Scalar potential analysis of the $\mathbb{Z}_5$ multi-component dark matter model

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[1] Wan-Lei Guo and Yue-Liang Wu. "The real singlet scalar dark matter model". In: *Journal of High Energy Physics* (2010).

[2] Geneviève Bélanger, Kristjan Kannike, Alexander Pukhov, and Martti Raidal. "$\mathbb{Z}_3$ scalar singlet dark matter". In: *Journal of Cosmology and Astroparticle Physics* (2013), 022–022.

[3] Geneviève Bélanger, Alexander Pukhov, Carlos Yaguna, and Oscar Zapata. "The $\mathbb{Z}_5$ model of two-component dark matter". In: *Journal of High Energy Physics* 2020.30 (2020).

[4] Geneviève Bélanger, Kristjan Kannike, Alexander Pukhov, and Martti Raidal. "Minimal semi-annihilating $\mathbb{Z}_N$ scalar dark matter". In: *Journal of Cosmology and Astroparticle Physics* 2014.06 (2014), 021–021.

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This model have the minimum value of N which allows two **complex** fields.

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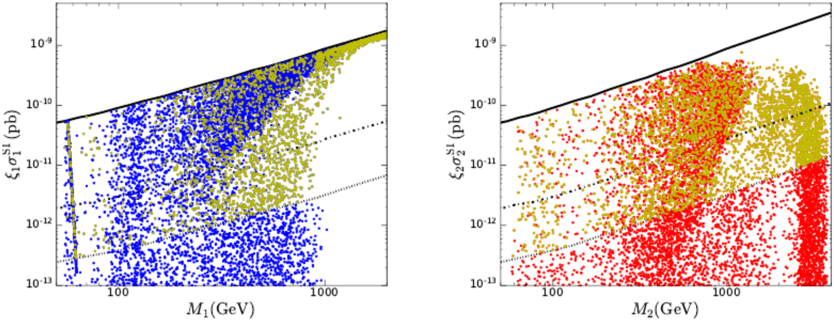

Figure: arXiv:2006.14922

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$$\begin{aligned}\mathcal{V}_{\mathbb{Z}_5} =& -\mu_H^2|H|^2 + \lambda_H|H|^4 + \mu_1^2|\phi_1|^2 + \lambda_{S1}|H|^2|\phi_1|^2 + \lambda_{41}|\phi_1|^4 \\ &+ \mu_2^2|\phi_2|^2 + \lambda_{42}|\phi_2|^4 + \lambda_{S2}|H|^2|\phi_2|^2 + \lambda_{412}|\phi_1|^2|\phi_2|^2 \\ &+ \frac{1}{2}(\mu_{S1}\phi_1^2\phi_2^* + \mu_{S2}\phi_2^2\phi_1 + \lambda_{31}\phi_1^3\phi_2 + \lambda_{32}\phi_1\phi_2^{*3} + \text{h.c.}),\end{aligned} \tag{2}$$

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where the charges under $\mathbb{Z}_5$ for the two new complex scalar fields are

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In the model, $\phi_{1,2}$ do not acquire VEV and $M_1 < M_2 < 2M_1$ so that both are stable. They are singlets under $\mathcal{G}_{\text{SM}}$ and the SM particles are singlets under $\mathbb{Z}_5$.

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Set of free parameters

$$M_i, \lambda_{4i}, \lambda_{Si}, \lambda_{412}, \mu_{Si}, \lambda_{3i}. \tag{4}$$

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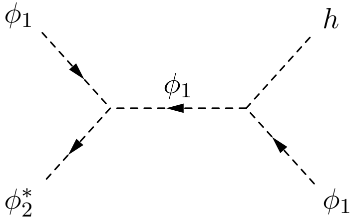

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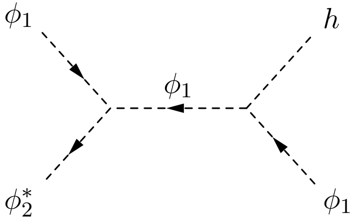

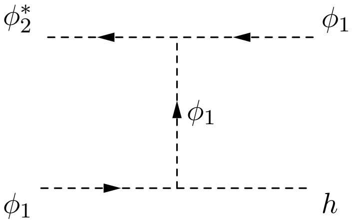

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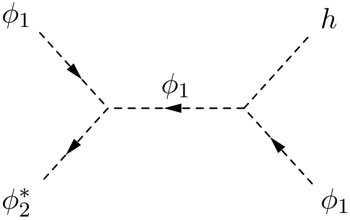

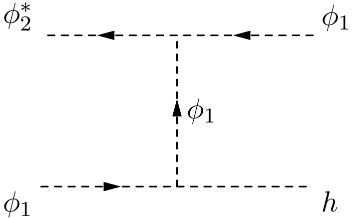

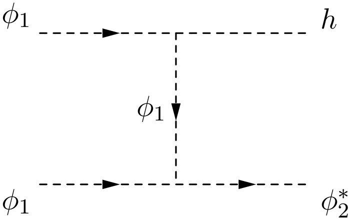

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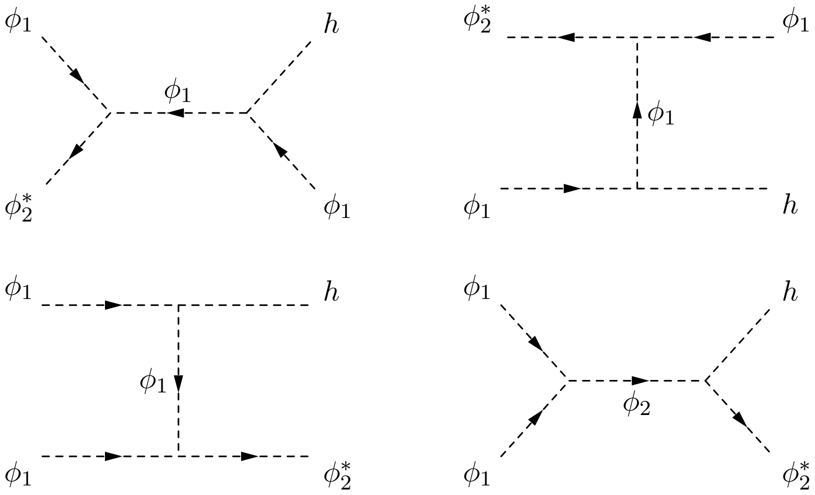

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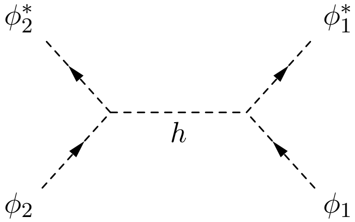

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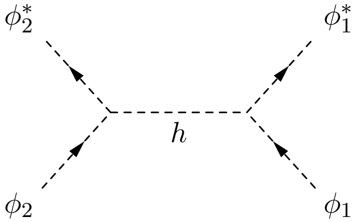

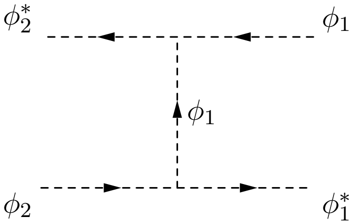

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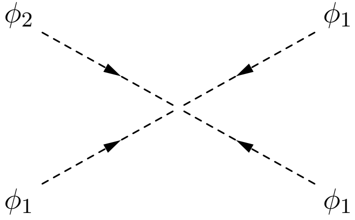

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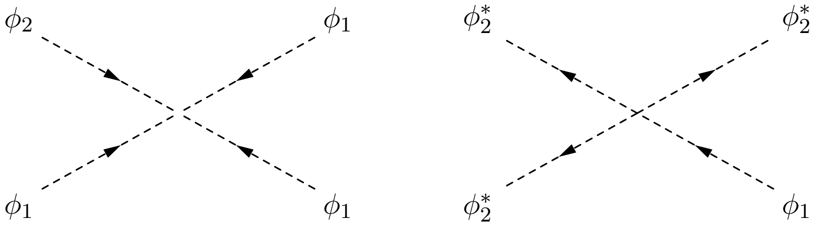

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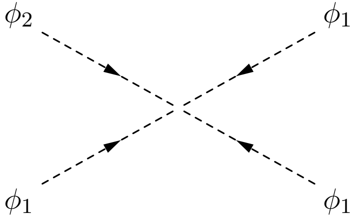

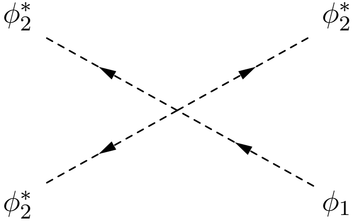

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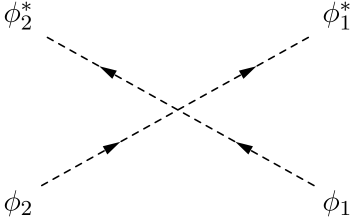

Initial thresholds

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$$\begin{aligned} 40\text{ GeV} \leq M_1 \leq 2\text{ TeV}, \\ M_1 < M_2 < 2M_1, \\ 10^{-4} \leq \lambda_{4i}, |\lambda_{412,Si,3i}| \leq \sqrt{4\pi}, \\ 100\text{ GeV} \leq |\mu_{Si}| \leq 10\text{ TeV}. \end{aligned} \tag{5}$$

CONSTRAINTS

Relic abundance and cross section

The DM relic abundance, must be bounded according to the reports from PLANCK collaboration,

$$\Omega_{\text{DM}} h^2 = 0.1198 \pm 0.0012. \tag{6}$$

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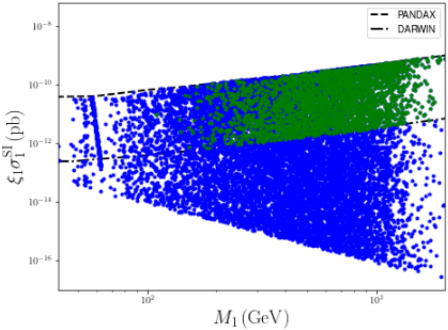
We calculate $\Omega_{\mathrm{DM}} h^2$ using `micromegas 5.2` and defining the scaling factor as $\xi_i = \Omega_i/(\Omega_1 + \Omega_2)$ for re-scaling Ω to each DM particle.

Direct detection bounds



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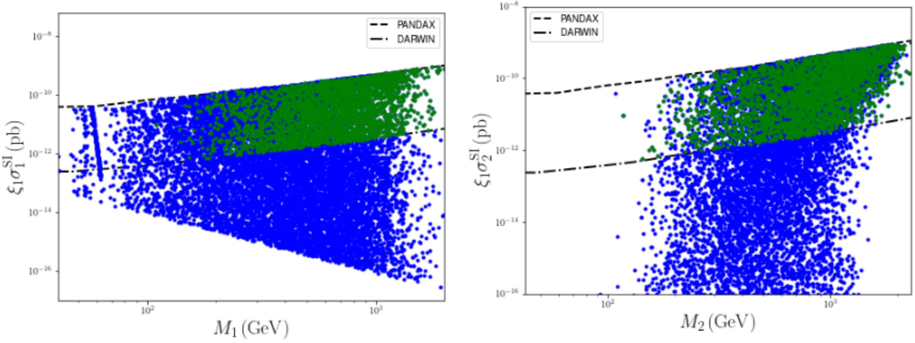

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All of the above up to energy scales $\gtrsim$ GUT.

$\mathbb{Z}_3$ constraints

We may see these analysis, e.g. for the $\mathbb{Z}_3$ model in Andi, Andrzej and Kristjan (2019), *Improved bounds on $\mathbb{Z}_3$ singlet dark matter* (arXiv:1901.08074).

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$$\begin{aligned}\mathcal{V}_{\mathbb{Z}_3} &= \mu_H^2|H|^2 + \lambda_H|H|^4 + \mu_S^2|S|^2 \\ &+ \lambda_S|S|^4 + \lambda_{SH}|S|^2|H|^2 \\ &+ \frac{\mu_3}{2}\left(S^3 + S^{\dagger 3}\right).\end{aligned}$$

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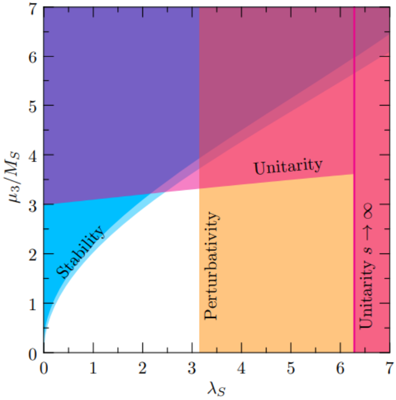

Figure: arXiv:1901.08074

RESULTS

Renormalization group equations

The renormalization group equations (RGEs) at two-loops, for $dx/d(\ln\mu) = \beta_x^{(2)}/(16\pi^2)$ read

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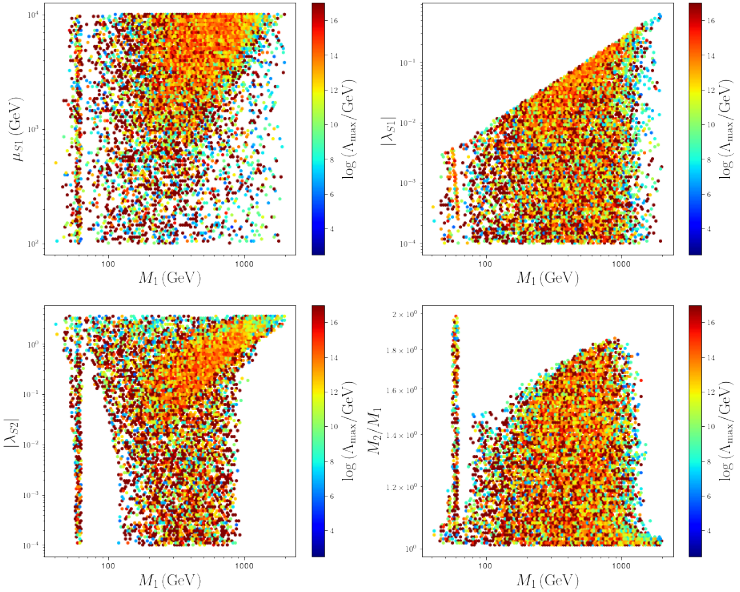
$$\beta_{\lambda_{4i}}^{(2)} = 3\lambda_{3j}^2\left(\lambda_{4i} - 3\lambda_{412}\right) - 9\lambda_{3i}^2\left(11\lambda_{4i} + 3\lambda_{412}\right) - \frac{2}{5}\left(-6g_1^2\lambda_{Si}^2 - 30g_2^2\lambda_{Si}^2 + 600\lambda_{4i}^3 + 25\lambda_{412}^2\lambda_{4i} + 10\lambda_{412}^3 + 50\lambda_{4i}\lambda_{Si}^2 + 20\lambda_{Si}^3\right), \tag{7}$$

$$\beta_{\lambda_{Si}}^{(2)} = \frac{72}{5}g_1^2\lambda_H\lambda_{Si} + 72g_2^2\lambda_H\lambda_{Si} + \frac{1671}{400}g_1^4\lambda_{Si} + \frac{3}{5}g_1^2\lambda_{Si}^2 + \frac{9}{8}g_2^2g_1^2\lambda_{Si} + 3g_2^2\lambda_{Si}^2 - \frac{145}{16}g_2^4\lambda_{Si} - 72\lambda_H\lambda_{Si}^2 - 60\lambda_H^2\lambda_{Si} - 11\lambda_{Si}^3 - 48\lambda_{4i}\lambda_{Si}^2 - 40\lambda_{4i}^2\lambda_{Si} - \lambda_{412}^2\lambda_{Si} - \lambda_{Si}\lambda_{Sj}^2 - 8\lambda_{412}\lambda_{Si}\lambda_{Sj} - \frac{9}{2}\lambda_{3i}^2\left(3\lambda_{Si} + 2\lambda_{Sj}\right) + \frac{3}{2}\lambda_{3j}^2\left(\lambda_{Si} - 6\lambda_{Sj}\right) - 4\lambda_{412}\lambda_{Sj}^2 - 4\lambda_{412}^2\lambda_{Sj}, \tag{8}$$

and so on.

Real parameters bounds



Perturbative unitarity

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$$a_J^{ba} \equiv \frac{1}{32\pi}\sqrt{\frac{2|\boldsymbol{p}^b||\boldsymbol{p}^a|}{2^{\delta_{12}}2^{\delta_{34}}s}}\int_{-1}^{1} d(\cos\theta) \;\; \mathcal{M}_{ba}(\cos\theta)P_J(\cos\theta), \tag{10}$$

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The S-Matrix was calculated by using `SARAH 4.14.4`.

Perturbative unitarity at $s \to \infty$

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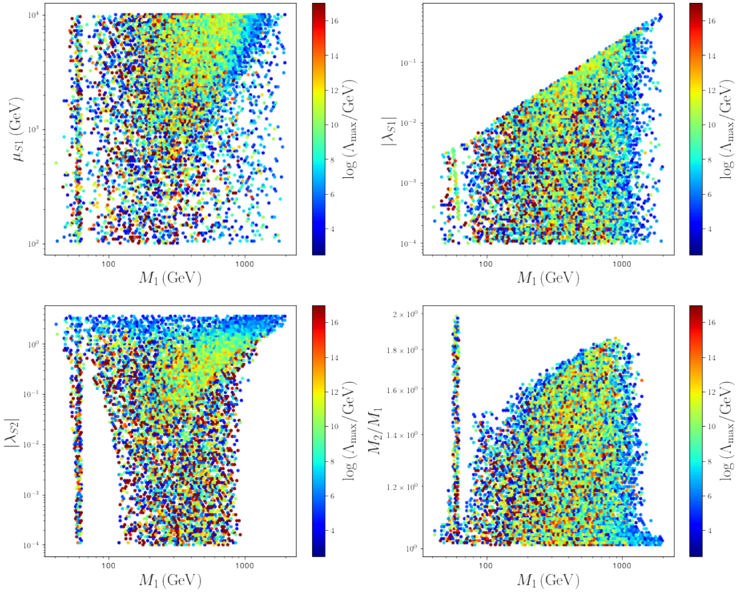
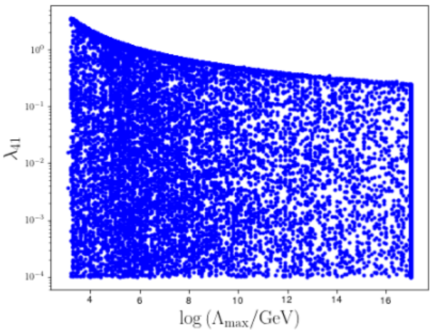
$$\begin{aligned} &\lambda_{Si} < 8\pi, \\ &\left| 2\lambda_{4i} + \lambda_{412} \pm \sqrt{18\lambda_{3i}^2 + (2\lambda_{4i} - \lambda_{412})^2} \right| < 16\pi, \\ &|\alpha_{1,2,3}| \leq 1/2, \end{aligned} \qquad (11)$$

where α_i are the roots of the polynomial $c_3 x^3 + c_2 x^2 + c_1 x + c_0$ with

$$\begin{aligned} c_0 &= 2v_H^2 \left(-3\lambda_{412}^2 \lambda_H + \lambda_{41} \left(48\lambda_{42}\lambda_H - 4\lambda_{S2}^2 \right) \right. \\ &\quad \left. -4\lambda_{42}\lambda_{S1}^2 + 2\lambda_{412}\lambda_{S1}\lambda_{S2} \right), \\ c_1 &= 6\pi v_H^2 \left(24 \left(\lambda_{41} + \lambda_{42} \right) \lambda_H - \lambda_{412}^2 + 16\lambda_{41}\lambda_{42} \right. \\ &\quad \left. -2 \left(\lambda_{S1}^2 + \lambda_{S2}^2 \right) \right), \\ c_2 &= 512\pi^2 v_H^2 \left(3\lambda_H + 2 \left(\lambda_{41} + \lambda_{42} \right) \right), \\ c_3 &= 4096\pi^3 v_H^2. \end{aligned} \qquad (12)$$

Perturbative unitarity at $s > 4M_2^2$ (finite)





λ_{41}
λ_{42}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
4
6
8
10
12
14
16
$\log\left(\Lambda_{\max}/\mathrm{GeV}\right)$
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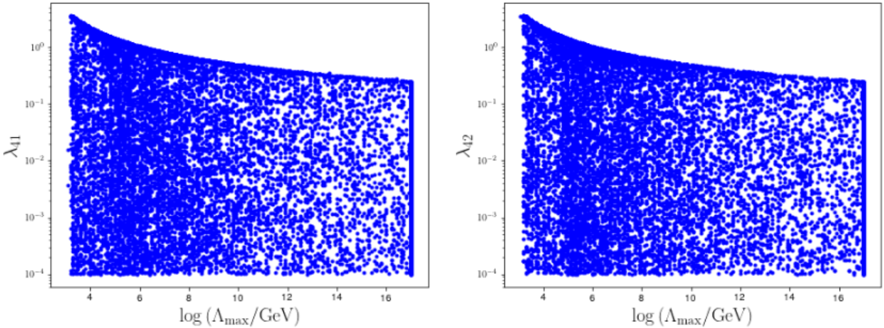

$$10^{-4} \leq \lambda_{4i} \lesssim 0.1 \tag{13}$$

Positivity

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$$\mathcal{V}_4 = \frac{1}{2} \begin{pmatrix} h^2 & \varphi_1^2 & \varphi_2^2 \end{pmatrix} \begin{pmatrix} 2\lambda_H & \lambda_{S1} & \lambda_{S2} \\ \lambda_{S1} & 2\lambda_{41} & \lambda_{412} \\ \lambda_{S2} & \lambda_{412} & 2\lambda_{42} \end{pmatrix} \begin{pmatrix} h^2 \\ \varphi_1^2 \\ \varphi_2^2 \end{pmatrix}. \quad (15)$$

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- $2\sqrt{\lambda_H \lambda_{41} \lambda_{42}} + \lambda_{S1}\sqrt{\lambda_{42}} + \lambda_{S2}\sqrt{\lambda_{41}} + \lambda_{412}\sqrt{\lambda_H} + \sqrt{\Lambda_1 \Lambda_2 \Lambda_3} \geq 0$.

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$$\lambda_{4k} > 0, D > 0 \wedge (Q > 0 \vee R > 0), \tag{16}$$

$$\begin{aligned}
D = & -27\lambda_{42}^2|\lambda_{31}|^4 - 4|\lambda_{32}|^3|\lambda_{31}|^3 + 18|\lambda_{32}|\lambda_{42}\lambda_{412}|\lambda_{31}|^3 \\
& - 4\lambda_{42}\lambda_{412}^3|\lambda_{31}|^2 + |\lambda_{32}|^2\lambda_{412}^2|\lambda_{31}|^2 - 6|\lambda_{32}|^2\lambda_{41}\lambda_{42}|\lambda_{31}|^2 \\
& + 144\lambda_{41}\lambda_{42}^2\lambda_{412}|\lambda_{31}|^2 - 192|\lambda_{32}|\lambda_{41}^2\lambda_{42}^2|\lambda_{31}| \\
& - 80|\lambda_{32}|\lambda_{41}\lambda_{42}\lambda_{412}^2|\lambda_{31}| + 18|\lambda_{32}|^3\lambda_{41}\lambda_{412}|\lambda_{31}| + 16\lambda_{41}\lambda_{42}\lambda_{412}^4 \\
& + 256\lambda_{41}^3\lambda_{42}^3 - 4|\lambda_{32}|^2\lambda_{41}\lambda_{412}^3 - 27|\lambda_{32}|^4\lambda_{41}^2 - 128\lambda_{41}^2\lambda_{42}^2\lambda_{412}^2 \\
& + 144|\lambda_{32}|^2\lambda_{41}^2\lambda_{42}\lambda_{412}, \\
Q = & \; 8\lambda_{41}\lambda_{412} - 3|\lambda_{31}|^2, \\
R = & -3|\lambda_{31}|^4 + 16\lambda_{41}\lambda_{412}|\lambda_{31}|^2 + 64\lambda_{41}^3\lambda_{42} \\
& - 16\lambda_{41}^2\left(\lambda_{412}^2 + |\lambda_{31}||\lambda_{32}|\right).
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\end{aligned}$$

Allowing λ_{Si} take negative values, arise further conditions. They are not shown here by their analytical extension.

Negative values of λ_{Si} are allowed due some values of the quartic couplings. So, fixing momentarily $\lambda_{412} = \lambda_{3i} = \mu_{Si} = 0$, we obtain:

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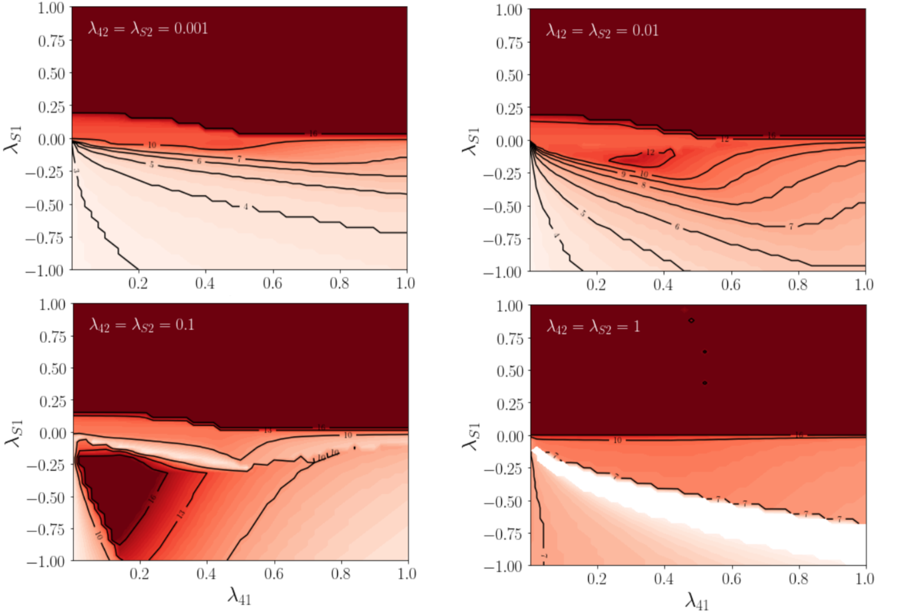
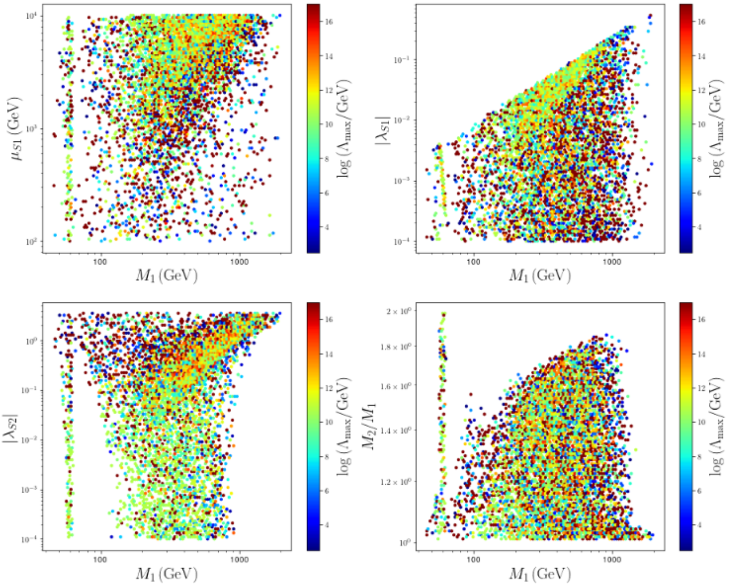


Positivity bounds



Vacuum stability implications

Many points are killed since the vacuum stability is broken at $\Lambda \simeq 10^{11}$ GeV as is expected.

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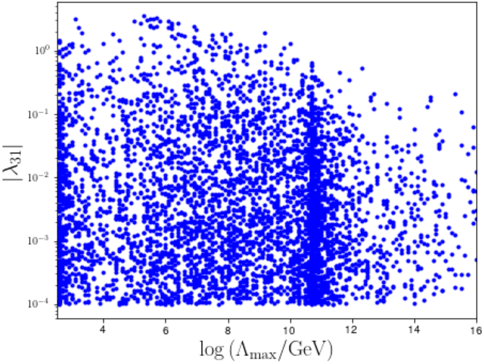

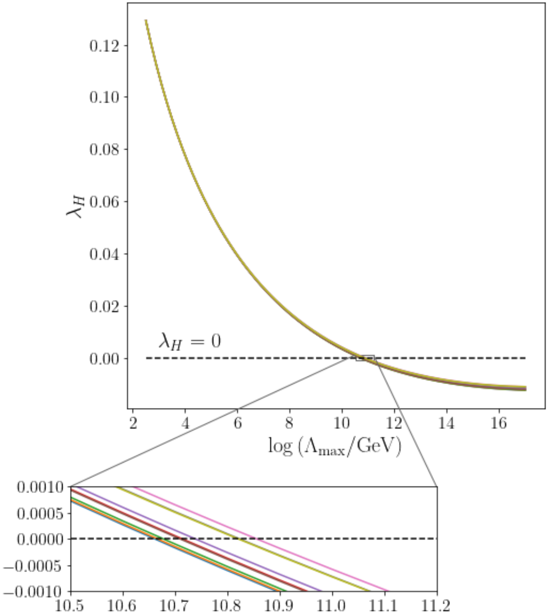
0.12
0.10
0.08
0.06
0.04
0.02
0.00
λ_H
$\lambda_H = 0$
2
4
6
8
10
12
14
16
log $(\Lambda_{max}/\text{GeV})$
0.0010
0.0005
0.0000
−0.0005
−0.0010
10.5
10.6
10.7
10.8
10.9
11.0
11.1
11.2

Stability

We have different minima of the scalar potential, for the cases in that EW symmetry be broken or the $\mathbb{Z}_5$ be broken, i.e., $\langle\phi_1\rangle = v_1 \neq 0$ or $\langle\phi_2\rangle = v_2 \neq 0$:

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- $\mathcal{M}_A: \quad v_H^2 = 0, \quad v_i^2 = 0.$
- $\mathcal{M}_B: \quad v_H^2 = 0, \quad v_i^2 \neq 0, \quad v_j^2 = 0,$ for $i \neq j$.
- $\mathcal{M}_C: \quad v_H^2 = 0, \quad v_i^2 \neq 0.$
- $\mathcal{M}_D: \quad v_H^2 \neq 0, \quad v_i^2 \neq 0, \quad v_j^2 = 0,$ for $i \neq j$.
- $\mathcal{M}_E: \quad v_H^2 \neq 0, \quad v_i^2 = 0.$
- $\mathcal{M}_F: \quad v_H^2 \neq 0, \quad v_i^2 \neq 0.$

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They must fulfill

$$\mathcal{V}_{\mathbb{Z}_5}\Big|_{\mathcal{M}_E} = -\frac{\mu_H^4}{4\lambda_{4H}} < \mathcal{V}_{\mathbb{Z}_5}\Big|_{\mathcal{M}_{A,B,C,D,F}} \tag{17}$$

Stability conditions

Here, we report the analytical minima of the potential,

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$$\mathcal{V}_{\mathbb{Z}_5}\Big|_{\mathcal{M}_A} = 0, \tag{18}$$

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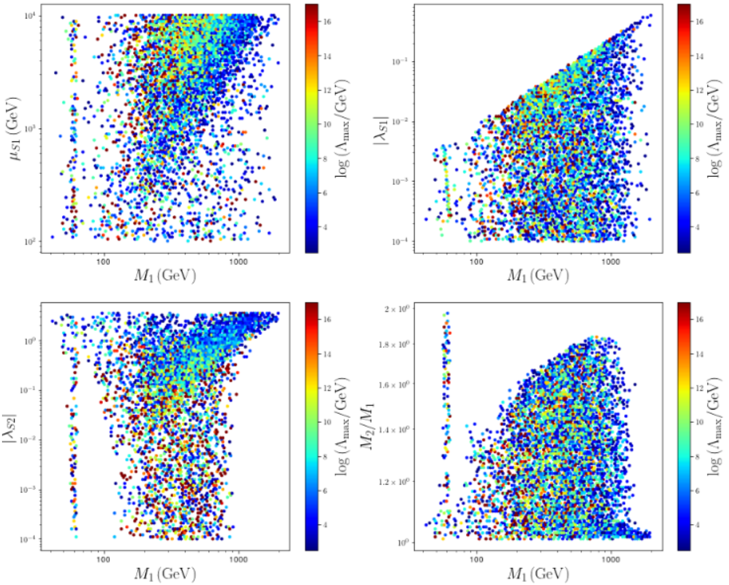
$$\mathcal{V}_{\mathbb{Z}_5}\Big|_{\mathcal{M}_D} = -\frac{\mu_i^4\lambda_H + \lambda_{4i}\mu_H^4 - \mu_i^2\mu_H^2\lambda_{Si}}{4\lambda_{4i}\lambda_H - \lambda_{Si}^2}. \tag{20}$$

The expressions for the remaining minima are quite involved

Stability bounds



Expected results

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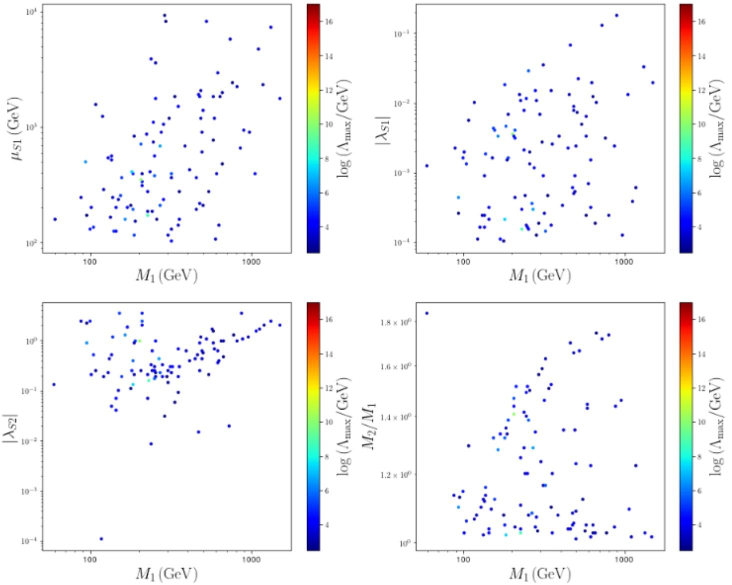
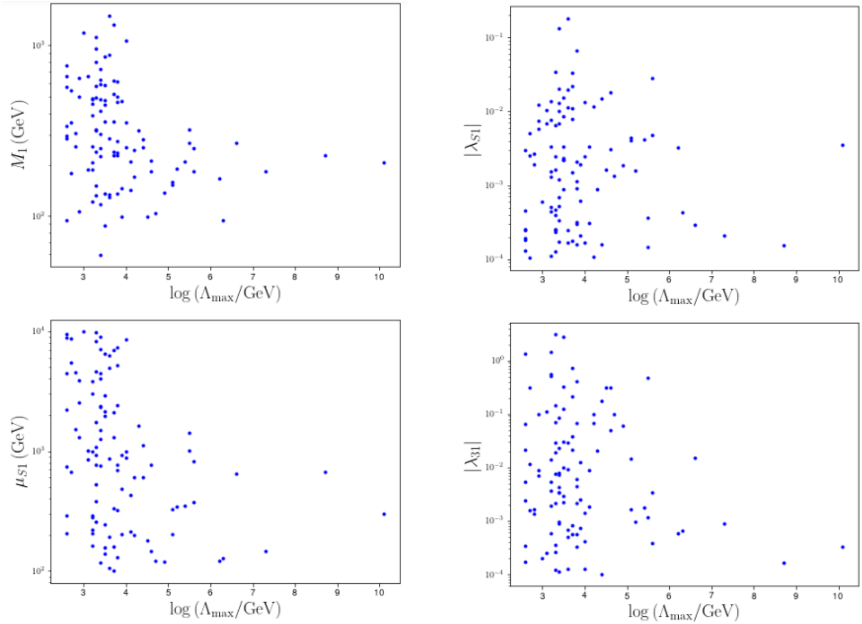
On the other hand, the implementation of all the restrictions will allow to us, observe how the parameters are bounded at each energy scale.

All the constraints





M_1 (GeV)
$\log(\Lambda_{max}/\text{GeV})$
$|\lambda_{S1}|$
$\log(\Lambda_{max}/\text{GeV})$
μ_{S1} (GeV)
$\log(\Lambda_{max}/\text{GeV})$
$|\lambda_{31}|$
$\log(\Lambda_{max}/\text{GeV})$

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- We observe a global restriction for the self-coupling of dark matter: $\lambda_{4i} \lesssim 0.1$

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Thanks.