# No more standard scaling Solutions in Generalized Proca Theory


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# Motivation

## New scaling solutions in cubic Horndeski theories

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arxiv.org/pdf/1807.09800.pdf

## Scaling solutions in quintessential inflation

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arxiv.org/pdf/1908.01516.pdf

# The model

$$S = \int d^4x \sqrt{-g} \left( \sum_{i=2}^{5} \mathcal{L}_i + \mathcal{L}_F + \mathcal{L}_M \right)$$

$$\tilde{F} = -\frac{1}{4} \tilde{F}^{\mu\nu} F_{\mu\nu} \quad \mathcal{L}_M = \mathcal{L}_r + \mathcal{L}_b + f(X)\mathcal{L}_c \qquad X = -\frac{1}{2} A_\mu A^\mu \quad F = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

$$\mathcal{L}_F = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

$$\mathcal{L}_2 = G_2\left(X, F, \tilde{F}\right)$$

$$\mathcal{L}_3 = G_3(X) \nabla_\mu A^\mu$$

$$\mathcal{L}_4 = G_4(X) R + G_{4X} \left[ (\nabla_\mu A^\mu)^2 + c_2 \nabla_\rho A_\sigma \nabla^\rho A^\sigma - (1 + c_2) \nabla_\rho A_\sigma \nabla^\sigma A^\rho \right]$$

$$\begin{aligned} \mathcal{L}_5 = {} & G_5(X) G_{\mu\nu} \nabla^\mu A^\nu - \frac{1}{6} G_{5X} \left[ (\nabla_\mu A^\mu)^3 \right. \\ & - 3 d_2 \nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\rho A^\sigma \\ & - 3 (1 - d_2) \nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\sigma A^\rho \\ & + (2 - 3 d_2) \nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma \\ & \left. + 3 d_2 \nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla_\gamma A^\sigma \right] \end{aligned}$$

# On the background of FLRW

The profile of the vector field compatible with the FLRW metric is $A_\mu \equiv (A_0(t), 0, 0, 0)$ then $F = \tilde{F} = 0$ and computing the tensor perturbations the function $G_4$ and $G_5$ are constrained due to the speed of gravitational waves such that

$$c_T = \frac{2G_4 - A_0^2 A_0' G_{5X}}{2G_4 - 2A_0^2 G_{4X} - H A_0^3 G_{5X}} = 1 \longrightarrow G_4 \to \text{const and } G_5 \to \text{const}$$

$$\mathcal{L} = \frac{m_{\rm p}^2}{2} R + \mathcal{L}_{\rm GP} + \mathcal{L}_M \quad \text{, where } \mathcal{L}_{\rm GP} = G_2(X) + G_3(X)\nabla_\mu A^\mu$$

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# The Scaling condition

The Scaling condition

$$\frac{\rho_\Lambda}{\rho_c} = \text{const} \quad w_\Lambda = \frac{p_\Lambda}{\rho_\Lambda} = \text{const} \qquad \rho_\Lambda = -G_2 \quad p_\Lambda = G_2 + 2X\dot{A}_0 G_{3,X}$$

$$\frac{dX}{dN} = -\frac{3\Omega_\Lambda w_\Lambda}{f_{,X}/f}$$

$$\Omega \equiv \frac{\rho}{3m_{\rm p}^2 H^2}$$

$$\frac{d\rho_\Lambda}{dN} + 3\rho_\Lambda(1+w_\Lambda) = -\frac{f_{,X}}{f}\rho_c\frac{dX}{dN}$$

$$\frac{d\rho_c}{dN} + 3\rho_c = \frac{f_{,X}}{f}\rho_c\frac{dX}{dN}$$

Then the GP lagrangian satisfies

$$\frac{\partial \,\mathrm{Ln}\,\mathcal{L}_{GP}}{\partial x}\frac{dx}{dN} + \frac{\partial \,\mathrm{Ln}\,\mathcal{L}_{GP}}{\partial \,\mathrm{Ln}\,\dot{A}_0}\frac{d\,\mathrm{Ln}\,\dot{A}_0}{dN} = -3\left(1+w_{\rm eff}\right)$$

$$\mathcal{L}_{GP} = L_\alpha \dot{A}^{\lambda\alpha}\left(A_0\frac{f_{,X}}{f}\right)^{\alpha\lambda} f^{-\lambda}$$

$$\lambda = -\frac{1+w_{\rm eff}}{\Omega_\Lambda w_\Lambda} = \frac{1}{\gamma}$$

$$\alpha = 0 \qquad \alpha = 1/\lambda$$

# The Generalized Proca Lagrangian

GP Lagrangian:

$$\mathcal{L}_{\rm GP} = L_0 f^{-\lambda} + \mathcal{L}_{1/\lambda} \dot{A}_0 A_0 \frac{f_{,X}}{f} f^{-\lambda}$$

The Free functions:

$$\boxed{G_2 = L_0 f^{-\lambda} = -\rho_\Lambda} \qquad G_{3,X} = \frac{L_{1/\lambda}}{\sqrt{2X} L_0} \frac{f_{,X}}{f} G_2$$

EOM:

$$X G_{2,X} + X \frac{f_{,X}}{f} \mathcal{L}_c = 3\sqrt{2X} X H G_{3,X} \Longrightarrow X \frac{f_{,X}}{f} \left( \lambda \rho_\Lambda - \rho_c + \frac{L_{1/\lambda}}{L_0} 3H\rho_\Lambda \right) = 0$$

The continuity equations of the DE or CDM both imply

$$\frac{d\rho_\Lambda}{dN} + 3\rho_\Lambda(1+w_\Lambda) = -\frac{f_{,X}}{f}\rho_c \frac{dX}{dN} \qquad \Longrightarrow \frac{f_{,X}}{f} X' \lambda \rho_\Lambda = \frac{f_{,X}}{f} X' \rho_c$$

$$\frac{d\rho_c}{dN} + 3\rho_c = \frac{f_{,X}}{f}\rho_c \frac{dX}{dN}$$

the only compatible solution is when f=0, and this solutions is exactly $\Lambda$CDM

# The generalized proca lagrangia $\alpha = 0$

The free functions: $G_2 = L_0 f^{-\lambda}$, $G_3 = 0$

The EOM: $$A_0 \frac{f_x}{f}\left(\lambda\rho_\Lambda - \rho_c\right) = 0$$

If f is an arbitrary function the scaling condition is really strong $\lambda\rho_\Lambda(N) = \rho_c(N)$ it is not a viable cosmological solution due only baryons there are not enough to reproduce the data growth that we observe, if we assume a weak scaling condition ( $\lambda\rho'_\Lambda(N) = \rho'_c(N)$ ) we could get

$$\rho_r = \rho_{r0}a^{-4},\ \rho_b = \rho_{b0}a^{-3},\ \rho_c = \rho_{c0}a^{-\frac{3\lambda}{\lambda+1}},\ \rho_\Lambda = \Lambda + \frac{\rho_{c0}}{\lambda}\left(a^{-\frac{3\lambda}{\lambda+1}} - 1\right)$$

$$\rho_\Lambda > 0 \qquad \lambda \geq \rho_{c0}/\Lambda$$

$$\frac{\rho_\Lambda}{\rho_c} = \frac{\Lambda}{\rho_{c0}}a^{\frac{3\lambda}{1+\lambda}} + \frac{1}{\lambda} - \frac{1}{\lambda}a^{\frac{3\lambda}{1+\lambda}} \quad \xrightarrow{\text{Scaling condition}} \quad a \ll \left(\frac{\Lambda}{\rho_{c0}} - \frac{1}{\lambda}\right)^{\frac{1+\lambda}{3\lambda}}$$

# The generalized proca lagrangian

How need to be f in order to get this solution?

Continuity equation:

$$\rho'_\Lambda(N) = -\frac{f_{,X}}{f}X'(N)\rho_c$$

$$\rho'_c(N) + 3\rho_c(N) = +\frac{f_{,X}}{f}X'(N)\rho_c$$

$$\longrightarrow \quad \frac{f'(N)}{f} = \frac{3}{1+\lambda} \qquad f(N) = e^{\frac{3N}{1+\lambda}}$$

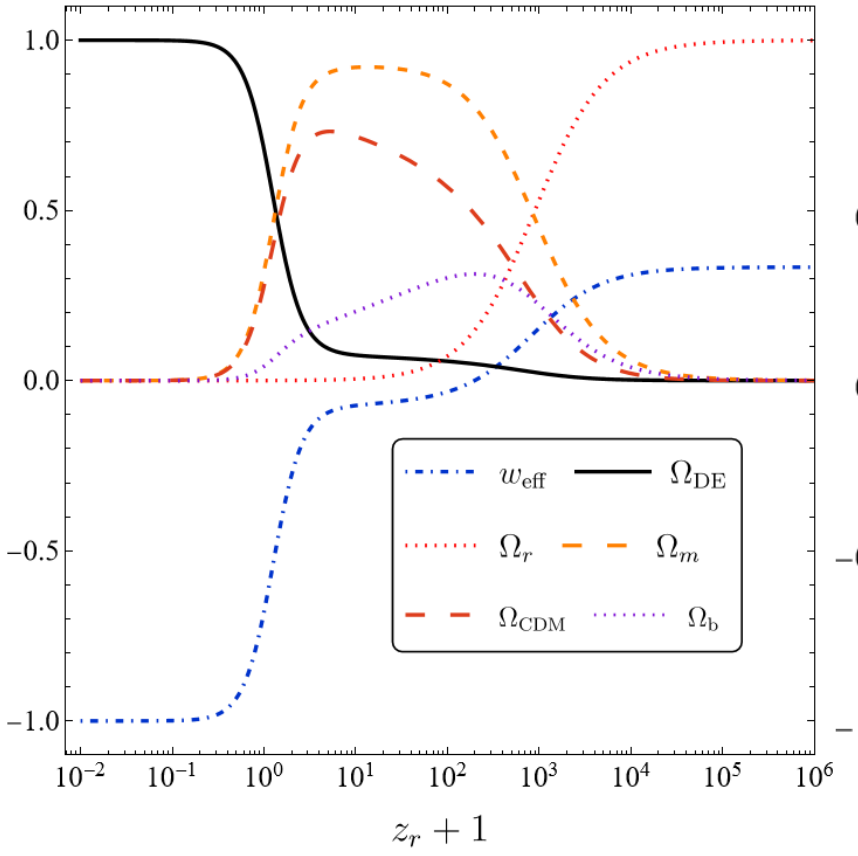
Equation of motion: $A_0\frac{f_{,X}}{f}(\lambda\Lambda - \rho_{c0}) = 0 \rightarrow f_{,X} = 0 \text{ or } \lambda = \frac{\rho_{c0}}{\Lambda}$

Then the solution is also a eternal scaling solution. $\rho_\Lambda = \Lambda\left(f(N)\right)^{-\lambda} = \frac{\Lambda}{\rho_{c0}}\rho_c$

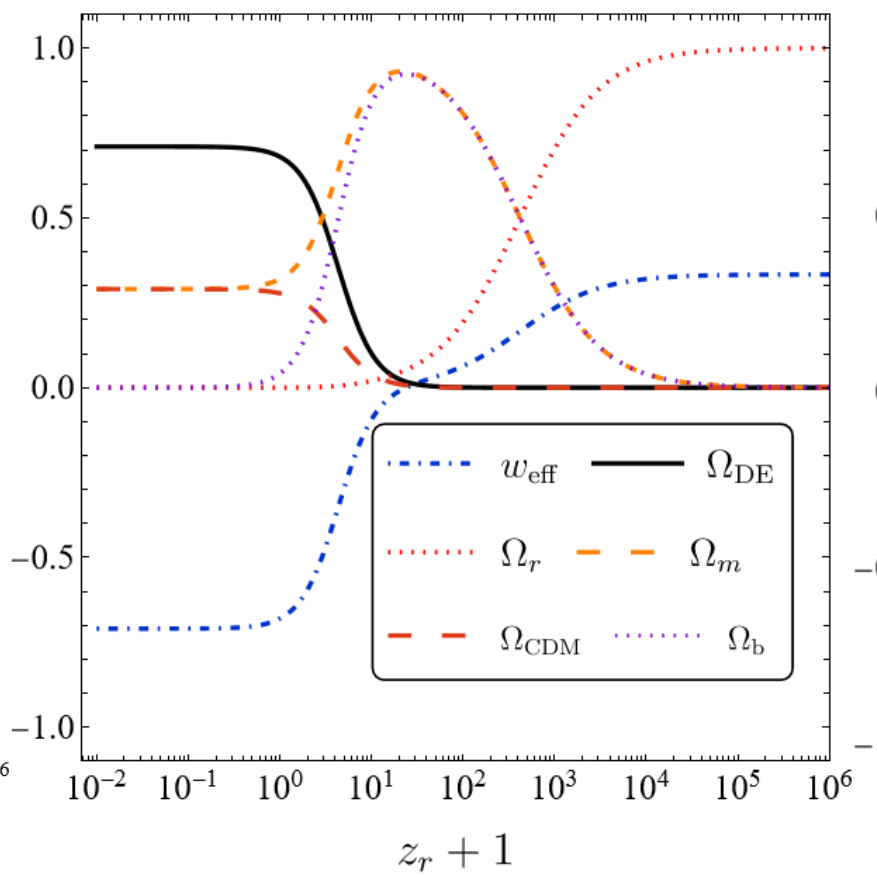
Hence proca with only G2 cannot produce a weak scaling solution compatible with observations.

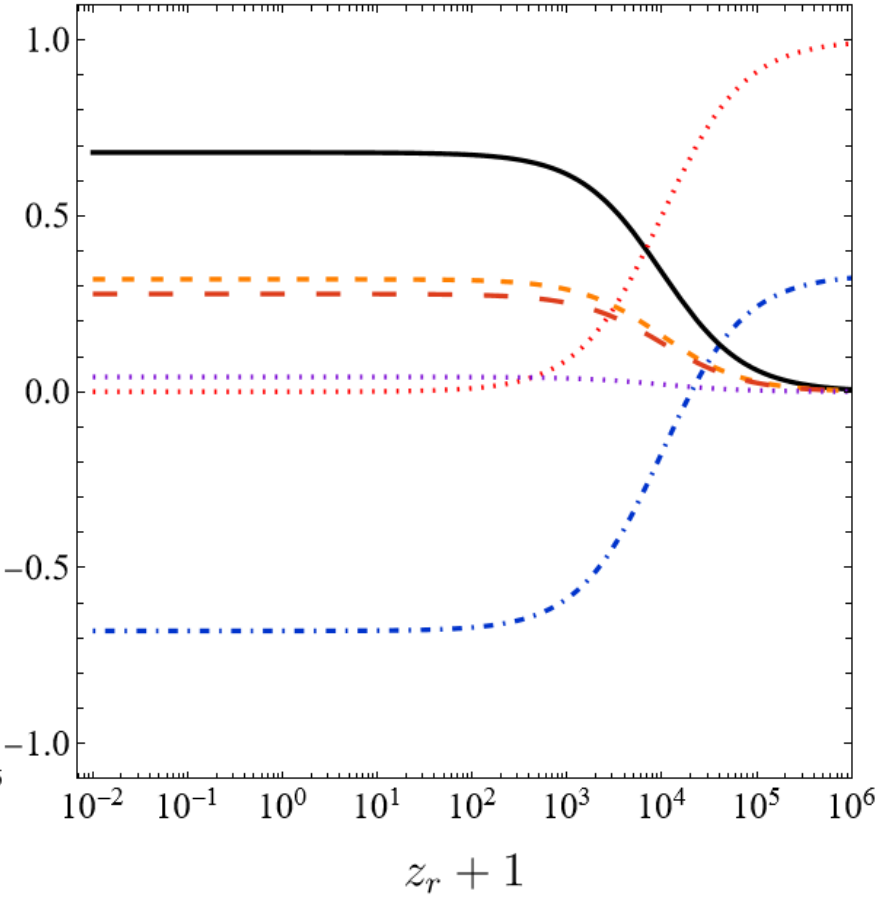
# Background

The ideal case




Real case for this model / Eternal scaling condition



# Conclusions and possible alternatives

## Conclusions

- Following the standard procedure in the scaling cosmological scenario it is not possible to find a compatible case with the observations
- The eternal scaling condition is not compatible because it is not possible to describe both the accelerated expansion today and the growth of structure at time, DE and CDM are equivalent to a new effective fluid.

## Alternatives

- The standard scaling condition is so restricted for the proca model, so a possible betted description is consider a weak scaling condition, this happen when we consider the scaling condition in the derivative of the densities and not directly in the fluid densities.

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