Gravitational theories in the context of non-Riemannian geometries on the edge of Ockham's razor

Third workshop on current challenges in cosmology

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Newton's gravity – Newton Cartan

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What do we have?

Newton's second law

$$\frac{d^2x^j}{dt^2} + \frac{\partial \Phi}{\partial x^j} = 0$$

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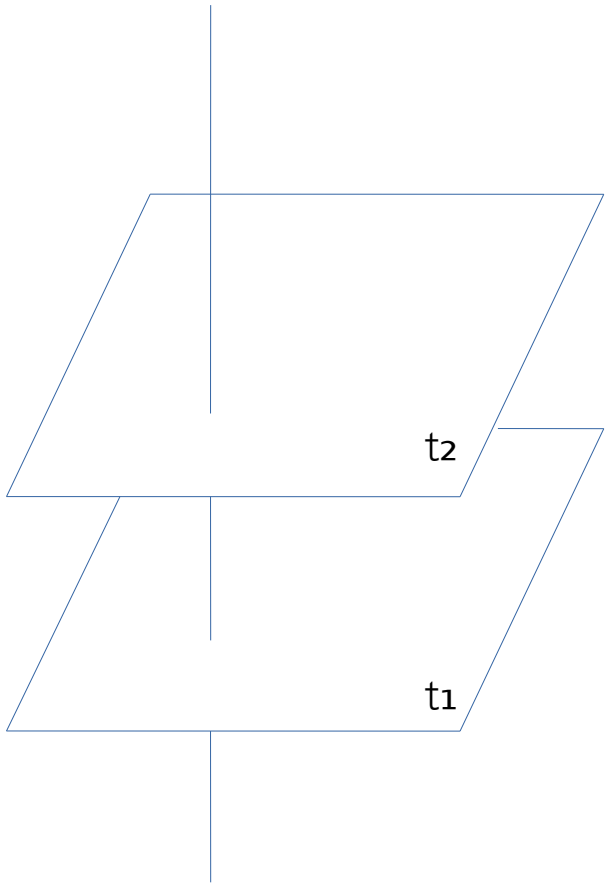
Ricci scalar

$$R_{00} = 4\pi\rho$$

Newton's gravity – Newton Cartan (Spacetime's structure)



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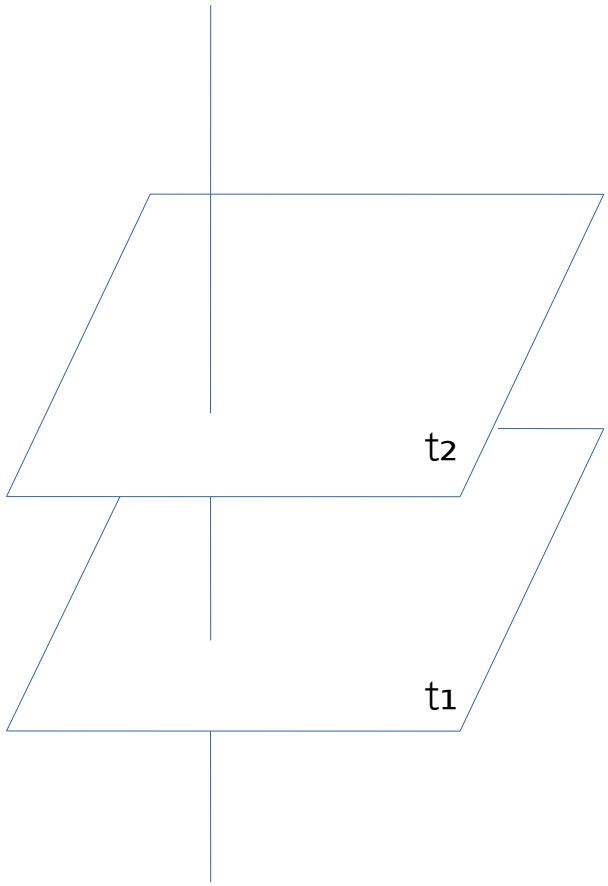

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$$t = t(\mathcal{P})$$

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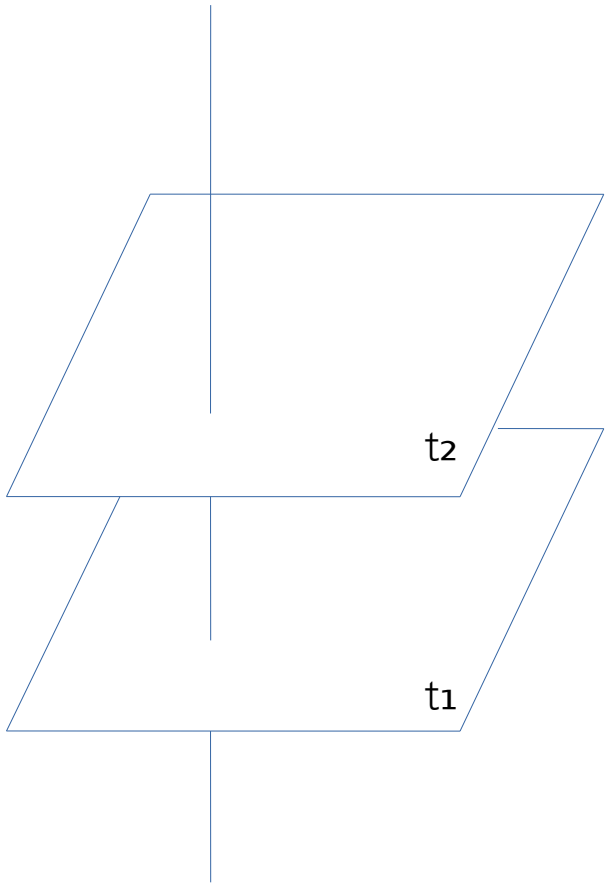

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3-dimensional "absolute space" is flat and has an orthonormal Galilean coordinate base (Euclidean)

$$\mathbf{e}_i \cdot \mathbf{e}_j = \frac{\partial}{\partial x^i} \cdot \frac{\partial}{\partial x^j} = \delta_{ij}$$

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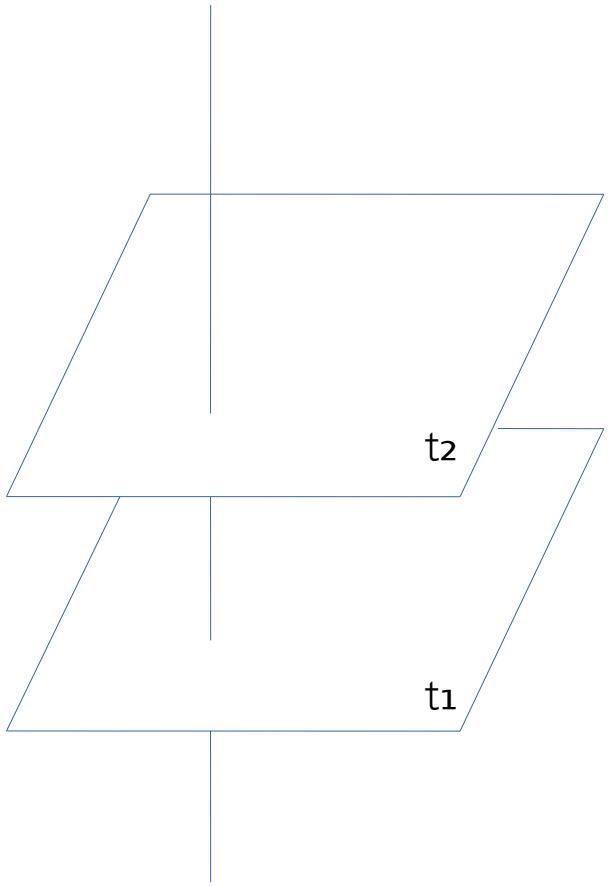

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Spacetime is NOT flat

$$\Gamma^j_{00} \neq 0$$

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Coordinate transformations

$$x^{\mu'} = A^{\mu'}{}_{\lambda} x^{\lambda} + a^{\mu'}$$

$A^{\mu'} \rightarrow$ Rotations

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$$\Gamma^{j'}_{0'k'} = \Gamma^{j'}_{k'0'} = A^{j'}{}_{i} \dot{A}^{i}{}_{k'}$$

$$\Gamma^{j'}_{00} = \frac{\partial \Phi}{\partial x^{j'}} + A^{j'}_{i} \left(\ddot{A}^{i}_{l'} x^{l'} - \ddot{a}^{i} \right)$$

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The transformation goes from a Galilean coordinate basis to a Galilean coordinate basis if and only if

$$\dot{A}^{i}{}_{k'} = 0$$

$$\Phi' = \Phi - \ddot{a}_k x^k$$

$$\lim_{r \to \infty} \Phi = 0, \qquad r = (x^j x_j)^{1/2}$$

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"Absolute" Galilean coordinates

Requirements for Newton - Cartan vs General Relativity

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What geometric structures should spacetime be equipped with?	Newton - Cartan	General Relativity
	1) A universal scalar function t 2) A covariant derivative 3) A flat spatial metric (but a temporal metric can not be defined) 4) Zero torque and metricity	1) A Lorentzian spacetime metric 2) Zero torque and metricity

Requirements for Newton - Cartan vs General Relativity

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What coordinate systems are preferred?	1) Galilean coordinate systems 2) Absolute Galilean coordinate systems	1) None

Requirements for Newton - Cartan vs General Relativity

	Newton - Cartan	General relativity
What information is required to select a preferential coordinate system?	1) Three constant Euler angles (there should be no rotation of the frame) 2) World line and a tetrad (observer).	The four position functions $x^{\alpha}(\mathcal{P})$

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Under what conditions can it be said that two events occur in the same place?	Only after choosing Galilean coordinates	Only after choosing spatial coordinates.

Requirements for Newton - Cartan vs General Relativity

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Under what conditions is an "invariant distance" between two different events well defined?	Only if the two events live in the same spatial slice.	Only if the two are close enough, or if they are part of the same preferred worldline.

Geometric derivation of Newton - Cartan vs General Relativity

Newton - Cartan

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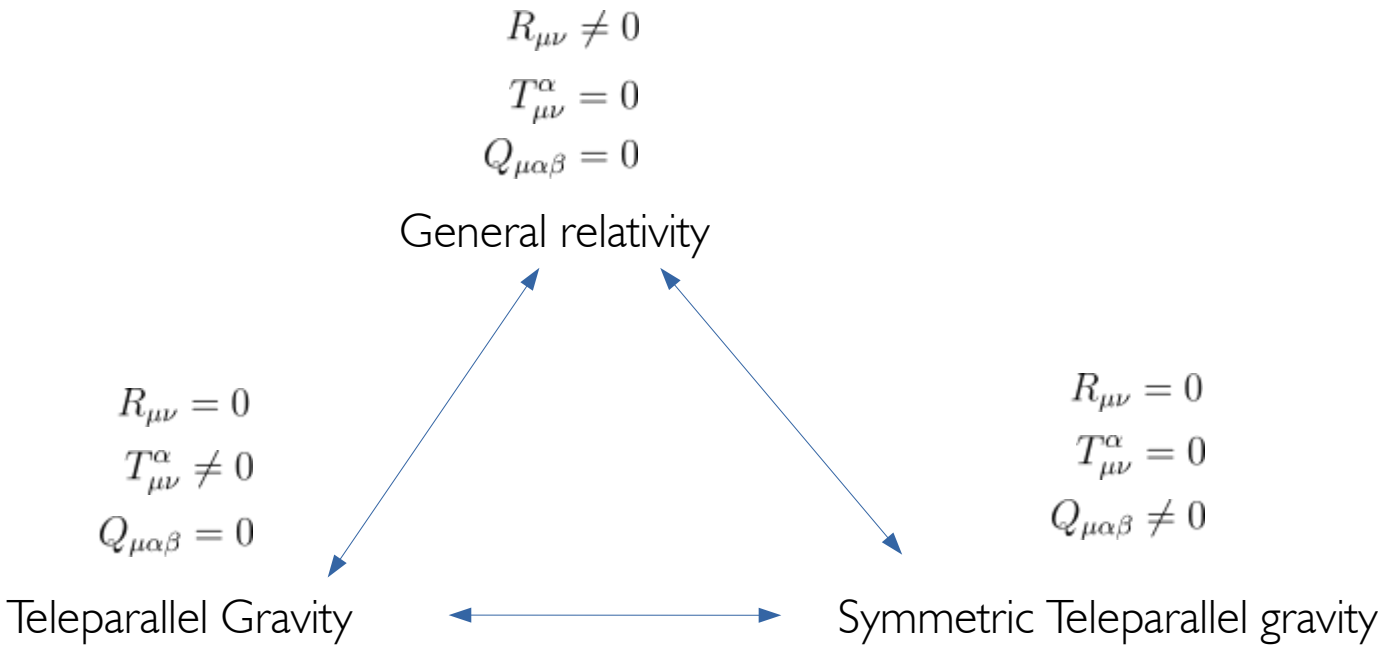
3) All special relativistic laws of physics are valid in local lorentz frames of metric.

Ockham's Razor

Entia non sunt multiplicanda praeter necessitatem
Entities must not be multiplied beyond necessity

The trinity of gravity



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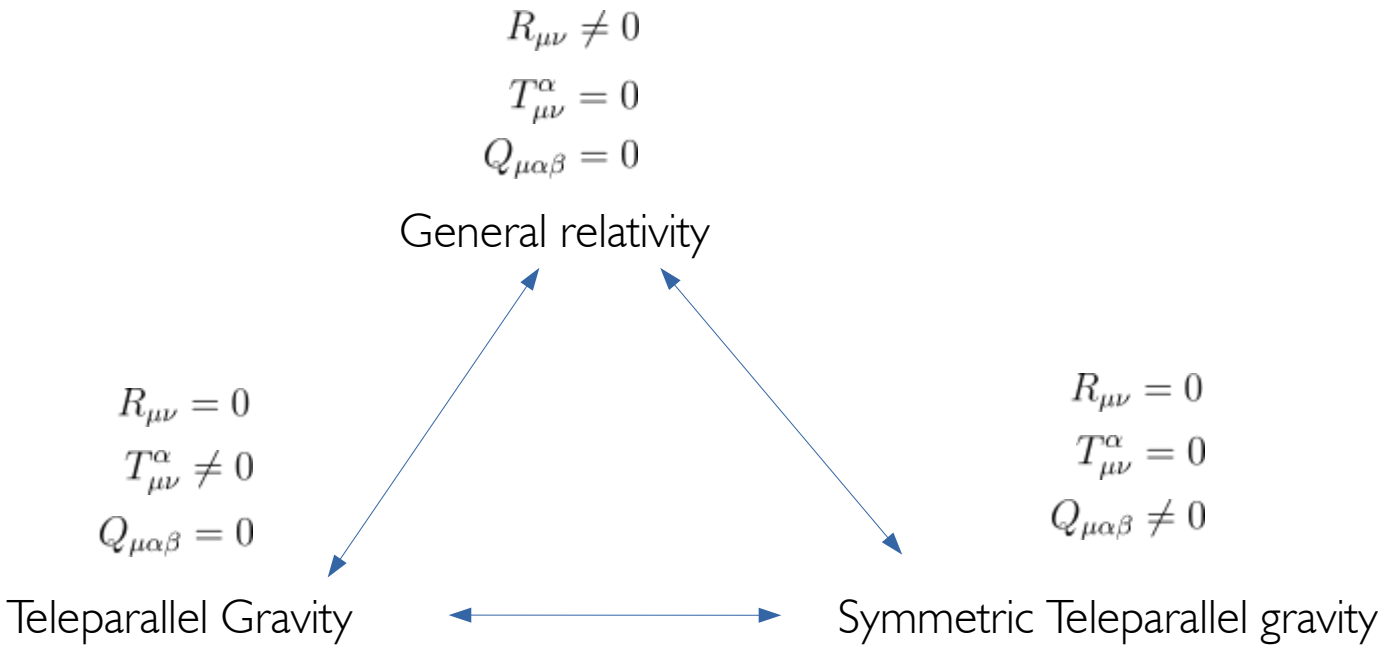

Which one is simpler?

Thank you