



# Potential Energy Surface Emulation and Impact on Fission Trajectories

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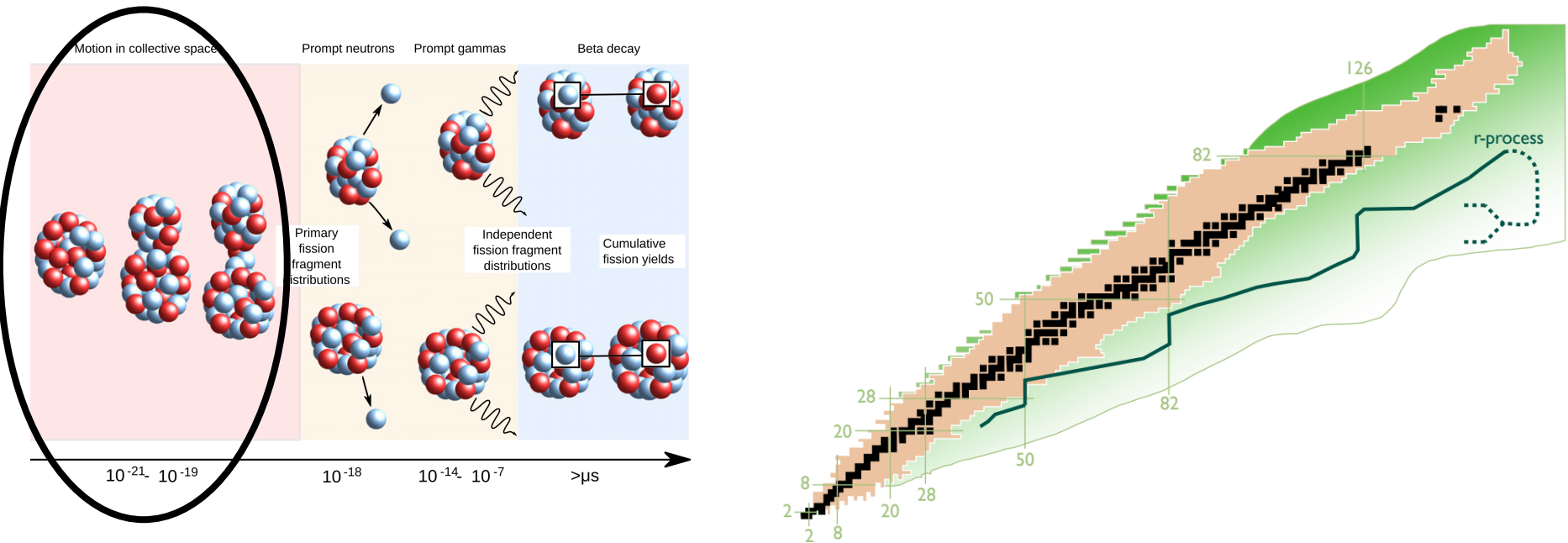
# Outline

- **Physics overview**
  - Nuclear fission
  - Density functional theory
  - WKB approach to tunneling
- **Neural network overview**
- **Results**



# Nuclear Fission

- Splitting of heavy nucleus into lighter fragments
- Important for understanding *r*-process nucleosynthesis, through fission cycling



Left: M. Bender, et al. "Future of nuclear fission theory." J. Phys. G Nucl. Part. Phys. 47.11 (2020): 113002.

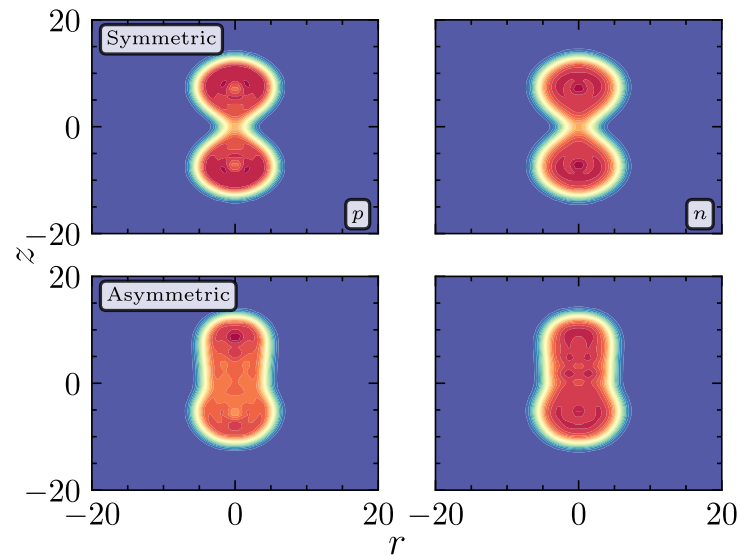
Right: [https://cococubed.com/pix\\_pages/nuclide\\_chart.shtml](https://cococubed.com/pix_pages/nuclide_chart.shtml)



**Facility for Rare Isotope Beams**  
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# Density Functional Theory (DFT)

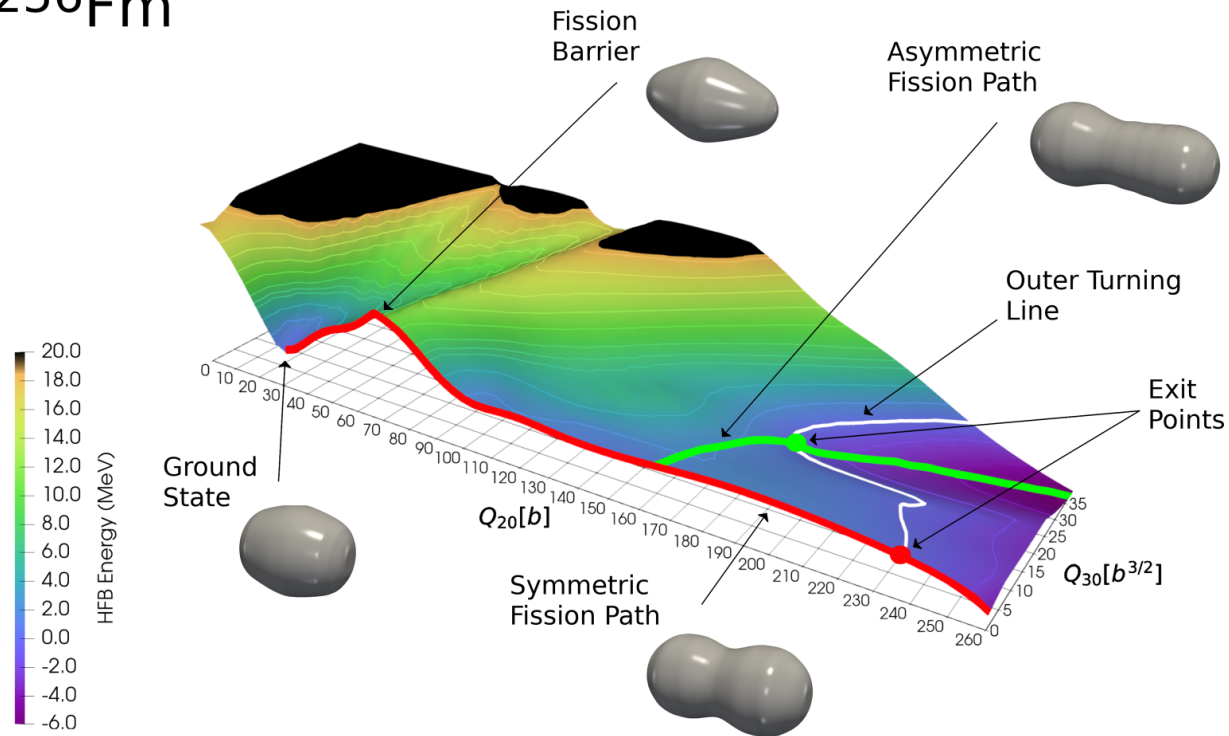
- Solve Hartree-Fock-Bogoliubov (HFB) equations with a density-dependent energy density functional
  - Nucleons move in a self-consistent mean field
  - Describes collective motion well
  - Static description of the nucleus
- Use constrained HFB equations to examine nuclear shape



# DFT and Fission

- For grid of deformations, compute:
  - HFB energy, for potential energy surface (PES)
  - Collective inertia tensor  $M_{\mu\nu}$

$^{256}\text{Fm}$

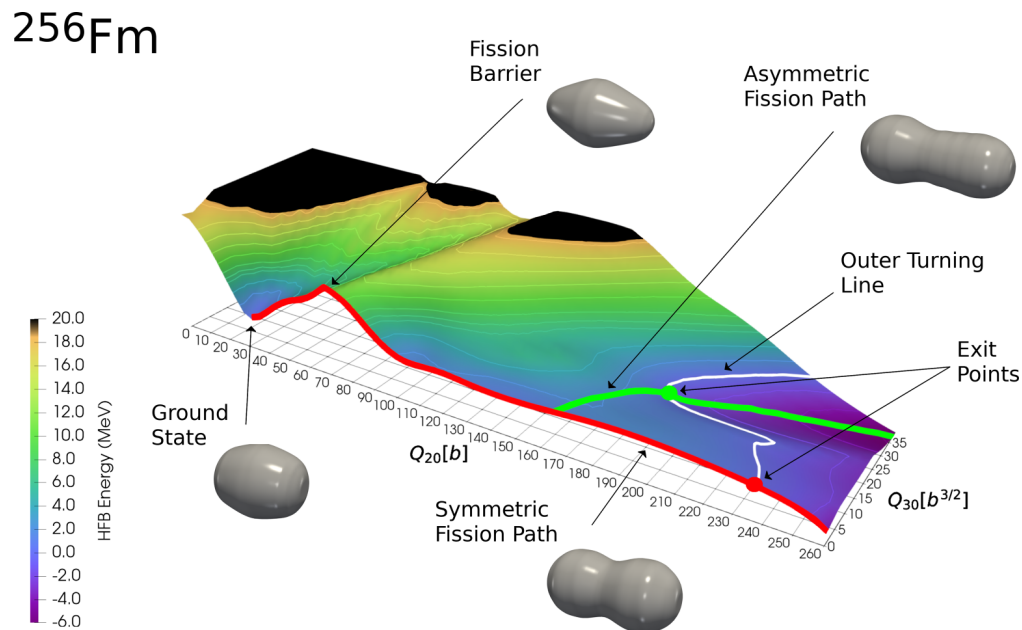


# DFT and Fission

- Fission described as tunneling through a barrier
  - Solve in WKB approximation

$$L(s) = \int_{s_0}^{s_1} ds \sqrt{2V_{\text{eff}}(s) M_{\mu\nu}(s) \dot{q}_\mu \dot{q}_\nu}$$

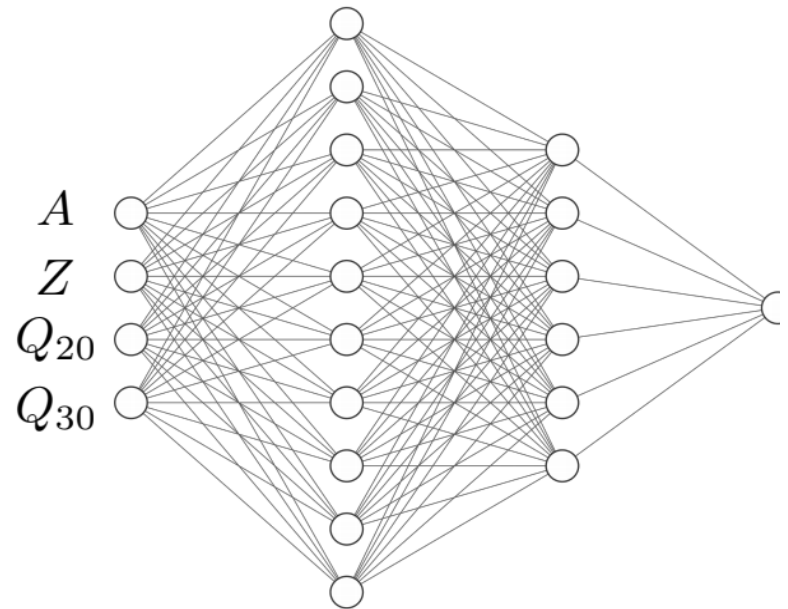
- Fragment yields determined by exit point location [1]



[1] J. Sadhukhan et. al., Phys. Rev. C 101, 065803 (2020)

# Neural Network Emulator

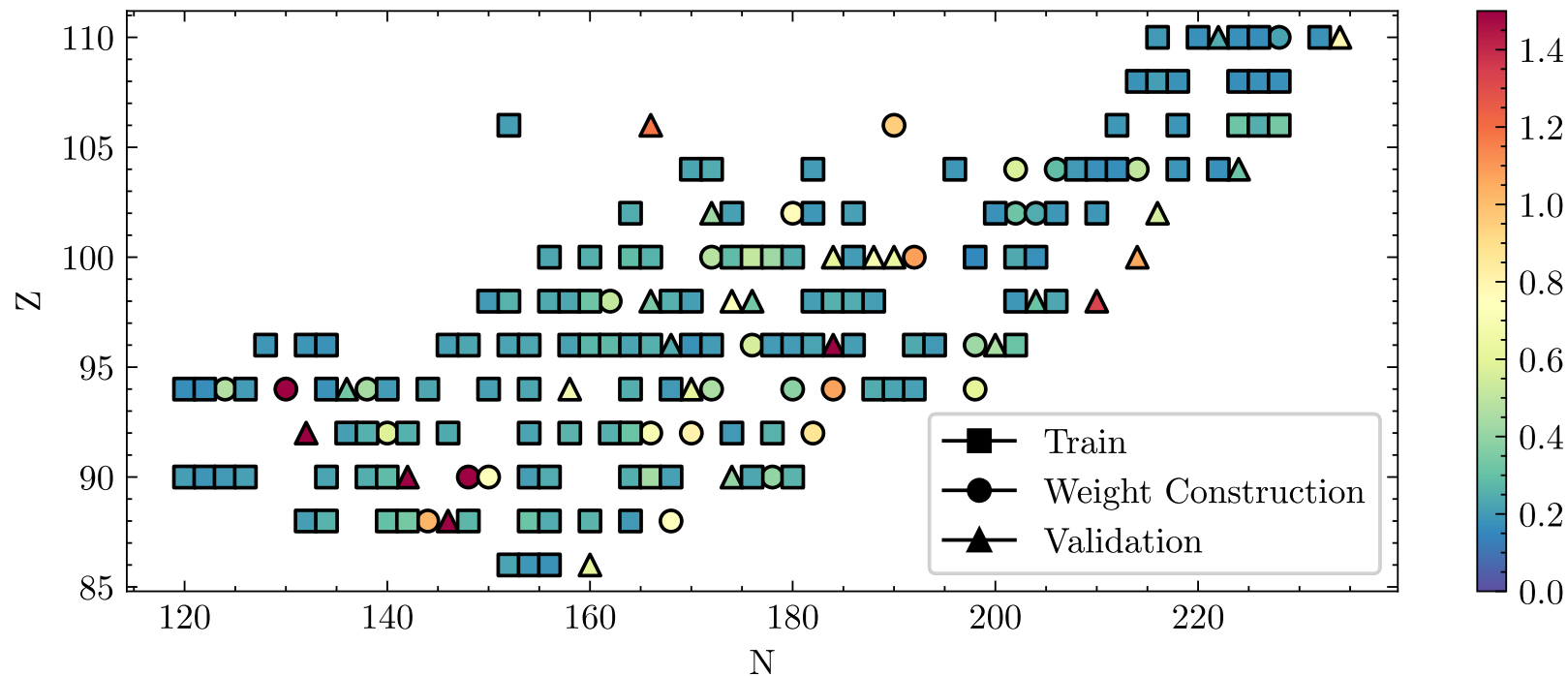
- Use committee of neural networks (NNs) to emulate PES and collective inertia across the nuclear chart, similar to [1]
- Collective inertia must be positive semi-definite matrix
  - Emulate eigenvalue decomposition of inertia
  - Components vary on multiple scales, so emulate log of values
- Dataset is 200 nuclei, with ~700 points per nucleus
  - From D1S Gogny interaction [2]
- Vary depth of NN



- [1] R.-D. Lasserri et. al., Phys. Rev. Lett. 124, 162502 (2020)
- [2] J. Berger et. al., Nucl. Phys. A 428, 23 (1984)

# Results: PES Quality

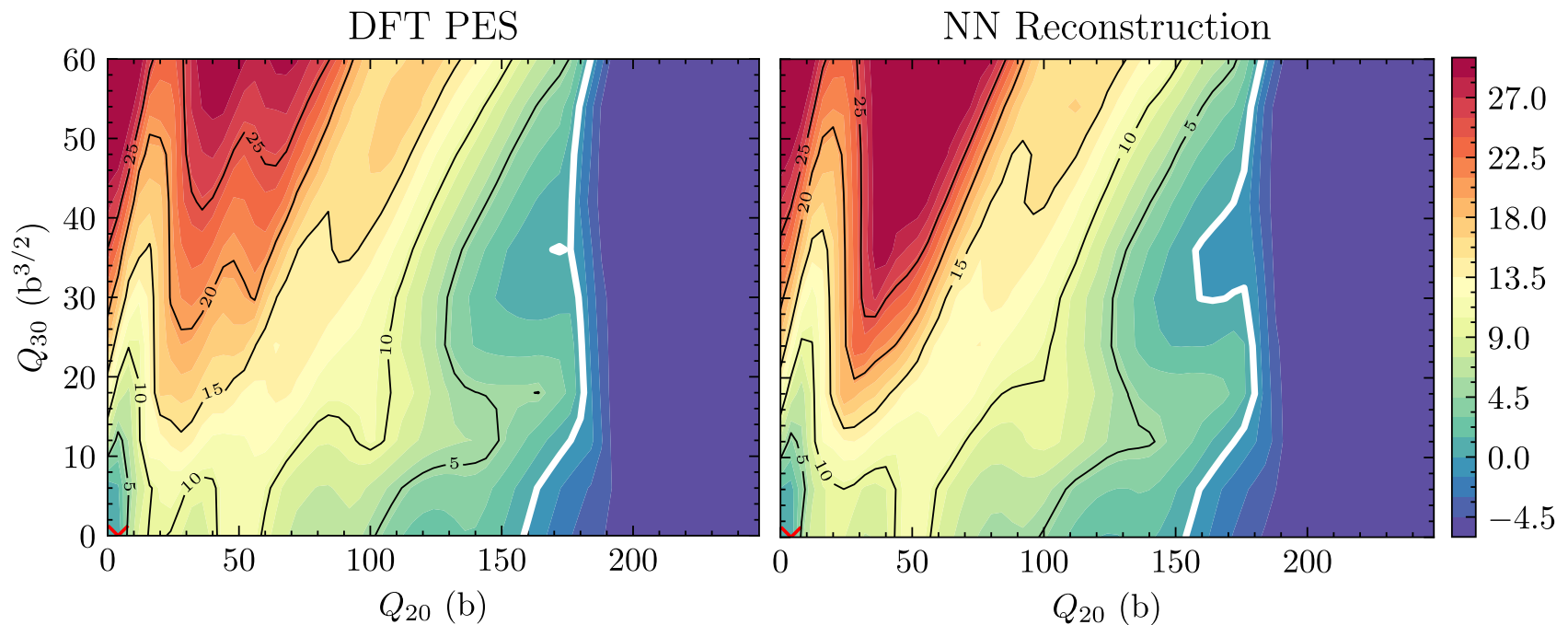
- Root-mean-squared error across all nuclei considered
  - PES varies on order of 10 MeV
- PES of most nuclei reproduced well
  - Order of 400 keV RMSE





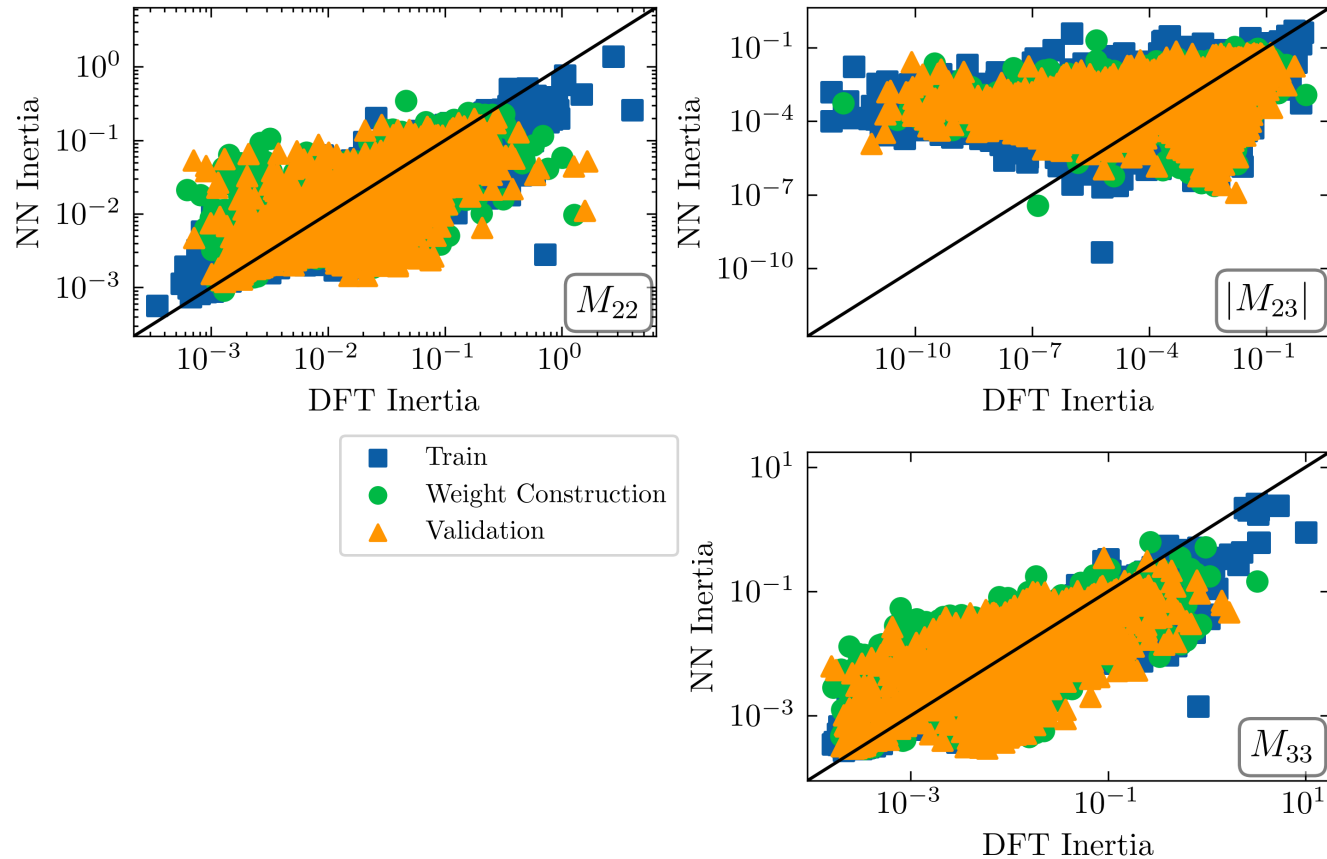
# Results: PES Quality

- $^{280}\text{Cm}$  has largest RMSE, at 2 MeV
- Qualitative features reproduced
  - Ground state and outer turning line locations
- Non-accessible regions are dominant source of error



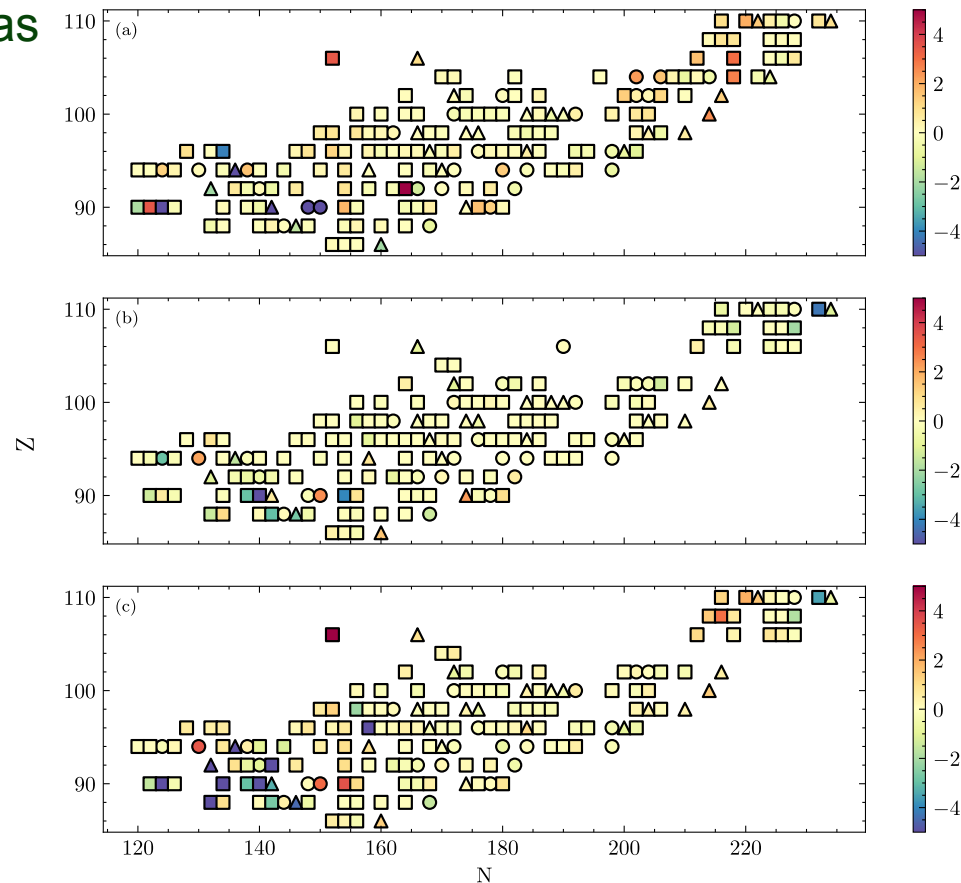
# Results: Collective Inertia

- NN captures diagonal components well across multiple orders of magnitude
- Small off-diagonal components usually overestimated



# Results: Exit Points

- Octupole moment of exit point shown as proxy for fragment yields
  - Quadrupole moment behaves similarly
  - Take weighted average over different fission modes
- Both PES and inertia emulated well enough to accurately predict fragment yields
  - Most important to emulate PES correctly
  - $^{280}\text{Cm}$  reproduced almost exactly



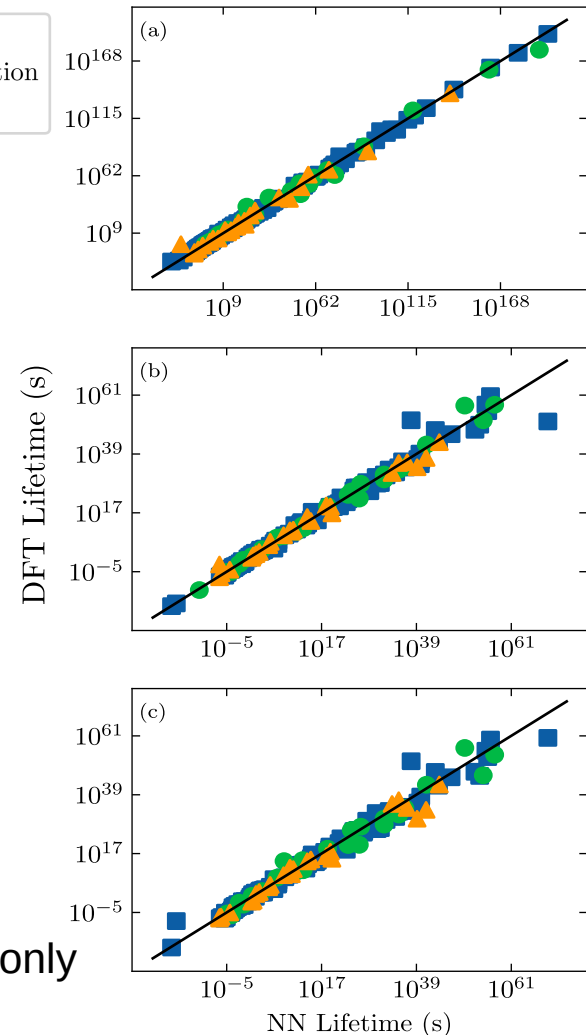
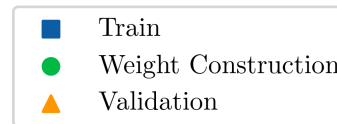
(top): impact of PES only

(middle): impact of collective inertia only

(bottom): impact of both

# Results: Lifetimes

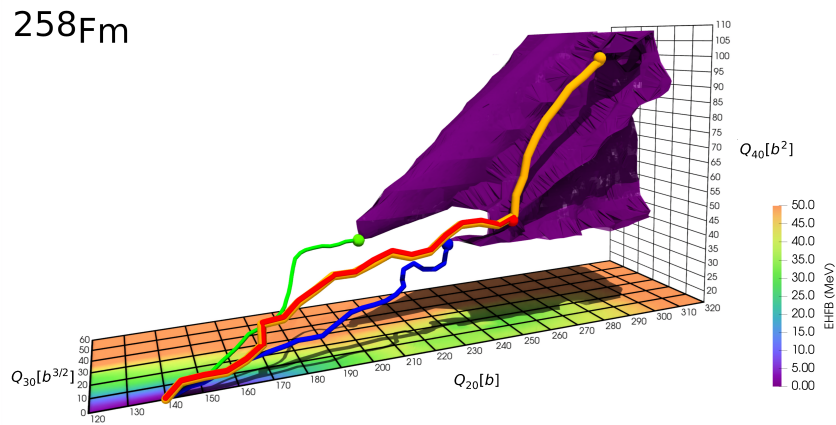
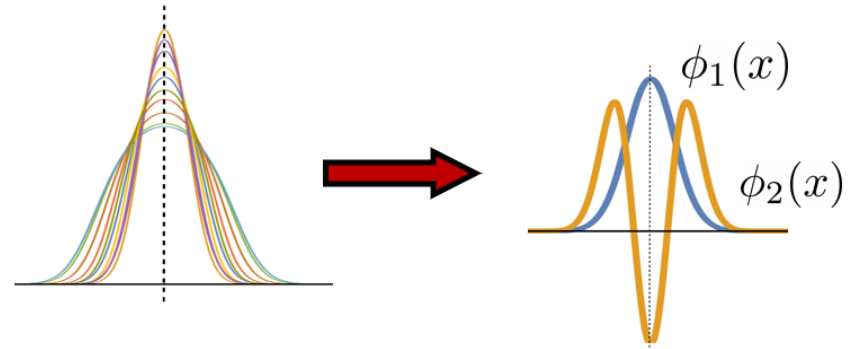
- Show lifetime ratio
  - Lifetimes systematically incorrect in current space of collective coordinates
- DFT lifetimes typically reproduced by NNs within 1-3 orders of magnitude
  - Collective inertia critically important
- Nuclei with long spontaneous fission lifetime have larger error
  - Expected, and not problematic



(top): impact of PES only  
(middle): impact of collective inertia only  
(bottom): impact of both

# Future Directions

- Other emulation methods
  - Reduced basis methods
  - Gaussian processes
- Expansion to more collective coordinates
  - Dynamic pairing fluctuations, higher multipole moments
- Large-scale predictions and model calibration





# Questions?