Structure Formation in an Anisotropic Universe

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Structure formation in an anisotropic universe: Eulerian perturbation theory

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Λ CDM model

Predictions	Problems:
• Cosmic Microwave Background. • Age of the universe. • Evolution of the universe.	• The cosmological constant. • The Hubble tension. • CMB anomalies.

ASSUMPTIONS

01 We assume a small shear approximation.

02 We assume that the metric perturbation are small, but space derivatives are large.

The Background

We use the Bianchi-I geometry, described by the line element

$$ds^2 = a^2(\eta)[-d\eta^2 + \gamma_{ij}(\eta)dx^i dx^j]$$

The anisotropic expansion of the universe is fully characterized by the shear tensor $\sigma_{ij} = \frac{1}{2}\gamma'_{ij}$

$$3\mathcal{H}^2 = (\rho_m + \rho_{de})a^2 + \frac{1}{2}\sigma^2\,,$$

$$(\sigma^i{}_j)' + 2\mathcal{H}\sigma^i{}_j = a^2\pi^i{}_j\,,$$

$$\rho'_m + 3\mathcal{H}\rho_m = 0\,,$$

$$\rho'_{de} + 3\mathcal{H}(\rho_{de} + p_{de}) = -\sigma_{ij}\pi^{ij}\,,$$

Newtonian limit and perturbed variables

We adopt a perturbative expansion scheme in the parameter $\epsilon = \left(\frac{aH}{k}\right)^2$

	Variable	n in $\mathcal{O}(\epsilon^n)$
Derivatives	∂_i/aH	$-1/2$
Fluid	$\rho,\ \delta,\ \theta$	0
	v	1
	v_i	$1/2$
Metric	$\phi,\ \psi$	1
	ω_i	$3/2$
	τ_{ij}	1

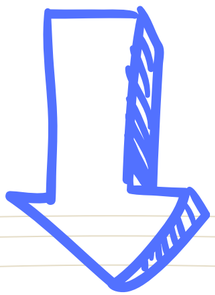
Table 1. Order of fluid and metric perturbations

Newtonian Limit: we define the Newtonian limit as the leading order equations in the parameter ϵ

Continuity and Euler equation

$$\delta' + \theta(1+\delta) + v^i \partial_i \delta = 0\,,$$

$$\theta' + \mathcal{H}\theta + \partial_i(v^j \partial_j v^i) + \frac{3}{2}\Omega_m \mathcal{H}^2 \delta = -2\sigma^{ij}\partial_i v_j\,,$$



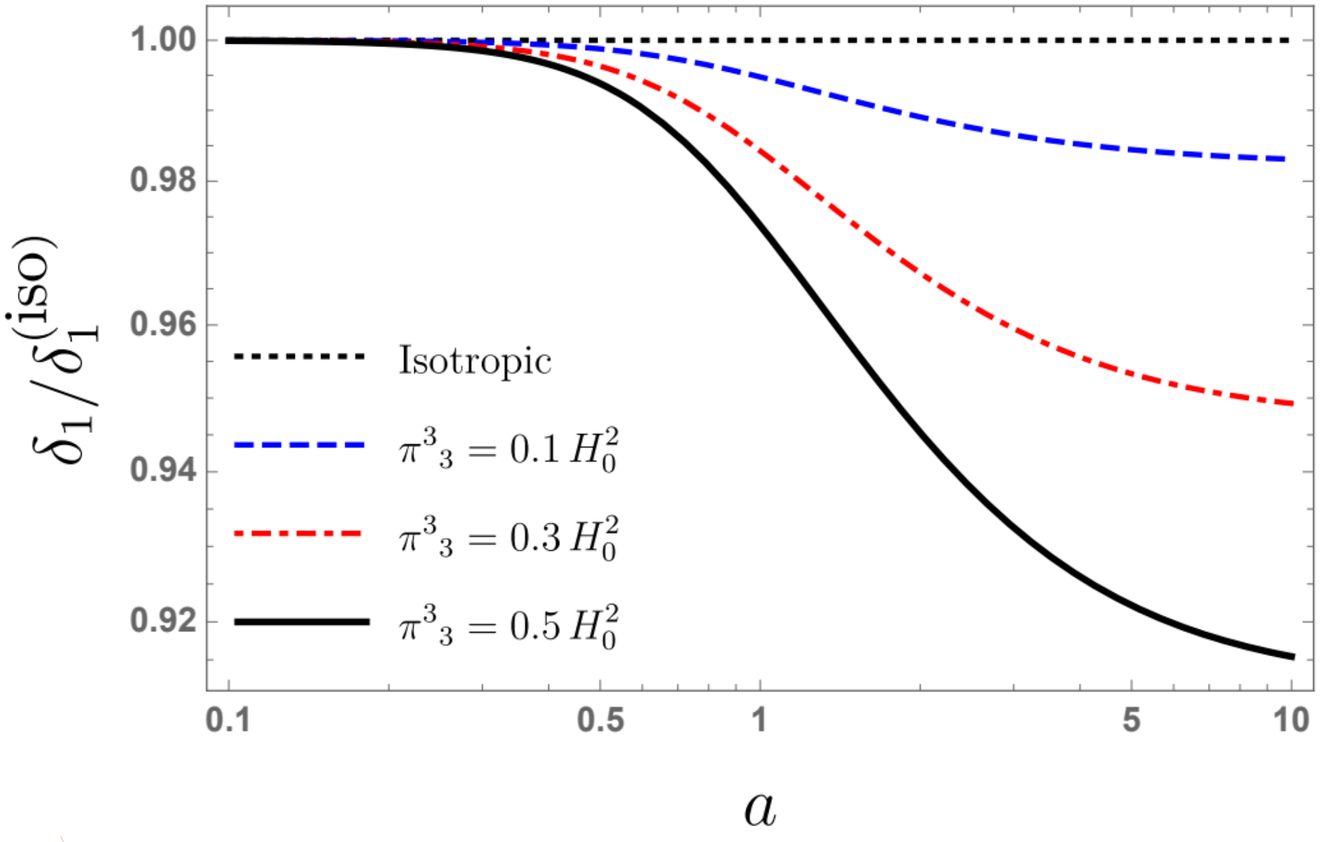
$$\delta_1''(\eta,\mathbf{k}) + \mathcal{H}\delta_1'(\eta,\mathbf{k}) - \frac{3}{2}\mathcal{H}^2\Omega_m\delta_1(\eta,\mathbf{k}) = -2\sigma_{\parallel}(\eta,\mathbf{k})\delta_1'(\eta,\mathbf{k})$$

Linear solutions

$$\delta_1(a,\mathbf{k}) = D_+(a)\left(1 + Q_{ij}(a)\hat{k}^i\hat{k}^j\right)\delta_l(\mathbf{k})$$

$$Q_{ij}(a) \equiv -\frac{2}{D_+(a)}\int_{a_i}^{a} ds\,\mathcal{G}(a,s)\frac{\sigma_{ij}(s)\dot{D}_+(s)}{s^2 H(s)}$$

$$\mathcal{G}(a,s) = \frac{2s^3}{5\Omega_m^0}\frac{H(s)}{H_0}[D_+(a)D_-(s) - D_-(a)D_+(s)]$$



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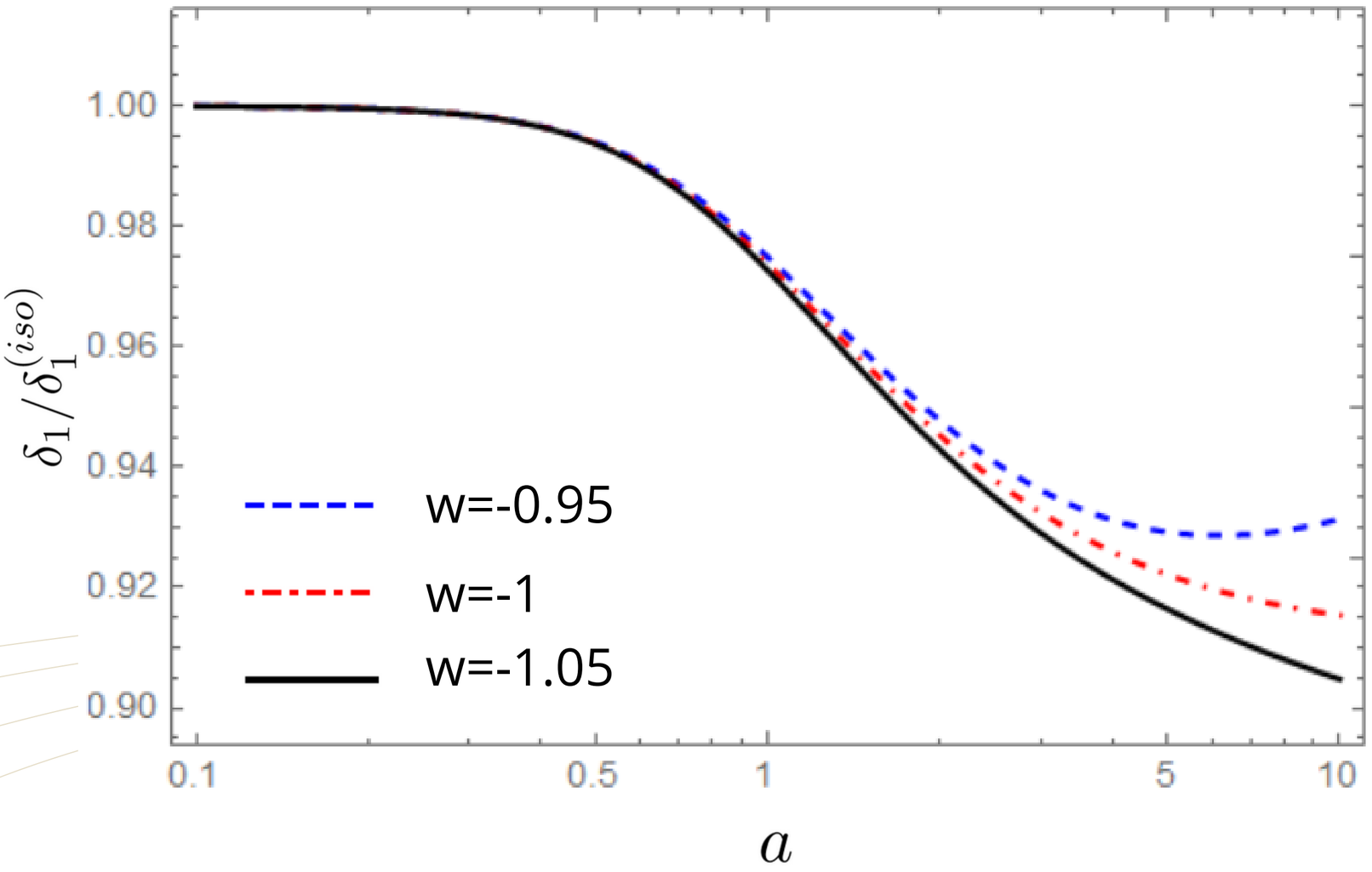
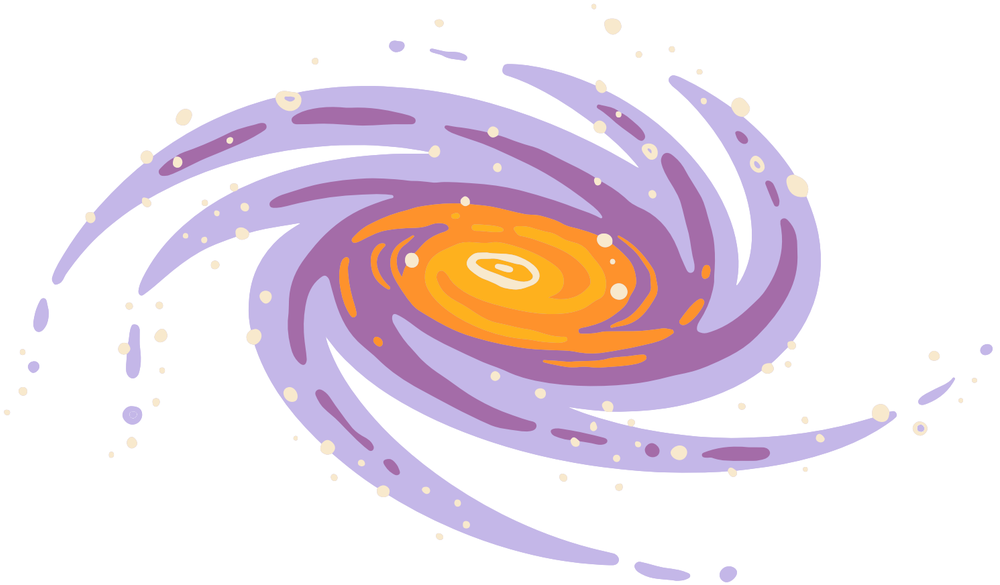


Future works

- Dynamic shear.

- Non-linear solutions.



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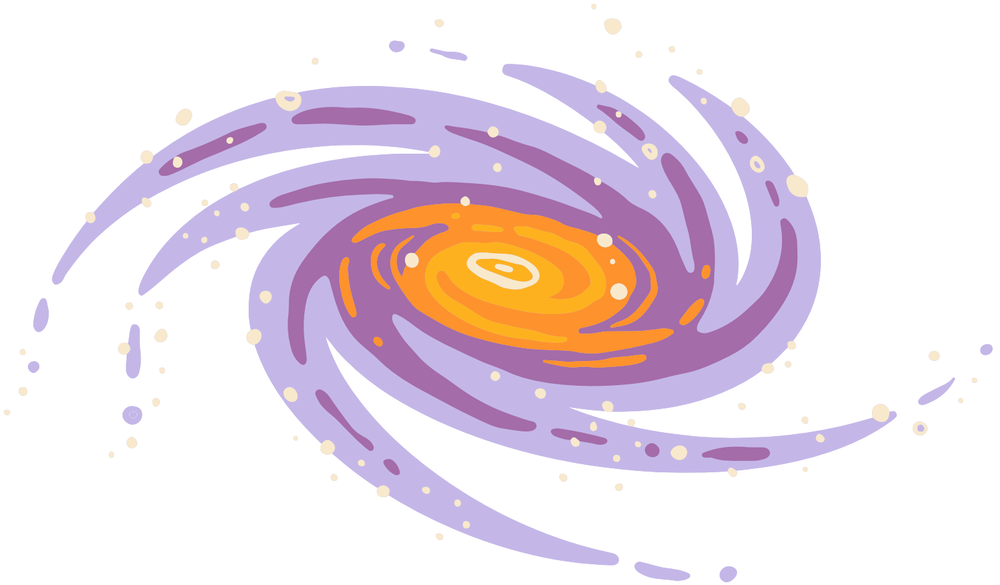

Thank you!