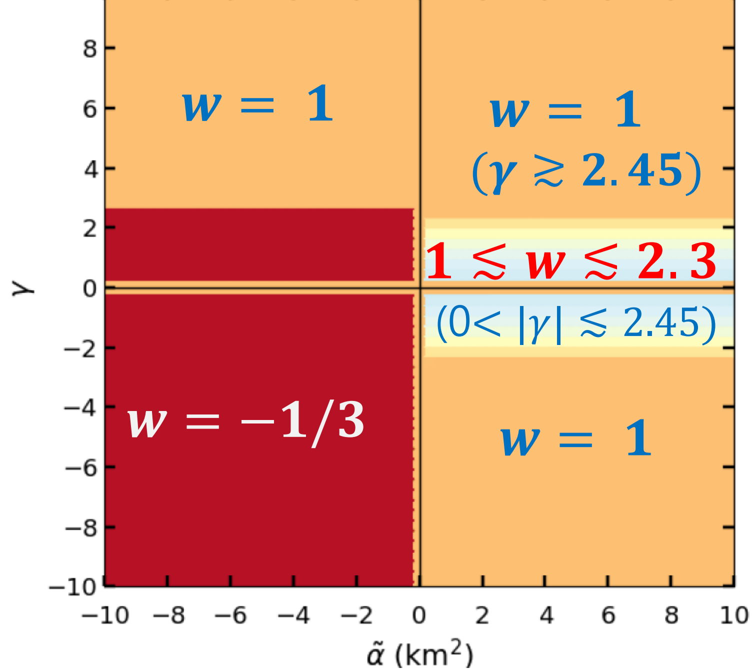
- The dEGB scenario presents an interesting twist to the picture. In fact, in a "slow–roll regime" with w = −1/3 where the energy density of the Universe is dominated by ρ_{rad}, $|\rho_{GB}| \propto T^4$ while at the same time $\rho_{rad} + \rho_{GB} \propto T^2$, with a large cancellation between ρ_{rad} and ρ_{GB}.
- dEGB allows to have and epoch when the relativistic plasma dominates the energy while at the same time the rate of dilution with T of ρ_{tot} is slower than what usually expected during rad dom.

- This strongly enhances the GW expected signal compared to the standard case and allows to put bounds on $T_{RH} \simeq 10^9$ GeV $\ll 10^{16}$ GeV in the the "slow roll" asymptotic behaviour regions.



The peak value of the SGWB ($\max(\Omega_{GW}h^2)$), for $T_{max} = 10^{16}$ GeV, normalised to the BBN upper limit ($\Omega_{GW,lim}h^2 \simeq 10^{-6}$).

The hatched areas are ruled out by the detection of GW from compact binary mergers.



In the "slow roll" asymptotic behaviour parameter space of the dEGB, the GW signal strongly enhances compared to the standard case and allows to put sensible bounds on $T_{RH} \simeq 10^9$ GeV $\ll 10^{16}$ GeV.

The SGWB can set a meaningful bound on $T_{max} < 10^{16}$GeV only for $\tilde{\alpha} < 0$, when the slow-roll attractor solution is achieved (w = −1/3).

V. Summary

Modified Gravity beyond Einstein needed?

Theoretical Aspect

- an **effective theory** below UV cut-off, $M_{Pl}{\sim}10^{19}GeV \rightarrow$ Einstein Grav + **higher curvature** terms
- Is Standard Cosmology (**ΛCDM**) satisfactory? extremely fine-tuned ($\Lambda = 2{,}888\text{x}10^{-122}\ell_P^{-2}$)
- Holography

Observational Aspect - $\boldsymbol{H_0}$ **tension**, etc.

Modification of GR - needs to introduce additional d.o.f.

- higher than 2nd order theories have generically, ghosts & Ostrogradsky instability :

Horndeski theory is the most general scalar-tensor theory w/ 2nd-order field eqn in 4 dim. (no ghost or instability, as a result), classified by 4 arbitrary functions $\{G_i(\phi, X),\ i = 2,3,4,5\}$.

(d=4) the Dilaton-Einstein-Gauss-Bonnet (dEGB) Gravity belongs to Horndeski theory

$$S_{dEGB} = \int d^4x\,\sqrt{-g}\left[\frac{1}{2\kappa}R + f(\phi)R_{GB}^2 - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_m\right]$$

We chose $f(\phi) = \alpha e^{\gamma\phi}$

In dim>4, GB term is dynamical as well as allowing 2nd order e.o.m.

V.Summary (continued)

In dim>4, consider the Einstein-Gauss-Bonnet (EGB)- Λ Gravity (GB-AdS)

$$S_{EGB-\Lambda} = \int d^d x \sqrt{-g}\left[\frac{1}{2\kappa}(R - 2\Lambda + \alpha R^2_{GB}) + \mathcal{L}_m^{matt}\right]$$

$$\Lambda = -\frac{(d-1)(d-2)}{2\ell^2}$$

$$\kappa = 8\pi G, \quad g = \det g_{\mu\nu}$$

We systematically study the black hole solutions, thermodynamics, and phases:

- Schwarzschild BH
- AdS Schwarzchild BH,
- RN AdS BH,
- AdS GB Black Holes
- charged GB AdS BH, etc.

In dim=4,

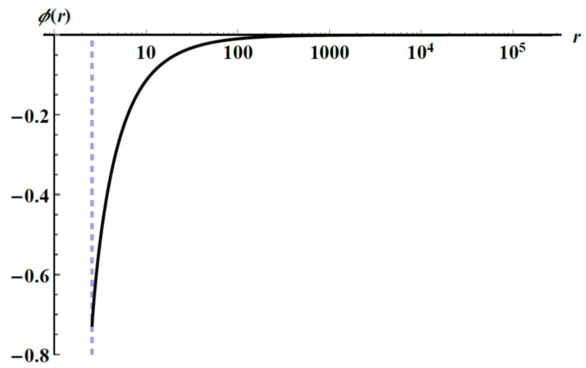
We study the Dilaton-Einstein-Gauss-Bonnet (DEGB) Gravity $\quad f(\phi) = \alpha e^{\gamma\phi}$

$$S_{dEGB} = \int d^4 x \sqrt{-g}\left[\frac{1}{2\kappa}\left(R - 2\Lambda e^{\lambda\phi(r)} + f(\phi)R^2_{GB}\right) + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_m^{matt}\right]$$

V.Summary (continued)

DEGB hairy Black Hole

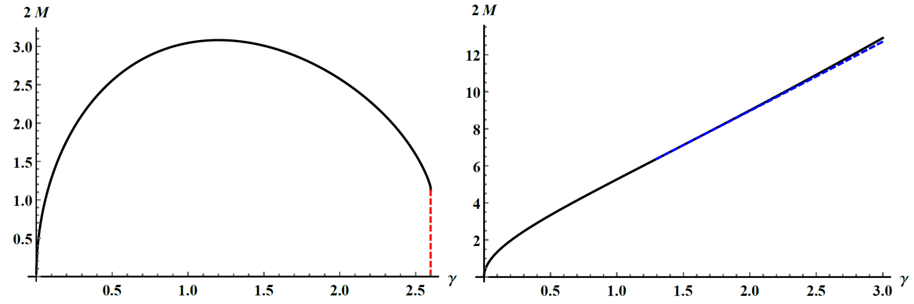
(with Dilaton, Gauss-Bonnet term and cosmological constant)



(b) $\phi(r)$ vs. r

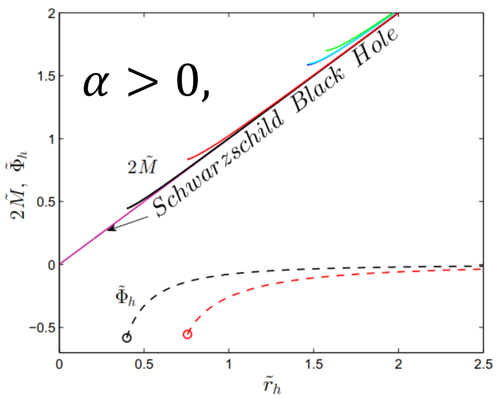
- There exists **minimum mass**.
- BHs have hairs (shown to be consistent with the no hair theorem).

- The BH solution & its properties are strongly dependent on the signature of the Gauss-Bonnet term (as well as Λ).



(a) The black hole mass vs. γ with $\alpha = -1$.

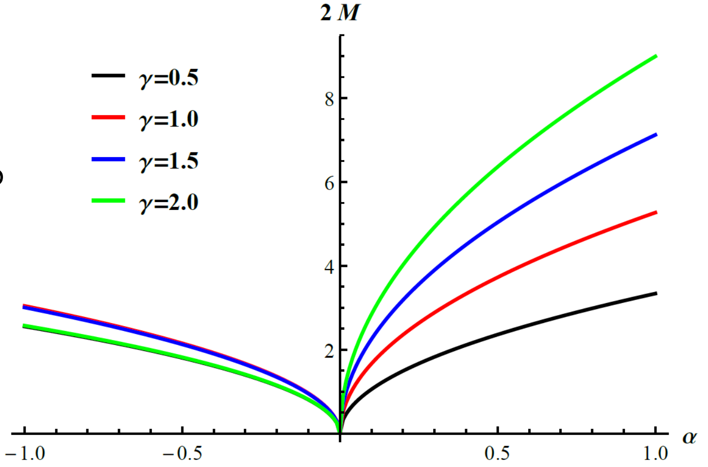
(b) The black hole mass vs. γ with $\alpha = 1$.



When the scalar field on the horizon is the maximum, the DGB black hole solution has the minimum horizon size.

The amount of black hole hair decreases as the DGB black hole mass increases.

"attractive" "repulsive"



- With Cos. Const :

$\Phi(r) \rightarrow \Phi_\infty$(Constant) $\quad$ *only when* $\gamma + \lambda = 0$, & $\Lambda = \frac{3\lambda}{8\kappa\alpha\gamma} e^{-(\gamma+\lambda)\Phi_\infty}$

Fragmentation instability of black holes:

For some parameter range, the dEGB BH is unstable under fragmentation, even if these phases are stable under perturbation.

5.Summary

Ex) The String theory at low Energy
→ Einstein Grav + higher curvature terms

Modified Gravity beyond Einstein needed?

Theoretical Aspect

- an **effective theory** below UV cut-off, $M_{Pl} \sim 10^{19} GeV \rightarrow$ Einstein Grav + higher curvature terms
- Is Standard Cosmology (**ΛCDM**) satisfactory? extremely fine-tuned ($\Lambda = 2{,}888\text{x}10^{-122} \ell_P^{-2}$)
- Holography

Observational Aspect - H_0 tension, Cosmological Birefringence etc.

Modification of GR - needs to introduce additional d.o.f.

- higher derivatives is one way of introducing additional d.o.f.
  Genirically, ghosts & Ostrogradsky instability :

Horndeski theory is the most general scalar-tensor theory w/ 2nd-order field eqn in 4 dim. (no ghost or instability, as a result), classified by 4 arbitrary functions $\{G_i(\phi, X),\ i = 2,3,4,5\}$.

the Dilaton-Einstein-Gauss-Bonnet (dEGB) Gravity belongs to Horndeski theory

$$S_{dEGB} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa} R + f(\phi) R_{GB}^2 - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) + \mathcal{L}_m \right]$$

We chose

$$f(\phi) = \alpha e^{\gamma\phi}$$

5. Summary (continued)

Cosmological implications of dEGB gravity
-Inflation, reheating, rad-dom period, etc

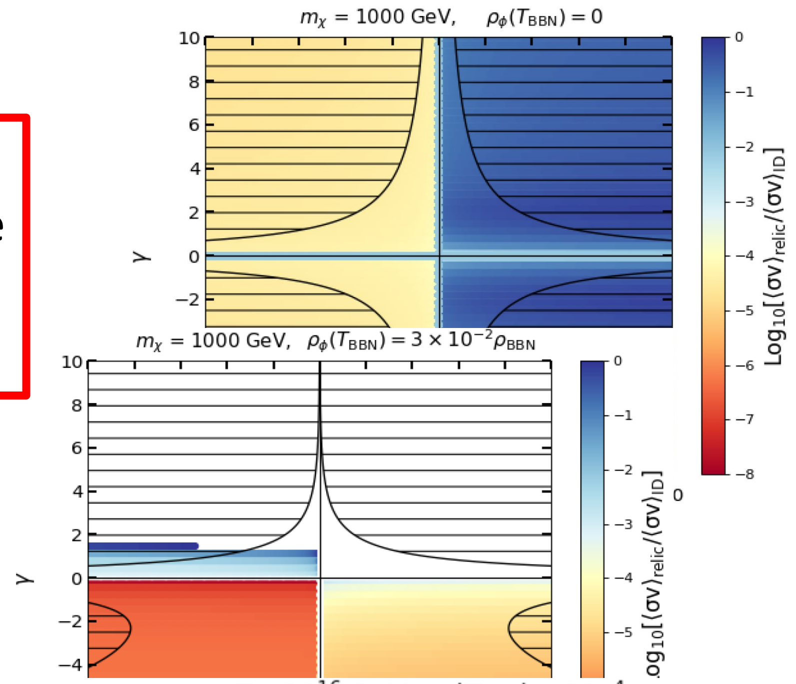
- **WIMPs** indirect detection put some constraints

- **Bounds from GWs of BH-BH & BH-NS mergers**

The WIMP indirect detection bounds are complementary to late-time BBH merger constraints.

Hatched areas are disallowed by the BBHs

White regions in the figures are disfavored.

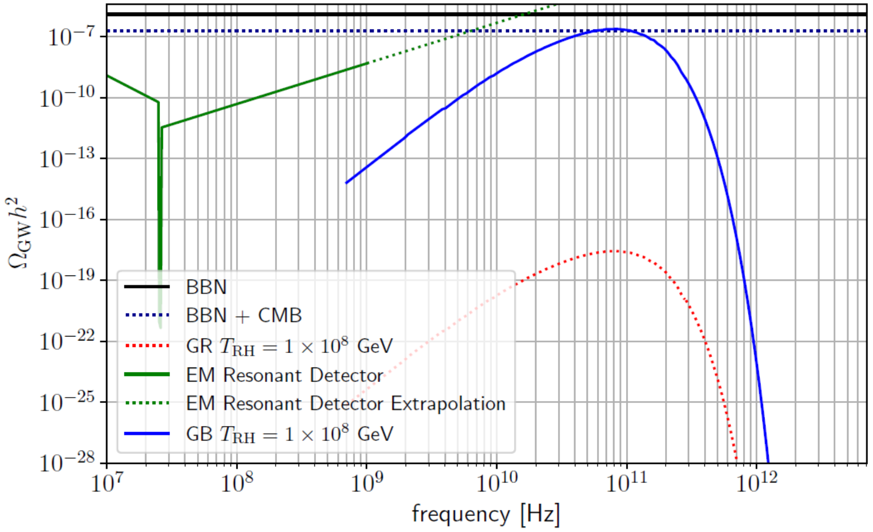


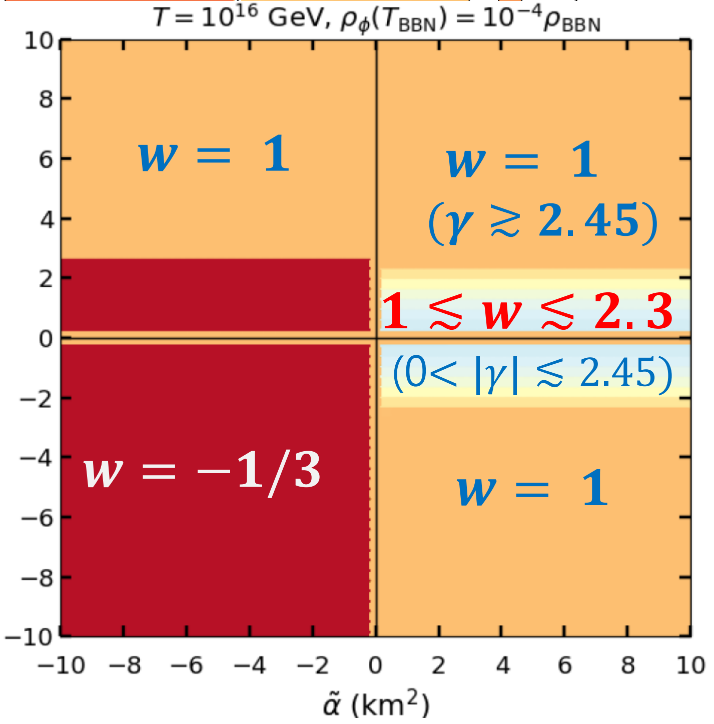
New Phases exists at high enough temperature

NEW PHASEs⟶ | ⟵Rad Dom⟶ | ⟵ Matt ⟶ | ⟵ Λ(DE) ⟶

- the regions w = −1/3 imply a strong enhancement of the expected GWSG produced by the primordial plasma of relativistic particles.

- This allows to put bounds on $T_{RH} \simeq 10^8 - 10^9$ GeV $\ll 10^{16}$ GeV.





Thank you!