Theoretical Particle Physics and Cosmology

QCD and Magnetic Gluon Condensation

High Order Corrections and Monte Carlo Simulations

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Brainstorming for NCSR-D participation at CERN
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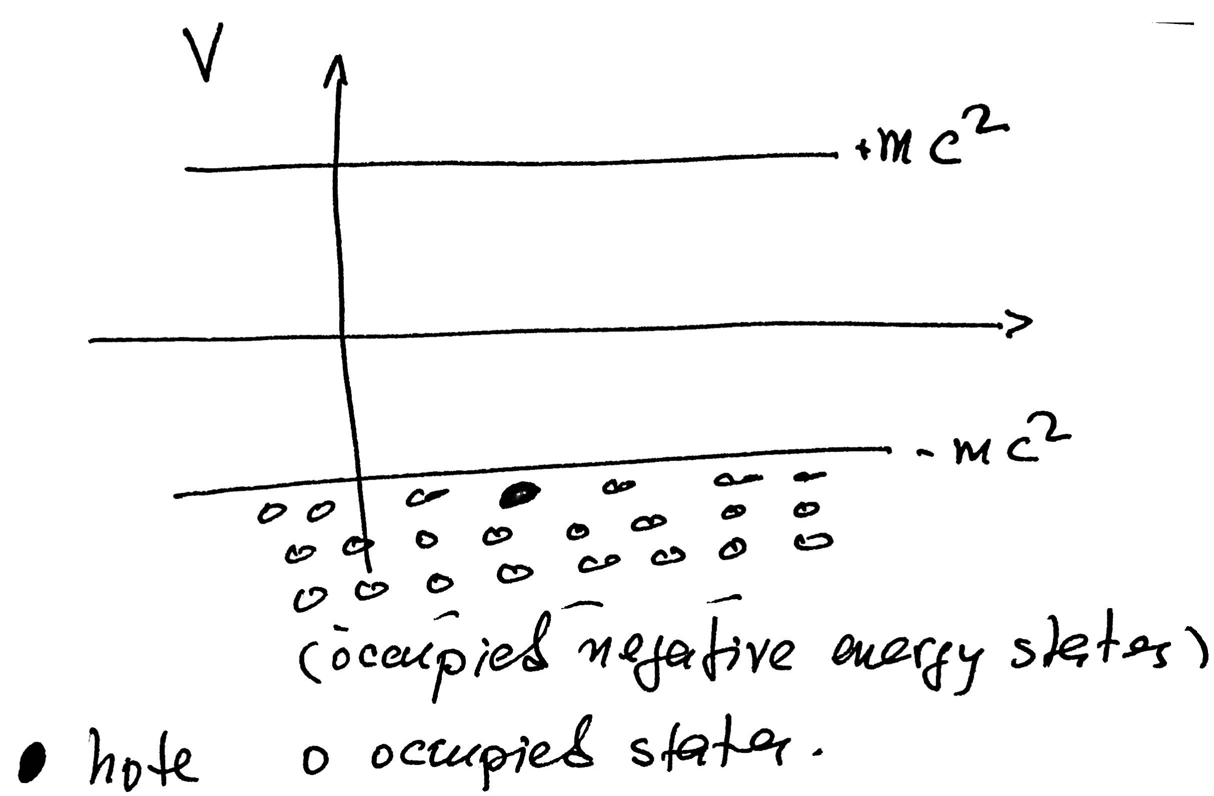
Dirac Sea and the Vacuum

Dirac suggested that the "vacuum" is the state in which all electron states of negative energy $E \leq -mc^2$ are occupied and all states of positive energy $E \geq mc^2$ are empty.

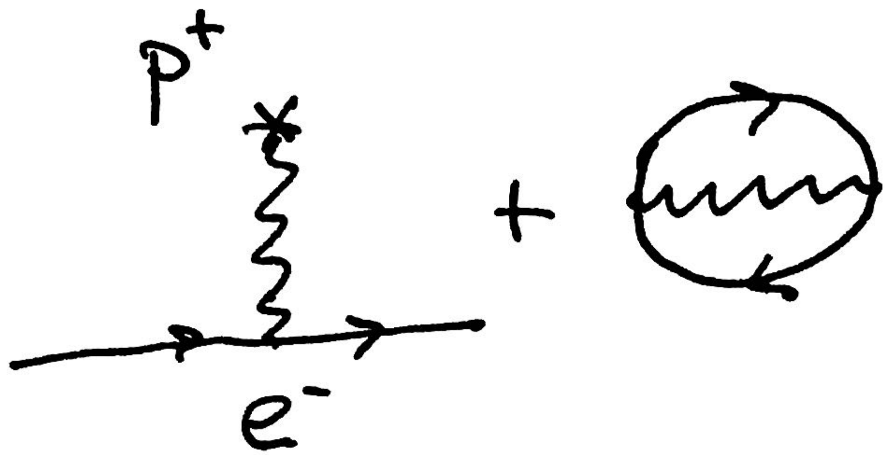
Only departure from the vacuum distribution will have influence on the electromagnetic current.

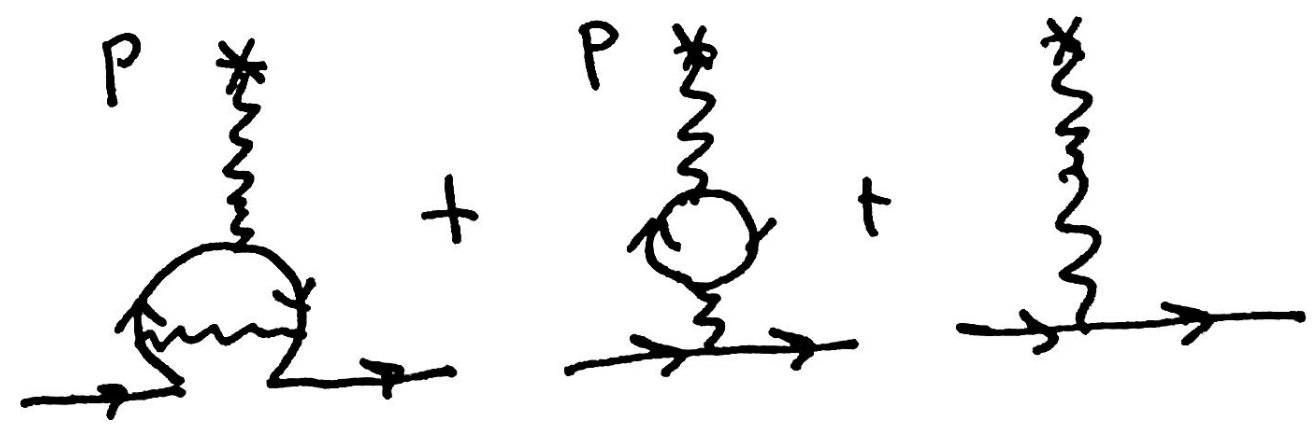


Vacuum Fluctuations

$$\Delta t \cdot \Delta U \sim \hbar$$

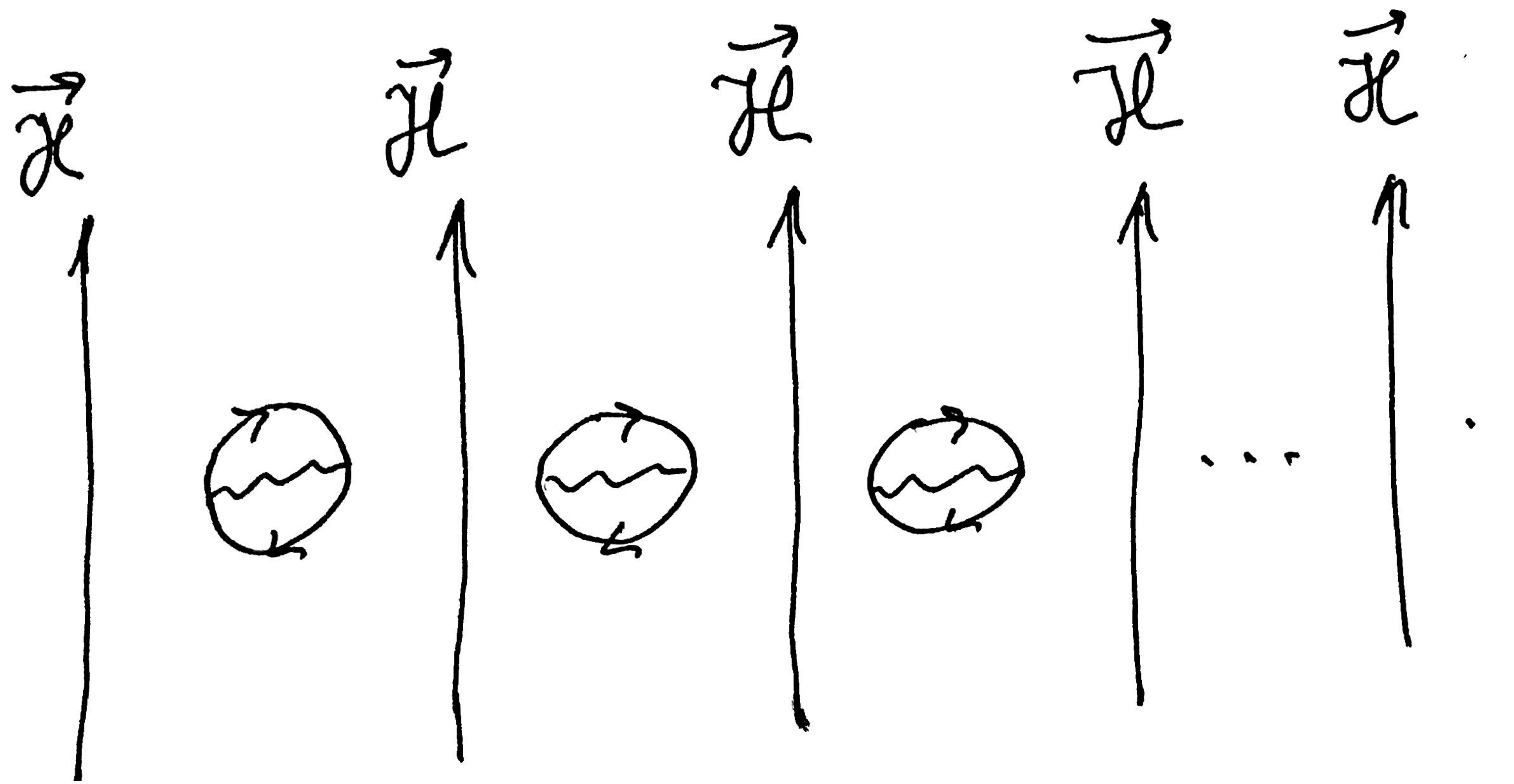
$$\Delta U \sim \frac{\hbar}{\Delta t}$$





Lamb Shift $2S_{1/2}$ and $2P_{1/2}$ levels of hydrogen

Vacuum Polarisation



Sauter Solution in Electric Field and Pair Creation

Arnold Sommerfeld   Werner Heisenberg

Bohr thought that the large transmission coefficient that Klein found was because the Klein potential step was so abrupt.

He discussed this with Heisenberg and Sommerfeld and as a result Sommerfeld's assistant Sauter in Munich calculated the transmission coefficient for a potential of the form

$$V = e\, E\, x$$

Sauter Solution in Electric Field and Pair Creation

The solution in constant electric field was of the following form

$$\psi = e^{-\frac{i}{\hbar}(Et - p_y y - p_z z)}\chi(\xi), \qquad \chi(\xi) = \begin{pmatrix} f(\xi) \\ g(\xi) \end{pmatrix}$$

$$(\frac{d}{d\xi} - i\xi)f + \kappa g = 0$$
$$(\frac{d}{d\xi} - i\xi)g + \kappa f = 0$$

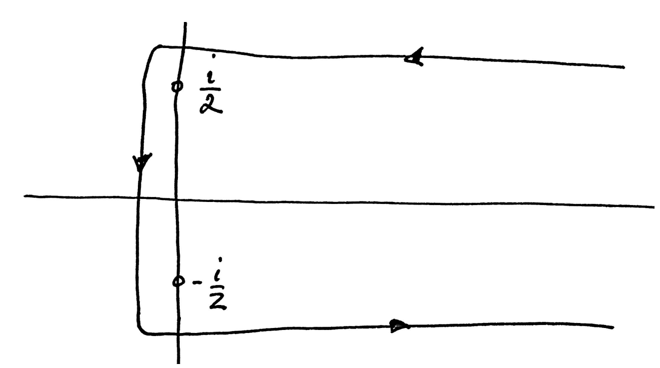
$$\xi = \sqrt{\frac{1}{\hbar c\ e\mathcal{E}}}(e\mathcal{E}x - E), \qquad \kappa = \sqrt{\frac{m^2c^4 + (p_y^2 + p_z^2)c^2}{\hbar c\ e\mathcal{E}}}$$

$$f_1 = -\frac{1}{2\sqrt{\pi}}|\xi| \int e^{-\xi^2 s}\Big(s + \frac{i}{2}\Big)^{-\frac{\kappa^2}{4i} - \frac{1}{2}}\Big(s - \frac{i}{2}\Big)^{\frac{\kappa^2}{4i}} ds$$

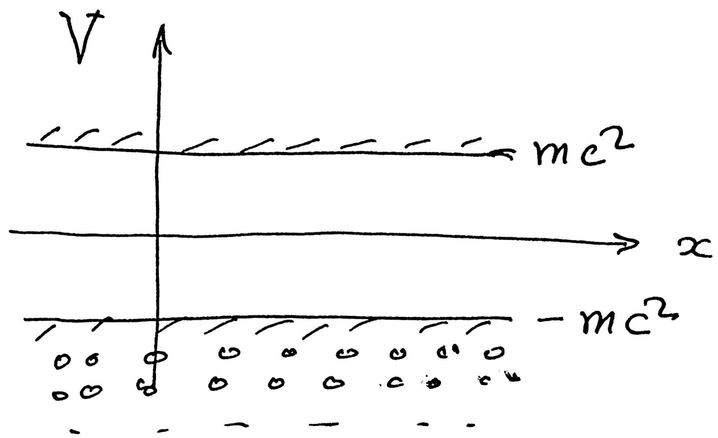
$$g_1 = -\frac{1}{2\sqrt{\pi}}\frac{\kappa|\xi|}{2\xi} \int e^{-\xi^2 s}\Big(s + \frac{i}{2}\Big)^{-\frac{\kappa^2}{4i} - \frac{1}{2}}\Big(s - \frac{i}{2}\Big)^{\frac{\kappa^2}{4i} - 1} ds$$

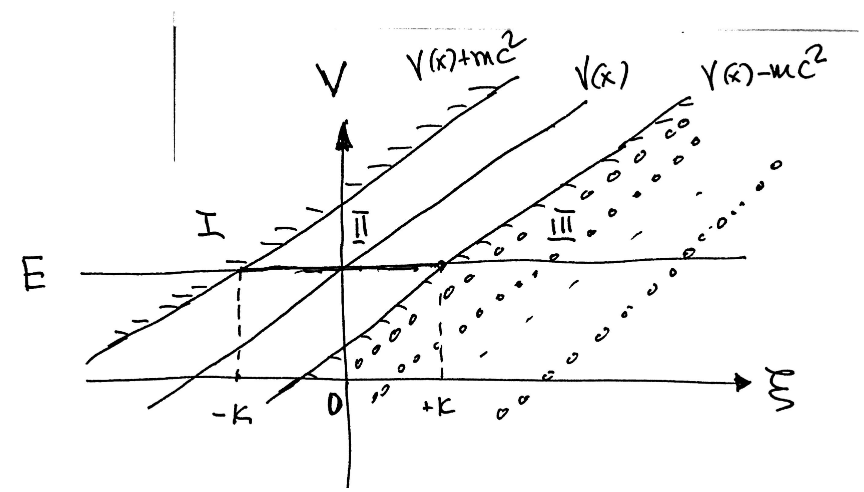
$$f_2 = -\frac{1}{2\sqrt{\pi}}\frac{\kappa|\xi|}{2\xi} \int e^{-\xi^2 s}\Big(s + \frac{i}{2}\Big)^{-\frac{\kappa^2}{4i} - 1}\Big(s - \frac{i}{2}\Big)^{\frac{\kappa^2}{4i} - \frac{1}{2}} ds$$

$$g_2 = -\frac{1}{2\sqrt{\pi}}|\xi| \int e^{-\xi^2 s}\Big(s + \frac{i}{2}\Big)^{-\frac{\kappa^2}{4i}}\Big(s - \frac{i}{2}\Big)^{\frac{\kappa^2}{4i} - \frac{1}{2}} ds,$$



Sauter Solution in Electric Field and Pair Creation



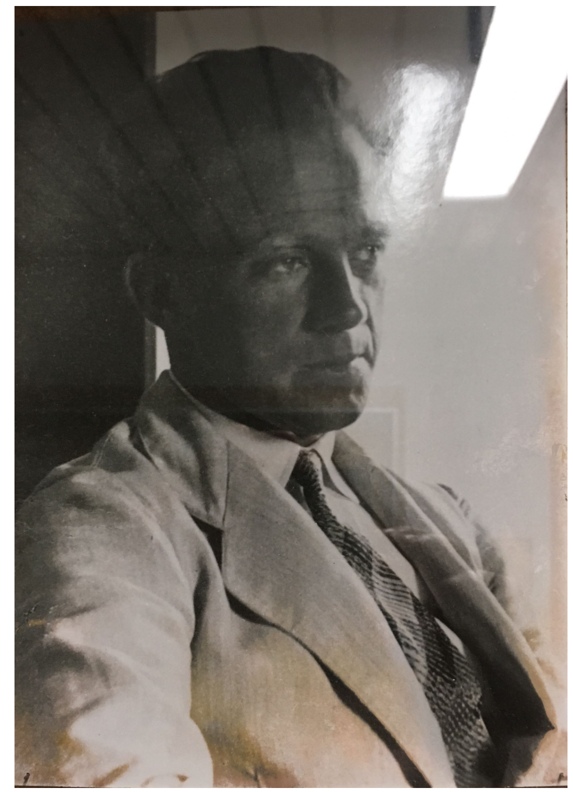


$$\kappa = \sqrt{\frac{m^2c^3}{e\hbar}\frac{1}{\mathcal{E}}} = \sqrt{\frac{\mathcal{E}_c}{\mathcal{E}}}, \quad \text{where the critical field is} \ \ \mathcal{E}_c = \frac{m^2c^3}{e\hbar} \sim 10^{16}\ Volt/cm$$

For the fields which are much smaller than the critical value the k >> 1 the decay of the wave functions towards the point ξ =0 is exponential

$$P \ \sim \ e^{-\pi \frac{m^2c^3}{e\hbar\mathcal{E}}}.$$

Similar phenomena is a black hole Hawking radiation $T = \dfrac{\hbar c^3}{8\pi G M k_B}$



Hans Euler

Werner Heisenberg

Werner Heisenberg in Demokritos National Research Center
Athens, 1956 -1965

Heisenberg-Euler Effective Lagrangian

$$\vec{D} = \vec{\mathcal{E}} + 4\pi \vec{P}_{vac}$$
$$\vec{B} = \vec{\mathcal{H}} - 4\pi \vec{M}_{vac}$$

The important step was the introduction of the effective Lagrangian

$$\mathcal{L}_{eff} = \frac{\vec{\mathcal{E}}^2 - \vec{\mathcal{H}}^2}{2} + \mathcal{L}_{vac}(\vec{\mathcal{E}}, \vec{\mathcal{H}})$$

$$D_i = \frac{\partial \mathcal{L}_{eff}}{\partial \mathcal{E}_i} = \mathcal{E}_i + \frac{\partial \mathcal{L}_{vac}}{\partial \mathcal{E}_i}, \quad B_i = -\frac{\partial \mathcal{L}_{eff}}{\partial \mathcal{H}_i} = \mathcal{H}_i - \frac{\partial \mathcal{L}_{vac}}{\partial \mathcal{H}_i}.$$

Heisenberg-Euler Effective Lagrangian

In the case of week electric field one can ignore the pair creation and the energy density of the vacuum will take the form

$\mathcal{E} \ll \mathcal{E}_c$

$$U = -mc^2(\frac{mc}{\hbar})^3(\frac{e\hbar\mathcal{E}}{m^2c^3})(\frac{e\hbar\mathcal{H}}{m^2c^3})\sum_{n=0}^{\infty}\sum_{\sigma=\pm1}\int_{-\infty}^{+\infty}\frac{d\xi}{8\pi^2}|\xi|\,\Big(|f_1|^2+|g_1|^2+|f_2|^2+|g_2|^2\Big)e^{-\frac{\kappa^2\pi}{2}-\epsilon(\xi^2-\frac{1}{a})}$$

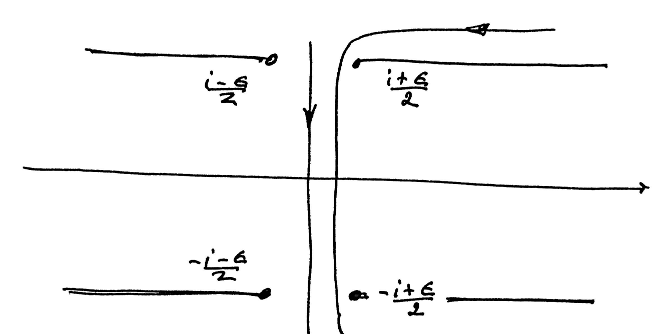
Substituting the Sauter wave functions one can get the vacuum energy density

$$U = -mc^2(\frac{mc}{\hbar})^3(\frac{e\hbar\mathcal{E}}{m^2c^3})(\frac{e\hbar\mathcal{H}}{m^2c^3})\sum_{n=0}^{\infty}\sum_{\sigma_x=\pm1}\int_{-\infty}^{+\infty}\frac{d\xi}{8\pi^2}|\xi|e^{-\frac{\kappa^2\pi}{2}-\epsilon(\xi^2-\frac{1}{a})}$$

$$\int ds_1\int ds_2\;\frac{e^{-(s_1+s_2)\xi^2}}{2\pi\sqrt{(s_1+\frac{i}{2})(s_2-\frac{i}{2})}}\;\Big(\xi^2+\frac{\kappa^2}{4(s_1-\frac{i}{2})(s_2+\frac{i}{2})}\Big)\Big(\frac{(s_1+\frac{i}{2})(s_2+\frac{i}{2})}{(s_1-\frac{i}{2})(s_2-\frac{i}{2})}\Big)^{-\frac{\kappa^2}{4i}}$$

where ε is the regularisation parameter

$$a = \frac{e\hbar\mathcal{E}}{m^2c^3}, \qquad b = \frac{e\hbar\mathcal{H}}{m^2c^3}$$



Heisenberg-Euler Effective Lagrangian

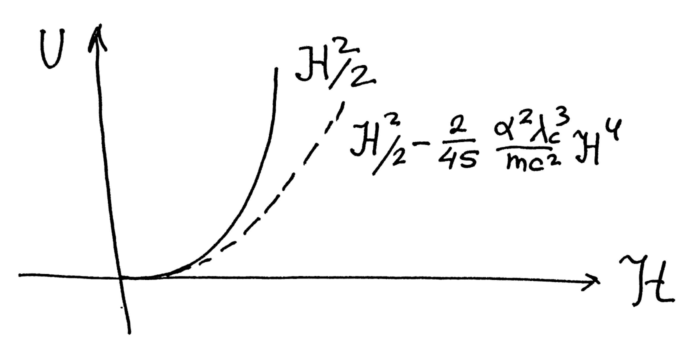
$$\mathcal{L}_{eff} = \frac{\mathcal{E}^2 - \mathcal{H}^2}{2} - 4\pi^2 mc^2 (\frac{mc}{\hbar})^3 \int_0^\infty \frac{ds}{s^3} e^{-s} \{ \frac{as\cos(as)}{\sin(as)} \; \frac{bs\cosh(bs)}{\sinh(bs)} - 1 + \frac{a^2 - b^2}{3} s^2 \}$$

where
$$a = \frac{e\hbar\mathcal{E}}{m^2c^3}, \qquad b = \frac{e\hbar\mathcal{H}}{m^2c^3}$$

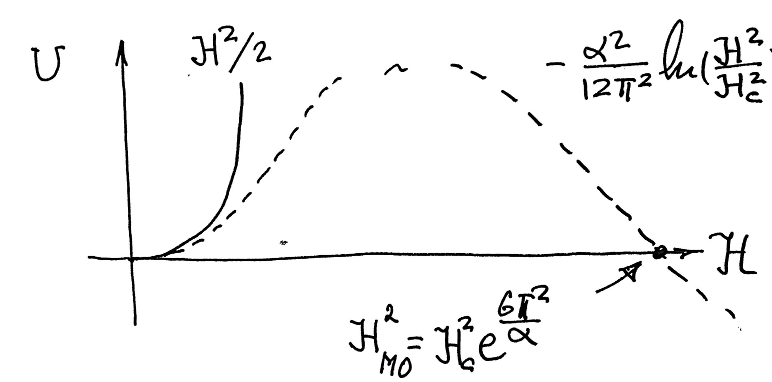
$$mc^2 = 8.2 \cdot 10^{-7} \ \frac{g\ cm^2}{s^2} \qquad \lambda_c = \frac{\hbar}{mc} = 3.86 \cdot 10^{-11} cm \qquad \frac{mc^2}{(\frac{\hbar}{mc})^3} = 1.43 \cdot 10^{25} \frac{g}{cm\ s^2}$$

For strong and week magnetic fields the quantum correction have the form

$$\mathcal{L}_{eff} \approx -\frac{\mathcal{H}^2}{2} + \frac{2}{45} (\frac{e^2}{\hbar c})^2 (\frac{\hbar}{mc})^3 \frac{1}{mc^2} (\vec{\mathcal{H}}^2)^2$$



$$\mathcal{L}_{eff} \approx -\frac{\mathcal{H}^2}{2} + (\frac{e^2}{\hbar c}) \frac{\mathcal{H}^2}{12\pi^2} \ln(\frac{e\hbar\mathcal{H}}{m^2c^3})^2$$



The QED vacuum is paramagnetic !

Heisenberg-Euler Effective Lagrangian

1. The zeta function regularisation was introduced and used to express the finale result
2. The renormalisation of Quantum Electrodynamics was clearly performed
3. The results represent infinite sum of the electromagnetic coupling constant expansion
4. The asymptotic behaviour of the effective Lagrangian at week and strong fields was derived
5. Weak expansion coincides with the Euler-Kockel Scattering of Light by Light
6. Clear understanding the tunnelling production of electron-positron pairs by strong electric field
7. The strong field behaviour demonstrates the vacuum instability known as Moscow zero

$$\mathcal{E}_c = 10^{16}\ Volt/cm \qquad U_{elec} = 0.8\ 10^{26} \frac{g}{cm\ s^2}$$

$$\mathcal{H}_c = 4.4 \cdot 10^{13}\ Gauss \qquad U_{magnet} = 0.8 \cdot 10^{26} \frac{g}{cm\ s^2}$$

$$\mathcal{E}_b = 3 \cdot 10^{4}\ Volt/cm \qquad U_{elec} = 4 \cdot 10^{2} \frac{g}{cm\ s^2}$$

$$\mathcal{H}_{neutron\ star} = 10^{15}\ Gauss \qquad U_{magnet} = 4 \cdot 10^{28} \frac{g}{cm\ s^2}$$

Schwinger Effective Lagrangian and Anomalies

A pseudoscalar interaction between the spinless neutral meson field and the proton field is described by the term

$$\mathcal{L} = g\phi < \frac{1}{2}[\bar{\psi}, \gamma_5 \psi] >= -g\phi \partial \mathcal{L}^{(1)}/\partial M = -g\phi M \int_0^\infty ds e^{-iM^2 s} tr(x|\gamma_5 U(x)|x)$$

$$= -\frac{g\phi M}{(4\pi)^2} \int_0^\infty \frac{ds}{s^2} e^{-M^2 s} e^{-l(s)} tr(\gamma_5 e^{\frac{1}{2} e\sigma F s}) = g\phi \frac{e^2}{4\pi^2} M \int_0^\infty ds e^{-M^2 s} \mathcal{G} = \frac{e^2}{4\pi^2} \frac{g}{M} \phi \vec{\mathcal{E}} \vec{\mathcal{H}}$$

This effective coupling term implies the decay of a neutral meson, into two perpendicularly polarised photons. This is chiral anomaly corresponding to the famous "triangle diagram", contributing to the pion decays.

Spontaneous Symmetry Breaking by Radiative Correction

Dimensional Transmutation

Let us consider the theory of a single real scalar field

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{\lambda}{4!}\phi^4$$

The effective Lagrangian became

$$\mathcal{L} = -\frac{\lambda}{4!}\phi^4 - \frac{\lambda^2 \phi^4}{256\pi^2}(\ln \frac{\phi^2}{M^2} - \frac{25}{6})$$

and the new minimum occurs at nonzero value

$$< \phi >^2 = M^2 e^{-\frac{32\pi^2}{3\lambda}}.$$

In massless scalar electrodynamics a similar computation will give

$$\mathcal{L} = -\frac{\lambda}{4!}\phi^4 - \frac{3e^4}{64\pi^2}\phi^4(\ln \frac{\phi^2}{< \phi >^2} - \frac{25}{6})$$

Instead of two initial dimensionless parameters λ and e there now one have the dimensionless coupling constant e and dimensional one $< \phi >$. This phenomena was called by Coleman-Weinberg *dimensional transmutation*. The scalar and vector particles will get the masses

$$m_S^2 = \frac{3e^4}{8\pi^2} < \phi >^2, \qquad m_V^2 = e^2 < \phi >^2$$

and finally

$$\frac{m_S^2}{m_V^2} = \frac{3}{2\pi}\frac{e^2}{4\pi}.$$

Effective Lagrangian in Quantum Chromodynamics

$$\Gamma(A) = S_{YM}(A) + \frac{i}{2} Tr \ln[\frac{\delta^2 S_{YM}(A)}{\delta A \; \delta A}] - iTr \ln[\nabla_\mu(A)\nabla_\mu(A)]$$

$$S_{YM}(A) = -\frac{1}{4}\int d^4x \; tr G_{\mu\nu}G_{\mu\nu}, \quad G_{\mu\nu} =$$
$$H_{\mu\nu}(\alpha) = \frac{\delta^2 S_{YM}(A)}{\delta A \; \delta A} = \eta_{\mu\nu}\nabla_\sigma(A)\nabla_\sigma(A) - 2gG_{\mu\nu} + (\alpha-1)\nabla_\mu(A)\nabla_\nu(A),$$
$$H_{FP} = \nabla_\mu(A)\nabla_\mu(A).$$

The effective Lagrangian take the following form

$$\mathcal{L}^{(1)} = -\frac{1}{8\pi^2}\int \frac{ds}{s^3} e^{-i\mu^2 s} \frac{(gF_1 s)\;(gF_2 s)}{\sinh(gF_1 s)\;\sinh(gF_2 s)} -$$
$$-\frac{1}{4\pi^2}\int \frac{ds}{s^3} e^{-i\mu^2 s}(gF_1 s)\;(gF_2 s)[\frac{\sinh(gF_1 s)}{\sinh(gF_2 s)} + \frac{\sinh(gF_2 s)}{\sinh(gF_1 s)}]$$

$$F_1^2 = -\mathcal{F} - (\mathcal{F}^2 + \mathcal{G}^2)^{1/2}, \qquad F_2^2 = -\mathcal{F} + (\mathcal{F}^2 + \mathcal{G}^2)^{1/2}$$

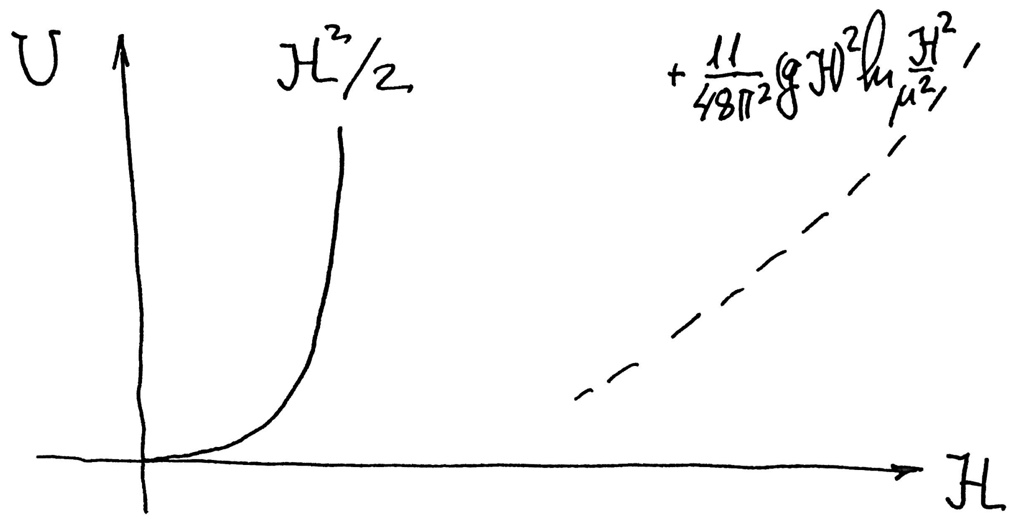
Effective Lagrangian in Quantum Chromodynamics

The asymptotic behaviour for strong magnetic fields is in QCD

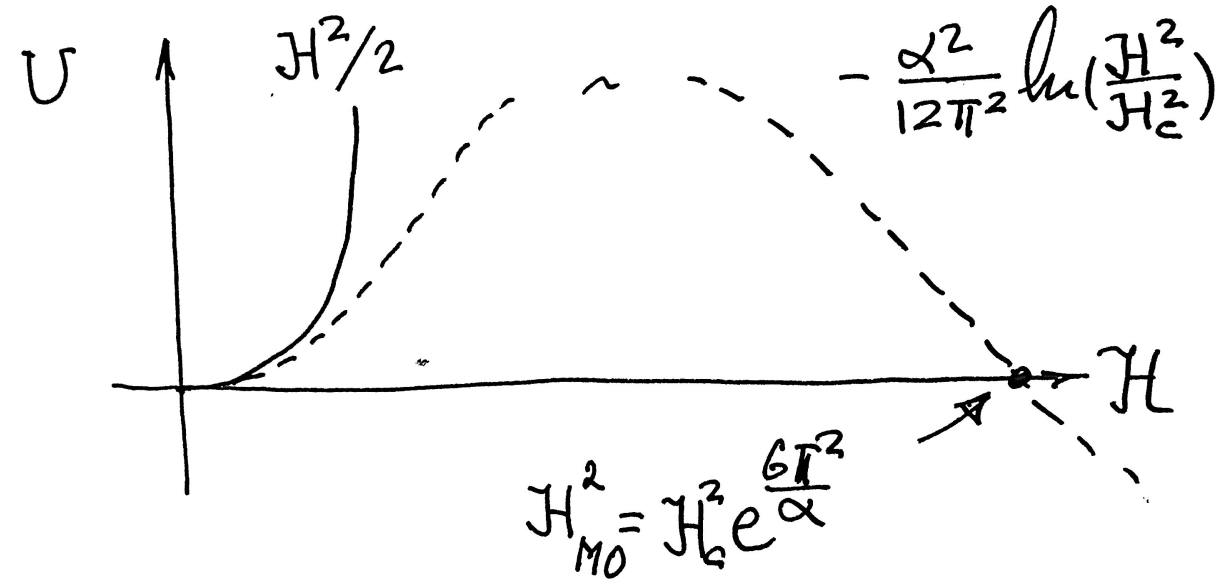
$$\mathcal{L}^{(1)} \approx -\frac{11}{48\pi^2}(g\mathcal{H})^2 \ln \frac{g\mathcal{H}}{\mu^2}$$

As one can clearly see in QED the asymptotic is different

$$\mathcal{L}^{(1)} \approx +\frac{1}{24\pi^2}(e\mathcal{H})^2 \ln(\frac{e\mathcal{H}}{m^2})$$



The QCD vacuum is diamagnetic !



The QED vacuum is paramagnetic !

Gluon Condensation in Quantum Chromodynamics

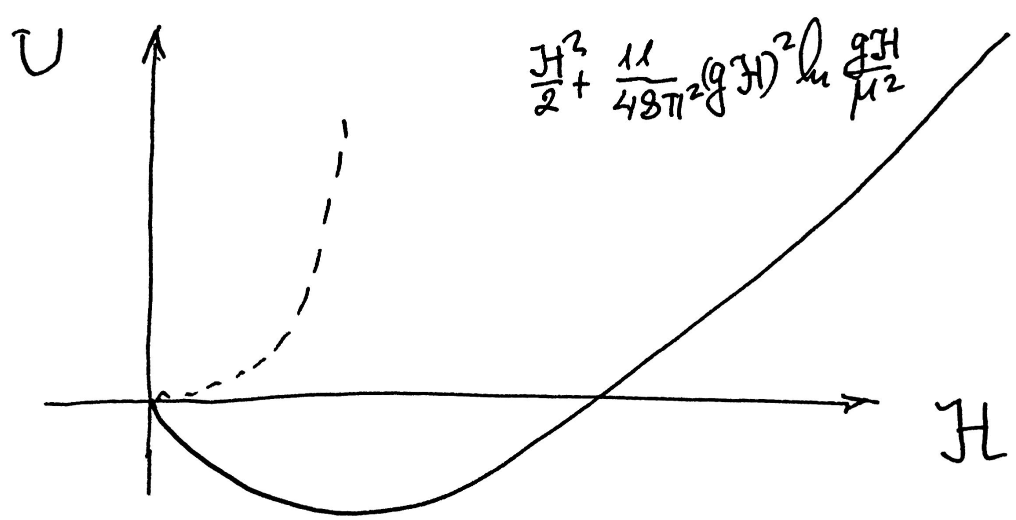
$$\frac{\partial \mathcal{L}}{\partial \mathcal{F}}\Big|_{t=\ln(\frac{2\mathcal{F}}{\mu^4})=\mathcal{G}=0} = -1$$

this leads to the renormalised

$$\mathcal{L}^{(1)} = \frac{g^2\mu^4}{8\pi^2}\int_0^\infty \frac{ds}{s^3}\Big(\frac{as}{\sinh as} - \frac{a^2 s}{2}(\frac{1}{\sinh s} - \frac{s\cosh s}{\sinh^2 s})\Big) + \\ + \frac{g^2\mu^4}{4\pi^2}\int_0^\infty \frac{ds}{s^3}\Big(as\sin(as) - \frac{a^2 s}{2}(\sin s + s\cos s)\Big)$$

The energy density of the vacuum will take the form

$$U(\mathcal{H}) = \frac{\mathcal{H}^2}{2} + \frac{11}{48\pi^2}(g\mathcal{H})^2[\ln\frac{g\mathcal{H}}{\mu^2} - \frac{1}{2}]$$



$$< gH_{vac} >= \mu^2 \exp\big(-\frac{24\pi^2}{11g^2}\big).$$

Reducing full one-loop amplitudes to scalar integrals at the integrand level

G. Ossola, C. Papadopoulos and R. Pittau

With the ongoing evolution of the experimental programs of the LHC and the International Linear Collider, high precision predictions for multi-particle processes are urgently needed. In the last years we have seen a remarkable progress in the theoretical description of multi-particle processes at tree-order, thanks to very efficient recursive algorithms. Nevertheless the current need of precision goes beyond tree order and therefore a similar description at the one loop level is more than desirable.

High Order Corrections and Monte Carlo Simulations

It was shown how computing the integrand of any one-loop amplitude at special values of the integration momentum allows the one-shot reconstruction of all the coefficients of the scalar loop functions and of the rational terms. Then, by simply multiplying those coefficients by the known scalar integrals, the computation of the amplitude becomes trivial. The method should be particularly useful in the case when recursive techniques are used to numerically compute the integrand.

Cited by 578 records

MIXMAX random number generator for MC simulations

The Monte Carlo method is widely used in many areas of science and applications:
Physical sciences, Engineering, Climate change and radiative forcing, Computational biology, Computer graphics, Applied statistics, Artificial intelligence for games, Design and visuals, Search and rescue, Finance and business

1. The Consortium has developed a cutting-edge theory of the MIXMAX generator. The MIXMAX code in C and C++ was developed by Konstantin Savvidy. The MIXMAX code generates 64-bit high quality random sequences and it is one of the fastest generators on the market:

   https://mixmax.hepforge.org

2. The MIXMAX generator has been implemented into Geant4/CLHEP and ROOT software as default generator. "Geant4 is a toolkit for the simulation of the passage of particles through matter. Its areas of application include high energy, nuclear and accelerator physics, as well as studies in medical and space science".

   http://geant4.cern.ch

   http://proj-clhep.web.cern.ch/proj-clhep/

   https://gitlab.cern.ch/CLHEP/CLHEP/blob/master/Random/Random/MixMaxRng.h

3. The code of the MIXMAX generator has been implemented into PYTHIA 8 software, which is widely used in high energy experiments at CERN:

   http://home.thep.lu.se/~torbjorn/pythia81html/Welcome.html

   http://home.thep.lu.se/~torbjorn/doxygen/MixMax_8h_source.html

   http://home.thep.lu.se/~torbjorn/doxygen/dir_f0ac0583067f8579d45895eb993b0618.html