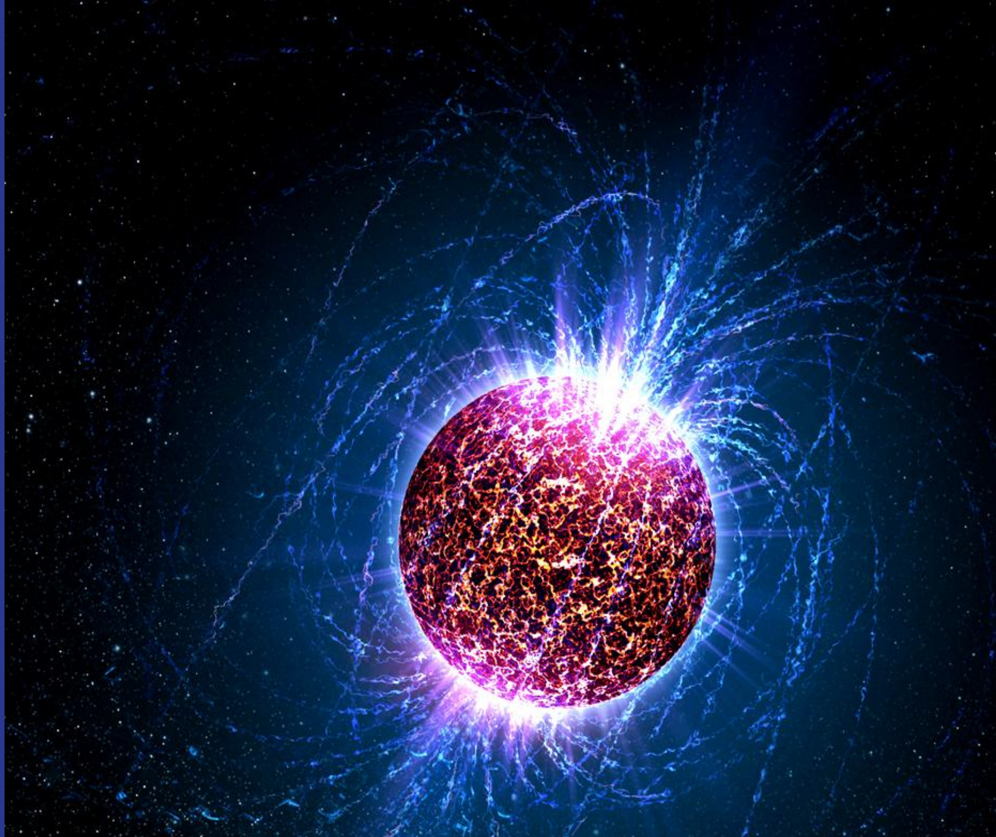
Compact objects in the Generalized SU(2) Proca Theory

Jose Fernando Rodriguez Ruiz
Universidad Industrial de Santander



Martínez J, Rodríguez J, Becerra L, Rodríguez Y, Gómez G, Phys. Rev. D 110, 104070 (2024)

J.N. Martínez, J. F. Rodríguez, Y. Rodríguez, and G. Gómez, J. Cosmol. Astropart. Phys. 04 (2023) 032.

Gómez G. & Rodríguez, J.F. Phys. Rev. D 108, 024069 (2023)

Introduction

- Einstein's General Relativity is a very successful theory.
- General relativity is an effective theory of the gravitational interaction.
- Questions: What is dark energy? What is dark matter?
- Modified gravity models could answer these questions.
- Metric theories of gravity: additional degrees of freedom.
- This work: additional vectorial degrees of freedom → generalized SU(2) Proca theory (GSU2P).

GSU2P Theory

- Main components: contractions of the Riemann tensor, SU(2) vector field,

$$F^a{}_{\mu\nu} = 2\nabla_{[\mu}B^a{}_{\nu]} + g_c\epsilon^a{}_{bc}B^b{}_\mu B^c{}_\nu \quad A^a{}_{\mu\nu} = 2\nabla_{[\mu}B^{a'}{}_{\nu]} \quad S^a{}_{\mu\nu} = 2\nabla_{(\mu}B^a{}_{\nu)}$$

- Global SU(2) symmetry.
- Equations are maximum of second order
- Correct number of propagating degrees of freedom
- Action principle A. G. Cadavid, Y. Rodríguez, and L. G. Gómez, Phys. Rev. D 102, 104066 (2020)

$$S = \int \sqrt{-g}d^4x \frac{1}{16\pi}\left(R + F_{a\mu\nu}F^{a\mu\nu} + 2\mu B_{a\mu}B^{a\mu} + \sum_{n=0}^{6}\alpha_n \mathscr{L}^n_{4,2} + \sum_{m=1}^{7}\chi_m \mathscr{L}^m_2\right)$$

GSU2P Lagrangian

- α's Lagrangian pieces

$$\mathcal{L}^1_{4,2} = B_{b\alpha}B^{b\alpha}\left(S^{a\mu}{}_{\mu}S_{a\nu}{}^{\nu} - S^{a\mu}{}_{\nu}S_{a\mu}{}^{\nu}\right) + 2B_{a\alpha}B^{\alpha}_{b}\left(S^{\mu a}_{\mu}S^{\nu b}_{\nu} - S^{\mu a}_{\nu}S^{\nu b}_{\mu}\right),$$

$$\mathcal{L}^2_{4,2} = A^{a}_{\mu\nu}S^{\nu b}_{\sigma}B^{\nu}_{a}B^{\sigma}_{b} - A^{a}_{\mu\nu}S^{\mu b}_{\sigma}B^{\nu}_{b}B^{\sigma}_{a} + A^{a}_{\mu\nu}S^{\sigma b}_{\sigma}B^{\mu}_{a}B^{\nu}_{b},$$

$$\mathcal{L}^3_{4,2} = B^{a\mu}R^{\alpha}{}_{\sigma\rho\mu}B_{a\alpha}B^{c\rho}B_{c}{}^{\sigma} + \frac{3}{4}B^{a}_{\mu}B^{\mu}_{a}B^{\nu}_{b}B^{b}_{\nu}R,$$

$$\mathcal{L}^4_{4,2} = \left(B^{a\mu}B_{a\mu}B^{b\nu}B_{b\nu} + 2B^{a}{}_{\mu}B^{b\mu}B_{a\nu}\,B_{b}{}^{\nu}\right)R,$$

$$\mathcal{L}^5_{4,2} = G_{\mu\nu}B^{\mu a}B_{a}{}^{\nu}B^{b\alpha}B_{b\alpha},$$

$$\mathcal{L}^6_{4,2} = G_{\mu\nu}B^{\mu a}B^{\nu b}B_{a\alpha}B_{b}{}^{\alpha}.$$

GSU2P Lagrangian

- χ's Lagrangian pieces

$$\mathcal{L}_2^1 = B_{a\mu}B^{a\mu}B_{b\nu}B^{b\nu},$$

$$\mathcal{L}_2^2 = B^a{}_\mu B_b{}^\mu B^b{}_\nu B_a{}^\nu,$$

$$\mathcal{L}_2^3 = B^b{}_\mu B_b{}^\rho A^{a\mu\nu} A_{a\rho\nu},$$

$$\mathcal{L}_2^4 = B^b{}_\mu B_a{}^\rho A^{a\mu\nu} A_{b\rho\nu},$$

$$\mathcal{L}_2^5 = B_{\mu a}B^{b\rho} A^{a\mu\nu} A_{\rho\nu b},$$

$$\mathcal{L}_2^6 = B^{b\rho}B^{b\rho} A_{a\mu\nu} A^{a\mu\nu},$$

$$\mathcal{L}_2^7 = B^{b\rho}B_{a\rho} A_{b\mu\nu} A^{a\mu\nu}.$$

Conserved charges

- The topological charge is defined from the Bianchi Identity,

$$d(*\mathbf{F}) = 0 \qquad Q^a_M = \int \nabla_\mu (*F^{a\mu 0})\sqrt{-g}\, d^3x = \int \partial_\mu (\epsilon^{\mu 0 \alpha\beta} F^a{}_{\alpha\beta}) d^3x = \int dS_k \sqrt{-g} * F^{k0}$$

- Noether charge from the global SU(2) symmetry,

$$\nabla_\mu J^{a\mu} = 0, \qquad J^{a\mu} = \frac{\delta \mathscr{L}}{\delta(B_{b\nu;\mu})} \epsilon^a{}_{bc} B^c{}_\nu$$

Cosmological implications

- Vector fields isotropy? Cosmic triad, emerges naturally in SU(2)
- GW propagation speed on a FRW background is luminal? This is a very stringent condition to modified gravity models.
- Perturbations: the tensorial sector is the same as GR,

$$\alpha_2 = 2\alpha_3, \alpha_4 = \frac{7\alpha_3}{20} - 2\alpha_1, \alpha_5 = \frac{14\alpha_3}{3} - \frac{20\alpha_1}{3}, \alpha_6 = -8\alpha_3,$$

$$\chi_3 = 0, \chi_7 = 5\alpha_1 + \alpha_3 - \frac{\chi_4}{2} - 3\chi_6$$

- Early inflation: constant roll J. C. Garnica, L. G. Gomez, A. A. Navarro, and Y. Rodriguez, Annalen der Physik, 534, 2100453 (2021)

Yang-Mills vector fields + Gravity

- Strong field regime in compact objects.
- Explore solution to the breakdown of GR at the singularity of black hole
- Einstein + Yang-Mills (EYM) , static, localized solutions, asymptotically flat: soliton, particlelike.
- Equilibrium: Attractive (gravity) and repulsive (Yang-Mills) interactions.
- EYM Good: particlelike and BHs. Bad: unstable.
- Einstein + Skyrme: Particlelike, BHs. Stable solutions.
- Are there particlelike, BH and NS solutions in GSU2P theory?
- Can we constrain GSU2P theory with compact objects?

Field equations

- Field equations from varying the metric $G_{\mu\nu} = 8\pi T^{\text{eff}}_{\mu\nu}\Big|_{\text{GW}}$,

$$T^{\text{eff}}_{\mu\nu} = -\frac{1}{8\pi\sqrt{-g}}\frac{\delta}{\delta g_{\mu\nu}}\left[F_{a\mu\nu}F^{a\mu\nu} + \sum_{i=0}^{6}\alpha_n \mathcal{L}^n_{4,2} + \sum_{m=1}^{7}\chi_m \mathcal{L}^m_2\right]$$

- Field equations from varying the vector fields,

$$\frac{\delta}{\delta B_{a\mu}}\left[F_{a\mu\nu}F^{a\mu\nu} + \sum_{i=0}^{6}\alpha_n \mathcal{L}^n_{4,2} + \sum_{m=1}^{7}\chi_m \mathcal{L}^m_2\right] = 0$$

- Magnetic monopole ansatz

$$A_0 = A_1 = \phi_1 = 0,\ \phi_2 = w$$

Stationary and spherical symmetric solution

- Regular solutions: all algebraic curvature invariants, energy density and pressure are finite.
- Line element,

$$ds^2 = -e^{2\Phi}dt^2 + (1-2m/r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

$$e^{2\Phi} \equiv e^{-2\delta}(1-2m/r)$$

- SU(2) spherical configuration,

$$\mathbf{B} = \frac{\tau^i}{g_c}\left[A_0\frac{x_i}{r}dt + A_1\frac{x_i x_j}{r^2}dx^j + \frac{\phi_1}{r}\left(\delta_{ij} - \frac{x_i x_j}{r^2}\right)dx^j - \epsilon_{ijk}x^j\frac{(1-w)}{r^2}dx^k\right],$$

Analytic solution: Generalized Non-abelian charged BH

Gomez G. & Rodriguez, J.F. Phys. Rev. D 108, 024069 (2023)

- We found an analytic solution for the parameters $\chi_{12} = 2\chi_1 + \chi_2$, with the ansatz $w = w_c = $ conts.
- Reissner-Nordstrom black hole,

$$w_c = w_{\mathrm{I,II}},\ m = M - \frac{Q^2_{\mathrm{I,II}}}{2r},\ \Phi = \frac{1}{2}\ln\left(1 - \frac{2M}{r} + \frac{Q^2_{\mathrm{I,II}}}{r^2}\right) \qquad w_{\mathrm{I,II}} = \frac{1 + 2\chi_{12} \pm \sqrt{1 + 8\chi_{12}}}{2 - 2\chi_{12}},$$

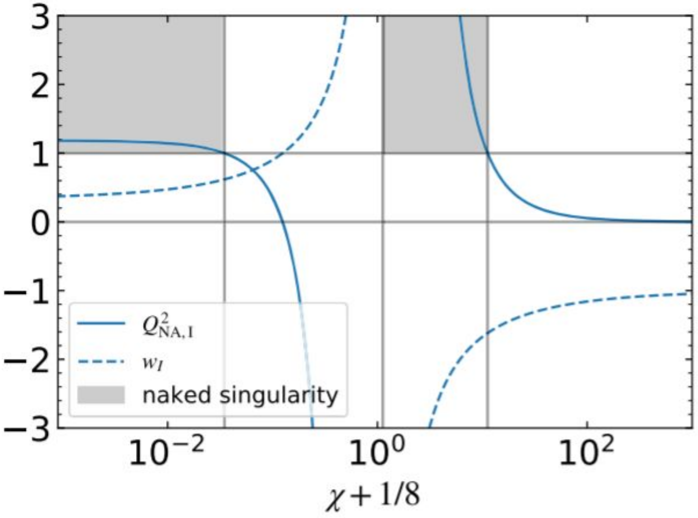
$$Q^2_{\mathrm{I,II}} = \frac{1 - 4\chi_{12}(5 + 2\chi_{12}) \mp (1 + 8\chi_{12})\sqrt{1 + 8\chi_{12}}}{2(1 - \chi_{12})^3},$$

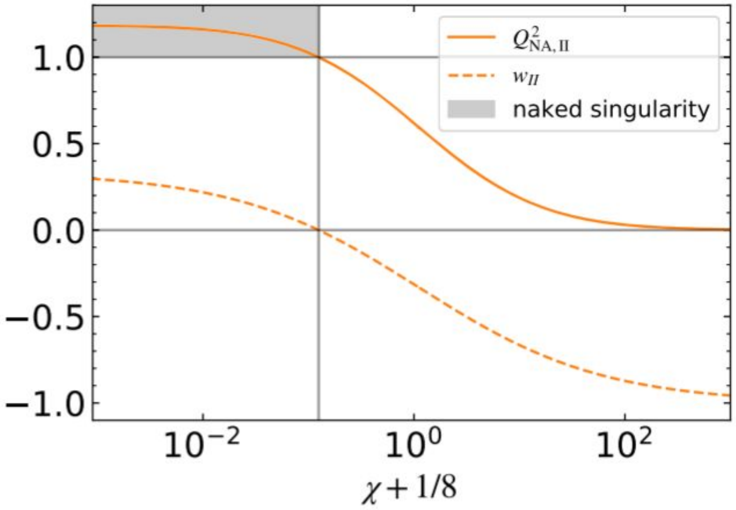
- Effective charge, $Q^2 \equiv 2r(M - m)$ $\quad Q^a_M \propto (1 - w^2_\infty)$ $\quad Q = Q^a_M + q$ $\quad \chi_{12} > -1/8$

- For the solution I the charge can be imaginary in $\chi_{12} \in (0,1)$

Non-abelian charged black hole

Gomez G. & Rodriguez, J.F. Phys. Rev. D 108, 024069 (2023)





Shadow of non-abelian charged black hole

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- Photon sphere in a static and spherical spacetime,

$$r_{\rm ph} g'_{tt}(r_{\rm ph}) - 2 g_{tt}(r_{\rm ph}) = 0$$

- Shadow radius,

$$r_{\rm sh} = \frac{r_{\rm ph}}{\sqrt{g_{tt}(r_{\rm ph})}}$$

- Non-abelian charged black hole, $q_{\rm NA} = Q_{\rm NA}/M$

$$\frac{r_{\rm sh}}{M} = x_{\rm sh} = \frac{\sqrt{2}\left(\sqrt{9 - 8q_{\rm NA}^2} + 3\right)}{\sqrt{\frac{4q_{\rm NA}^2 + \sqrt{9 - 8q_{\rm NA}^2} - 3}{q_{\rm NA}^2}}}$$

EHT constraints

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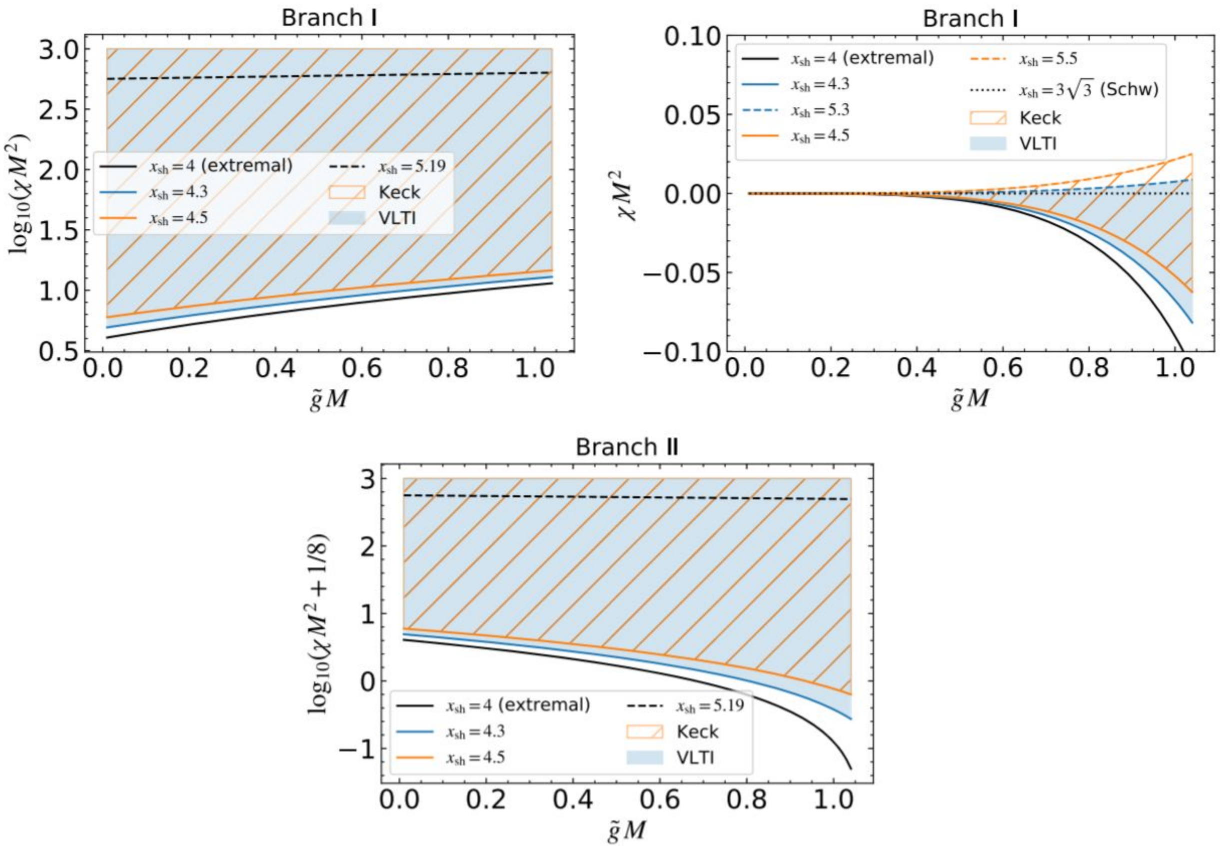

Particlelike case: asymptotic series solutions r→0

J.N. Martínez, J. F. Rodríguez, Y. Rodríguez, and G. Gómez, J. Cosmol. Astropart. Phys. 04 (2023) 032.

- Power series expansion,

$$m = a_0 + a_1 r + a_2 r^2 + a_3 r^3 + a_4 r^4 + a_5 r^5 + \mathcal{O}(r^6)$$
$$w = b_0 + b_1 r + b_2 r^2 + b_3 r^3 + b_4 r^4 + \mathcal{O}(r^5),$$
$$\Phi = c_1 r + c_2 r^2 + c_3 r^3 + c_4 r^4 + \mathcal{O}(r^5)\,.$$

- Curvature invariants are finite at r = 0 when $a_1 = a_2 = c_1 = 0$. Effective energy density and pressure are finite when $b_0 = -1$, $b_1 = 0$.

$$a_3 = 2b_2^2,$$
$$a_5 = \frac{3}{5}\mu^2 b_2^2 - \frac{8b_2^3}{5} + \frac{172\alpha_1 b_2^4}{3} + \frac{7\alpha_3 b_2^4}{15} - 4\chi_6 b_2^4,$$
$$b_4 = \frac{\mu^2 b_2}{10} - \frac{3b_2^2}{10} + \frac{4b_2^3}{5} + \alpha_1 b_2^3$$
$$c_2 = 2b_2^2,$$
$$c_4 = \frac{\mu^2 b_2^2}{5} - \frac{4b_2^3}{5} + \frac{12b_2^4}{5} - 8\alpha_1 b_2^4 + \frac{9\alpha_3 b_2^4}{10} - \frac{2\chi_5 b_2^4}{5} - 2\chi_6 b_2^4,$$

Asymptotic solutions r→∞

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Solution as a series of inverse powers of r,

$$m = M + \frac{\tilde{a}_1}{r} + \frac{\tilde{a}_2}{r^2} + \frac{\tilde{a}_3}{r^3} + \mathcal{O}\left(\frac{1}{r^4}\right),$$

$$w = w_\infty + \frac{\tilde{b}_1}{r} + \frac{\tilde{b}_2}{r^2} + \frac{\tilde{b}_3}{r^3} + \mathcal{O}\left(\frac{1}{r^4}\right),$$

$$\Phi = \Phi_\infty + \frac{\tilde{c}_1}{r} + \frac{\tilde{c}_2}{r^2} + \frac{\tilde{c}_3}{r^3} + \mathcal{O}\left(\frac{1}{r^4}\right),$$

Asymptotic flatness.

Asymptotic solutions r→∞

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	$w_\infty = -1$	$w_\infty = 1$	$w_\infty = w_{\rm I,II}$
$\tilde{a}_1$	0	0	$-Q_{\rm I,II}^2/2$
$\tilde{a}_2$	0	0	0
$\tilde{a}_3$	$-\tilde{b}_1^2$	$64(25\alpha_1 - 36\alpha_3 + 5\chi_6)/15 - \tilde{b}_1^2$	$\frac{2}{3}\tilde{b}_2(w_\infty+1)[\chi_{12} + (\chi_{12}-1)w_\infty^2 + 2\chi_{12}w_\infty + w_\infty] + \frac{4}{15}(w_\infty+1)^4(25\alpha_1 - 36\alpha_3 + 5\chi_6)$
$\tilde{b}_1$	Free	Free	0
$\tilde{b}_2$	$3(2M - \tilde{b}_1)\tilde{b}_1/4$	$16\alpha_1 + 6\alpha_3 - 16\chi_6 + 3(2M + \tilde{b}_1)\tilde{b}_1/4$	$\frac{(w_\infty+1)^3(8\alpha_1 + 3\alpha_3 - 8\chi_6)}{3\chi_{12}(w_\infty+1)^2 - 3w_\infty^2 + 7}$
$\tilde{b}_3$	$\tilde{b}_1[48M^2 - 42M\tilde{b}_1 + (11 - 2\chi_{12})\tilde{b}_1^2]/20$	$(512\alpha_1 M + 96\alpha_3 - 512\chi_6 M)/20$ $+16\tilde{b}_1(26\alpha_1 + 13\alpha_3 + 2\chi_5 - 26\chi_6)/20$ $+(48M^2 - 42M\tilde{b}_1 + 11\tilde{b}_1^2)/20$	$\frac{2M\left(8\tilde{b}_2 - 3\alpha_3(w_\infty+1)^3\right)}{3\chi_{12}(w_\infty+1)^2 - 3w_\infty^2 + 13}$
$\tilde{c}_1$	$-M$	$-M$	$-M$
$\tilde{c}_2$	$-M^2$	$-M^2$	$-M^2 + Q_{\rm I,II}^2/2$
$\tilde{c}_3$	$-4M^3/3$	$-4M^3/3$	$-4M^3/3 + Q_{\rm I,II}^2 M/2$

Asymptotic charged solution

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Series of inverse powers of r with noninteger exponent,

$$m = M - \frac{Q_\mathrm{I}^2}{2r} - \frac{d^2 D}{r^{2\beta+1}} + \mathcal{O}\left(\frac{1}{r^{3\beta+1}} + \frac{1}{r^{2\beta+2}}\right)$$

$$w = w_\mathrm{I} + \frac{d}{r^\beta} + \mathcal{O}\left(\frac{1}{r^{\beta+1}} + \frac{1}{r^{2\beta}}\right)$$

$$\sigma = -\frac{d^2\beta^2}{(1+\beta)}\frac{1}{r^{2\beta+2}} + \mathcal{O}\left(\frac{1}{r^{2\beta+3}} + \frac{1}{r^{3\beta+2}}\right),$$

$$\beta = \frac{1}{2}\left(1 - \sqrt{\frac{3 + 15\chi_{12} + 6\sqrt{1 + 8\chi_{12}}}{\chi_{12} - 1}}\right),$$

$$D = \frac{1 + 8\chi_{12} + \sqrt{1 + 8\chi_{12}} - 2\beta^2(\chi_{12} - 1)}{2(1 + 2\beta)(\chi_{12} - 1)}.$$

Generalized Non-Abelian Boson Star

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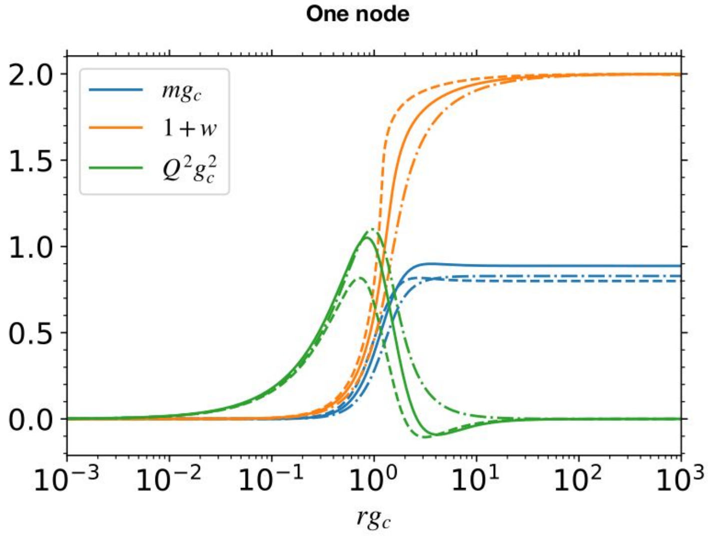
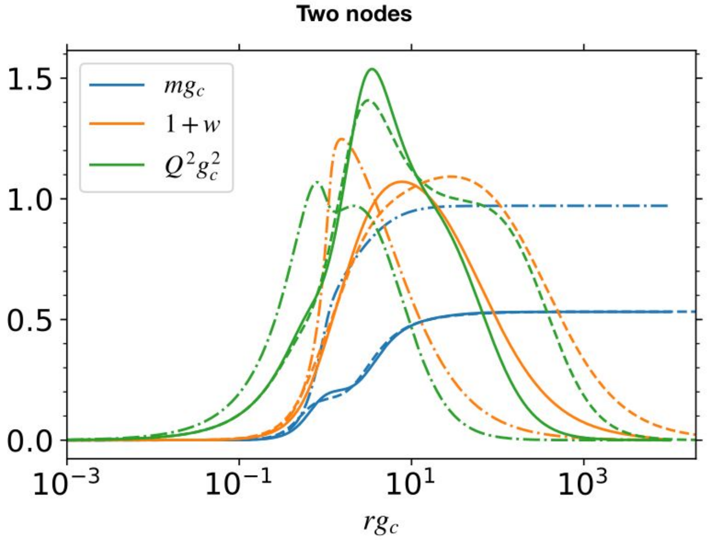
Case α_1

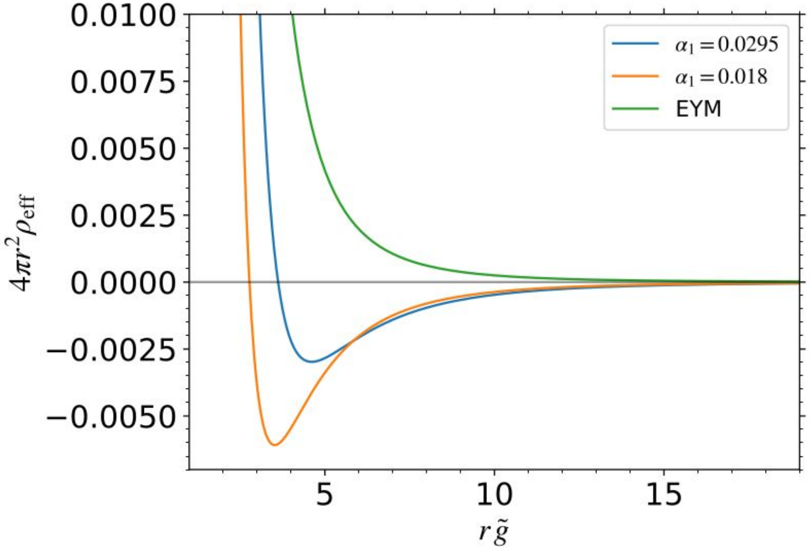


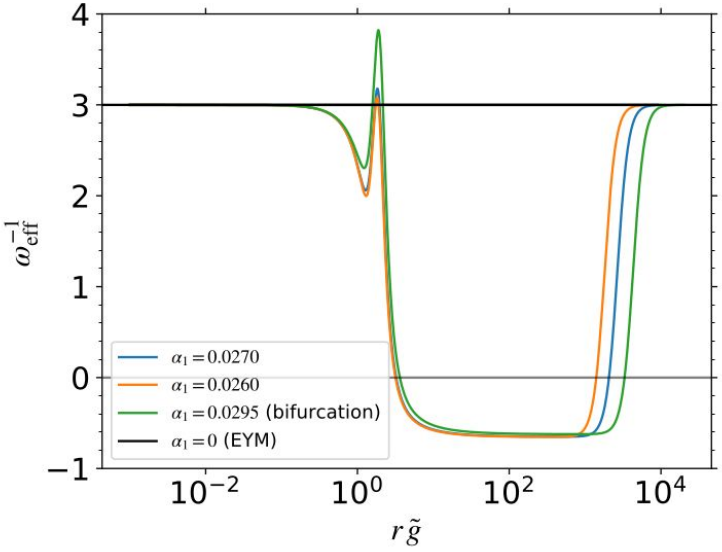
Continuous $\alpha_1 = 0.029519$, dashed $\alpha_1 = 0.017$, dotted-dashed EYM



Continuous and dashed $\alpha_1 = -1$, dotted-dashed EYM

Energy density and parameter of state α_1





Equilibrium sequence α_1

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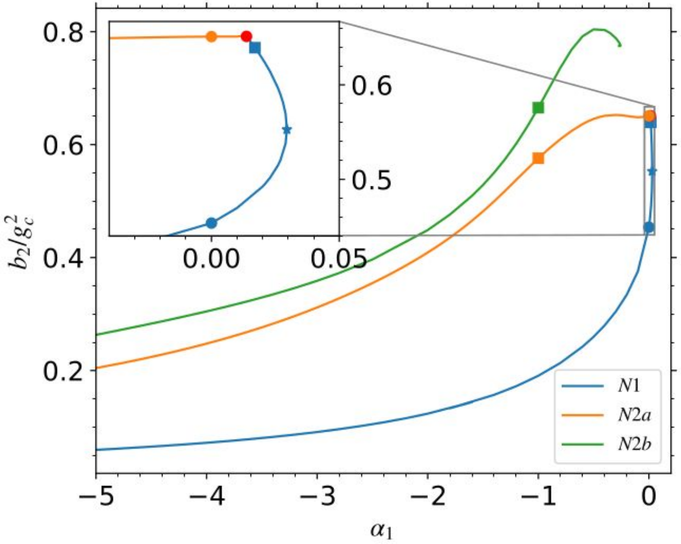
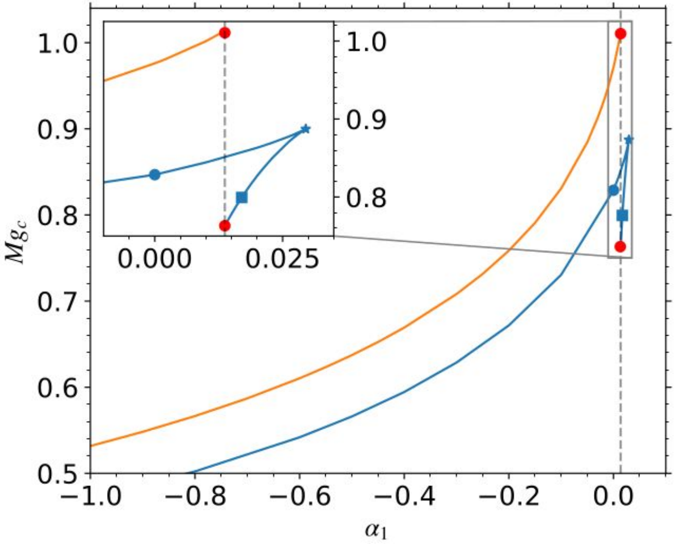
α_1 control parameter





Generalized Non-Abelian Boson Star χ_6

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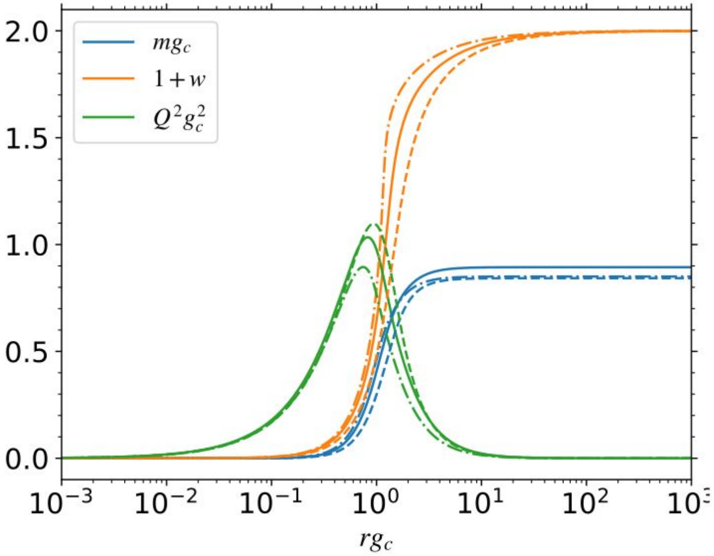

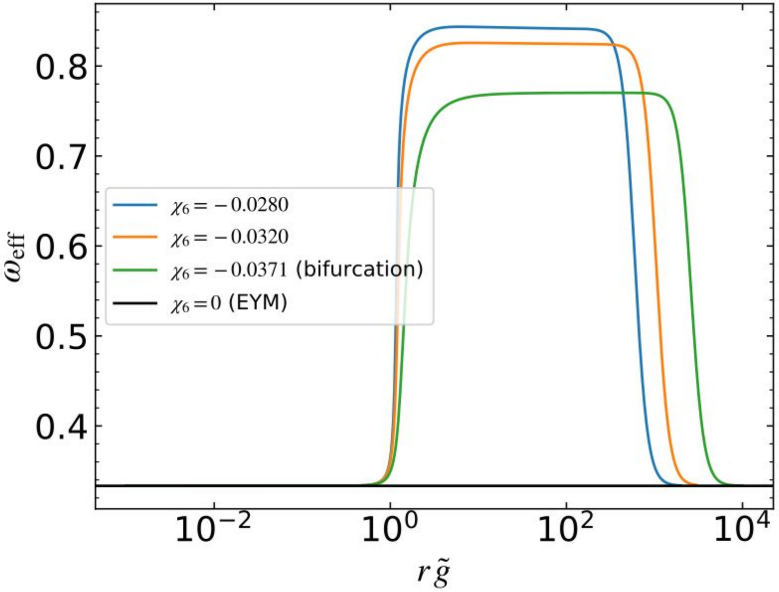

Equilibrium sequences χ_6

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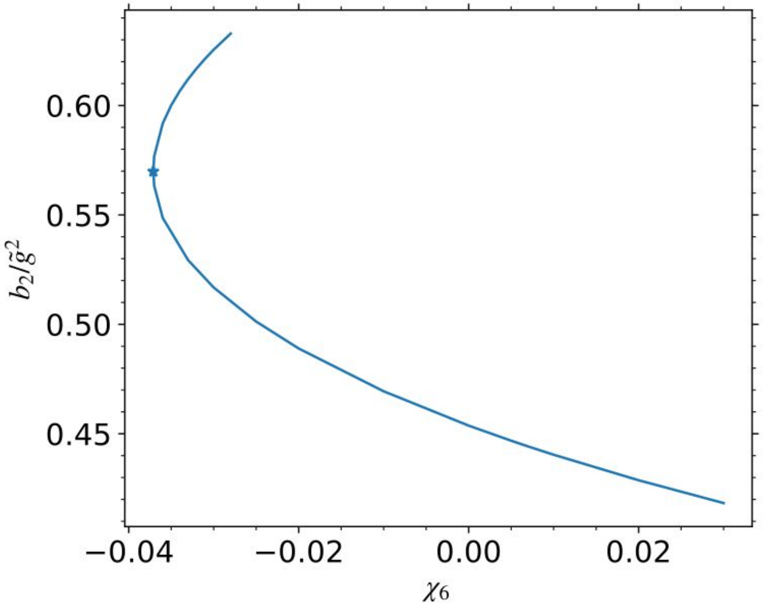

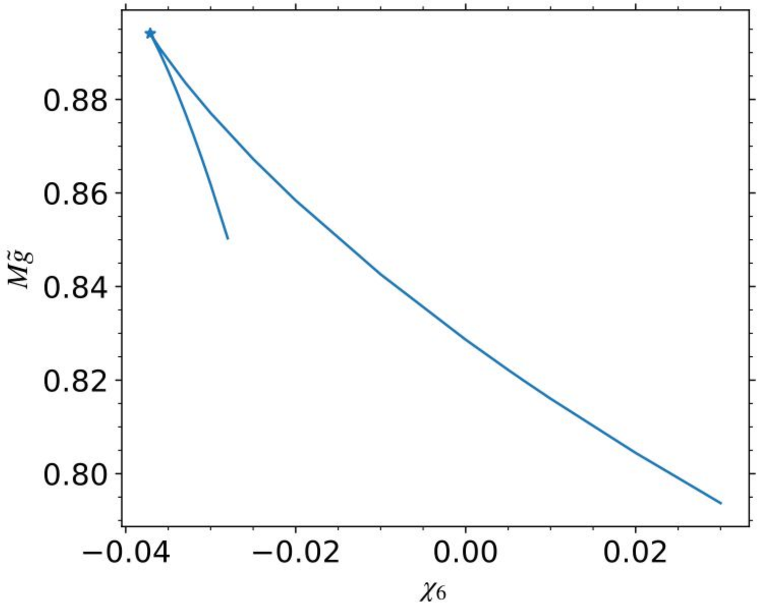

Globally charged solutions

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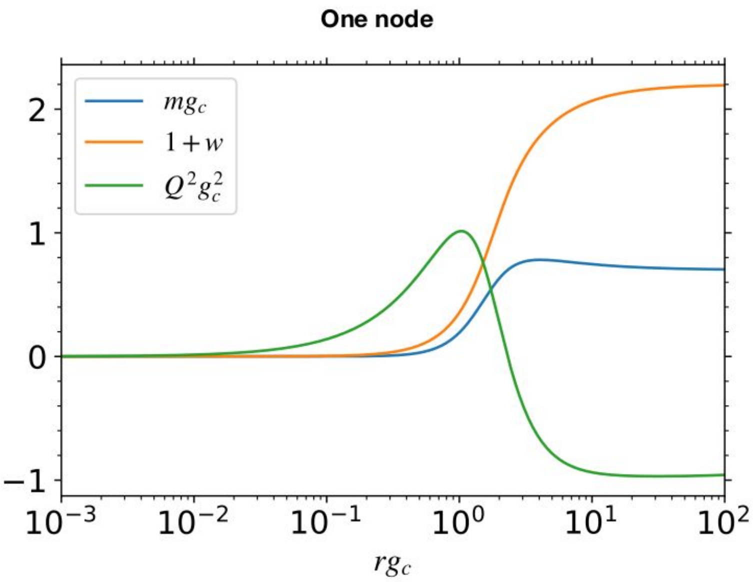
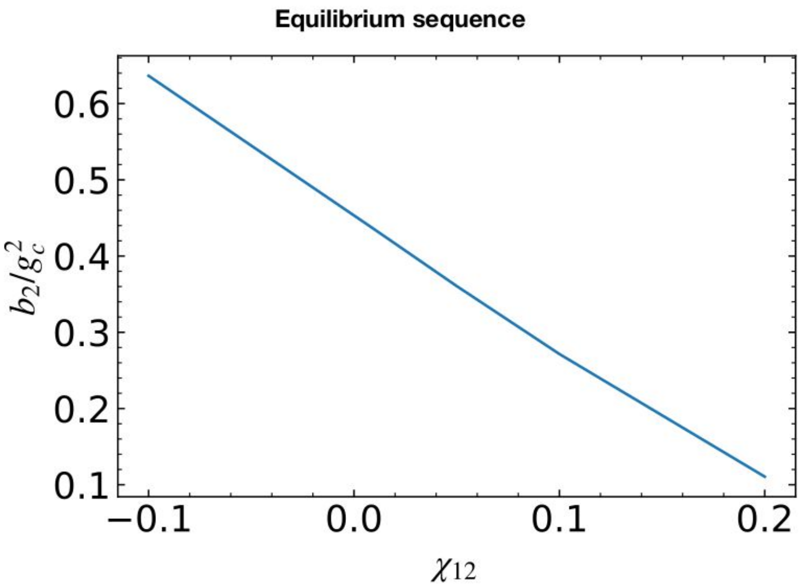
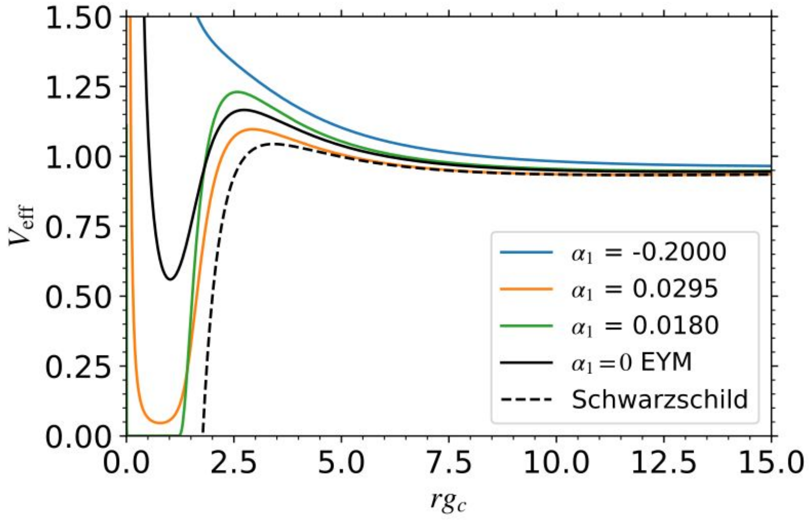


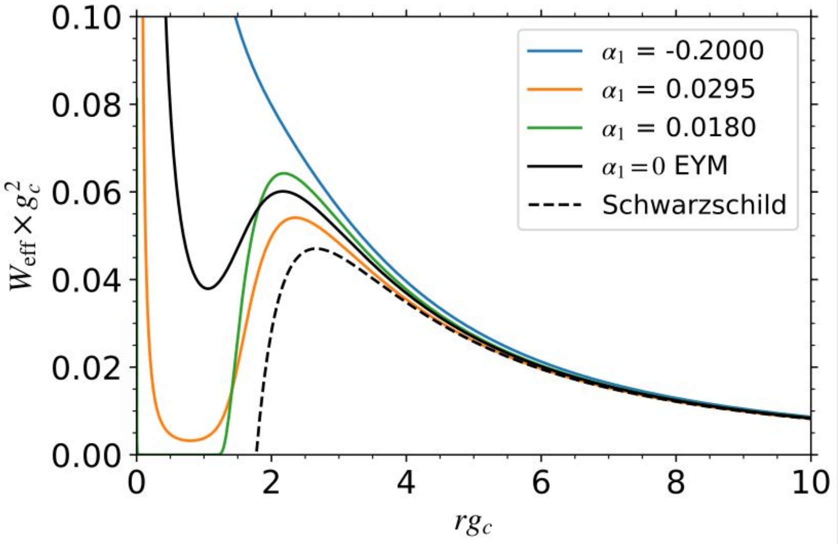
$\chi_{12} = -1$



No bifurcations, no change in stability (?)

Effective potential α_1





Neutron Stars

Martínez J, Rodríguez J, Becerra L, Rodríguez Y, Gómez G, Phys. Rev. D 110, 104070 (2024)

Field equations are modified by presence of baryonic matter,

$$G_{\mu\nu} = 8\pi(T^{\text{V}}_{\mu\nu} + T^{m}_{\mu\nu}),$$

$$T^{\text{V}}_{\mu\nu} \equiv -\frac{1}{8\pi\sqrt{-g}}\frac{\delta(\mathcal{L}\sqrt{-g})}{\delta g^{\mu\nu}},$$

$$T^{m}_{\mu\nu} \equiv -\frac{1}{8\pi\sqrt{-g}}\frac{\delta(\mathcal{L}_m\sqrt{-g})}{\delta g^{\mu\nu}}.$$

$$\nabla_\nu T^{\mu\nu}_{m} = 0$$

$$(T^m)^\mu{}_\nu = \text{diag}(-\epsilon(r), P(r), P(r), P(r))$$

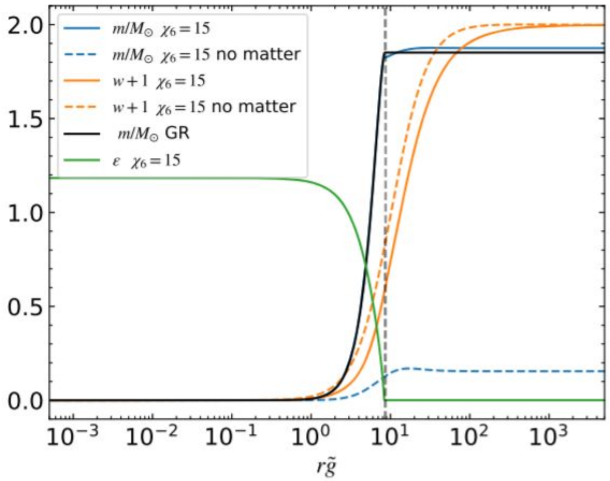
Equation of State

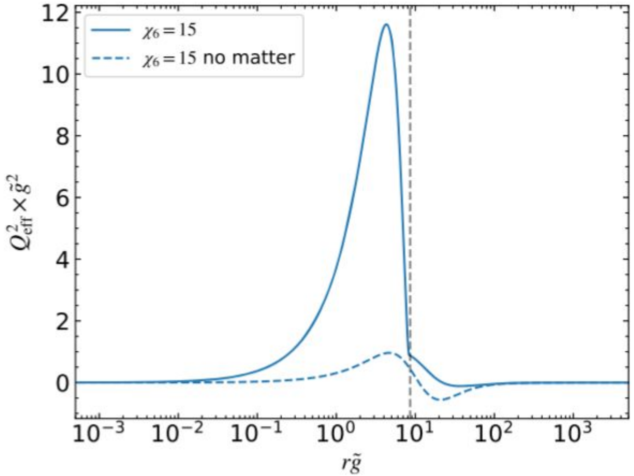
$$P(\rho) = K_i\rho^{\Gamma_i} + \Lambda_i,$$

$$\epsilon(\rho) = \frac{K_i}{\Gamma_i - 1}\rho^{\Gamma_i} + (1 + a_i)\rho - \Lambda_i.$$

EOS	$\log\rho_1$ (g cm^{-3})	$\log\rho_2$ (g cm^{-3})	$\log\rho_3$ (g cm^{-3})	Γ_1	Γ_2	Γ_3
H4	14.99	14.87	13.49	2.51	2.33	1.56
GM1Y6	14.99	14.87	13.75	2.96	1.62	1.63

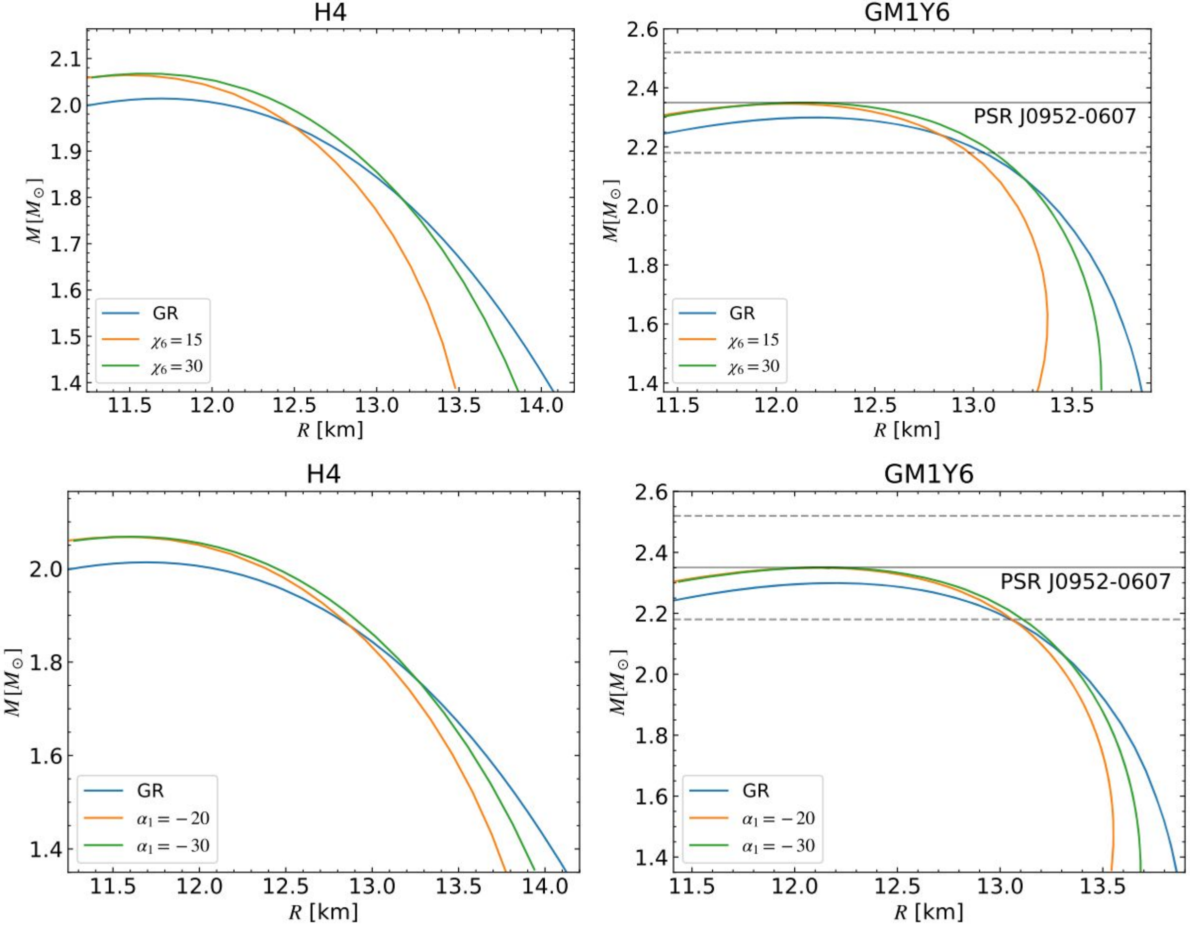
Numerical solution minimal coupling χ_6



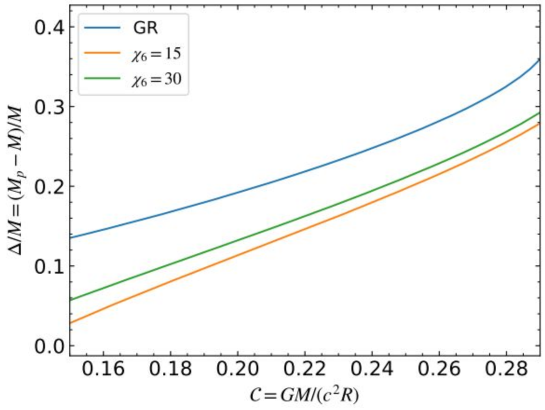


Martínez J, Rodríguez J, Becerra L, Rodríguez Y, Gómez G, Phys. Rev. D 110, 104070 (2024)

Equilibrium sequences

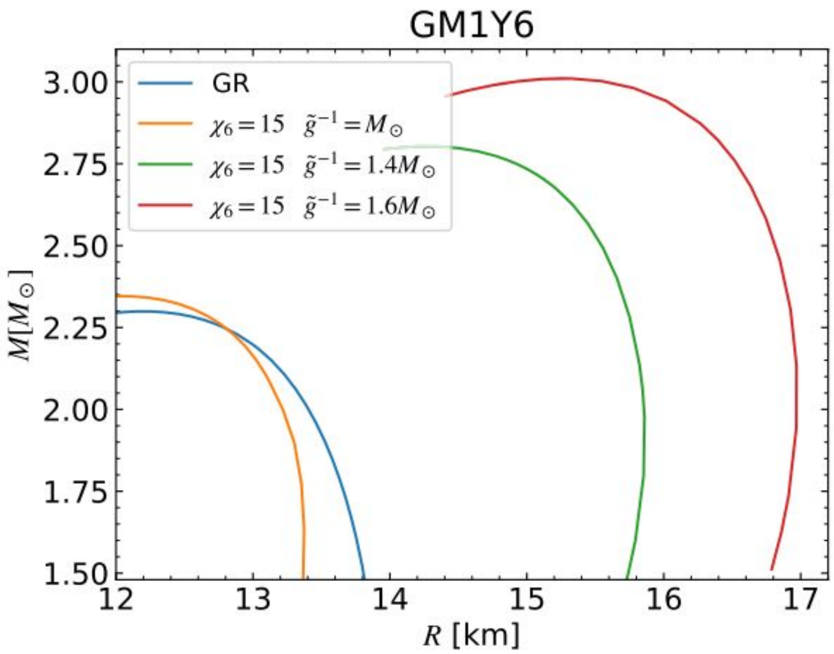


Martínez J, Rodríguez J, Becerra L, Rodríguez Y, Gómez G, Phys. Rev. D 110, 104070 (2024)



Mass-gap hints

Martínez J, Rodríguez J, Becerra L, Rodríguez Y, Gómez G, Phys. Rev. D 110, 104070 (2024)



Conclusions

- We have found particlelike solutions in the GSU2P theory.
- Generalization of the EYM case. New charged solutions with negative energy density regions.
- We have constructed equilibrium sequences and found bifurcations points for in the cases α_1, α_3, χ_6, which hints towards the existence of stable solutions.
- Boson stars are highly compact objects with photon sphere.
- We have found NS in the GSU2P theory (There are no NS in EYM).
- The NS in the GSU2P theory are more compact than in GR.
- In the GSU2P theory the NS are more massive even reaching the lower mass gap.

Future work

- Stability, perturbations: radial (ongoing) and angular.
- 1+1 Full Numerical integration (ongoing)
- GW near these compact objects
- Rotating solutions. (William).
- Static black holes with axial symmetry.
- Ringdown of solitons and BH solutions (GW).
- NS with anisotropic EOS (ongoing)
- Post-Newtonian and Post-Minkowskian expansion.