CP violation in mixing and oscillations for leptogenesis with quasi-degenerate neutrinos

Juan Racker

Instituto de Astronomía Teórica y Experimental (IATE), Observatorio Astronómico de Córdoba (OAC)
Universidad Nacional de Córdoba (UNC) -
Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET)

Outline (an overview of):

- CP violation from mixing - Real intermediate state subtraction
- CP violation from oscillations
- What has been said
- Our approach

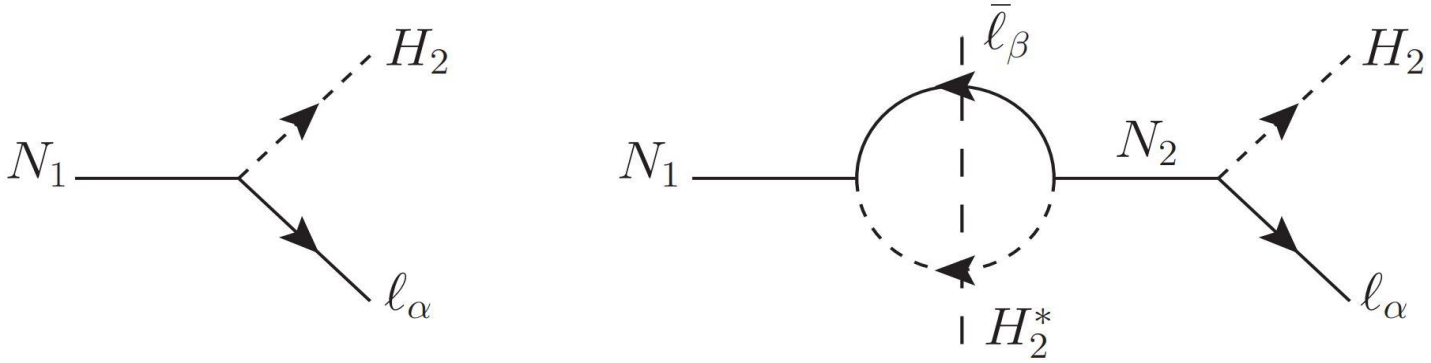
Ingredients for CP violation: CP odd and even phases

$$A(i \to f) = |A_1|\, e^{i\theta_1} e^{i\phi_1} + |A_2|\, e^{i\theta_2} e^{i\phi_2}$$
$$A(\bar{i} \to \bar{f}) = |A_1|\, e^{i\theta_1} e^{-i\phi_1} + |A_2|\, e^{i\theta_2} e^{-i\phi_2}$$

$$|A(i \to f)|^2 - \left|A(\bar{i} \to \bar{f})\right|^2 = 4\,|A_1|\,|A_2| \sin(\theta_1 - \theta_2) \sin(\phi_1 - \phi_2)$$

$\Rightarrow$ Interference of at least two amplitudes contributing to $i \to f$, with a relative CP even and a relative CP odd phase.

CP even phase from the absorptive part of the propagator



$$\epsilon_i = \frac{1}{(\lambda^\dagger\lambda)_{ii}} \frac{\mathrm{Im}[(\lambda^\dagger\lambda)^2_{21}]}{8\pi} \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + A^2}$$

Boltzmann equations - Real intermediate state subtraction

$$\frac{\mathrm{d}Y_{B-L}}{\mathrm{d}z} = \frac{-1}{zHs}\left\{\epsilon_\ell^N\left(\frac{Y_N}{Y_N^{eq}} - 1\right)\gamma_D - \frac{Y_L}{Y_L^{eq}}\left(\frac{\gamma_D}{2} + 2\gamma_{Ntu} + 2\gamma'_{Ns}\right)\right\}$$

$$\ell h \Longleftrightarrow N$$

$$\ell h \Longleftrightarrow \bar{\ell}\,\bar{h}$$ (only off-shell contribution)

$$\ell h \Longleftrightarrow \ell\, h$$

CP even oscillating phases due to the coherent propagation of different mass eigenstates

$$e^{-iM_j t/\gamma} \qquad \gamma \equiv E_0/M$$

$$i\frac{d\rho}{dt} = [H,\rho] - \frac{i}{2}\{\Gamma,\rho\} + \frac{i}{2}\{\Gamma^p, 1-\rho\}$$

Baryogenesis via neutrino oscillations

E. Kh. Akhmedov[(a,b)] V. A. Rubakov[(c,a,d)] and A. Yu. Smirnov[(a,c)]

November 1996

Baryogenesis from mixed particle decays

Laura Covi[1] and Esteban Roulet[2]

September 1993

Unstable particle mixing and CP violation in weak decays

Jiang Liu and Gino Segrè

$$(VH^2V^{-1})_{ab} = \omega_a^2 \delta_{ab}. \tag{4}$$

This matrix V will then transform the initial 'flavour' states $|\Phi_a\rangle$ into the 'propagation' eigenstates[2] $|\Phi'_c\rangle$, i.e.

$$|\Phi'_c\rangle = V_{ac}^{-1}|\Phi_a\rangle.$$

$$|\Phi'_c(t)\rangle = \mathrm{e}^{-i\omega_c t}|\Phi'_c(0)\rangle \qquad T_{fa}(t) = \sum_{b,c} T_{fb} V_{bc}^{-1} V_{ca} \mathrm{e}^{-i\omega_c t}$$

$$T_{fa} = \langle f|H_{int}|\Phi_a\rangle \qquad \bar{T}_{fa}(t) = \sum_{b,c} T_{fb}^{*} V_{cb} V_{ac}^{-1} \mathrm{e}^{-i\omega_c t}.$$

$$\Delta_{fa}(t) \equiv |T_{fa}(t)|^2 - |\bar{T}_{fa}(t)|^2$$

$$\epsilon^w_{fa} = N_f \Omega_a \int_0^\infty dt \Delta_{fa}(t)$$

$$\epsilon^w_{fa} = 2N_f \Omega_a \mathrm{Im}\left\{T_{f1} T^*_{f2} \Gamma^2_{12}\right\} F_a$$

In the case $M_2^2 \gg M_1^2 \gg |\Gamma^2_{ab}|$, one has

$$F_a \to \frac{2}{\gamma_a (M_2^2 - M_1^2)}, \tag{22}$$

where in this limit $\gamma_a \simeq \Gamma^2_{aa}/M_a$ becomes just the total width of particle Φ_a. Hence, the results obtained in the literature in the limit of large mass splittings can be recovered.

April 2014

Flavour Covariant Transport Equations: an Application to Resonant Leptogenesis

P. S. Bhupal Dev[a], Peter Millington[a,b], Apostolos Pilaftsis[a], Daniele Teresi[a]

$$\frac{H_N\, n^\gamma}{z}\,\frac{\mathrm{d}[\underline{\eta}^N]_\alpha{}^\beta}{\mathrm{d}z} = -\,i\,\frac{n^\gamma}{2}\left[\mathcal{E}_N,\,\delta\eta^N\right]_\alpha^{\ \beta} + \left[\widetilde{\mathrm{Re}}(\gamma^N_{L\Phi})\right]_\alpha^{\ \beta} - \frac{1}{2\,\eta^N_{\mathrm{eq}}}\left\{\underline{\eta}^N,\,\widetilde{\mathrm{Re}}(\gamma^N_{L\Phi})\right\}_\alpha^{\ \beta}, \tag{4.82}$$

$$\frac{H_N\, n^\gamma}{z}\,\frac{\mathrm{d}[\delta\eta^N]_\alpha{}^\beta}{\mathrm{d}z} = -\,2\,i\,n^\gamma\left[\mathcal{E}_N,\,\underline{\eta}^N\right]_\alpha^{\ \beta} + 2\,i\left[\widetilde{\mathrm{Im}}(\delta\gamma^N_{L\Phi})\right]_\alpha^{\ \beta} - \frac{i}{\eta^N_{\mathrm{eq}}}\left\{\underline{\eta}^N,\,\widetilde{\mathrm{Im}}(\delta\gamma^N_{L\Phi})\right\}_\alpha^{\ \beta} - \frac{1}{2\,\eta^N_{\mathrm{eq}}}\left\{\delta\eta^N,\,\widetilde{\mathrm{Re}}(\gamma^N_{L\Phi})\right\}_\alpha^{\ \beta}, \tag{4.83}$$

$$\begin{aligned}\frac{H_N\, n^\gamma}{z}\,\frac{\mathrm{d}[\delta\eta^L]_l{}^m}{\mathrm{d}z} = &-\left[\delta\gamma^N_{L\Phi}\right]_l^{\ m} + \frac{[\underline{\eta}^N]_\beta{}^\alpha}{\eta^N_{\mathrm{eq}}}\left[\delta\gamma^N_{L\Phi}\right]_l^{\ m}{}_\alpha^{\ \beta} + \frac{[\delta\eta^N]_\beta{}^\alpha}{2\,\eta^N_{\mathrm{eq}}}\left[\gamma^N_{L\Phi}\right]_l^{\ m}{}_\alpha^{\ \beta}\\ &- \frac{1}{3}\left\{\delta\eta^L,\,\gamma^{L\Phi}_{L^{\bar c}\Phi^{\bar c}} + \gamma^{L\Phi}_{L\Phi}\right\}_l^{\ m} - \frac{2}{3}\left[\delta\eta^L\right]_k^{\ n}\left(\left[\gamma^{L\Phi}_{L^{\bar c}\Phi^{\bar c}}\right]_n^{\ k}{}_l^{\ m} - \left[\gamma^{L\Phi}_{L\Phi}\right]_n^{\ k}{}_l^{\ m}\right)\\ &- \frac{2}{3}\left\{\delta\eta^L,\,\gamma_{\mathrm{dec}}\right\}_l^{\ m} + \left[\delta\gamma^{\mathrm{back}}_{\mathrm{dec}}\right]_l^{\ m}.\end{aligned} \tag{4.84}$$

(i) Resonant mixing between heavy neutrinos, described by the resummed Yukawa couplings in $\gamma^N_{L\Phi}$ and $\delta\gamma^N_{L\Phi}$. This provides a flavour-covariant generalization of the mixing effects discussed earlier in [29].

(ii) Coherent oscillations between heavy neutrinos, described by the commutators in (4.82) and (4.83), and transferred to the lepton asymmetry via the *new* rank-4 term $[\gamma^N_{L\Phi}]_l{}^m{}_\alpha{}^\beta$ in the first line of (4.84). We should stress here that this phenomenon of coherent oscillations is an $\mathcal{O}(h^4)$ effect on the *total* lepton asymmetry, and so differs from the $\mathcal{O}(h^6)$ mechanism proposed in [87] (see Section 4.2).

(iii) Decoherence effects due to charged-lepton Yukawa couplings, described by the last line of (4.84). Our description of these effects goes along the lines of [93], which has been generalized here to an arbitrary flavour basis.

Finally, we note that taking the limit in which the number densities are diagonal, i.e. $[\overline{\eta^N}]_\alpha{}^\beta = \delta^\beta_\alpha\,\eta^N_\beta$ and $[\delta\eta^L]_l{}^m = \delta^m_l\,\delta\eta^L_m$ in (4.82)–(4.84), we recover the flavour-diagonal Boltzmann equations (2.12) and (2.13).

[Submitted on 29 Dec 2015 (v1), last revised 12 Jun 2016 (this version, v2)]

Lepton asymmetry from mixing and oscillations

Alexander Kartavtsev, Peter Millington, Hendrik Vogel

In this article, we illustrate how the mixing and oscillation sources of lepton asymmetry can be identified unambiguously in the Kadanoff-Baym formalism by means of the spectral structure of the resummed heavy-neutrino propagators and independently of the definition of particle number densities. In the context of a toy two-flavour model, we will show that this spectral structure contains *three* distinct shells: two of these shells correspond to mixing and can be associated with the quasi-particle mass shells, whereas the third, which can be identified with oscillations, lies at an intermediate energy. In addition, we identify terms lying on the oscillation shell that can be interpreted as the interference between oscillation and mixing. In so doing, we provide a further illustration of the interplay of these two effects in the generation of lepton asymmetry. Most significantly, we find that this interference is destructive. With respect to the "benchmark" of the

Nevertheless, in spite of this destructive interference and in the weak-washout regime, we find that the oscillation and mixing sources can be of equal magnitude and contribute additively to the final asymmetry in agreement with the conclusions of refs. [9–11].

CP violation in mixing and oscillations in a toy model for leptogenesis with quasi-degenerate neutrinos

J. Racker

Journal of High Energy Physics **2021**, Article number: 290 (2021)

$$\mathcal{L} = \frac{1}{2}\partial^\mu \psi_i \, \partial_\mu \psi_i - \frac{1}{2}\psi_i M_i^2 \psi_i + \partial^\mu \bar{b} \, \partial_\mu b - m^2 \, \bar{b} b - \frac{h_i}{2}\psi_i \, bb - \frac{h_i^*}{2}\psi_i \, \bar{b}\bar{b} - \frac{\lambda}{2 \cdot 2}(\bar{b} b)^2$$

The one-loop renormalized inverse propagator matrix $\mathbf{G}^{-1}$ is given by

$$i\mathbf{G}^{-1}(p^2) = p^2\mathbf{1} - \mathbf{M^2}(p^2),$$

with

$$\mathbf{M^2}(p^2) = \begin{pmatrix} M_1^2 + \Sigma_{11}(p^2) & \Sigma_{12}(p^2) \\ \Sigma_{21}(p^2) & M_2^2 + \Sigma_{22}(p^2) \end{pmatrix},$$

and

$$\Sigma_{ii}(p^2) = \frac{|h_i|^2}{(4\pi)^2}\left[1 + \ln\frac{p^2}{M_i^2} - \frac{p^2}{M_i^2} - i\pi\theta(p^2)\right],$$

$$\Sigma_{12}(p^2) = \Sigma_{21}(p^2) = \frac{\mathrm{Re}\,[h_1^* h_2]}{(4\pi)^2}\left[\frac{M_2^2 \ln\frac{p^2}{M_1^2} - M_1^2 \ln\frac{p^2}{M_2^2} - p^2 \ln\frac{M_2^2}{M_1^2}}{M_2^2 - M_1^2} - i\pi\theta(p^2)\right]$$

Next we will make an expansion of the full propagator around the two poles. Following [29] (see eq. 76) we write:

$$\mathbf{G} \simeq \mathbf{Z}^T \, \mathbf{\Delta}^{\rm BW} \, \mathbf{Z} \,,$$

[29] E. Fuchs and G. Weiglein, *Breit-Wigner approximation for propagators of mixed unstable states*, *JHEP* **09** (2017) 079, [arXiv:1610.06193].

with

$$\mathbf{\Delta}^{\rm BW} = i \begin{pmatrix} (p^2 - \mathcal{M}_a^2)^{-1} & 0 \\ 0 & (p^2 - \mathcal{M}_b^2)^{-1} \end{pmatrix}, \tag{2.10}$$

$\mathcal{M}_{a,b}^2$ the poles of the propagator (given by the roots of the determinant of $\mathbf{G}^{-1}$), and

$$\mathbf{Z} = \begin{pmatrix} \sqrt{Z_1} & \sqrt{Z_1} Z_{12} \\ \sqrt{Z_2} Z_{21} & \sqrt{Z_2} \end{pmatrix}, \tag{2.11}$$

with

$$Z_{12} = \frac{\mathbf{G}_{12}(\mathcal{M}_a^2)}{\mathbf{G}_{11}(\mathcal{M}_a^2)} = \frac{\Sigma_{12}(\mathcal{M}_a^2)}{\mathcal{M}_a^2 - M_2^2 - \Sigma_{22}(\mathcal{M}_a^2)} \,,$$

$$Z_{21} = \frac{\mathbf{G}_{21}(\mathcal{M}_b^2)}{\mathbf{G}_{22}(\mathcal{M}_b^2)} = \frac{\Sigma_{12}(\mathcal{M}_b^2)}{\mathcal{M}_b^2 - M_1^2 - \Sigma_{11}(\mathcal{M}_b^2)} \,,$$

at first order in the quantities

$$\eta_{ij} \equiv \frac{|h_i h_j| / (4\pi)^2}{M_2^2 - M_1^2}.$$

$$\boxed{\theta \equiv \frac{\Sigma_{12}}{M_2^2 - M_1^2}}$$

$$\mathbf{G}^{(1,\mathrm{p})}(p^2) = i \begin{pmatrix} 1 & \theta \\ -\theta & 1 \end{pmatrix} \begin{pmatrix} (p^2 - \mathcal{M}_a^2)^{-1} & 0 \\ 0 & (p^2 - \mathcal{M}_b^2)^{-1} \end{pmatrix} \begin{pmatrix} 1 & -\theta \\ \theta & 1 \end{pmatrix}$$

$$-\mathcal{M}(\bar{b}\bar{b} \to bb) = \left(h_1^{*2} - 2h_1^* h_2^* \theta\right) \Delta_1 + \left(h_2^{*2} + 2h_1^* h_2^* \theta\right) \Delta_2 \,,$$

$$-\mathcal{M}(bb \to \bar{b}\bar{b}) = \left(h_1^2 - 2h_1 h_2 \theta\right) \Delta_1 + \left(h_2^2 + 2h_1 h_2 \theta\right) \Delta_2 \,,$$

$$\Delta_j \equiv \frac{1}{p^2 - M_j^2 + i M_j \Gamma_j} \,.$$

These amplitudes can be used in a quantum field theory model of oscillations. We will consider an external wave packet model [32, 33] following the detailed review and analysis of [25]. In this model the initial and final states of a given process are described by localized wave packets. Assuming that the factors related to coherence and localization which could destroy oscillations can be neglected, the probabilities of the lepton number violating processes are given by

$$|A|^2(L) = N\left|\left(h_1^{*2} - 2h_1^* h_2^* \theta\right) e^{-i\left(M_1 - i\frac{\Gamma_1}{2}\right)\frac{ML}{p_0}} + \left(h_2^{*2} + 2h_1^* h_2^* \theta\right) e^{-i\left(M_2 - i\frac{\Gamma_2}{2}\right)\frac{ML}{p_0}}\right|^2$$

$$\left|\bar{A}\right|^2(L) = N\left|\left(h_1^{2} - 2h_1 h_2 \theta\right) e^{-i\left(M_1 - i\frac{\Gamma_1}{2}\right)\frac{ML}{p_0}} + \left(h_2^{2} + 2h_1 h_2 \theta\right) e^{-i\left(M_2 - i\frac{\Gamma_2}{2}\right)\frac{ML}{p_0}}\right|^2 .$$

M. Beuthe, *Oscillations of neutrinos and mesons in quantum field theory*, *Phys. Rept.* **375** (2003) 105–218, [hep-ph/0109119].

$$\frac{|A|^2(t) - |\bar{A}|^2(t)}{N} = M(t) + O(t) + I(t) \qquad \begin{aligned} A &\equiv A(\bar{b}\bar{b} \to bb) \\ \bar{A} &\equiv A(bb \to \bar{b}\bar{b}) \end{aligned}$$

$$\begin{aligned}
M(t) &= 8\,\mathrm{Im}\left[h_1 h_2^*\right]\theta_\mathrm{I}\left(|h_1|^2\, e^{-\Gamma_1 t/\gamma} + |h_2|^2\, e^{-\Gamma_2 t/\gamma}\right), \\
O(t) &= 8\,\mathrm{Im}\left[h_1 h_2^*\right]\mathrm{Re}\left[h_1 h_2^*\right]\mathrm{Im}\left[e^{i(M_2-M_1)t/\gamma}\right] e^{-\Gamma t/\gamma}, \\
I(t) &= 8\,\mathrm{Im}\left[h_1 h_2^*\right]\left(|h_1|^2\,\mathrm{Im}\left[e^{i(M_2-M_1)t/\gamma}\,\theta^*\right] - |h_2|^2\,\mathrm{Im}\left[e^{i(M_2-M_1)t/\gamma}\,\theta\right]\right) e^{-\Gamma t/\gamma} \\
&\simeq -8\,\mathrm{Im}\left[h_1 h_2^*\right]\theta_\mathrm{I}\left(|h_1|^2 + |h_2|^2\right)\mathrm{Re}\left[e^{i(M_2-M_1)t/\gamma}\right] e^{-\Gamma t/\gamma},
\end{aligned}$$

$\Gamma \equiv (\Gamma_1 + \Gamma_2)/2$ and $\theta = \theta_\mathrm{R} + i\theta_\mathrm{I}$, so that

$$\theta_\mathrm{I} = -\frac{\mathrm{Re}\left[h_1 h_2^*\right]\pi}{(4\pi)^2\left(M_2^2 - M_1^2\right)}\,.$$

Unitarity & CPT

$$\sum_j |A(i \to j)|^2 = \sum_j |A(\bar{i} \to \bar{j})|^2$$

$$\int_0^\infty |A|^2 (t)\, \mathrm{d}t = \int_0^\infty \left|\bar{A}\right|^2 (t)\, \mathrm{d}t + \mathcal{O}\left(h^8\right),$$

and therefore unitarity is verified up to the order we have been working, i.e up to $\mathcal{O}\left(h^6\right)$.

Source term

For our purposes it is enough to consider a static universe and, to keep things simple, that all the neutrinos mediating the processes in eqs. 3.3 have the same average momentum p_0, so that momentum integrals are avoided (the procedure can be generalized to include the expansion of the universe and more realistic momentum distributions, which is work in progress). Finite density effects will also not be included.

$$\frac{\mathrm{d}n_L}{\mathrm{d}t} = S(t) - W(t)$$

$$S(T) = 2\int_0^T \frac{n^{\mathrm{eq}}(t)}{(32\pi E_0)^2}\Big[M(T-t) + O(T-t) + I(T-t)\Big]$$

$$S(T) = 2\int_0^T \frac{n^{\rm eq}(t)}{(32\pi E_0)^2}\, 8\,{\rm Im}\left[h_1 h_2^*\right]\Big\{\theta_I\Big[|h_1|^2\, e^{-\Gamma_1(T-t)/\gamma} + |h_2|^2\, e^{-\Gamma_2(T-t)/\gamma} - \big(\,|h_1|^2 + |h_2|^2\,\big)\cos\big((M_2-M_1)(T-t)/\gamma\big)\, e^{-\Gamma(T-t)/\gamma}\Big] + {\rm Re}\left[h_1 h_2^*\right]\sin\big((M_2-M_1)(T-t)/\gamma\big)\, e^{-\Gamma(T-t)/\gamma}\Big\}\,{\rm d}t\ .$$

$n^{\rm eq}(t)$ is the equilibrium density of a scalar particle of mass $\tilde{M}$. Although in realistic calculations $n^{\rm eq}(t)$ would be a function of the time dependent temperature, in the examples given below for a static universe we will artificially vary $n^{\rm eq}(t)$ and equilibrium will simply correspond to constancy over time. In equilibrium, i.e. when $n^{\rm eq}(t)$ remains constant for a time period larger than the other time scales (the oscillation period and lifetimes of neutrinos), the unitarity condition 3.14 ensures that $S(t)$ becomes null.

density of ψ_i calculated with the classical BE in the absence of oscillations. In the standard classical treatment, appropriate for large enough mass splittings, the source term can be built considering the production of leptons by the density rates $\gamma(\psi_i \to bb)$ and $\gamma'(\bar{b}\bar{b} \to bb)$, where the prime in the second rate means that a real intermediate state subtraction must be performed to be consistent with unitarity. Subtracting the corresponding production terms for antileptons, the classical source term $S^{\rm cl}(T)$ reads

$$S^{\rm cl}(T) = 2\,\epsilon_1 \frac{\Gamma_1}{\gamma}\left[n_{\psi_1}(T) - n^{\rm eq}(T)\right] + 2\,\epsilon_2 \frac{\Gamma_2}{\gamma}\left[n_{\psi_2}(T) - n^{\rm eq}(T)\right], \tag{4.9}$$

where the terms proportional to $n_{\psi_i}(T)$ come from the production of leptons and antileptons via ψ_i decays, and the ones proportional to $n^{\rm eq}(T)$ come from the off-shell lepton and antilepton annihilations. Therefore, the contribution from mixing in the source term $S(T)$ (first line of eq. 4.7), exactly matches the contribution from decays to the standard classical source $S^{\rm cl}(T)$. Both sources satisfy the unitarity requirement that their total integral over time is zero if the population of neutrinos is null at the beginning and the end. In $S(T)$ the mixing contribution is canceled by the oscillation contribution, while in $S^{\rm cl}(T)$ the off-shell annihilations cancel the production form decays. Indeed, next we show with some plots that the lepton asymmetry obtained integrating $S(t)$, tends to the one obtained from $S^{\rm cl}(T)$, as the mass splitting $\Delta M/M \equiv (M_2 - M_1)/M$ increases.

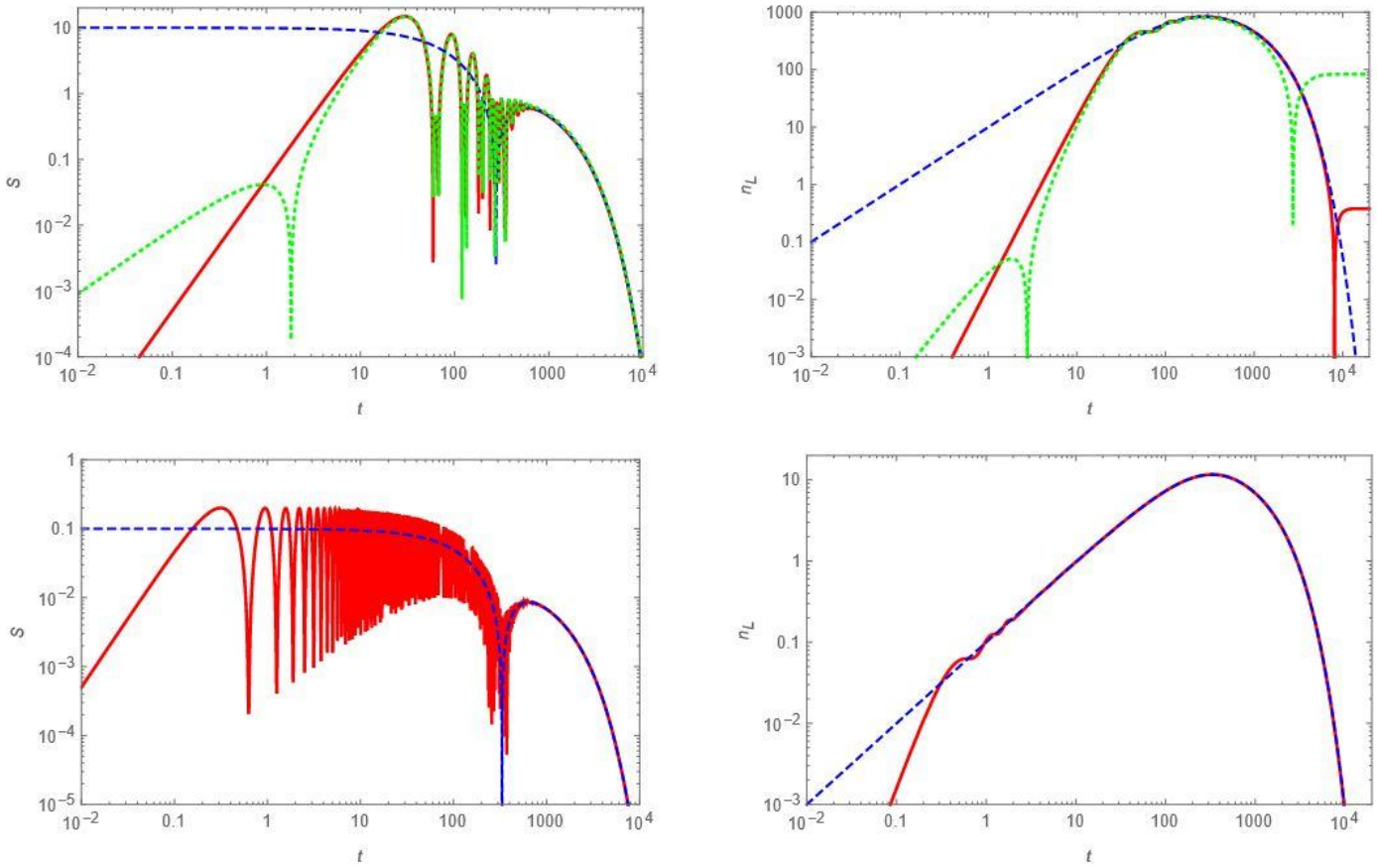
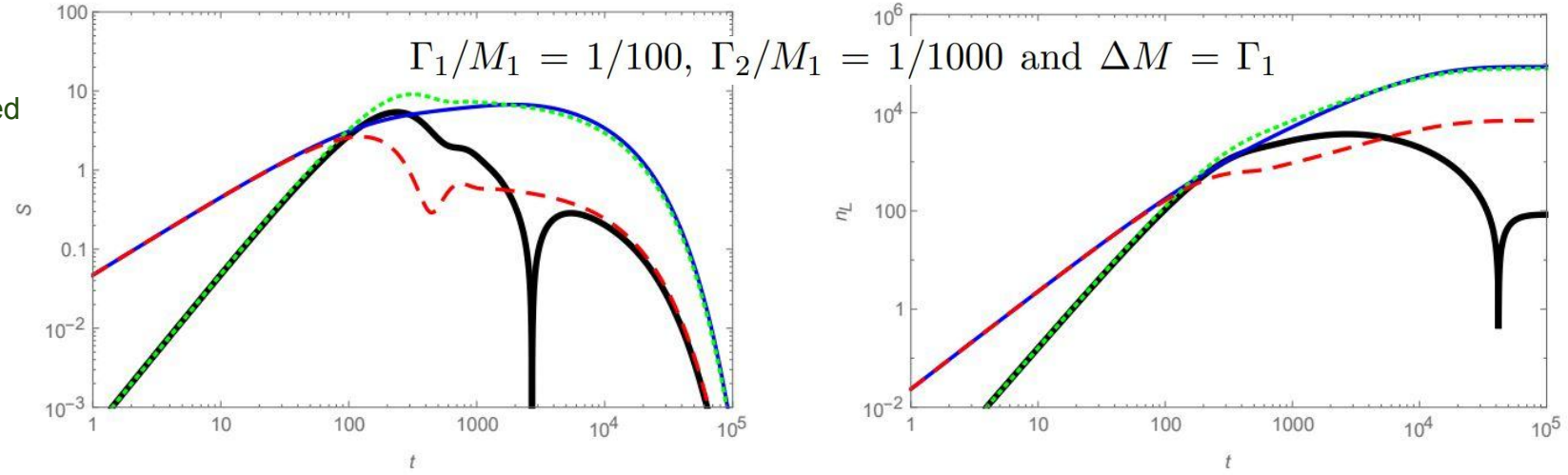
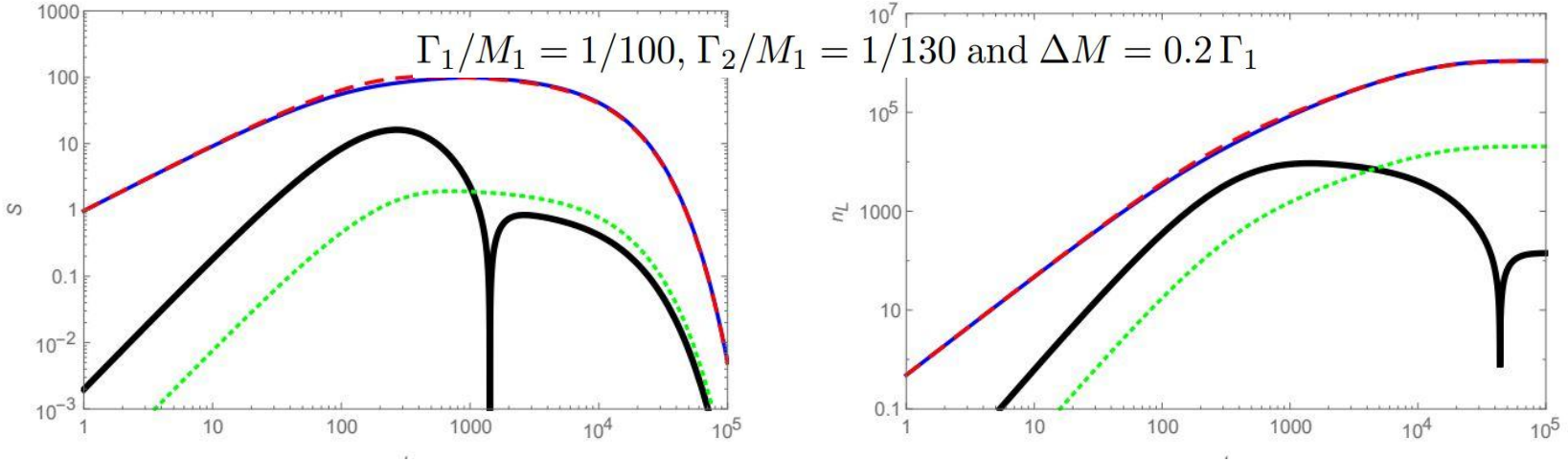


Figure 1: Source term (left plots), and lepton asymmetry (right plots), as a function of time normalized to γ/M, with $\Gamma_1/M = 1/100$, $\Gamma_2/M = 1/120$, and $\Delta M/M = 0.1\,(10)$

To be submitted soon.





Conclusions (highly degenerate case):

The source term has contributions that can be identified with CP violation from mixing, oscillations and interference between both. For the highly-degenerate case, i.e. when the mass difference between two neutrinos is similar to their decay widths, we find that: the mixing and oscillation terms contribute with opposite signs to the source term and the contribution of the interference term is typically very relevant and crucial to ensure unitarity is satisfied. Moreover, the expressions we obtain are finite in the double degenerate limit of equal masses and couplings.