The mirage of luminal modified gravitational-wave propagation

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Partially based on work in collaboration
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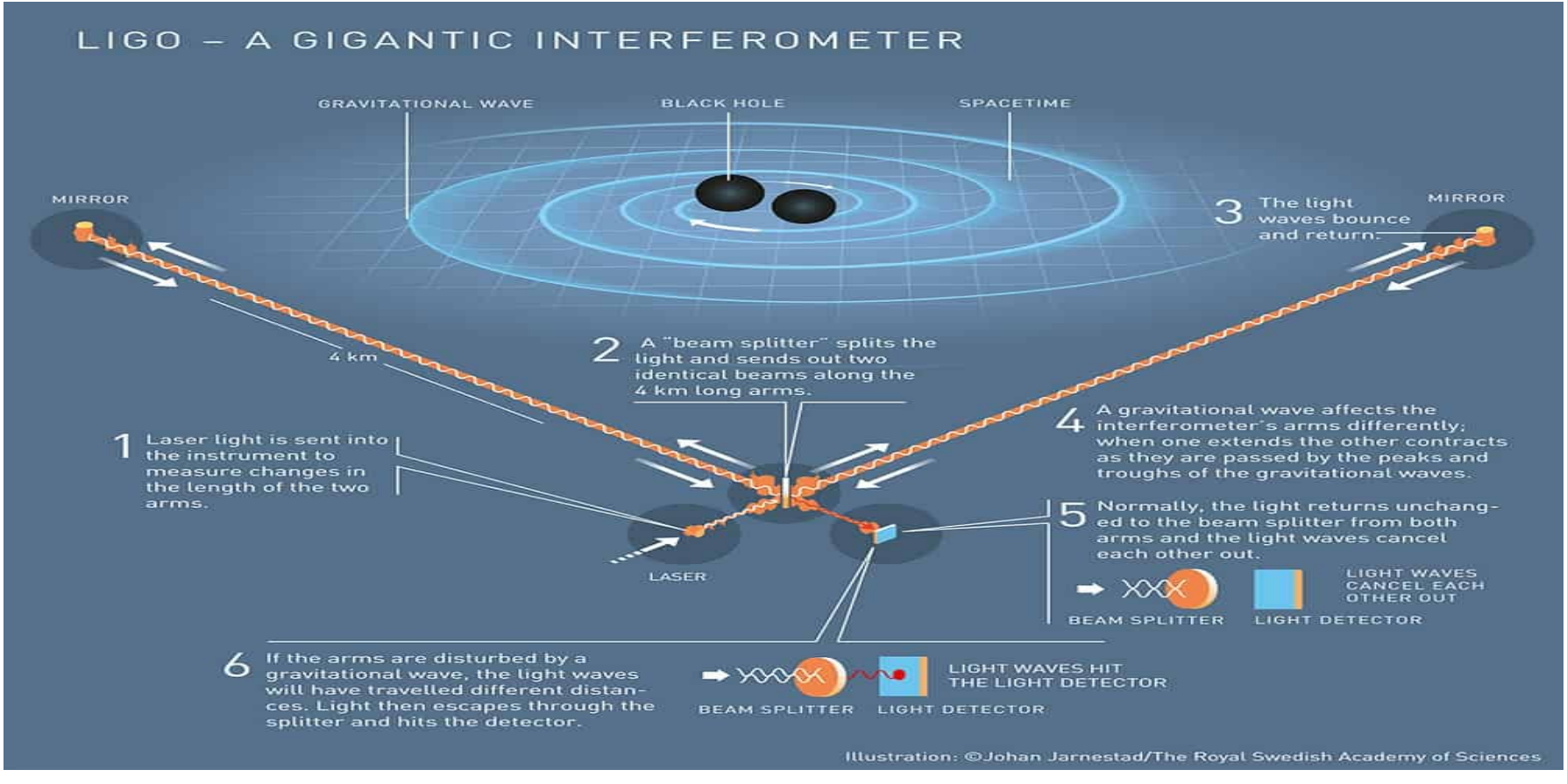
GW observatory



International Gravitational-Wave Observatory Network (IGWN)

Cosmology with gravitational waves

- Strain

$$\frac{2\mathcal{M}_z}{d_L}\left(1+\cos^2(\iota)\right)\left(\frac{5}{256}\frac{\mathcal{M}_z}{(T-t)}\right)^{1/4}\cos\left(-2\left(\frac{(T-t)}{5\mathcal{M}_z}\right)^{5/8}+\Psi\right)$$

- Chirp mass

$$\mathcal{M}_z=(1+z)\frac{(m_1m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

- We can measure d_L but not z because the strain depends on the chirp mass, not just on z
- **Bright sirens** have an **EM counterpart** so $d_L(z)$ can be constrained directly
- **Dark sirens** have **no EM counterpart** but cross-correlation with galaxies catalogs allows a statistical estimation of the red-shift, based on a joint fitting of the rates and population models
- In **General relativity (GR)** the electromagnetic and gravitational $d_L(z)$ are predicted to be equal

Gravitational waves detection implications

Cosmology

- Bright Sirens : Binary systems with EM counter part, only GW170817
  D with GW + z with EM = D(z), few events, assumes $D_{GW}=D_{EM}$

- Dark Sirens : No EM counterpart, statistical inference of the redshift by comparison with Galaxies catalogues, many events

Test of General Relativity

- GW170817 has been confidently associated to GRB 170817A, allowing to use the time delay to tightly constrain $c-c_{T,}$ ruling out many MGTs

GW propagation in General Relativity

Field equations: $$G_{\mu\nu} = T^{DE}_{\mu\nu} + T^{mat}_{\mu\nu} = T^{eff}_{\mu\nu} \, ;$$

Linearized l.h.s $$h''_A + 2\mathcal{H}h'_A + \nabla^2 h_A = a^2 \Pi^{eff}_A$$

r.h.s. Is not expanded, it just comes from **Einstein's tensor structure** (TT tensor part), and is valid ad any order, can **include higher order effects** neglected by the quadratic action for h, at least **cubic** action is required to obtain the **space/polarization** dependent anisotropic **source term**

Gravity-Matter coupling frame

Jordan (JMC) Einstein(EMC)

Calculation frame

Jordan

$$\mathcal{L}_{\rm JMC} = \sqrt{g_{\rm J}}\left[\Omega^2 R_{\rm J} + L_{\rm J}^{\rm MG} + L_{\rm J}^{\rm matter}(g_{\rm J})\right] \qquad \mathcal{L}_{\rm EMC} = \sqrt{g_{\rm J}}\left[\Omega^2 R_{\rm J} + L_{\rm J}^{\rm MG} + L_{\rm J}^{\rm matter}(\Omega^2 g_{\rm J})\right]$$

Einstein

$$\mathcal{L}_{\rm JMC} = \sqrt{g_{\rm E}}\left[R_{\rm E} + L_{\rm E}^{\rm MG} + L_{\rm E}^{\rm matter}(\Omega^{-2} g_{\rm E})\right] \qquad \mathcal{L}_{\rm EMC} = \sqrt{g_{\rm E}}\left[R_{\rm E} + L_{\rm E}^{\rm MG} + L_{\rm E}^{\rm matter}(g_{\rm E})\right]$$

A modified gravity theory is defined by two ingredients

- Free Lagrangian of the metric and the additional MGT degrees of freedom
- Interaction Lagrangian with matter coupling
- 2 Different families for each free Lagrangian: JMC and EMC
- The interaction Lagrangian affects the source term for Gws
- Ω is the real effective Planck mass, not $\mathbf{M_*}=\Omega/c_T$

General Relativity GW propagation in Jordan and Einstein frame : mirage of modified luminal propagation

Einstein	Jordan
$g_{\rm E} = \Omega^2 g_{\rm J}$, $a_{\rm E} = \Omega\, a_{\rm J}$	
$\mathcal{L}_{\rm GR} = a_{\rm E}^2 \left[h'^2 - (\nabla h)^2 \right]$	$\mathcal{L}_{\rm GR} = \Omega^2 a_{\rm J}^2 \left[h'^2 - (\nabla h)^2 \right]$
$h'' + 2\mathcal{H}_{\rm E} h' - \nabla^2 h = 0$	$h'' + 2\mathcal{H}_{\rm J}\left(1 - \frac{\Omega'}{\mathcal{H}_{\rm J}\Omega}\right) h' - \nabla^2 h = 0$
$\mathcal{H}_{\rm J}\left(1 - \frac{\Omega'}{\mathcal{H}_{\rm J}\Omega}\right) = \mathcal{H}_{\rm E}$	$\mathcal{H}_{\rm J} = a'_{\rm J}/a_{\rm J}$.

- Gws are conformally invariant, but the friction term-scale factor relation no
- The friction term is the same function of space and time in the two frames
- The scale factor-friction relation is not in itself and indicator of gravity modification

Redshift scale factor relation

$$
\begin{aligned}
P_\mu P^\mu &= P^\mu P^\nu g^{\mathrm{J}}_{\mu\nu} \\
&= E^2 - \delta_{ij} p^i p^j a_{\mathrm{J}}^2 \\
&= E^2 - \delta_{ij} p^i p^j \Omega^{-2} a_{\mathrm{E}}^2
\end{aligned}
\qquad \longrightarrow \qquad E = p\, a_{\mathrm{J}} = p\, a_{\mathrm{E}} \Omega^{-1}
$$

$$
(1+z) = \frac{E(z)}{E(0)} = \left[\frac{a(0)}{a(z)}\right]_{\mathrm{J}} = \frac{\Omega(z)}{\Omega(0)} \left[\frac{a(0)}{a(z)}\right]_{\mathrm{E}}
$$

Einstein frame formulation of generic MGTs

- There is no physical frame: any theory can be studied in any frame, without any observable consequence
- The interaction between GWs and the detectors is currently computed assuming minimal coupling between matter and metric, since detectors were built to test GR
- Local observational constraints on gravity imply that $\Omega(z=0)\approx 1$, but at higher redshift the non minimal coupling of matter to the Einstein frame metric can be tested to constrain $\Omega(z)$ with other observables
- The Friedman equation is modified, due to the time varying effective Planck mass, so the scale factor evolution can be also modified, in particular a(z), but the from of the GW propagation equation is the same that in GR if c=cT, because the gravity modification is just an effective DE fluid in the Einstein frame

Effective field theory confirmation

Jordan Frame

1210.0201,1304.4840

$$\mathcal{L}^{\rm eff}_{\rm DE} = \sqrt{g_{\rm J}}\left[\Omega^2 R_{\rm J} + L^{(2)}_{\rm J}\right]$$

$$\mathcal{L}^{\rm eff}_{h} = \frac{a^2_{\rm J}\Omega^2}{c^2_{\rm T}}\left[h'^2 - c^2_{\rm T}(\nabla h)^2\right]$$

Einstein frame

1407.8439,1705.09290

$$\mathcal{L}^{\rm eff}_{\rm DE} = \sqrt{g_{\rm E}}\left[R_{\rm E} + L^{(2)}_{\rm E}\right]$$

$$\mathcal{L}^{\rm eff}_{h} = \frac{a^2_{\rm E}}{c^2_{\rm T}}\left[h'^2 - c^2_{\rm T}(\nabla h)^2\right]$$

Propagation equation

Jordan Frame

$$h'' + 2\mathcal{H}_{\rm J}\left(1 - \frac{c_{\rm T}'}{\mathcal{H}_{\rm J} c_{\rm T}} - \frac{\Omega'}{\mathcal{H}_{\rm J}\Omega}\right)h' - c_{\rm T}^2\nabla^2 h = 0$$

Einstein frame

$$h'' + 2\left(\frac{a_{\rm E}'}{a_{\rm E}} - \frac{c_{\rm T}'}{c_{\rm T}}\right)h' - c_{\rm T}^2\nabla^2 h = 0$$

$$h'' + 2\frac{\alpha'}{\alpha}h' - c_{\rm T}^2\nabla^2 h = 0$$

$$\alpha = \frac{a_{\rm E}}{c_{\rm T}} = \frac{\Omega a_{\rm J}}{c_{\rm T}}$$

$$\mathcal{H}_{\rm J}\left(1 - \frac{\Omega'}{\mathcal{H}_{\rm J}\Omega}\right) = \mathcal{H}_{\rm E}$$

GW and EM luminosity distance

$$h = \chi/\alpha \longrightarrow \chi_k'' + \left(c_{\rm T} k^2 - \frac{\alpha''}{\alpha}\right)\chi_k = 0 \longrightarrow h_k \propto \frac{1}{\alpha}$$

For GW emitted by binary $\quad h_k \propto \frac{1}{\alpha\, r} \propto \frac{1}{d_{\rm L}^{\rm GW}}$

For EM wave $\quad d_{\rm L}^{\rm EM} = r(1+z)$

The matter-gravity coupling has no effect on GW propagation in vacuo

$$\alpha = \frac{a_{\rm E}}{c_{\rm T}} = \frac{\Omega a_{\rm J}}{c_{\rm T}} \longrightarrow d_{\rm L}^{\rm GW}|_{\rm E} = d_{\rm L}^{\rm GW}|_{\rm J} = r\,\frac{\alpha(\eta_o)}{\alpha(\eta_s)} = \frac{c_T(\eta_s)}{c_T(\eta_o)} d_{\rm L}^{GR}$$

Observable relations derived using EFT

	GR	MGT - Einstein frame	MGT - Jordan frame
Scale factor	$a_{\rm E}$	$a_{\rm E}$	$a_{\rm J} = \left(M_* c_{\rm T}\right)^{-1} a_{\rm E}$
Coupling	$g_{\rm E}$	$g_{\rm E}$	$g_{\rm J}$
$(1+z)$	$\frac{a_{\rm E}(0)}{a_{\rm E}(z)}$	$\frac{a_{\rm E}(0)}{a_{\rm E}(z)}$	$\frac{a_{\rm J}(0)}{a_{\rm J}(z)} = \frac{M_*(z) c_{\rm T}(z) a_{\rm E}(0)}{M_*(0) c_{\rm T}(0) a_{\rm E}(z)}$
$d_L^{\rm GW}$	$d_{\rm GR}^{\rm GW} = r \frac{a_{\rm E}(0)}{a_{\rm E}(z)}$	$d_{\rm MGT}^{\rm GW} = \frac{c_{\rm T}(z)}{c_{\rm T}(0)} d_{\rm GR}^{\rm GW}$	$d_{\rm MGT}^{\rm GW} = \frac{c_{\rm T}(z)}{c_{\rm T}(0)} d_{\rm GR}^{\rm GW}$
$d_L^{\rm EM}$	$d_{\rm GR}^{\rm EM} = r \frac{a_{\rm E}(0)}{a_{\rm E}(z)}$	$d_{\rm MGT}^{\rm EM} = d_{\rm GR}^{\rm EM}$	$d_{\rm MGT}^{\rm EM} = r \frac{a_{\rm J}(0)}{a_{\rm J}(z)} = \frac{M_*(z) c_{\rm T}(z)}{M_*(0) c_{\rm T}(0)} d_{\rm GR}^{\rm EM}$
$\frac{d_{\rm L}^{\rm GW}}{d_{\rm L}^{\rm EM}}$	1	$\frac{c_{\rm T}(z)}{c_{\rm T}(0)}$	$\frac{M_*(0)}{M_*(z)}$

Friction term and dark energy relation in JMC theories

- In the Jordan frame the friction term is not independent from the effective dark energy associated to the MGT, but the relation is theory dependent
- Other cosmological observations constrain DE, and consequently the friction, so in general LCDM parametrization cannot be used independently of friction
- Let's consider the example of Horndeski theories, 2210.12174

$$H^2 = \rho_{\rm m} + \rho_{\rm DE} \qquad h_A^{''} + [2 + \alpha_M(t)]\,\mathcal{H} h_A^{'} + k^2 c_T^2 h_A = 0, \qquad \alpha_M \equiv \frac{d\ln(M_*/M_P)^2}{d\ln a} = \frac{2}{\mathcal{H}}\frac{M_*^{'}}{M_*}$$

$$\rho_{\rm DE} \equiv -\frac{1}{3}G_2 + \frac{2}{3}X\left(G_{2X} - G_{3\phi}\right) - \frac{2H^3\phi' X}{3a}\left(7G_{5X} + 4XG_{5XX}\right) + H^2\left[1 - (1-\alpha_{\rm B})\,M_*^2 - 4X\left(G_{4X} - G_{5\phi}\right) - 4X^2\left(2G_{4XX} - G_{5\phi X}\right)\right]$$

$$M_*^2 \equiv 2\left(G_4 - 2XG_{4X} + XG_{5\phi} - \frac{H\phi'}{a}XG_{5X}\right)$$

$$p_{\rm DE} \equiv \frac{1}{3}G_2 - \frac{2}{3}X\left(G_{3\phi} - 2G_{4\phi\phi}\right) + \frac{4H\phi'}{3a}\left(G_{4\phi} - 2XG_{4\phi X} + XG_{5\phi\phi}\right) - \frac{(\phi'' - aH\phi')}{3\phi' a}HM_*^2\alpha_{\rm B} - \frac{4}{3}H^2X^2G_{5\phi X} - \left(H^2 + \frac{2H'}{3a}\right)\left(1 - M_*^2\right) + \frac{2H^3\phi' XG_{5X}}{3a},$$

Jordan frame “Friedman equation” caveats

Jordan frame Friedman eq.

$$3M_*^2\mathcal{H}^2 = a^2\left(\rho_{\rm m} + \hat{\mathcal{E}}_{\rm S}\right)$$
$$M_*^2\left(2\mathcal{H}' + \mathcal{H}^2\right) = -a^2\left(p_{\rm m} + \hat{\mathcal{P}}_{\rm S}\right)$$
$$\hat{\mathcal{E}}_{\rm S}' + 3\mathcal{H}\left(\hat{\mathcal{E}}_{\rm S} + \hat{\mathcal{P}}_{\rm S}\right) = \left(\rho_{\rm m} + \hat{\mathcal{E}}_{\rm S}\right)\mathcal{H}\alpha_{\rm M}$$
$$\rho_{\rm m}' + 3\mathcal{H}\left(\rho_{\rm m} + p_{\rm m}\right) = 0\,,$$

$$\hat{\mathcal{E}}_{\rm S} \equiv \mathcal{E}_{\rm S} + 3\left(M_*^2 - M_{\rm Pl}^2\right)\frac{\mathcal{H}^2}{a^2}$$
$$\hat{\mathcal{P}}_{\rm S} \equiv \mathcal{P}_{\rm S} - \frac{1}{a^2}\left(M_*^2 - M_{\rm Pl}^2\right)\left(2\mathcal{H}' + 2\mathcal{H}^2\right)$$

Effective Jordan frame Friedman eq.

$$3M_{\rm Pl}^2\mathcal{H}^2 = a^2\left(\rho_{\rm m} + \mathcal{E}_{\rm S}\right)$$
$$M_{\rm Pl}^2\left(2\mathcal{H}' + \mathcal{H}^2\right) = -a^2\left(p_{\rm m} + \mathcal{P}_{\rm S}\right)$$
$$\rho_{\rm m}' + 3\mathcal{H}\left(\rho_{\rm m} + p_{\rm m}\right) = 0$$
$$\mathcal{E}_{\rm S}' + 3\mathcal{H}\left(\mathcal{E}_{\rm S} + \mathcal{P}_{\rm S}\right) = 0\,,$$

M_* is hidden inside the effective DE fluid to facilitate the modification of existing LCDM numerical codes, 2210.12174, 1605.06102
This, and other common functions,
makes the friction and DE interdependent

GW propagation in General Relativity

Field equations: $$G_{\mu\nu} = T^{DE}_{\mu\nu} + T^{mat}_{\mu\nu} = T^{eff}_{\mu\nu} \, ;$$

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Effective action and metric encoding higher order interaction

The effective speed is Frequency/Polarization dependence

2211.05760,2301.05679

$$\frac{a^2}{c_{T,A}^2}\left[h_A'^2 - c_{T,A}^2(\nabla h_A)^2\right] = \frac{\Omega^2\tilde{a}^2}{c_{T,A}^2}\left[h_A'^2 - c_{T,A}^2(\nabla h_A)^2\right]$$

$$ds_A^2 = a^2\left[c_{T,A}d\eta^2 - \frac{\delta_{ij}}{c_{T,A}}dx^i dx^j\right]$$

Higher order effects

Matter coupling

Distance ratio

Jordan

$$d_{L,A}^{GW}(z) = \frac{\tilde{M}_A(0,k)}{\tilde{M}_A(z,k)} d_L^{EM}(z)$$

Einstein

$$\frac{d_L^{GW}}{d_L^{EM}}(z) = \frac{a(z)}{\tilde{\alpha}_A(z)} \frac{\tilde{\alpha}_A(0)}{a(0)} = \frac{\tilde{c}_{T,A}(z,k)}{\tilde{c}_{T,A}(0,k)}$$

Conclusions

- GWs propagating in the vacuo at the speed of light follows GR
- Any apparent modification of the friction term can be removed by moving to the Einstein frame
- For luminal MGTs the modification of the GW-EMW distance ratio is due to the non minimal matter-gravity coupling, and consequent change in scale factor-redshift relation, not to the modification of the GW propagation
- The calculation frame and the matter-gravity frame should be clearly distinguished
- Observable quantities should be used to test MGTs, while the friction term of the GW propagation equation is not invariant under conformal transformations, and can be miss-leading in defining the effects of the modification of gravity
- MGTs with matter coupled to the Einstein frame are an independent (neglected) family with a distinct observational signatures

Difference between ‘calculation’ and matter-gravity coupling frames

- Matter follows the geodesics of the metric to which is minimally coupled
- For Jordan metric matter-gravity coupling matter follows the Jordan metric geodesics, while in the Einstein frame a ‘fifth’ force determines a deviation from the Einstein metric geodesics, but the two descriptions are completely equivalent, at least classically.
- Any MGT with matter coupled to the Jordan metric can be studied in the Einstein frame introducing a non minimal coupling to the Einstein metric
- The Einstein frame formulation of MGTs it allows to apply the results obtained in relativistic cosmological perturbations theory, encoding the effects of the gravity modification in the effective energy-stress-tensor (EST), leading to new source terms in the perturbations equations
- The calculation and matter-gravity coupling metrics should be distinguished

EFT GW propagation equation

- EFT

$$h'' + 2\mathcal{H}_{\rm J}\left(1 - \frac{c_{\rm T}'}{\mathcal{H}_{\rm J} c_{\rm T}} - \frac{\Omega'}{\mathcal{H}_{\rm J}\Omega}\right)h' - c_{\rm T}^2\nabla^2 h = 0$$

- EFT c=1

$$h'' + 2\mathcal{H}_{\rm J}\left(1 - \frac{\Omega'}{\mathcal{H}_{\rm J}\Omega}\right)h' - \nabla^2 h = 0$$

- Lagrangian

$$\mathcal{L}_{\rm GR} = a_{\rm J}^2\Omega^2\left[h'^2 - (\nabla h)^2\right] = a_{\rm E}\left[h'^2 - (\nabla h)^2\right]$$

- The EFT confirms the model independent argument

What defines General Relativity and its modifications?

- Einstein's equations are derived from the Hilbert action + matter action
- In GR the metric is minimally coupled to matter fields
- The matter lagrangian determines the form of the energy-stress-tensor (EST)
- The Hilbert action is the free gravity action, describing gravity in the vacuum, i.e. in absence of interaction of the graviton with other fields
- In MGTs there are additional fields, which in the Einstein frame can be considered additional matter fields, changing the definition of vacuum
- Depending on the MGT matter can be coupled to the Einstein or Jordan frame
- The effects of MGTs on GW propagation in the Einstein frame are due to the interaction of these fields with the graviton.

$$\mathcal{L}_{\rm DE}^{\rm eff} = \sqrt{g_{\rm J}}\left[\Omega^2 R_{\rm J} + L_{\rm J}^{(2)}\right] \qquad \mathcal{L}_{\rm DE}^{\rm eff} = \sqrt{g_{\rm E}}\left[R_{\rm E} + L_{\rm E}^{(2)}\right]$$

Unified effective approach to cosmological perturbations

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Unified effective approach to scalar and tensor perturbations

- The equation for **tensor** perturbations and for **curvature** perturbations look very **similar** in FRW cosmological models
- Can similar **new equations** be derived in a **model independent** way to describe **any interaction** of perturbations with other fields in a **unified framework**?
- **Yes!** A set of **universal equations/actions** can be derived in terms of an appropriately **effective speed encoding** the effects of **interaction**
- **Unfied single model independent framework to study the effects of entropy and anisotropy**
- Useful for **phenomenological model independent** data analysis, including **modified gravity**, **dark/baryonic matter interaction**, **inflation/bounce**, **multi-fields effects**

Effective Lagrangian approach

$$S = \int d\eta dx \mathcal{L} = \int d\eta dx z^2 L \quad , \quad \mathcal{L} = z^2 L \quad , \quad z^2 = \epsilon a^2$$

$$\mathcal{L}_\zeta = \mathcal{L}_\zeta^{(2)} + \mathcal{L}_\zeta^{int} = z^2 \left[\zeta'^2 - (\nabla\zeta)^2 + L_\zeta^{int}(\zeta, \phi^i) \right] = z^2 \left[\zeta'^2 \left(1 + \frac{L_\zeta^{int}}{\zeta'^2} \right) - (\nabla\zeta)^2 \right]$$

Any order !

$$c_s^2(\eta, x^i) = \left(1 + \frac{L_\zeta^{int}}{\zeta'^2} \right)^{-1}$$

Propagation equation

$$\mathcal{L}_\zeta^{eff} = \frac{z^2}{c_s^2}\left[\zeta'^2 - c_s^2(\nabla\zeta)^2\right] = \alpha^2\left[\zeta'^2 - c_s^2(\nabla\zeta)^2\right] \quad , \quad \alpha^2 = \frac{z^2}{c_s^2} = \frac{\epsilon a^2}{c_s^2}$$

$$\zeta'' + 2\frac{\alpha'}{\alpha}\zeta' - c_s^2\nabla^2\zeta = 0$$

$$\zeta'' + 2\left(\frac{z'}{z} - \frac{c_s'}{c_s}\right)\zeta' - c_s^2\nabla^2\zeta = 0$$

Field equation approach

$$\delta P_c(\eta, x^i) = c_a(\eta)^2 \delta\rho_c(\eta, x^i) + \Gamma(\eta, x^i)$$

$$\mathcal{S} = -\frac{c_a^2}{\epsilon}\nabla^2\Pi - \frac{1}{2a^2 z_a^2}\left[\frac{a^3}{c_a^2 H}\left(\Gamma + \frac{2}{3}\nabla^2\Pi\right)\right]' \,, \quad z_a^2 = \frac{\epsilon a^2}{c_a^2}$$

$$\zeta'' + \frac{\partial_\eta z_a^2}{z_a^2}\zeta' - c_a^2\nabla^2\zeta = a^2\mathcal{S}$$

Absorbing the source in the SESS

$$g = \frac{1}{z_a^2} \int z_a^2 a^2 \mathcal{S}\, d\eta$$

$$\frac{(\zeta' z_a^2)'}{z_a^2} - \frac{(g z_a^2)'}{z_a^2} - c_a^2 \nabla^2 \zeta = \frac{\left[\zeta' z_a^2 (1 - g/\zeta')\right]'}{z_a^2} - c_a^2 \nabla^2 \zeta = 0$$

Effective sound speed

$$1+\delta(\eta,x^i)=\left(1-\frac{g}{\zeta'}\right)^{-1/2} \quad , \quad \alpha^2=\frac{z_a^2}{(1+\delta)^2}=\frac{\epsilon a^2}{c_a^2(1+\delta)^2} \, ,$$

$$c_s(\eta,x^i)=c_a(\eta)\left[1+\delta(\eta,x^i)\right] \qquad \alpha^2=\frac{\epsilon a^2}{c_s^2}$$

$$\zeta''+2\frac{\alpha'}{\alpha}\zeta'-c_s^2\nabla^2\zeta \;=\; 0$$

GW effective Lagrangian derivation in Einstein's frame

Consistent with

EFT of inflation

***Phys.Rev.Lett.* 113 (2014)**

$$\mathcal{L}_h = a^2\left[h_A'^2 - (\nabla h_A)^2 + \mathcal{S}(h_A, \phi^i)\right]$$

$$c_{T,A}^2 = \left(1 + \frac{\mathcal{S}}{h_A'^2}\right)^{-1} \qquad \mathcal{S}^{(2)} \propto \bar{M}_2(\eta)\delta K_{\mu\nu}\delta K^{\mu\nu} \propto \bar{M}_2(\eta)h_A'^2$$

$$\mathcal{L}_h^{eff} = \left[\alpha_A^2 h_A'^2 - a^2(\nabla h_A)^2\right] = \frac{a^2}{c_{T,A}^2}\left[h_A'^2 - c_{T,A}^2(\nabla h_A)^2\right]$$

GW propagation in the field equation approach

Field equations: $$G_{\mu\nu} = T^{DE}_{\mu\nu} + T^{mat}_{\mu\nu} = T^{eff}_{\mu\nu} \, ;$$

Linearized l.h.s $$h''_A + 2\mathcal{H}h'_A + \nabla^2 h_A = a^2 \Pi^{eff}_A$$

r.h.s. Is not expanded, it just comes from **Einstein's tensor structure** (TT tensor part), and is valid ad any order, can **include higher order effects** neglected by the quadratic action for h, at least **cubic** action is required to obtain the **space/polarization** dependent anisotropic **source term**

Space effective sound speeds: SESS and SEGS

	Gravitational waves	Curvature perturbations
Speed	$c^2_{T,A}(\eta, x^i) = \left(1+\frac{L^{int}_h}{h'^2_A}\right)^{-1} = \left(1-\frac{g_A}{h'_A}\right)^{-1}$	$c^2_s(\eta, x^i) = \left(1+\frac{L^{int}_\zeta}{\zeta'^2}\right)^{-1} = c^2_a(\eta)\left(1-\frac{g}{\zeta'}\right)^{-1}$
$\mathcal{L}^{eff}$, α	$\frac{a^2}{c^2_{T,A}}\left[h'^2_A - c^2_{T,A}(\nabla h_A)^2\right]$, $\alpha_A = \frac{a^2}{c^2_{T,A}}$	$\frac{z^2}{c^2_s}\left[\zeta'^2 - c^2_s(\nabla\zeta)^2\right]$, $\alpha = \frac{\epsilon a^2}{c^2_s} = \frac{z^2}{c^2_s}$
Eq.	$h''_A + 2\frac{\alpha'_A}{\alpha_A}h'_A - c^2_{T,A}\nabla^2 h_A = 0$	$\zeta'' + 2\frac{\alpha'}{\alpha}\zeta' - c^2_s\nabla^2\zeta = 0$
Eq.	$h''_A + 2\left(\frac{a'}{a} - \frac{c'_{T,A}}{c_{T,A}}\right)h'_A - c^2_{T,A}\nabla^2 h_A = 0$	$\zeta'' + 2\left(\frac{z'}{z} - \frac{c'_s}{c_s}\right)\zeta' - c^2_s\nabla^2\zeta = 0$

Momentum effective speeds: MESS and MEGS

	Gravitational waves	Curvature perturbations
Speed	$\tilde{c}^2_{T,A}(\eta,k)=\left(1+\frac{L^{int}_{\tilde{h}}}{\tilde{h}'^2_A}\right)^{-1}=\left(1-\frac{\tilde{g}_A}{\tilde{\tilde{h}}'_A}\right)^{-1}$	$\tilde{c}^2_s(\eta,k)=\left(1+\frac{L^{int}_{\zeta_k}}{\zeta'^2_k}\right)^{-1}=c_a(\eta)^2\left(1-\frac{\tilde{g}}{\zeta'_k}\right)^{-1}$
$\mathcal{L}^{eff}$, α	$\frac{a^2}{\tilde{c}^2_{T,A}}\left[\tilde{h}'^2_A+k^2\tilde{c}^2_{T,A}\tilde{h}^2_A\right]$, $\tilde{\alpha}_A=\frac{a^2}{\tilde{c}^2_{T,A}}$	$\frac{z^2}{\tilde{c}^2_s}\left[\zeta'^2_k+k^2\tilde{c}^2_s\zeta^2_k\right]$, $\tilde{\alpha}=\frac{\epsilon a^2}{\tilde{c}^2_s}=\frac{z^2}{\tilde{c}^2_s}$
Eq.	$\tilde{h}''_A+2\frac{\tilde{\alpha}'_A}{\tilde{\alpha}_A}\tilde{h}'_A+k^2\tilde{c}^2_{T,A}\tilde{h}^2_A=0$	$\zeta''_k+2\frac{\tilde{\alpha}'}{\tilde{\alpha}}\zeta'_k+k^2\tilde{c}^2_s\zeta^2_k=0$
Eq.	$\tilde{h}''_A+2\left(\frac{a'}{a}-\frac{\tilde{c}'_{T,A}}{\tilde{c}_{T,A}}\right)\tilde{h}'_A+k^2\tilde{c}^2_{T,A}\tilde{h}^2_A=0$	$\zeta''_k+2\left(\frac{z'}{z}-\frac{\tilde{c}'_s}{\tilde{c}_s}\right)\zeta'_k+k^2\tilde{c}^2_s\zeta^2_k=0$

Effective metric

$$ds_A^2 = a^2\left[c_{T,A}d\eta^2 - \frac{\delta_{ij}}{c_{T,A}}dx^i dx^j\right]$$

$$\mathcal{L}_h^{eff} = \sqrt{-g_A}\left(\partial_\mu h_A \partial^\mu h_A\right), \quad \Box h_A = \frac{1}{\sqrt{-g_A}}\partial_\mu\left(\sqrt{-g_A}\partial^\mu h_A\right) = 0$$

Consistency with previous results

In **agreement** with:

- **K-inflation** action
- **Effective field theory of inflation**
- **Frequency dependency** of **sound speed** in multi-fields models
- **Frequency of GW speed** in effective field theory of dark energy

New predictions

- **Polarization/time/frequency** dependency **of GW speed** during **inflation**
- **Polarization/time/frequency** dependency of **GW speed** due to **modified gravity**
- **TIme/Frequency dependency** of sound speed in inflation

$$b\mathcal{L}^{(3)}_{\zeta\zeta h} = b\,a^2 h_{ij}\partial^i\zeta\partial^j\zeta = b\,a^2 L^{(3)}_{\zeta\zeta h} = z^2\frac{b}{\epsilon}\left[h_+\left(\partial^x\zeta\partial^x\zeta-\partial^y\zeta\partial^y\zeta\right)+2h_\times\left(\partial^x\zeta\partial^y\zeta\right)\right] = z^2\frac{b}{\epsilon}\left[h_+\pi_+ + h_\times\pi_\times\right]$$

$$h''_A + 2\mathcal{H}h'_A + \nabla^2 h_A = a^2\,b\,\pi_A = a^2\Pi_A\,,$$

$$\zeta'' + 2\frac{z'}{z}\zeta' - c_s^2\nabla^2\zeta = a^2\,b\,h^{ij}\partial_i\partial_j\zeta = a^2\mathcal{S}$$

Effective description of tensor-scalar interaction

$$h_A'' + 2\frac{\alpha_A'}{\alpha_A} h_A' - c_{T,A}^2 \nabla^2 h_A = 0 \quad , \quad \alpha_A = \frac{a^2}{c_{T,A}^2} \, ,$$

$$\zeta'' + 2\frac{\alpha'}{\alpha} \zeta' - c_s^2 \nabla^2 \zeta = 0 \quad , \quad \alpha = \frac{\epsilon a^2}{c_s^2} = \frac{z^2}{c_s^2} \, .$$

Jordan-Einstein frame and eff. Plack mass

$$\mathcal{L}^J = a^2 q_s \left[\zeta'^2 - c_s^2 (\nabla\zeta)^2\right] + a^2 q_t \left[h'^2 - c_T^2 (\nabla h)^2\right]$$

$$\tilde{\mathcal{L}}^J = \tilde{a}^2 \tilde{q}_s \left[\tilde{\zeta}'^2 - \tilde{c}_s^2 (\nabla\tilde{\zeta})^2\right] + a^2 \tilde{q}_t \left[\tilde{h}'^2 - \tilde{c}_T^2 (\nabla\tilde{h})^2\right]$$

$$\tilde{a} = \Omega a \quad \rightarrow \quad \tilde{q}_s = \Omega^{-2} q_s \,,\; \tilde{q}_t = \Omega^{-2} q_t \,,\; c_s = \tilde{c}_s \,,\; c_T = \tilde{c}_T$$

Einstein frame

$$\mathcal{L}_E^{eff} = \tilde{\epsilon}\tilde{a}^2\frac{1}{c_s^2}\left[\zeta'^2 - c_s^2(\nabla\zeta)^2\right] + \tilde{a}^2\frac{1}{c_T^2}\left[h'^2 - c_T^2(\nabla h)^2\right] = \mathcal{L}_\zeta^{eff} + \mathcal{L}_h^{eff}$$

$$\tilde{q}_t = \frac{1}{c_T^2}\,, \qquad \Omega = c_T\sqrt{q_t}\,,$$

$$\tilde{q}_s = \frac{\tilde{\epsilon}}{c_s^2}\,. \qquad \tilde{\epsilon} = \frac{q_s c_s^2}{q_t c_T^2}\,,$$

Quntum correlators: wavefunction vs in-in formalism

$$\Psi[\varphi] = \int_{\substack{\phi(t)=\varphi \\ \phi(-\infty)=0}} \mathcal{D}\phi\, e^{iS[\phi]}\,.$$

$$\langle \phi_1 \cdots \phi_N \rangle = \int \mathcal{D}\phi\, \phi_1 \cdots \phi_N \left|\Psi[\phi]\right|^2$$

GW propagation in the Einstein's frame

Field equations: $$G_{\mu\nu} = T^{DE}_{\mu\nu} + T^{mat}_{\mu\nu} = T^{eff}_{\mu\nu} \, ;$$

Linearized l.h.s $$h''_A + 2\mathcal{H}h'_A + \nabla^2 h_A = a^2 \Pi^{eff}_A$$

r.h.s. Is not expanded, it just comes from **Einstein's tensor structure** (TT tensor part), and is valid ad any order, can **include higher order effects** neglected by the quadratic action for h, at least **cubic** action is required to obtain the **space/polarization** dependent anisotropic **source term**

GW propagation equation with effective speed

The effective sound speed and scale factor depends on

Time

Space

Polarization

$$c^2_{T,A}(\eta, x^i) \qquad \alpha^2_A = a^2/c^2_{T,A}$$

$$h''_A + 2\frac{\alpha'_A}{\alpha_A}h'_A - c^2_{T,A}\nabla^2 h_A = 0$$

$$h''_A + 2\mathcal{H}\left(1 - \frac{c'_{T,A}}{\mathcal{H}c_{T,A}}\right)h'_A - c^2_{T,A}\nabla^2 h_A = 0$$

GW propagation in momentum space

The effective sound speed and scale factor depends on :

Time

Momentum/Frequency

Polarization

$$\tilde{c}^2_{T,A}(\eta, k) \qquad \tilde{\alpha}^2_A = a^2/\tilde{c}^2_{T,A}$$

$$\tilde{h}''_A + 2\frac{\tilde{\alpha_A}'}{\tilde{\alpha_A}}\tilde{h}'_A + \tilde{c}^2_{T,A}k^2\tilde{h}_A$$

$$\tilde{h}''_A + 2\mathcal{H}\left(1 - \frac{\tilde{c}'_{T,A}}{\mathcal{H}\tilde{c}_{T,A}}\right)\tilde{h}'_A + \tilde{c}^2_{T,A}k^2\tilde{h}_A$$

Predicted effects on GW luminosity distance

Redshift, **Frequency**, **Polarization** dependent

$$\tilde{h}_A = \tilde{\chi}_A / \tilde{\alpha}_A \qquad \tilde{\chi}_A'' + \left(\tilde{c}_{T,A} k^2 - \frac{\tilde{\alpha}_A''}{\tilde{\alpha}_A} \right) \tilde{\chi}_A = 0$$

$$d_{L,A}^{GW}(z) = \frac{a(z)}{\tilde{\alpha}_A(z)} d_L^{EM}(z) = \tilde{c}_{T,A}(z) d_L^{EM}(z)$$

Advantages for data analysis

- **Single parametrization** for speed and luminosity distance

- **Model independent**

- Includes **higher order effects, momentum and polarization dependency,** not accounted for in quadratic calculation for modified gravity theories

Effective Larangian

$$\mathcal{L}_h^{eff} = \left[\alpha_A^2 h_A'^2 - a^2(\nabla h_A)^2\right] = \alpha_A^2\left[h_A'^2 - c_{T,A}^2(\nabla h_A)^2\right]$$

$$\mathcal{L}_h^{GR} = a^2\left[h_A'^2 - c^2(\nabla h_A)^2\right]$$

Consistent with

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$$a \to \alpha_A = \frac{a}{c_{T,A}} \quad , \quad c \to c_{T,A}$$

Check: Relation to Jordan's frame for Horndeski

For Horndeski's theory the quadratic action in the Jordan's frame is

$$\mathcal{L}_h^{Hor} = a^2\left[\mathcal{G}_T h_A'^2 - \mathcal{F}_T(\nabla h_A)^2\right] \quad a^2 M_*^2\left[h_A'^2 - c_T^2(\nabla h_A)^2\right]$$

And a **conformal transformation** takes to the Einstein's frame

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \quad \Omega = \mathcal{F}_T^{1/2} \quad \mathcal{F}_T = M_*^2 c_T^2 \quad c_T^2 = \mathcal{F}_T/\mathcal{G}_T$$

$$\alpha_A^2\left[h_A'^2 - c_{T,A}^2(\nabla h_A)^2\right]$$

Non perturbative change of frame

The field equation for **Horndeski's** theories in the Jordan frame can be written as

$$\begin{aligned} \mathcal{F}_T G_{\mu\nu} &= T^{MGT}_{\mu\nu} \\ \mathcal{F}_T &= G_4 + \frac{1}{2}\nabla_\lambda G_5 \nabla^\lambda \phi \end{aligned}$$

In unitary gauge, as appearing in the action-> disformal > conformal for h

$$\mathcal{F}_T \propto \left[G_4 - X(\ddot{\phi} G_{5\phi}) + G_\phi \right]$$

Effective Lagrangian includes higher order effects

After the change of frame the effective action and the Horndeski's action agree, but the **effective action** is **more general**, since it includes the **higher order effects** coming from the higher order coupling of h and the scalar field **ignored** in the **quadratic action**

Einstein's vs Jordan's frame

Since **tensor perturbations are gauge** invariant under conformal/disformal transformations, the two frames are **equivalent**

Hubble is **not invariant**!

In general relations between gauge invariant quantities will hold in both frames

Frame change

$$\frac{a^2}{c_{T,A}^2}\left[h_A'^2 - c_{T,A}^2(\nabla h_A)^2\right] = \frac{\Omega^2\tilde{a}^2}{c_{T,A}^2}\left[h_A'^2 - c_{T,A}^2(\nabla h_A)^2\right]$$

$$h_A'' + 2\tilde{\mathcal{H}}\left(1 - \frac{c_{T,A}'}{\tilde{\mathcal{H}}c_{T,A}} - \frac{\Omega'}{\tilde{\mathcal{H}}\Omega}\right)h_A' - c_{T,A}^2\nabla^2 h_A = 0$$

$$\tilde{d}_{L,A}^{GW}(z) = \frac{\tilde{a}(z)}{\alpha_A(z)}\tilde{d}_L^{EM}(z) = \frac{c_{T,A}(z)}{\Omega}\tilde{d}_L^{EM}(z)$$

Effective Lagrangian and metric

$$a \to \alpha_A = \frac{a}{c_{T,A}} \quad , \quad c \to c_{T,A} ;$$

$$\mathcal{L}_h^{eff} = \left[\alpha_A^2 h_A'^2 - a^2 (\nabla h_A)^2\right] = \frac{a^2}{c_{T,A}^2}\left[h_A'^2 - c_{T,A}^2 (\nabla h_A)^2\right]$$

$$\mathcal{L}_h^{GR} = a^2\left[h_A'^2 - c^2 (\nabla h_A)^2\right] = \sqrt{-g}(\partial_\mu h_A \partial^\mu h_A)$$

$$ds_A^2 = a^2\left[c_{T,A} d\eta^2 - \frac{\delta_{ij}}{c_{T,A}} dx^i dx^j\right] \quad \Box h_A = \frac{1}{\sqrt{-g_A}} \partial_\mu (\sqrt{-g_A} \partial^\mu h_A) = 0$$

Effective Lagrangian derivation in Einstein's frame

$$\mathcal{L}_h = a^2\left[h_A'^2 - (\nabla h_A)^2 + \mathcal{S}(h_A, \phi^i)\right]$$

$$c_{T,A}^2 = \left(1 + \frac{\mathcal{S}}{h_A'^2}\right)^{-1} \qquad \mathcal{S}^{(2)} \propto \bar{M}_2(\eta)\delta K_{\mu\nu}\delta K^{\mu\nu} \propto \bar{M}_2(\eta)h_A'^2$$

$$\mathcal{L}_h^{eff} = \left[\alpha_A^2 h_A'^2 - a^2(\nabla h_A)^2\right] = \frac{a^2}{c_{T,A}^2}\left[h_A'^2 - c_{T,A}^2(\nabla h_A)^2\right]$$

Effective Lagrangian derivation in Jordan's frame

$$\mathcal{L}_J = \tilde{a}^2 M_*^2 \left[h_A'^2 - c_T^2 (\nabla h_A)^2 + \mathcal{S}_J^{i>2} \right]$$

$$1 + \delta_A = \left(1 + \frac{\mathcal{S}_J^{i>2}}{h_A'^2} \right)^{-1/2}$$

$$\mathcal{L}_{h,J}^{eff} = \frac{\tilde{a}^2 \mathcal{F}_T}{c_{T,A}^2} \left[h_A'^2 - c_{T,A}^2 (\nabla h_A)^2 \right]$$

$$= \tilde{a}^2 \frac{M_*^2}{(1+\delta_A)^2} \left[h_A'^2 - c_T^2 (1+\delta_A)^2 (\nabla h_A)^2 \right]$$

Jordan frame

$$M_*^2 = \mathcal{G}_T \qquad M_A(\eta, x^i) = M_*/(1+\delta_A) \qquad a = \mathcal{F}_T^{1/2}\tilde{a}$$

$$\mathcal{F}_T = c_T^2 M_*^2 \qquad c_{T,A}(\eta, x^i) = [1 + \delta_A(\eta, x^i)]c_T(\eta)$$

$$h_A'' + 2\tilde{\mathcal{H}}\left(1 + \frac{M_A'}{\tilde{\mathcal{H}}M_A}\right)h_A' - c_{T,A}^2\nabla^2 h_A = 0$$

Conclusions

- A model independent action for cosmological perturbation allows model independent analysis
- The interaction with other fields produces an effective medium which can be characterized by the the effective speeds MESS and SESS
- Can be applied to any system, MGT, mutifields inflation

Constraints from large Scale Structure

The effects of the gravity modification are described by Einstein's frame eq.

$$\frac{k^2}{a}\psi = \frac{1}{2}G_{eff}\delta\rho \qquad G_{eff}(\mathcal{F}_T, \mathcal{G}_T)$$

For **Horndeski's** G_eff depends on

$$\mathcal{F}_T = M_*^2 c_T^2$$

- For **scalar** perturbations the **effective Planck plays a physical role,** contrary to GW
- **Combining GW and large scale** structure observations can **constrain the GW speed**.

F.E. Derivation:

$$h''_A + 2\mathcal{H}h'_A + \nabla^2 h_A = a^2 \Pi_A^{eff}$$

$$\frac{(h'_A a^2)'}{a^2} - \frac{(g a^2)'}{a^2} - \nabla^2 h_A = 0 \qquad g_A = \frac{1}{a^2}\int a^4 \Pi_A^{eff}\, d\eta$$

$$\alpha_A^2 = a^2 / c_{T,A}^2 \qquad c_{T,A}^2(\eta, x^i) = \left(1 - \frac{g_A}{h'_A}\right)^{-1} \qquad \frac{1}{a^2}(\alpha_A^2 h'_A)' - \nabla^2 h_A = 0$$

$$h''_A + 2\frac{\alpha'_A}{\alpha_A} h'_A - c_{T,A}^2 \nabla^2 h_A = 0$$

$$h''_A + 2\mathcal{H}\left(1 - \frac{c'_{T,A}}{\mathcal{H} c_{T,A}}\right) h'_A - c_{T,A}^2 \nabla^2 h_A = 0$$